Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/7.3.2-d-x-^m-a+b-arctanh-c-x^n-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [243]. This is test number [192].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 88.89 (216)	% 11.11 (27)
Mathematica	% 95.06 (231)	% 4.94 (12)
Maple	% 80.25 (195)	% 19.75 (48)
Maxima	% 63.37 (154)	% 36.63 (89)
Fricas	% 60.49 (147)	% 39.51 (96)
Sympy	% 34.16 (83)	% 65.84 (160)
Giac	% 52.26 (127)	% 47.74 (116)
Mupad	% 52.67 (128)	% 47.33 (115)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

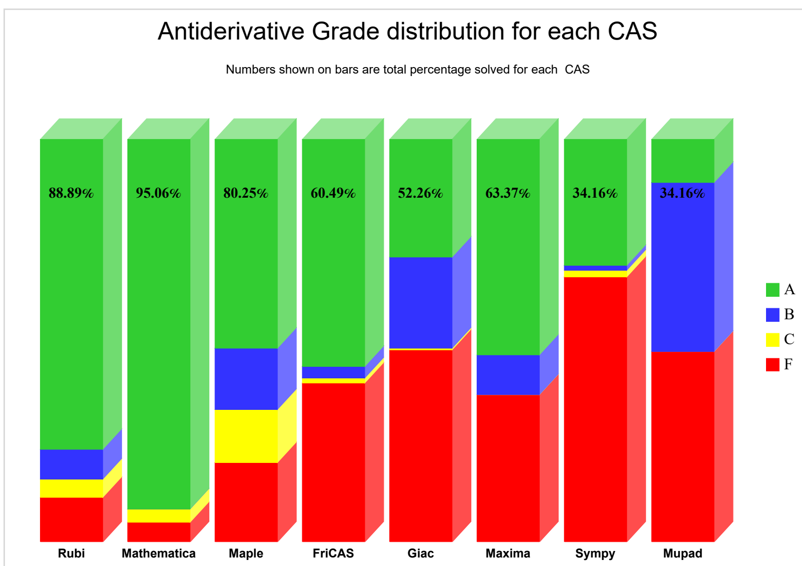
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

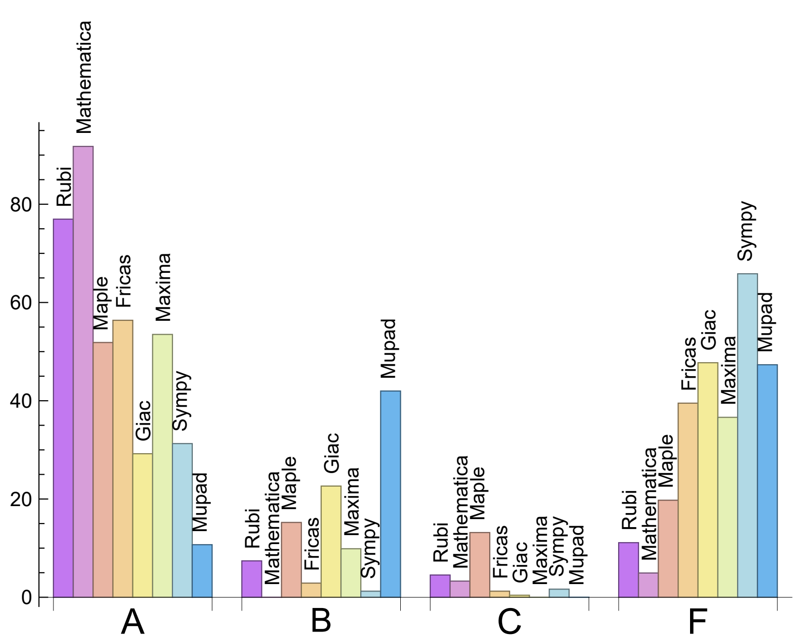
System	% A grade	% B grade	% C grade	% F grade
Rubi	76.95	7.41	4.53	11.11
Mathematica	91.77	0.00	3.29	4.94
Maple	51.85	15.23	13.17	19.75
Maxima	53.50	9.88	0.00	36.63
Fricas	56.38	2.88	1.23	39.51
Sympy	31.28	1.23	1.65	65.84
Giac	29.22	22.63	0.41	47.74
Mupad	10.70	41.98	0.00	47.33

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	27	100.00 %	0.00 %	0.00 %
Mathematica	12	100.00 %	0.00 %	0.00 %
Maple	48	81.25 %	2.08 %	16.67 %
Maxima	89	100.00 %	0.00 %	0.00 %
Fricas	96	98.96 %	1.04 %	0.00 %
Sympy	160	61.88 %	38.12 %	0.00 %
Giac	116	100.00 %	0.00 %	0.00 %
Mupad	115	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

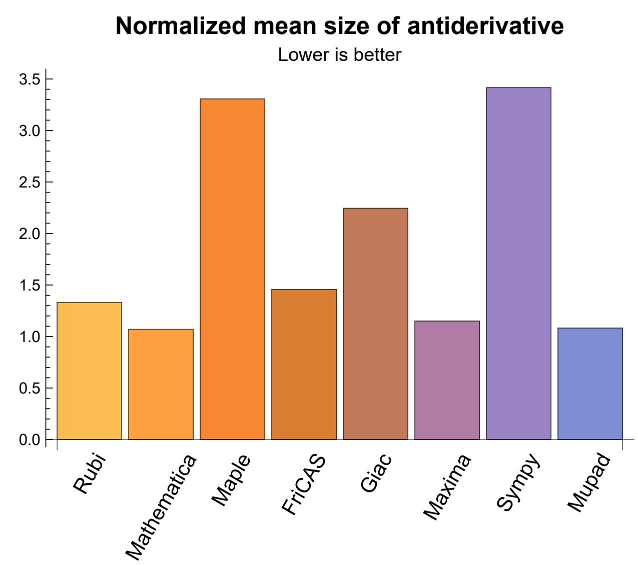
1.3 Performance

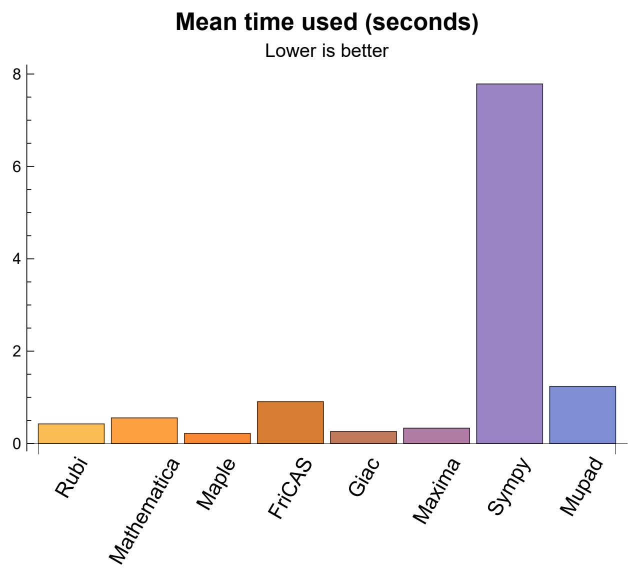
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.43	199.60	1.33	74.00	1.00
Mathematica	0.56	122.28	1.07	99.00	1.13
Maple	0.22	431.61	3.31	78.00	1.22
Maxima	0.33	127.16	1.15	62.00	1.02
Fricas	0.91	135.14	1.46	64.00	1.26
Sympy	7.79	207.42	3.42	70.00	1.26
Giac	0.26	159.84	2.24	109.00	1.41
Mupad	1.24	87.70	1.08	59.00	1.10

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{43, 44, 46, 47, 48, 49, 94, 95, 97, 98, 130, 131, 133, 134, 182, 183, 185, 186, 231, 232, 234, 235, 239, 240, 242, 243}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {64, 65, 66, 67, 69, 70, 77, 78, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 143, 144, 145, 146, 148, 149, 155, 171, 172, 174, 175}

Mathematica {14, 16, 18, 20, 22, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 65, 67, 69, 73, 74, 77, 78, 80, 81, 117, 119, 121, 123, 124, 125, 126, 128, 129, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 172, 173, 174, 178, 179, 202, 203, 204, 205, 207, 208}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

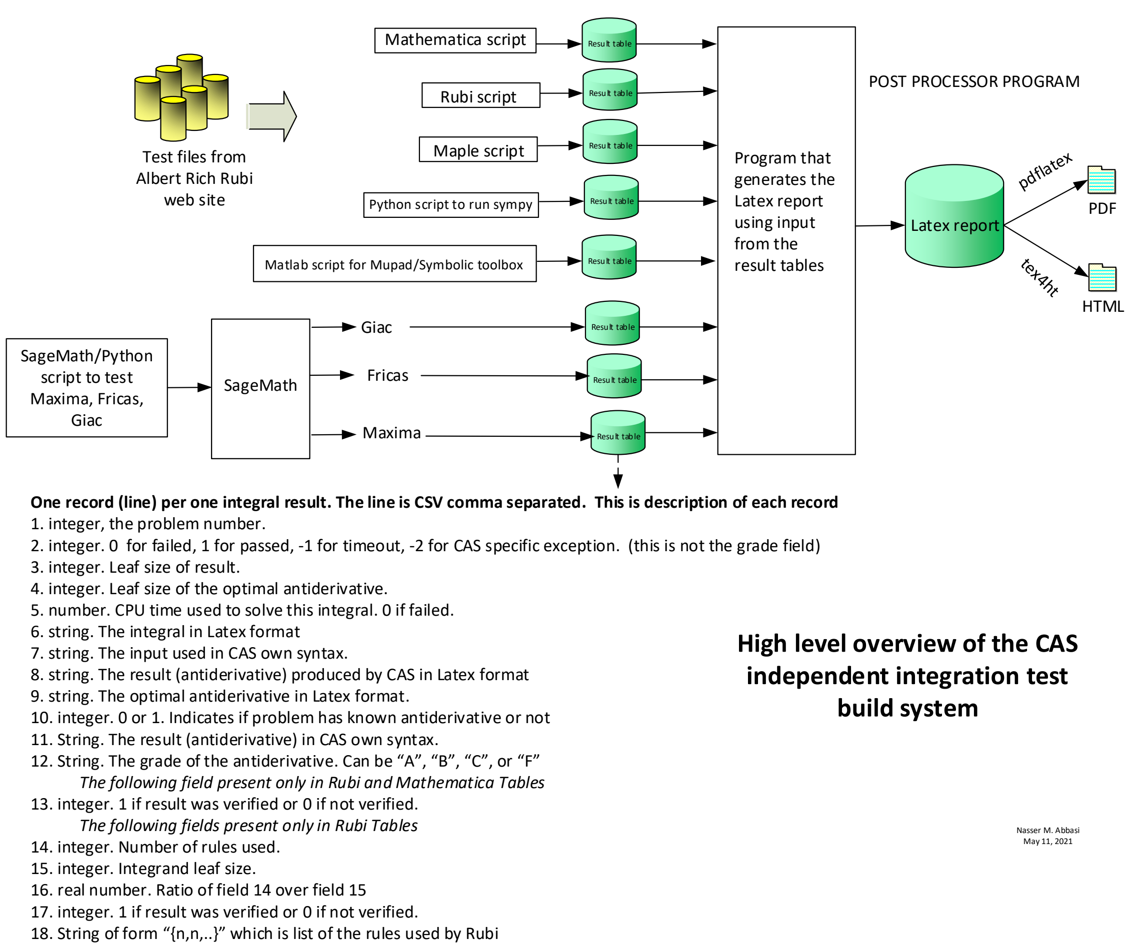
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 71, 72, 73, 74, 75, 76, 79, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 147, 154, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 173, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 199, 206, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243 }

B grade: { 65, 67, 69, 77, 78, 117, 119, 121, 123, 124, 125, 126, 144, 146, 148, 155, 172, 174 }

C grade: { 64, 66, 70, 116, 118, 122, 143, 145, 149, 171, 175 }

F grade: { 80, 81, 90, 91, 92, 93, 128, 129, 150, 151, 152, 153, 156, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 207, 208, 221, 223 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 178, 179, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243 }

B grade: { }

C grade: { 31, 33, 80, 128, 151, 153, 227, 236 }

F grade: { 71, 72, 75, 76, 90, 91, 92, 93, 176, 177, 180, 181 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 18, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 119, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 148, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 182, 183, 185, 186, 187, 188, 189, 190, 192, 193, 194, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 231, 232, 234, 235, 238, 239, 240, 242, 243 }

B grade: { 7, 13, 14, 15, 16, 17, 20, 21, 22, 23, 29, 54, 66, 70, 78, 118, 122, 126, 139, 143, 144, 145, 146, 149, 155, 161, 191, 195, 196, 197, 198, 200, 201, 217, 221, 227, 236 }

C grade: { 19, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 77, 103, 125, 147, 150, 151, 152, 153, 154, 156, 199, 202, 203, 204, 205, 206, 207, 208, 222, 233, 237 }

F grade: { 45, 64, 65, 68, 69, 71, 72, 73, 74, 75, 76, 79, 80, 81, 90, 91, 92, 93, 96, 116, 117, 120, 121, 123, 124, 127, 128, 129, 132, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 184, 223, 224, 225, 226, 228, 229, 230, 241 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 145, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 182, 183, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 231, 232, 234, 235, 238, 239, 240, 242, 243 }

B grade: { 17, 21, 23, 66, 70, 118, 122, 149, 175, 191, 197, 198, 200, 201, 202, 203, 204, 207, 208, 217, 221, 223, 236, 237 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 45, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 199, 205, 206, 222, 224, 225, 226, 227, 228, 229, 230, 233, 241 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 61, 62, 63, 64, 66, 70, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 122, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 175, 182, 183, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 201, 209, 210, 211, 212, 214, 218, 219, 220, 231, 232, 234, 235, 238, 239, 240, 242, 243 }

B grade: { 60, 198, 200, 221, 223, 227, 236 }

C grade: { 213, 215, 216 }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 45, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 191, 199, 202, 203, 204, 205, 206, 207, 208, 217, 222, 224, 225, 226, 228, 229, 230, 233, 237, 241 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 43, 44, 46, 47, 48, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 70, 94, 95, 97, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 159, 162, 163, 164, 165, 166, 167, 168, 169, 170, 175, 182, 183, 185, 192, 193, 200, 201, 210, 231, 232, 234, 235, 238, 242 }

B grade: { 209, 211, 212 }

C grade: { 37, 158, 160, 171 }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 45, 49, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 202, 203, 204, 205, 206, 207, 208, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 236, 237, 239, 240, 241, 243 }

2.1.7 Giac

A grade: { 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 62, 63, 64, 94, 95, 97, 98, 99, 100, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 130, 131, 133, 134, 157, 158, 162, 163, 164, 165, 166, 167, 168, 169, 170, 182, 183, 185, 186, 216, 218, 219, 220, 231, 232, 234, 235, 239, 240, 242, 243 }

B grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 52, 53, 60, 61, 66, 85, 86, 87, 88, 89, 101, 102, 118, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 159, 160, 171, 187, 188, 189, 190, 192, 193, 194, 209, 210, 211, 212, 214, 238 }

C grade: { 213 }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 45, 54, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 191, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 215, 217, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 236, 237, 241 }

2.1.8 Mupad

A grade: { 43, 44, 46, 47, 48, 49, 94, 95, 97, 98, 130, 131, 133, 134, 182, 183, 185, 186, 231, 232, 234, 235, 239, 240, 242, 243 }

B grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 70, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 122, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 175, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 200, 201, 209, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 223, 238 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 191, 199, 202, 203, 204, 205, 206, 207, 208, 210, 217, 222, 224, 225, 226, 227, 228, 229, 230, 233, 236, 237, 241 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	67	70	67	63	442	52
normalized size	1	1.00	1.37	1.14	1.19	1.14	1.07	7.49	0.88
time (sec)	N/A	0.033	0.011	0.008	0.313	1.333	1.621	0.312	0.839

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	62	60	55	69	68	403	53
normalized size	1	1.00	1.09	1.05	0.96	1.21	1.19	7.07	0.93
time (sec)	N/A	0.043	0.009	0.009	0.307	1.347	1.267	0.251	0.821

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	70	58	61	58	53	296	43
normalized size	1	1.00	1.46	1.21	1.27	1.21	1.10	6.17	0.90
time (sec)	N/A	0.029	0.010	0.008	0.308	0.802	0.914	0.165	0.773

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	51	51	44	58	58	258	44
normalized size	1	1.00	1.11	1.11	0.96	1.26	1.26	5.61	0.96
time (sec)	N/A	0.035	0.009	0.008	0.309	0.580	0.726	0.229	0.744

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	59	49	50	48	42	148	35
normalized size	1	1.00	1.59	1.32	1.35	1.30	1.14	4.00	0.95
time (sec)	N/A	0.017	0.008	0.007	0.303	0.580	0.492	0.291	0.727

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	30	42	27	156	27
normalized size	1	1.00	1.00	0.97	1.00	1.40	0.90	5.20	0.90
time (sec)	N/A	0.013	0.004	0.003	0.326	0.562	0.331	0.185	0.684

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	47	0	0	0	0	-1
normalized size	1	1.00	0.92	1.81	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.011	0.014	0.000	0.562	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	45	39	47	41	94	33
normalized size	1	1.00	1.08	1.25	1.08	1.31	1.14	2.61	0.92
time (sec)	N/A	0.026	0.008	0.010	0.321	0.696	0.726	0.133	0.700

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	59	49	45	43	36	135	46
normalized size	1	1.00	1.59	1.32	1.22	1.16	0.97	3.65	1.24
time (sec)	N/A	0.022	0.009	0.011	0.320	1.301	0.578	0.148	0.728

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	59	49	59	70	251	46
normalized size	1	1.00	1.09	1.09	0.91	1.09	1.30	4.65	0.85
time (sec)	N/A	0.036	0.010	0.012	0.317	0.592	1.219	0.144	0.730

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	70	58	60	52	46	292	59
normalized size	1	1.00	1.46	1.21	1.25	1.08	0.96	6.08	1.23
time (sec)	N/A	0.027	0.009	0.013	0.317	1.212	0.896	0.130	1.053

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	70	68	61	70	80	397	71
normalized size	1	1.00	1.08	1.05	0.94	1.08	1.23	6.11	1.09
time (sec)	N/A	0.041	0.010	0.013	0.327	0.628	1.911	0.158	0.911

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	164	314	215	193	211	889	171
normalized size	1	1.00	1.13	2.17	1.48	1.33	1.46	6.13	1.18
time (sec)	N/A	0.327	0.073	0.032	0.368	1.181	2.737	0.177	1.042

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	161	306	0	0	0	0	-1
normalized size	1	1.00	0.99	1.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.302	0.500	0.018	0.000	0.600	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	132	278	189	160	168	603	134
normalized size	1	1.00	1.17	2.46	1.67	1.42	1.49	5.34	1.19
time (sec)	N/A	0.223	0.064	0.017	0.331	0.574	1.679	0.168	0.916

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	122	270	0	0	0	0	-1
normalized size	1	1.00	0.94	2.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.276	0.013	0.000	0.444	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	90	239	158	122	114	301	89
normalized size	1	1.00	1.20	3.19	2.11	1.63	1.52	4.01	1.19
time (sec)	N/A	0.114	0.067	0.016	0.325	0.613	0.840	0.161	0.785

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	82	123	0	0	0	0	-1
normalized size	1	1.00	1.11	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.149	0.189	0.000	0.528	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	120	701	0	0	0	0	-1
normalized size	1	1.00	1.03	5.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	0.075	0.280	0.000	0.618	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	94	248	0	0	0	0	-1
normalized size	1	1.00	1.32	3.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.153	0.023	0.000	0.652	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	101	253	151	135	126	278	246
normalized size	1	1.00	1.26	3.16	1.89	1.69	1.58	3.48	3.08
time (sec)	N/A	0.133	0.072	0.019	0.323	0.707	1.144	0.158	1.490

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	145	339	0	0	0	0	-1
normalized size	1	1.00	1.12	2.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.379	0.023	0.000	0.460	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	164	290	224	173	184	612	303
normalized size	1	1.00	1.40	2.48	1.91	1.48	1.57	5.23	2.59
time (sec)	N/A	0.228	0.076	0.027	0.337	0.900	1.996	0.153	1.903

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	305	1330	0	0	0	0	-1
normalized size	1	1.00	1.23	5.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.964	0.782	2.326	0.000	0.514	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	383	1275	0	0	0	0	-1
normalized size	1	1.00	1.46	4.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.774	0.809	2.219	0.000	0.692	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	245	1245	0	0	0	0	-1
normalized size	1	1.00	1.32	6.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.569	0.600	1.412	0.000	1.143	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	250	1177	0	0	0	0	-1
normalized size	1	1.00	1.27	5.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.449	0.561	1.411	0.000	0.649	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	161	6097	0	0	0	0	-1
normalized size	1	1.00	1.31	49.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.302	0.636	0.000	0.519	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	161	261	0	0	0	0	-1
normalized size	1	1.00	1.49	2.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.288	0.243	0.000	0.642	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	178	1470	0	0	0	0	-1
normalized size	1	1.00	0.97	7.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.449	0.137	0.263	0.000	0.822	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	196	1583	0	0	0	0	-1
normalized size	1	1.00	1.92	15.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	0.344	0.535	0.000	1.595	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	192	5098	0	0	0	0	-1
normalized size	1	1.00	1.56	41.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.294	0.293	0.710	0.000	0.550	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	323	1838	0	0	0	0	-1
normalized size	1	1.00	1.62	9.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.498	0.937	1.943	0.000	0.627	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	295	1281	0	0	0	0	-1
normalized size	1	1.00	1.58	6.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.626	0.699	1.707	0.000	1.051	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	128	107	134	296	0	0	-1
normalized size	1	1.00	1.03	0.86	1.08	2.39	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.078	0.045	0.424	0.579	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	115	93	118	255	0	0	-1
normalized size	1	1.00	1.08	0.88	1.11	2.41	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.061	0.036	0.430	0.664	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	114	89	119	223	685	0	-1
normalized size	1	1.00	1.08	0.84	1.12	2.10	6.46	0.00	-0.01
time (sec)	N/A	0.061	0.053	0.034	0.430	0.750	11.725	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	98	70	103	211	0	88	-1
normalized size	1	1.00	1.15	0.82	1.21	2.48	0.00	1.04	-0.01
time (sec)	N/A	0.052	0.035	0.035	0.428	0.553	0.000	0.152	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	99	78	94	221	0	93	-1
normalized size	1	1.00	1.16	0.92	1.11	2.60	0.00	1.09	-0.01
time (sec)	N/A	0.055	0.046	0.031	0.434	0.654	0.000	0.172	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	107	94	101	243	0	117	-1
normalized size	1	1.00	1.00	0.88	0.94	2.27	0.00	1.09	-0.01
time (sec)	N/A	0.063	0.066	0.037	0.427	1.265	0.000	0.186	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	108	94	112	253	0	117	-1
normalized size	1	1.00	1.01	0.88	1.05	2.36	0.00	1.09	-0.01
time (sec)	N/A	0.066	0.053	0.034	0.434	0.647	0.000	0.202	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	122	108	130	272	0	135	-1
normalized size	1	1.00	0.98	0.86	1.04	2.18	0.00	1.08	-0.01
time (sec)	N/A	0.078	0.073	0.038	0.415	1.482	0.000	0.204	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	3.831	1.756	0.000	1.079	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	2.519	1.411	0.000	0.564	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	59	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.071	1.470	0.000	0.739	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	0.273	0.883	0.000	1.538	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	0.547	0.900	0.000	1.368	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.005	0.477	0.303	0.000	2.140	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	0.355	0.565	0.000	1.056	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	78	66	69	64	58	78	69
normalized size	1	1.00	1.44	1.22	1.28	1.19	1.07	1.44	1.28
time (sec)	N/A	0.039	0.018	0.030	0.315	0.713	17.367	0.161	1.064

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	45	46	62	85	57	61
normalized size	1	1.00	1.10	0.94	0.96	1.29	1.77	1.19	1.27
time (sec)	N/A	0.035	0.016	0.025	0.318	1.071	12.461	0.132	0.791

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	67	57	58	54	48	181	60
normalized size	1	1.00	1.56	1.33	1.35	1.26	1.12	4.21	1.40
time (sec)	N/A	0.030	0.014	0.031	0.337	1.466	8.412	0.162	0.939

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	37	37	50	71	188	52
normalized size	1	1.00	1.14	1.00	1.00	1.35	1.92	5.08	1.41
time (sec)	N/A	0.015	0.008	0.021	0.315	0.843	6.943	0.151	0.765

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	124	0	0	0	0	-1
normalized size	1	1.00	0.93	4.13	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.033	0.014	0.055	0.000	0.412	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	45	49	41	55	80	51	55
normalized size	1	1.00	1.12	1.22	1.02	1.38	2.00	1.28	1.38
time (sec)	N/A	0.026	0.012	0.032	0.312	0.698	11.509	0.148	0.855

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	65	55	51	49	41	67	52
normalized size	1	1.00	1.59	1.34	1.24	1.20	1.00	1.63	1.27
time (sec)	N/A	0.027	0.013	0.033	0.322	1.104	8.073	0.138	0.999

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	61	63	51	65	97	65	67
normalized size	1	1.00	1.09	1.12	0.91	1.16	1.73	1.16	1.20
time (sec)	N/A	0.034	0.013	0.033	0.310	0.892	22.255	0.150	0.884

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	93	53	69	197	238	73	72
normalized size	1	1.00	1.43	0.82	1.06	3.03	3.66	1.12	1.11
time (sec)	N/A	0.035	0.026	0.030	0.411	0.879	11.708	0.394	0.984

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	91	51	66	186	559	75	70
normalized size	1	1.00	1.44	0.81	1.05	2.95	8.87	1.19	1.11
time (sec)	N/A	0.033	0.020	0.032	0.408	0.921	8.538	0.296	0.851

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	57	37	55	160	178	83	55
normalized size	1	1.00	1.30	0.84	1.25	3.64	4.05	1.89	1.25
time (sec)	N/A	0.024	0.022	0.030	0.402	0.877	5.614	0.129	0.790

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	75	42	61	157	1292	79	62
normalized size	1	1.00	1.63	0.91	1.33	3.41	28.09	1.72	1.35
time (sec)	N/A	0.026	0.019	0.030	0.413	1.162	10.148	0.170	0.921

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	91	51	65	181	1822	93	71
normalized size	1	1.00	1.44	0.81	1.03	2.87	28.92	1.48	1.13
time (sec)	N/A	0.033	0.029	0.033	0.409	0.976	14.518	0.182	0.995

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	91	51	66	187	1867	91	71
normalized size	1	1.00	1.44	0.81	1.05	2.97	29.63	1.44	1.13
time (sec)	N/A	0.032	0.030	0.034	0.405	1.017	20.466	0.219	1.026

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F(-2)	A	A	A	A	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	636	146	0	217	176	206	175	335
normalized size	1	5.09	1.17	0.00	1.74	1.41	1.65	1.40	2.68
time (sec)	N/A	1.547	0.080	180.000	0.332	0.861	23.145	0.280	1.733

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F(-2)	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	536	132	0	0	0	0	0	-1
normalized size	1	3.67	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.296	0.303	180.000	0.000	0.882	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	B	B	A	A	B	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	524	106	247	186	138	163	361	275
normalized size	1	5.76	1.16	2.71	2.04	1.52	1.79	3.97	3.02
time (sec)	N/A	0.965	0.062	0.264	0.325	0.796	11.526	0.191	1.262

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	207	99	144	0	0	0	0	-1
normalized size	1	2.20	1.05	1.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.514	0.163	0.178	0.000	0.957	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	183	0	0	0	0	0	-1
normalized size	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.336	0.076	0.216	0.000	0.843	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F(-2)	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	237	119	0	0	0	0	0	-1
normalized size	1	2.72	1.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.632	0.164	180.000	0.000	0.733	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	B	B	A	A	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	360	111	257	175	151	175	0	278
normalized size	1	4.09	1.26	2.92	1.99	1.72	1.99	0.00	3.16
time (sec)	N/A	1.057	0.097	0.273	0.328	0.844	17.023	0.000	1.498

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1173	1173	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.334	5.620	0.383	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1129	1129	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.061	3.305	0.344	0.000	0.882	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	958	958	566	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.468	2.741	0.288	0.000	0.990	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	942	942	566	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.335	3.723	0.342	0.000	0.465	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1102	1102	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.842	2.769	0.290	0.000	2.145	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1176	1176	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.990	3.269	0.296	0.000	0.677	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	479	185	748	0	0	0	0	-1
normalized size	1	3.40	1.31	5.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	4.216	0.499	0.390	0.000	0.660	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	390	213	298	0	0	0	0	-1
normalized size	1	2.91	1.59	2.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	2.323	0.086	0.200	0.000	0.904	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	211	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.560	0.210	0.214	0.000	1.121	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	0	222	0	0	0	0	0	-1
normalized size	1	0.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.780	0.433	0.289	0.000	1.286	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	0	218	0	0	0	0	0	-1
normalized size	1	0.00	1.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.558	0.295	0.338	0.000	1.157	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	241	302	316	57	0	0	-1
normalized size	1	1.00	0.76	0.95	1.00	0.18	0.00	0.00	-0.00
time (sec)	N/A	0.334	0.131	0.056	0.422	1.309	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	240	292	310	49	0	0	-1
normalized size	1	1.00	0.76	0.92	0.98	0.15	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.100	0.038	0.427	2.031	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	227	280	301	34	0	0	-1
normalized size	1	1.00	0.75	0.93	1.00	0.11	0.00	0.00	-0.00
time (sec)	N/A	0.251	0.080	0.039	0.422	1.292	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	227	273	296	34	0	493	-1
normalized size	1	1.00	0.80	0.96	1.04	0.12	0.00	1.73	-0.00
time (sec)	N/A	0.242	0.065	0.038	0.419	1.961	0.000	0.164	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	268	272	296	38	0	505	-1
normalized size	1	1.00	0.94	0.95	1.04	0.13	0.00	1.77	-0.00
time (sec)	N/A	0.248	0.111	0.034	0.411	1.829	0.000	0.851	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	268	280	277	38	0	516	-1
normalized size	1	1.00	0.89	0.93	0.92	0.13	0.00	1.71	-0.00
time (sec)	N/A	0.250	0.112	0.037	0.437	0.891	0.000	0.732	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	275	292	298	45	0	532	-1
normalized size	1	1.00	0.87	0.92	0.94	0.14	0.00	1.68	-0.00
time (sec)	N/A	0.278	0.094	0.040	0.425	1.122	0.000	2.613	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	281	302	297	46	0	519	-1
normalized size	1	1.00	0.89	0.95	0.94	0.15	0.00	1.64	-0.00
time (sec)	N/A	0.287	0.109	0.038	0.413	0.702	0.000	7.783	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F(-1)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	6327	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.026	63.715	0.465	0.000	1.081	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	6177	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.026	62.666	0.386	0.000	0.645	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	6334	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.029	99.105	0.474	0.000	0.433	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	6520	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.030	52.225	0.476	0.000	0.583	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	1.854	0.183	0.000	0.545	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	1.250	0.179	0.000	0.495	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.059	0.179	0.000	0.563	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.029	0.356	0.110	0.000	1.024	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	0.372	0.114	0.000	0.928	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	78	66	69	64	0	78	69
normalized size	1	1.00	1.44	1.22	1.28	1.19	0.00	1.44	1.28
time (sec)	N/A	0.038	0.018	0.031	0.304	0.595	0.000	0.143	1.108

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	45	46	62	0	57	61
normalized size	1	1.00	1.10	0.94	0.96	1.29	0.00	1.19	1.27
time (sec)	N/A	0.036	0.015	0.023	0.306	0.568	0.000	0.158	0.824

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	67	57	58	54	0	181	60
normalized size	1	1.00	1.56	1.33	1.35	1.26	0.00	4.21	1.40
time (sec)	N/A	0.030	0.014	0.029	0.305	0.710	0.000	0.145	0.957

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	37	37	50	0	188	52
normalized size	1	1.00	1.14	1.00	1.00	1.35	0.00	5.08	1.41
time (sec)	N/A	0.020	0.008	0.021	0.300	0.659	0.000	0.160	0.780

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	92	0	0	0	0	-1
normalized size	1	1.00	0.93	3.07	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.034	0.014	0.098	0.000	0.733	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	45	49	41	55	0	51	55
normalized size	1	1.00	1.12	1.22	1.02	1.38	0.00	1.28	1.38
time (sec)	N/A	0.027	0.012	0.033	0.307	0.637	0.000	0.154	0.853

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	65	55	51	49	0	67	52
normalized size	1	1.00	1.59	1.34	1.24	1.20	0.00	1.63	1.27
time (sec)	N/A	0.027	0.012	0.033	0.306	0.562	0.000	0.148	1.022

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	61	63	51	65	0	65	67
normalized size	1	1.00	1.09	1.12	0.91	1.16	0.00	1.16	1.20
time (sec)	N/A	0.036	0.012	0.036	0.300	0.684	0.000	0.135	0.901

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	196	184	162	981	0	207	125
normalized size	1	1.00	1.13	1.06	0.93	5.64	0.00	1.19	0.72
time (sec)	N/A	0.220	0.044	0.035	0.402	0.753	0.000	0.199	1.574

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	136	99	90	260	0	109	107
normalized size	1	1.00	1.35	0.98	0.89	2.57	0.00	1.08	1.06
time (sec)	N/A	0.097	0.043	0.030	0.403	1.004	0.000	0.129	2.757

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	187	159	155	228	0	165	118
normalized size	1	1.00	1.13	0.96	0.94	1.38	0.00	1.00	0.72
time (sec)	N/A	0.197	0.056	0.032	0.405	0.652	0.000	0.211	1.470

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	196	172	100	151	0	125	135
normalized size	1	1.00	1.70	1.50	0.87	1.31	0.00	1.09	1.17
time (sec)	N/A	0.095	0.051	0.035	0.424	0.967	0.000	0.165	3.119

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	198	186	164	248	0	208	127
normalized size	1	1.00	1.12	1.06	0.93	1.41	0.00	1.18	0.72
time (sec)	N/A	0.284	0.031	0.032	0.414	0.644	0.000	0.288	1.259

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	198	114	103	166	0	126	124
normalized size	1	1.00	1.69	0.97	0.88	1.42	0.00	1.08	1.06
time (sec)	N/A	0.097	0.029	0.026	0.414	0.772	0.000	0.155	2.438

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	187	177	155	238	0	179	118
normalized size	1	1.00	1.13	1.07	0.94	1.44	0.00	1.08	0.72
time (sec)	N/A	0.249	0.026	0.031	0.412	0.614	0.000	0.440	1.248

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	183	105	94	117	0	106	117
normalized size	1	1.00	1.76	1.01	0.90	1.12	0.00	1.02	1.12
time (sec)	N/A	0.083	0.027	0.026	0.412	0.728	0.000	0.149	2.387

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	196	172	160	196	0	187	125
normalized size	1	1.00	1.13	0.99	0.92	1.13	0.00	1.07	0.72
time (sec)	N/A	0.267	0.053	0.035	0.418	0.578	0.000	0.436	1.276

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F(-2)	A	A	F(-1)	A	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	636	146	0	217	176	0	175	335
normalized size	1	5.09	1.17	0.00	1.74	1.41	0.00	1.40	2.68
time (sec)	N/A	1.546	0.080	180.000	0.324	0.572	0.000	0.298	1.596

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	536	132	0	0	0	0	0	-1
normalized size	1	3.67	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.309	0.287	180.000	0.000	0.660	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	B	B	A	F(-1)	B	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	524	106	247	186	138	0	361	275
normalized size	1	5.76	1.16	2.71	2.04	1.52	0.00	3.97	3.02
time (sec)	N/A	0.985	0.059	0.276	0.322	0.554	0.000	0.245	1.247

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	207	99	145	0	0	0	0	-1
normalized size	1	2.16	1.03	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.592	0.159	0.179	0.000	0.620	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	183	0	0	0	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.335	0.072	0.220	0.000	0.756	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	237	117	0	0	0	0	0	-1
normalized size	1	2.63	1.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.619	0.163	180.000	0.000	1.247	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	B	B	A	F(-1)	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	360	111	257	175	151	0	0	278
normalized size	1	4.09	1.26	2.92	1.99	1.72	0.00	0.00	3.16
time (sec)	N/A	1.060	0.088	0.283	0.329	1.363	0.000	0.000	1.543

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	420	159	0	0	0	0	0	-1
normalized size	1	2.92	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.311	0.382	0.273	0.000	0.626	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	231	1421	334	0	0	0	0	0	-1
normalized size	1	6.15	1.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	6.350	0.496	0.343	0.000	0.578	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	479	185	750	0	0	0	0	-1
normalized size	1	3.45	1.33	5.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	4.204	0.316	0.436	0.000	0.951	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	390	208	295	0	0	0	0	-1
normalized size	1	3.00	1.60	2.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	2.467	0.257	0.205	0.000	0.764	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	214	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.565	0.190	0.222	0.000	0.692	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	F	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	0	223	0	0	0	0	0	-1
normalized size	1	0.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.779	0.409	0.312	0.000	0.526	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	0	218	0	0	0	0	0	-1
normalized size	1	0.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.575	0.298	0.327	0.000	0.556	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	1.876	0.192	0.000	0.635	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	1.245	0.193	0.000	1.056	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.061	0.183	0.000	0.545	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	0.363	0.113	0.000	0.669	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	0.376	0.118	0.000	1.069	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	67	62	57	48	46	262	45
normalized size	1	1.00	1.34	1.24	1.14	0.96	0.92	5.24	0.90
time (sec)	N/A	0.032	0.011	0.041	0.312	0.667	0.561	0.174	0.779

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	67	42	49	49	227	42
normalized size	1	1.00	1.11	1.49	0.93	1.09	1.09	5.04	0.93
time (sec)	N/A	0.033	0.008	0.040	0.318	0.679	0.448	0.199	0.719

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	56	53	44	39	36	130	36
normalized size	1	1.00	1.44	1.36	1.13	1.00	0.92	3.33	0.92
time (sec)	N/A	0.020	0.008	0.037	0.319	0.617	0.336	0.377	0.735

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	48	29	35	24	150	27
normalized size	1	1.00	1.00	1.66	1.00	1.21	0.83	5.17	0.93
time (sec)	N/A	0.013	0.003	0.034	0.314	0.556	0.247	0.214	0.679

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	63	0	0	0	0	-1
normalized size	1	1.00	0.93	2.10	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.032	0.013	0.043	0.000	0.651	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	38	37	37	48	39	87	43
normalized size	1	1.00	1.09	1.06	1.06	1.37	1.11	2.49	1.23
time (sec)	N/A	0.021	0.009	0.023	0.314	0.657	0.792	0.198	0.745

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	60	57	52	46	44	123	49
normalized size	1	1.00	1.40	1.33	1.21	1.07	1.02	2.86	1.14
time (sec)	N/A	0.027	0.009	0.030	0.320	0.509	0.953	0.169	0.709

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	62	57	55	62	68	234	59
normalized size	1	1.00	1.09	1.00	0.96	1.09	1.19	4.11	1.04
time (sec)	N/A	0.041	0.010	0.034	0.317	1.118	1.288	0.310	0.750

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	B	A	A	A	B	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	812	131	328	189	149	158	552	142
normalized size	1	6.60	1.07	2.67	1.54	1.21	1.28	4.49	1.15
time (sec)	N/A	1.703	0.064	0.064	0.336	0.594	0.957	0.175	0.892

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	695	145	391	0	0	0	0	-1
normalized size	1	4.89	1.02	2.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.389	0.327	0.069	0.000	0.558	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	B	A	A	A	B	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	574	92	287	136	111	104	268	101
normalized size	1	6.92	1.11	3.46	1.64	1.34	1.25	3.23	1.22
time (sec)	N/A	1.041	0.050	0.062	0.331	0.641	0.518	0.581	0.782

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	370	97	282	0	0	0	0	-1
normalized size	1	5.00	1.31	3.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.402	0.126	0.059	0.000	1.695	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	780	0	0	0	0	-1
normalized size	1	1.00	0.86	5.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.314	0.095	0.201	0.000	0.665	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	205	101	144	0	0	0	0	-1
normalized size	1	2.36	1.16	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.507	0.188	0.181	0.000	0.965	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	B	B	A	A	B	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	707	119	284	165	130	124	255	235
normalized size	1	8.13	1.37	3.26	1.90	1.49	1.43	2.93	2.70
time (sec)	N/A	1.247	0.072	0.051	0.329	0.700	1.173	0.177	1.284

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	0	286	1410	0	0	0	0	-1
normalized size	1	0.00	1.41	6.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.390	0.675	0.707	0.000	0.615	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	0	316	2033	0	0	0	0	-1
normalized size	1	0.00	1.46	9.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.302	0.792	0.871	0.000	0.522	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	0	193	5536	0	0	0	0	-1
normalized size	1	0.00	1.43	41.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	2.245	0.336	0.596	0.000	0.487	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	0	198	1756	0	0	0	0	-1
normalized size	1	0.00	1.83	16.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.701	0.279	0.363	0.000	0.889	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	171	1631	0	0	0	0	-1
normalized size	1	1.00	0.82	7.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	0.199	0.129	0.000	0.719	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	387	206	298	0	0	0	0	-1
normalized size	1	3.07	1.63	2.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	2.237	0.117	0.209	0.000	0.542	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	0	195	6645	0	0	0	0	-1
normalized size	1	0.00	1.40	47.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	2.102	0.352	0.565	0.000	0.625	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	73	64	62	53	51	71	66
normalized size	1	1.00	1.35	1.19	1.15	0.98	0.94	1.31	1.22
time (sec)	N/A	0.042	0.015	0.043	0.319	0.532	9.895	0.152	1.010

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	65	42	52	75	52	56
normalized size	1	1.00	1.11	1.44	0.93	1.16	1.67	1.16	1.24
time (sec)	N/A	0.033	0.011	0.061	0.318	0.574	6.845	0.154	0.814

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	55	49	44	41	162	57
normalized size	1	1.00	1.44	1.28	1.14	1.02	0.95	3.77	1.33
time (sec)	N/A	0.030	0.011	0.040	0.315	0.549	4.599	0.180	0.865

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	39	52	34	43	61	184	47
normalized size	1	1.00	1.15	1.53	1.00	1.26	1.79	5.41	1.38
time (sec)	N/A	0.018	0.007	0.057	0.324	0.683	3.200	0.169	0.789

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	154	0	0	0	0	-1
normalized size	1	1.00	0.93	5.13	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.033	0.014	0.058	0.000	0.515	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	37	37	55	76	52	56
normalized size	1	1.00	1.14	1.00	1.00	1.49	2.05	1.41	1.51
time (sec)	N/A	0.020	0.010	0.023	0.318	0.554	11.523	0.189	0.842

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	64	57	56	52	49	66	59
normalized size	1	1.00	1.42	1.27	1.24	1.16	1.09	1.47	1.31
time (sec)	N/A	0.032	0.012	0.038	0.305	0.691	15.088	0.699	0.995

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	62	45	55	67	94	65	66
normalized size	1	1.00	1.09	0.79	0.96	1.18	1.65	1.14	1.16
time (sec)	N/A	0.040	0.013	0.037	0.322	0.637	23.618	0.239	0.894

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	88	53	62	170	717	67	67
normalized size	1	1.00	1.40	0.84	0.98	2.70	11.38	1.06	1.06
time (sec)	N/A	0.035	0.022	0.042	0.413	0.625	11.515	0.303	0.983

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	86	51	61	162	702	69	65
normalized size	1	1.00	1.41	0.84	1.00	2.66	11.51	1.13	1.07
time (sec)	N/A	0.033	0.019	0.040	0.420	0.646	7.790	0.207	0.895

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	54	39	51	138	520	57	52
normalized size	1	1.00	1.23	0.89	1.16	3.14	11.82	1.30	1.18
time (sec)	N/A	0.024	0.017	0.036	0.422	0.585	5.143	0.361	0.819

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	72	44	57	159	620	62	59
normalized size	1	1.00	1.57	0.96	1.24	3.46	13.48	1.35	1.28
time (sec)	N/A	0.031	0.018	0.037	0.423	0.534	9.307	0.224	0.960

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	90	55	64	189	733	72	69
normalized size	1	1.00	1.38	0.85	0.98	2.91	11.28	1.11	1.06
time (sec)	N/A	0.037	0.032	0.039	0.414	0.649	13.689	0.252	0.992

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	90	55	65	196	797	74	69
normalized size	1	1.00	1.38	0.85	1.00	3.02	12.26	1.14	1.06
time (sec)	N/A	0.038	0.025	0.040	0.416	0.530	20.045	0.183	1.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F(-2)	A	A	C	B	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	599	104	0	157	126	151	327	247
normalized size	1	6.37	1.11	0.00	1.67	1.34	1.61	3.48	2.63
time (sec)	N/A	1.296	0.064	180.000	0.334	0.424	6.124	0.290	1.209

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	F(-2)	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	404	107	0	0	0	0	0	-1
normalized size	1	4.30	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.696	0.141	180.000	0.000	0.729	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	183	0	0	0	0	0	-1
normalized size	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.324	0.070	0.711	0.000	0.485	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	207	114	144	0	0	0	0	-1
normalized size	1	2.09	1.15	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.529	0.184	0.193	0.000	0.556	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F(-1)	B	A	A	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	770	131	0	183	143	172	0	262
normalized size	1	7.94	1.35	0.00	1.89	1.47	1.77	0.00	2.70
time (sec)	N/A	1.530	0.085	180.000	0.328	0.500	16.628	0.000	1.541

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1214	1214	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.712	5.895	1.013	0.000	0.571	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1172	1172	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.208	3.172	0.843	0.000	0.642	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1549	1549	565	0	0	0	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.249	3.480	0.767	0.000	0.602	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1117	1117	568	0	0	0	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.150	3.006	0.888	0.000	0.897	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1263	1263	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.585	2.906	0.832	0.000	0.616	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1337	1337	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.790	2.773	0.932	0.000	0.656	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	2.668	0.921	0.000	0.966	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	1.721	0.405	0.000	0.515	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	68	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.064	0.311	0.000	0.541	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	0.409	0.181	0.000	0.857	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	0.411	0.203	0.000	0.720	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	114	84	86	89	0	359	86
normalized size	1	1.00	1.30	0.95	0.98	1.01	0.00	4.08	0.98
time (sec)	N/A	0.042	0.034	0.030	0.318	0.639	0.000	0.205	1.434

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	101	75	78	80	0	301	58
normalized size	1	1.00	1.35	1.00	1.04	1.07	0.00	4.01	0.77
time (sec)	N/A	0.035	0.024	0.030	0.318	0.678	0.000	0.197	1.217

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	88	66	69	70	0	239	49
normalized size	1	1.00	1.42	1.06	1.11	1.13	0.00	3.85	0.79
time (sec)	N/A	0.024	0.021	0.029	0.314	0.611	0.000	0.212	1.170

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	42	50	53	56	0	174	32
normalized size	1	1.00	1.08	1.28	1.36	1.44	0.00	4.46	0.82
time (sec)	N/A	0.020	0.026	0.027	0.317	1.213	0.000	0.197	0.901

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	63	61	0	0	0	-1
normalized size	1	1.00	1.00	2.17	2.10	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.032	0.012	0.042	0.464	0.528	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	67	55	51	53	231	168	52
normalized size	1	1.00	1.68	1.38	1.28	1.32	5.78	4.20	1.30
time (sec)	N/A	0.023	0.025	0.033	0.318	0.798	21.867	0.431	1.117

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	86	64	64	64	342	356	61
normalized size	1	1.00	1.43	1.07	1.07	1.07	5.70	5.93	1.02
time (sec)	N/A	0.026	0.028	0.035	0.310	0.563	89.354	0.195	1.363

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	99	73	72	74	0	534	69
normalized size	1	1.00	1.36	1.00	0.99	1.01	0.00	7.32	0.95
time (sec)	N/A	0.034	0.031	0.036	0.315	0.774	0.000	0.219	1.386

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	A	A	F	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	0	224	396	265	273	0	0	453
normalized size	1	0.00	1.06	1.88	1.26	1.29	0.00	0.00	2.15
time (sec)	N/A	0.025	0.117	0.060	0.336	1.521	0.000	0.000	4.867

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	A	A	F	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	0	194	358	241	241	0	0	185
normalized size	1	0.00	1.12	2.07	1.39	1.39	0.00	0.00	1.07
time (sec)	N/A	0.024	0.108	0.059	0.336	1.214	0.000	0.000	1.565

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	B	A	F	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	0	160	317	215	207	0	0	143
normalized size	1	0.00	1.24	2.46	1.67	1.60	0.00	0.00	1.11
time (sec)	N/A	0.014	0.092	0.052	0.330	0.919	0.000	0.000	1.281

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	B	B	F	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	115	272	175	165	0	0	94
normalized size	1	0.00	1.35	3.20	2.06	1.94	0.00	0.00	1.11
time (sec)	N/A	0.006	0.065	0.053	0.339	1.132	0.000	0.000	1.058

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	164	742	0	0	0	0	-1
normalized size	1	1.00	1.13	5.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.317	0.089	0.227	0.000	1.157	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	B	B	A	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	129	292	174	157	680	0	278
normalized size	1	0.00	1.52	3.44	2.05	1.85	8.00	0.00	3.27
time (sec)	N/A	0.024	0.114	0.067	0.335	0.716	22.586	0.000	1.792

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	B	A	A	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	0	178	332	234	201	972	0	341
normalized size	1	0.00	1.34	2.50	1.76	1.51	7.31	0.00	2.56
time (sec)	N/A	0.024	0.132	0.071	0.335	0.736	91.043	0.000	2.707

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	0	418	1518	1972	0	0	0	-1
normalized size	1	0.00	1.12	4.06	5.27	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.023	1.267	3.658	1.281	0.609	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	304	0	351	1423	1579	0	0	0	-1
normalized size	1	0.00	1.15	4.68	5.19	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.024	0.861	0.579	1.143	1.716	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	0	285	1339	1184	0	0	0	-1
normalized size	1	0.00	1.22	5.72	5.06	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.014	0.559	0.419	1.040	0.710	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	0	201	6235	0	0	0	0	-1
normalized size	1	0.00	1.42	43.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.006	0.287	0.320	0.000	1.347	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	248	1542	0	0	0	0	-1
normalized size	1	1.00	1.11	6.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.512	0.207	0.183	0.000	1.224	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	0	230	5199	528	0	0	0	-1
normalized size	1	0.00	1.62	36.61	3.72	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.323	0.634	1.730	0.628	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	0	333	1365	703	0	0	0	-1
normalized size	1	0.00	1.42	5.83	3.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.023	0.724	0.593	1.959	0.728	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	35	24	36	121	170	24
normalized size	1	1.00	0.82	0.92	0.63	0.95	3.18	4.47	0.63
time (sec)	N/A	0.016	0.016	0.025	0.307	0.785	5.349	0.204	0.862

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	30	19	31	39	121	-1
normalized size	1	1.00	0.81	0.97	0.61	1.00	1.26	3.90	-0.03
time (sec)	N/A	0.013	0.012	0.024	0.316	1.005	1.065	0.154	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	25	87	72	14
normalized size	1	1.00	1.00	0.85	0.80	1.25	4.35	3.60	0.70
time (sec)	N/A	0.009	0.008	0.025	0.309	0.579	0.539	0.242	0.801

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	29	18	37	126	72	22
normalized size	1	1.00	1.00	1.21	0.75	1.54	5.25	3.00	0.92
time (sec)	N/A	0.010	0.020	0.031	0.308	0.960	1.375	0.177	0.786

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	222	194	172	1803	0	227	231
normalized size	1	1.00	1.17	1.02	0.91	9.49	0.00	1.19	1.22
time (sec)	N/A	0.301	0.057	0.036	0.425	3.706	0.000	0.369	13.380

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	75	57	58	64	0	97	110
normalized size	1	1.00	1.53	1.16	1.18	1.31	0.00	1.98	2.24
time (sec)	N/A	0.034	0.027	0.025	0.316	0.674	0.000	0.165	1.757

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	222	194	172	1848	0	0	247
normalized size	1	1.00	1.17	1.02	0.91	9.73	0.00	0.00	1.30
time (sec)	N/A	0.238	0.036	0.036	0.421	3.947	0.000	0.000	11.967

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	141	179	158	1682	0	186	107
normalized size	1	1.00	0.83	1.05	0.93	9.89	0.00	1.09	0.63
time (sec)	N/A	0.287	0.106	0.033	0.421	3.596	0.000	0.249	5.100

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	63	62	0	0	0	-1
normalized size	1	1.00	0.94	1.85	1.82	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.033	0.020	0.042	0.509	0.930	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	205	167	163	234	0	172	220
normalized size	1	1.00	1.19	0.97	0.95	1.36	0.00	1.00	1.28
time (sec)	N/A	0.221	0.035	0.036	0.421	0.745	0.000	0.329	7.456

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	220	180	168	214	0	194	228
normalized size	1	1.00	1.17	0.96	0.89	1.14	0.00	1.03	1.21
time (sec)	N/A	0.287	0.060	0.040	0.421	0.634	0.000	0.342	7.375

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	73	55	51	59	0	67	114
normalized size	1	1.00	1.55	1.17	1.09	1.26	0.00	1.43	2.43
time (sec)	N/A	0.032	0.030	0.035	0.307	0.705	0.000	0.233	1.358

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	B	B	F(-1)	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	0	122	284	186	179	0	0	105
normalized size	1	0.00	1.21	2.81	1.84	1.77	0.00	0.00	1.04
time (sec)	N/A	0.024	0.100	0.054	0.335	0.837	0.000	0.000	1.307

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	167	785	0	0	0	0	-1
normalized size	1	1.00	1.07	5.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.320	0.137	0.248	0.000	0.698	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	B	B	F(-1)	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	0	123	0	175	173	0	0	281
normalized size	1	0.00	1.28	0.00	1.82	1.80	0.00	0.00	2.93
time (sec)	N/A	0.024	0.142	0.432	0.332	1.459	0.000	0.000	2.037

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	73	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.056	0.138	0.000	0.973	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	73	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.047	0.277	0.000	0.664	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.032	0.220	0.000	0.503	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	76	0	141	0	0	-1
normalized size	1	1.00	1.08	2.11	0.00	3.92	0.00	0.00	-0.03
time (sec)	N/A	0.035	0.080	0.045	0.000	0.669	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	66	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.065	0.122	0.000	0.766	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.044	0.127	0.000	0.733	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	73	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.044	0.131	0.000	0.699	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	13.830	0.194	0.000	0.637	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.006	1.973	0.233	0.000	0.980	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	183	880	0	0	0	0	-1
normalized size	1	1.00	1.24	5.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.313	0.097	0.241	0.000	1.252	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	17.796	0.123	0.000	1.136	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	17.653	0.129	0.000	1.180	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	33	61	147	129	0	0	-1
normalized size	1	1.00	1.10	2.03	4.90	4.30	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.043	0.042	0.416	1.316	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	85	104	0	0	0	-1
normalized size	1	1.00	0.92	3.54	4.33	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.021	0.013	0.104	0.311	0.512	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	30	15	22	15	101	15
normalized size	1	1.00	0.89	1.58	0.79	1.16	0.79	5.32	0.79
time (sec)	N/A	0.006	0.002	0.046	0.309	0.635	0.194	0.164	0.071

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	8.422	0.246	0.000	0.685	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	5.973	0.234	0.000	0.527	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	88	77	0	0	0	0	0	-1
normalized size	1	1.05	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.090	0.286	0.000	0.812	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	0.382	0.064	0.000	1.074	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	1.830	0.296	0.000	0.588	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [178] had the largest ratio of [2.417]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	12	0.250
2	A	4	3	1.00	12	0.250
3	A	4	3	1.00	12	0.250
4	A	4	3	1.00	12	0.250
5	A	3	3	1.00	10	0.300
6	A	3	2	1.00	8	0.250
7	A	1	1	1.00	12	0.083
8	A	5	5	1.00	12	0.417
9	A	3	3	1.00	12	0.250
10	A	4	3	1.00	12	0.250
11	A	4	3	1.00	12	0.250
12	A	4	3	1.00	12	0.250
13	A	16	7	1.00	14	0.500
14	A	14	9	1.00	14	0.643
15	A	11	7	1.00	14	0.500
16	A	9	8	1.00	14	0.571
17	A	6	5	1.00	12	0.417
18	A	5	5	1.00	10	0.500
19	A	6	5	1.00	14	0.357
20	A	4	4	1.00	14	0.286
21	A	8	7	1.00	14	0.500
22	A	8	7	1.00	14	0.500
23	A	13	8	1.00	14	0.571
24	A	33	11	1.00	14	0.786
25	A	24	11	1.00	14	0.786
26	A	18	10	1.00	14	0.714
27	A	12	9	1.00	14	0.643
28	A	8	8	1.00	12	0.667
29	A	5	6	1.00	10	0.600
30	A	8	6	1.00	14	0.429
31	A	5	6	1.00	14	0.429
32	A	7	6	1.00	14	0.429
33	A	14	11	1.00	14	0.786
34	A	16	8	1.00	14	0.571
35	A	7	6	1.00	16	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	6	6	1.00	16	0.375
37	A	6	6	1.00	16	0.375
38	A	5	5	1.00	16	0.312
39	A	5	5	1.00	16	0.312
40	A	6	6	1.00	16	0.375
41	A	6	6	1.00	16	0.375
42	A	7	6	1.00	16	0.375
43	A	0	0	0.00	0	0.000
44	A	0	0	0.00	0	0.000
45	A	2	2	1.00	14	0.143
46	A	0	0	0.00	0	0.000
47	A	0	0	0.00	0	0.000
48	A	0	0	0.00	0	0.000
49	A	0	0	0.00	0	0.000
50	A	5	4	1.00	14	0.286
51	A	4	3	1.00	14	0.214
52	A	4	4	1.00	14	0.286
53	A	2	2	1.00	12	0.167
54	A	2	2	1.00	14	0.143
55	A	5	5	1.00	14	0.357
56	A	4	4	1.00	14	0.286
57	A	4	3	1.00	14	0.214
58	A	5	5	1.00	14	0.357
59	A	5	5	1.00	14	0.357
60	A	5	4	1.00	10	0.400
61	A	4	4	1.00	14	0.286
62	A	5	5	1.00	14	0.357
63	A	5	5	1.00	14	0.357
64	C	62	19	5.09	16	1.187
65	B	53	19	3.67	16	1.187
66	C	44	16	5.76	16	1.000
67	B	28	12	2.20	14	0.857
68	A	7	6	1.00	16	0.375
69	B	24	13	2.72	16	0.812
70	C	46	23	4.09	16	1.438
71	A	102	26	1.00	16	1.625

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	86	26	1.00	16	1.625
73	A	69	21	1.00	12	1.750
74	A	47	21	1.00	16	1.313
75	A	64	24	1.00	16	1.500
76	A	77	24	1.00	16	1.500
77	B	155	30	3.40	16	1.875
78	B	82	23	2.91	14	1.643
79	A	9	7	1.00	16	0.438
80	F	0	0	N/A	0	N/A
81	F	0	0	N/A	0	N/A
82	A	17	14	1.00	18	0.778
83	A	17	14	1.00	18	0.778
84	A	16	13	1.00	18	0.722
85	A	16	13	1.00	18	0.722
86	A	16	13	1.00	18	0.722
87	A	16	13	1.00	18	0.722
88	A	17	14	1.00	18	0.778
89	A	17	14	1.00	18	0.778
90	F	0	0	N/A	0	N/A
91	F	0	0	N/A	0	N/A
92	F	0	0	N/A	0	N/A
93	F	0	0	N/A	0	N/A
94	A	0	0	0.00	0	0.000
95	A	0	0	0.00	0	0.000
96	A	3	3	1.00	16	0.188
97	A	0	0	0.00	0	0.000
98	A	0	0	0.00	0	0.000
99	A	5	4	1.00	14	0.286
100	A	4	3	1.00	14	0.214
101	A	4	4	1.00	14	0.286
102	A	2	2	1.00	14	0.143
103	A	2	2	1.00	14	0.143
104	A	5	5	1.00	14	0.357
105	A	4	4	1.00	14	0.286
106	A	4	3	1.00	14	0.214
107	A	12	8	1.00	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	9	8	1.00	10	0.800
109	A	11	7	1.00	14	0.500
110	A	9	9	1.00	14	0.643
111	A	12	8	1.00	14	0.571
112	A	9	9	1.00	14	0.643
113	A	11	7	1.00	12	0.583
114	A	8	8	1.00	14	0.571
115	A	12	8	1.00	14	0.571
116	C	62	19	5.09	16	1.187
117	B	53	19	3.67	16	1.187
118	C	44	16	5.76	16	1.000
119	B	28	12	2.16	16	0.750
120	A	7	6	1.00	16	0.375
121	B	24	13	2.63	16	0.812
122	C	46	23	4.09	16	1.438
123	B	59	24	2.92	16	1.500
124	B	239	32	6.15	16	2.000
125	B	155	30	3.45	16	1.875
126	B	82	23	3.00	16	1.438
127	A	9	7	1.00	16	0.438
128	F	0	0	N/A	0	N/A
129	F	0	0	N/A	0	N/A
130	A	0	0	0.00	0	0.000
131	A	0	0	0.00	0	0.000
132	A	3	3	1.00	16	0.188
133	A	0	0	0.00	0	0.000
134	A	0	0	0.00	0	0.000
135	A	5	4	1.00	14	0.286
136	A	5	4	1.00	14	0.286
137	A	4	4	1.00	12	0.333
138	A	4	3	1.00	10	0.300
139	A	2	2	1.00	14	0.143
140	A	2	2	1.00	14	0.143
141	A	4	4	1.00	14	0.286
142	A	5	4	1.00	14	0.286
143	C	88	34	6.60	16	2.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	B	73	34	4.89	16	2.125
145	C	58	32	6.92	14	2.286
146	B	31	14	5.00	12	1.167
147	A	7	6	1.00	16	0.375
148	B	28	12	2.36	16	0.750
149	C	66	23	8.13	16	1.438
150	F	0	0	N/A	0	N/A
151	F	0	0	N/A	0	N/A
152	F	0	0	N/A	0	N/A
153	F	0	0	N/A	0	N/A
154	A	9	7	1.00	16	0.438
155	B	82	23	3.07	16	1.438
156	F	0	0	N/A	0	N/A
157	A	6	5	1.00	14	0.357
158	A	5	4	1.00	14	0.286
159	A	5	5	1.00	14	0.357
160	A	3	3	1.00	12	0.250
161	A	2	2	1.00	14	0.143
162	A	2	2	1.00	14	0.143
163	A	5	5	1.00	14	0.357
164	A	5	4	1.00	14	0.286
165	A	6	6	1.00	14	0.429
166	A	6	6	1.00	14	0.429
167	A	6	5	1.00	10	0.500
168	A	5	5	1.00	14	0.357
169	A	6	6	1.00	14	0.429
170	A	6	6	1.00	14	0.429
171	C	59	33	6.37	16	2.063
172	B	34	19	4.30	14	1.357
173	A	7	6	1.00	16	0.375
174	B	28	12	2.09	16	0.750
175	C	67	23	7.94	16	1.438
176	A	97	33	1.00	16	2.063
177	A	79	33	1.00	16	2.063
178	A	99	29	1.00	12	2.417
179	A	71	29	1.00	16	1.812

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	104	30	1.00	16	1.875
181	A	129	30	1.00	16	1.875
182	A	0	0	0.00	0	0.000
183	A	0	0	0.00	0	0.000
184	A	4	4	1.00	16	0.250
185	A	0	0	0.00	0	0.000
186	A	0	0	0.00	0	0.000
187	A	7	4	1.00	16	0.250
188	A	6	4	1.00	16	0.250
189	A	5	4	1.00	14	0.286
190	A	5	4	1.00	12	0.333
191	A	2	2	1.00	16	0.125
192	A	4	4	1.00	16	0.250
193	A	5	4	1.00	16	0.250
194	A	6	4	1.00	16	0.250
195	F	0	0	N/A	0	N/A
196	F	0	0	N/A	0	N/A
197	F	0	0	N/A	0	N/A
198	F	0	0	N/A	0	N/A
199	A	7	6	1.00	18	0.333
200	F	0	0	N/A	0	N/A
201	F	0	0	N/A	0	N/A
202	F	0	0	N/A	0	N/A
203	F	0	0	N/A	0	N/A
204	F	0	0	N/A	0	N/A
205	F	0	0	N/A	0	N/A
206	A	9	7	1.00	18	0.389
207	F	0	0	N/A	0	N/A
208	F	0	0	N/A	0	N/A
209	A	3	2	1.00	12	0.167
210	A	3	2	1.00	12	0.167
211	A	2	2	1.00	12	0.167
212	A	4	4	1.00	12	0.333
213	A	13	9	1.00	16	0.562
214	A	5	5	1.00	16	0.312
215	A	13	9	1.00	14	0.643

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	13	8	1.00	12	0.667
217	A	2	2	1.00	16	0.125
218	A	12	8	1.00	16	0.500
219	A	13	9	1.00	16	0.562
220	A	5	5	1.00	16	0.312
221	F	0	0	N/A	0	N/A
222	A	7	6	1.00	18	0.333
223	F	0	0	N/A	0	N/A
224	A	2	2	1.00	14	0.143
225	A	2	2	1.00	12	0.167
226	A	3	2	1.00	10	0.200
227	A	2	2	1.00	14	0.143
228	A	2	2	1.00	14	0.143
229	A	2	2	1.00	14	0.143
230	A	2	2	1.00	14	0.143
231	A	0	0	0.00	0	0.000
232	A	0	0	0.00	0	0.000
233	A	7	6	1.00	16	0.375
234	A	0	0	0.00	0	0.000
235	A	0	0	0.00	0	0.000
236	A	2	2	1.00	10	0.200
237	A	2	2	1.00	10	0.200
238	A	3	3	1.00	4	0.750
239	A	0	0	0.00	0	0.000
240	A	0	0	0.00	0	0.000
241	A	3	3	1.05	16	0.188
242	A	0	0	0.00	0	0.000
243	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int x^5 \left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=59

$$\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)-\frac{b\tanh^{-1}(cx)}{6c^6}+\frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{bx^5}{30c}$$

```
[Out] 1/6*b*x/c^5+1/18*b*x^3/c^3+1/30*b*x^5/c-1/6*b*arctanh(c*x)/c^6+1/6*x^6*(a+b
*arctanh(c*x))
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5916, 302, 206}

$$\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{bx^3}{18c^3}+\frac{bx}{6c^5}-\frac{b\tanh^{-1}(cx)}{6c^6}+\frac{bx^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(6*c^5) + (b*x^3)/(18*c^3) + (b*x^5)/(30*c) - (b*ArcTanh[c*x])/(6*c^6
) + (x^6*(a + b*ArcTanh[c*x]))/6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^5 \left(a+b\tanh^{-1}(cx)\right) dx = \frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{6}(bc)\int\frac{x^6}{1-c^2x^2}dx$$
$$=\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{6}(bc)\int\left(-\frac{1}{c^6}-\frac{x^2}{c^4}-\frac{x^4}{c^2}+\frac{1}{c^6\left(1-c^2x^2\right)}\right)dx$$
$$=\frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{bx^5}{30c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)-\frac{b\int\frac{1}{1-c^2x^2}dx}{6c^5}$$
$$=\frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{bx^5}{30c}-\frac{b\tanh^{-1}(cx)}{6c^6}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)$$

Mathematica [A] time = 0.01, size = 81, normalized size = 1.37

$$\frac{ax^6}{6}+\frac{b\log(1-cx)}{12c^6}-\frac{b\log(cx+1)}{12c^6}+\frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{1}{6}bx^6\tanh^{-1}(cx)+\frac{bx^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(6*c^5) + (b*x^3)/(18*c^3) + (b*x^5)/(30*c) + (a*x^6)/6 + (b*x^6*ArcT
anh[c*x])/6 + (b*Log[1 - c*x])/(12*c^6) - (b*Log[1 + c*x])/(12*c^6)
```

fricas [A] time = 1.33, size = 67, normalized size = 1.14

$$\frac{30\,ac^6x^6+6\,bc^5x^5+10\,bc^3x^3+30\,bcx+15\left(bc^6x^6-b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*x^6 + 6*b*c^5*x^5 + 10*b*c^3*x^3 + 30*b*c*x + 15*(b*c^6*x^6
 - b)*log(-(c*x + 1)/(c*x - 1)))/c^6
```

giac [B] time = 0.31, size = 442, normalized size = 7.49

$$\frac{1}{45}c\left(\frac{15\left(\frac{3\,(cx+1)^5b}{(cx-1)^5}+\frac{10\,(cx+1)^3b}{(cx-1)^3}+\frac{3\,(cx+1)b}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^6c^7}{(cx-1)^6}-\frac{6\,(cx+1)^5c^7}{(cx-1)^5}+\frac{15\,(cx+1)^4c^7}{(cx-1)^4}-\frac{20\,(cx+1)^3c^7}{(cx-1)^3}+\frac{15\,(cx+1)^2c^7}{(cx-1)^2}-\frac{6\,(cx+1)c^7}{cx-1}+c^7}+\frac{\frac{90\,(cx+1)^5a}{(cx-1)^5}+\frac{300\,(cx+1)^3a}{(cx-1)^3}+\frac{90\,(cx+}{cx-}}{\frac{(cx+1)^6c^7}{(cx-1)^6}-\frac{6\,(cx+1}{(cx-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/45*c*(15*(3*(c*x + 1)^5*b/(c*x - 1)^5 + 10*(c*x + 1)^3*b/(c*x - 1)^3 + 3*
(c*x + 1)*b/(c*x - 1))*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6*c^7/(c*x - 1)
^6 - 6*(c*x + 1)^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4 - 20*(c
*x + 1)^3*c^7/(c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x + 1)*c^
7/(c*x - 1) + c^7) + (90*(c*x + 1)^5*a/(c*x - 1)^5 + 300*(c*x + 1)^3*a/(c*x
 - 1)^3 + 90*(c*x + 1)*a/(c*x - 1) + 45*(c*x + 1)^5*b/(c*x - 1)^5 - 135*(c*
x + 1)^4*b/(c*x - 1)^4 + 230*(c*x + 1)^3*b/(c*x - 1)^3 - 210*(c*x + 1)^2*b/
(c*x - 1)^2 + 93*(c*x + 1)*b/(c*x - 1) - 23*b)/((c*x + 1)^6*c^7/(c*x - 1)^6
 - 6*(c*x + 1)^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4 - 20*(c*x
 + 1)^3*c^7/(c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x + 1)*c^7/
(c*x - 1) + c^7))
```

maple [A] time = 0.01, size = 67, normalized size = 1.14

$$\frac{x^6 a}{6}+\frac{b\,x^6 \operatorname{arctanh}\left(cx\right)}{6}+\frac{b\,x^5}{30c}+\frac{b\,x^3}{18c^3}+\frac{bx}{6c^5}+\frac{b\ln\left(cx-1\right)}{12c^6}-\frac{b\ln\left(cx+1\right)}{12c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctanh(c*x)+1/30*b*x^5/c+1/18*b*x^3/c^3+1/6*b*x/c^5+1/
12/c^6*b*ln(c*x-1)-1/12/c^6*b*ln(c*x+1)
```

maxima [A] time = 0.31, size = 70, normalized size = 1.19

$$\frac{1}{6}\,ax^6+\frac{1}{180}\left(30\,x^6 \operatorname{artanh}\left(cx\right)+c\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\left(cx+1\right)}{c^7}+\frac{15\,\log\left(cx-1\right)}{c^7}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/180*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x
)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b
```

mupad [B] time = 0.84, size = 52, normalized size = 0.88

$$\frac{\frac{b\,c^3\,x^3}{18}-\frac{b\,\mathrm{atanh}(c\,x)}{6}+\frac{b\,c^5\,x^5}{30}+\frac{b\,c\,x}{6}}{c^6}+\frac{a\,x^6}{6}+\frac{b\,x^6\,\mathrm{atanh}\left(c\,x\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x)),x)
```

```
[Out] ((b*c^3*x^3)/18 - (b*atanh(c*x))/6 + (b*c^5*x^5)/30 + (b*c*x)/6)/c^6 + (a*x
^6)/6 + (b*x^6*atanh(c*x))/6
```

sympy [A] time = 1.62, size = 63, normalized size = 1.07

$$\begin{cases} \frac{ax^6}{6}+\frac{bx^6 \operatorname{atanh}\left(cx\right)}{6}+\frac{bx^5}{30c}+\frac{bx^3}{18c^3}+\frac{bx}{6c^5}-\frac{b \operatorname{atanh}\left(cx\right)}{6c^6} & \text{for } c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atanh(c*x)/6 + b*x**5/(30*c) + b*x**3/(18*c**3
) + b*x/(6*c**5) - b*atanh(c*x)/(6*c**6), Ne(c, 0)), (a*x**6/6, True))
```

3.2 $\int x^4 \left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=57

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{bx^2}{10c^3}+\frac{b\log\left(1-c^2x^2\right)}{10c^5}+\frac{bx^4}{20c}$$

```
[Out] 1/10*b*x^2/c^3+1/20*b*x^4/c+1/5*x^5*(a+b*arctanh(c*x))+1/10*b*ln(-c^2*x^2+1
)/c^5
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5916, 266, 43}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{bx^2}{10c^3}+\frac{b\log\left(1-c^2x^2\right)}{10c^5}+\frac{bx^4}{20c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) + (b*x^4)/(20*c) + (x^5*(a + b*ArcTanh[c*x]))/5 + (b*Log[1
 - c^2*x^2])/(10*c^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+b\tanh^{-1}(cx)\right)dx &= \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{5}(bc)\int\frac{x^5}{1-c^2x^2}\,dx\\
&= \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{10}(bc)\,\text{Subst}\left(\int\frac{x^2}{1-c^2x}\,dx,x,x^2\right)\\
&= \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{10}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^4}-\frac{x}{c^2}-\frac{1}{c^4\left(-1+c^2x\right)}\right)dx,x,x^2\right)\\
&= \frac{bx^2}{10c^3}+\frac{bx^4}{20c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{b\log\left(1-c^2x^2\right)}{10c^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 62, normalized size = 1.09

$$\frac{ax^5}{5}+\frac{bx^2}{10c^3}+\frac{b\log\left(1-c^2x^2\right)}{10c^5}+\frac{1}{5}bx^5\tanh^{-1}(cx)+\frac{bx^4}{20c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) + (b*x^4)/(20*c) + (a*x^5)/5 + (b*x^5*ArcTanh[c*x])/5 + (b
*Log[1 - c^2*x^2])/(10*c^5)
```

fricas [A] time = 1.35, size = 69, normalized size = 1.21

$$\frac{2\,bc^5x^5\log\left(-\frac{cx+1}{cx-1}\right)+4\,ac^5x^5+bc^4x^4+2\,bc^2x^2+2\,b\log\left(c^2x^2-1\right)}{20\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/20*(2*b*c^5*x^5*log(-(c*x + 1)/(c*x - 1)) + 4*a*c^5*x^5 + b*c^4*x^4 + 2*b
*c^2*x^2 + 2*b*log(c^2*x^2 - 1))/c^5
```

giac [B] time = 0.25, size = 403, normalized size = 7.07

$$\frac{1}{5}c\left(\frac{\left(\frac{5\,(cx+1)^4b}{(cx-1)^4}+\frac{10\,(cx+1)^2b}{(cx-1)^2}+b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5c^6}{(cx-1)^5}-\frac{5\,(cx+1)^4c^6}{(cx-1)^4}+\frac{10\,(cx+1)^3c^6}{(cx-1)^3}-\frac{10\,(cx+1)^2c^6}{(cx-1)^2}+\frac{5\,(cx+1)c^6}{cx-1}-c^6}+\frac{2\left(\frac{5\,(cx+1)^4a}{(cx-1)^4}+\frac{10\,(cx+1)^2a}{(cx-1)^2}+a+\frac{2\,(cx+1)^4b}{(cx-1)^4}-\cdots\right.}{\frac{(cx+1)^5c^6}{(cx-1)^5}-\frac{5\,(cx+1)^4c^6}{(cx-1)^4}+\frac{10\,(cx+1)^3c^6}{(cx-1)^3}-\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/5*c*((5*(c*x + 1)^4*b/(c*x - 1)^4 + 10*(c*x + 1)^2*b/(c*x - 1)^2 + b)*log
(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^6/(c*x - 1)^5 - 5*(c*x + 1)^4*c^6/(c*
x - 1)^4 + 10*(c*x + 1)^3*c^6/(c*x - 1)^3 - 10*(c*x + 1)^2*c^6/(c*x - 1)^2
+ 5*(c*x + 1)*c^6/(c*x - 1) - c^6) + 2*(5*(c*x + 1)^4*a/(c*x - 1)^4 + 10*(c
*x + 1)^2*a/(c*x - 1)^2 + a + 2*(c*x + 1)^4*b/(c*x - 1)^4 - 4*(c*x + 1)^3*b
/(c*x - 1)^3 + 4*(c*x + 1)^2*b/(c*x - 1)^2 - 2*(c*x + 1)*b/(c*x - 1))/((c*x
 + 1)^5*c^6/(c*x - 1)^5 - 5*(c*x + 1)^4*c^6/(c*x - 1)^4 + 10*(c*x + 1)^3*c^
6/(c*x - 1)^3 - 10*(c*x + 1)^2*c^6/(c*x - 1)^2 + 5*(c*x + 1)*c^6/(c*x - 1)
- c^6) - b*log(-(c*x + 1)/(c*x - 1) + 1)/c^6 + b*log(-(c*x + 1)/(c*x - 1))/
c^6)
```

maple [A] time = 0.01, size = 60, normalized size = 1.05

$$\frac{a\,x^5}{5}+\frac{b\,x^5\,\mathrm{arctanh}\,(cx)}{5}+\frac{b\,x^4}{20c}+\frac{b\,x^2}{10c^3}+\frac{b\ln\left(cx-1\right)}{10c^5}+\frac{b\ln\left(cx+1\right)}{10c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/5*a*x^5+1/5*b*x^5*arctanh(c*x)+1/20*b*x^4/c+1/10*b*x^2/c^3+1/10/c^5*b*ln(
c*x-1)+1/10/c^5*b*ln(c*x+1)
```

maxima [A] time = 0.31, size = 55, normalized size = 0.96

$$\frac{1}{5}\,ax^5+\frac{1}{20}\left(4\,x^5\,\mathrm{artanh}\,(cx)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2
*x^2 - 1)/c^6))*b
```

mupad [B] time = 0.82, size = 53, normalized size = 0.93

$$\frac{a\,x^5}{5}+\frac{\frac{b\,\ln\left(c^2\,x^2-1\right)}{10}+\frac{b\,c^2\,x^2}{10}+\frac{b\,c^4\,x^4}{20}}{c^5}+\frac{b\,x^5\,\mathrm{atanh}\left(c\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x)),x)
```

```
[Out] (a*x^5)/5 + ((b*log(c^2*x^2 - 1))/10 + (b*c^2*x^2)/10 + (b*c^4*x^4)/20)/c^5
 + (b*x^5*atanh(c*x))/5
```

sympy [A] time = 1.27, size = 68, normalized size = 1.19

$$\begin{cases} \frac{ax^5}{5}+\frac{bx^5\,\mathrm{atanh}\,(cx)}{5}+\frac{bx^4}{20c}+\frac{bx^2}{10c^3}+\frac{b\log\left(x-\frac{1}{c}\right)}{5c^5}+\frac{b\,\mathrm{atanh}\,(cx)}{5c^5} & \text{for } c\neq 0 \\ \frac{ax^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atanh(c*x)/5 + b*x**4/(20*c) + b*x**2/(10*c**3
) + b*log(x - 1/c)/(5*c**5) + b*atanh(c*x)/(5*c**5), Ne(c, 0)), (a*x**5/5,
True))
```

3.3 $\int x^3 \left(a+b \tanh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=48

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)-\frac{b\tanh^{-1}(cx)}{4c^4}+\frac{bx}{4c^3}+\frac{bx^3}{12c}$$

```
[Out] 1/4*b*x/c^3+1/12*b*x^3/c-1/4*b*arctanh(c*x)/c^4+1/4*x^4*(a+b*arctanh(c*x))
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5916, 302, 206}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{bx}{4c^3}-\frac{b\tanh^{-1}(cx)}{4c^4}+\frac{bx^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) + (b*x^3)/(12*c) - (b*ArcTanh[c*x])/(4*c^4) + (x^4*(a + b*Arc
Tanh[c*x]))/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tanh^{-1}(cx)\right)\,dx &= \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{4}(bc)\int\frac{x^4}{1-c^2x^2}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{4}(bc)\int\left(-\frac{1}{c^4}-\frac{x^2}{c^2}+\frac{1}{c^4\left(1-c^2x^2\right)}\right)dx\\
&= \frac{bx}{4c^3}+\frac{bx^3}{12c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)-\frac{b\int\frac{1}{1-c^2x^2}\,dx}{4c^3}\\
&= \frac{bx}{4c^3}+\frac{bx^3}{12c}-\frac{b\tanh^{-1}(cx)}{4c^4}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 70, normalized size = 1.46

$$\frac{ax^4}{4}+\frac{b\log(1-cx)}{8c^4}-\frac{b\log(cx+1)}{8c^4}+\frac{bx}{4c^3}+\frac{1}{4}bx^4\tanh^{-1}(cx)+\frac{bx^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) + (b*x^3)/(12*c) + (a*x^4)/4 + (b*x^4*ArcTanh[c*x])/4 + (b*Lo
g[1 - c*x])/(8*c^4) - (b*Log[1 + c*x])/(8*c^4)
```

fricas [A] time = 0.80, size = 58, normalized size = 1.21

$$\frac{6\,ac^4x^4+2\,bc^3x^3+6\,bcx+3\,\left(bc^4x^4-b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*x^4 + 2*b*c^3*x^3 + 6*b*c*x + 3*(b*c^4*x^4 - b)*log(-(c*x + 1
)/(c*x - 1)))/c^4
```

giac [B] time = 0.16, size = 296, normalized size = 6.17

$$\frac{1}{3}\,c\left(\frac{3\,\left(\frac{(cx+1)^3b}{(cx-1)^3}+\frac{(cx+1)b}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4c^5}{(cx-1)^4}-\frac{4\,(cx+1)^3c^5}{(cx-1)^3}+\frac{6\,(cx+1)^2c^5}{(cx-1)^2}-\frac{4\,(cx+1)c^5}{cx-1}+c^5}+\frac{\frac{6\,(cx+1)^3a}{(cx-1)^3}+\frac{6\,(cx+1)a}{cx-1}+\frac{3\,(cx+1)^3b}{(cx-1)^3}-\frac{6\,(cx+1)^2b}{(cx-1)^2}+\frac{5\,(cx+1)b}{cx-1}-2\,b}{\frac{(cx+1)^4c^5}{(cx-1)^4}-\frac{4\,(cx+1)^3c^5}{(cx-1)^3}+\frac{6\,(cx+1)^2c^5}{(cx-1)^2}-\frac{4\,(cx+1)c^5}{cx-1}+c^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*c*(3*((c*x + 1)^3*b/(c*x - 1)^3 + (c*x + 1)*b/(c*x - 1))*log(-(c*x + 1)
/(c*x - 1))/((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - 1)^3 +
6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^5/(c*x - 1) + c^5) + (6*(c*x
+ 1)^3*a/(c*x - 1)^3 + 6*(c*x + 1)*a/(c*x - 1) + 3*(c*x + 1)^3*b/(c*x - 1)^
3 - 6*(c*x + 1)^2*b/(c*x - 1)^2 + 5*(c*x + 1)*b/(c*x - 1) - 2*b)/((c*x + 1)
^4*c^5/(c*x - 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - 1)^3 + 6*(c*x + 1)^2*c^5/(c*x
 - 1)^2 - 4*(c*x + 1)*c^5/(c*x - 1) + c^5))
```

maple [A] time = 0.01, size = 58, normalized size = 1.21

$$\frac{x^4a}{4}+\frac{b\,x^4\operatorname{arctanh}\left(cx\right)}{4}+\frac{b\,x^3}{12c}+\frac{bx}{4c^3}+\frac{b\ln\left(cx-1\right)}{8c^4}-\frac{b\ln\left(cx+1\right)}{8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x)+1/12*b*x^3/c+1/4*b*x/c^3+1/8/c^4*b*ln(c*x-
1)-1/8/c^4*b*ln(c*x+1)
```

maxima [A] time = 0.31, size = 61, normalized size = 1.27

$$\frac{1}{4}\,ax^4+\frac{1}{24}\left(6\,x^4\operatorname{artanh}\left(cx\right)+c\left(\frac{2\left(c^2x^3+3\,x\right)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x
 + 1)/c^5 + 3*log(c*x - 1)/c^5))*b
```

mupad [B] time = 0.77, size = 43, normalized size = 0.90

$$\frac{a\,x^4}{4}+\frac{\frac{b\,c^3\,x^3}{12}-\frac{b\,\mathrm{atanh}(c\,x)}{4}+\frac{b\,c\,x}{4}}{c^4}+\frac{b\,x^4\,\mathrm{atanh}\left(c\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x)),x)
```

```
[Out] (a*x^4)/4 + ((b*c^3*x^3)/12 - (b*atanh(c*x))/4 + (b*c*x)/4)/c^4 + (b*x^4*at
anh(c*x))/4
```

sympy [A] time = 0.91, size = 53, normalized size = 1.10

$$\begin{cases} \frac{ax^4}{4}+\frac{bx^4\,\mathrm{atanh}\left(cx\right)}{4}+\frac{bx^3}{12c}+\frac{bx}{4c^3}-\frac{b\,\mathrm{atanh}\left(cx\right)}{4c^4} & \text{for } c\neq 0\\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atanh(c*x)/4 + b*x**3/(12*c) + b*x/(4*c**3) -
b*atanh(c*x)/(4*c**4), Ne(c, 0)), (a*x**4/4, True))
```

3.4 $\int x^2 \left(a + b \tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=46

$$\frac{1}{3}x^3 \left(a + b\tanh^{-1}(cx)\right) + \frac{b\log\left(1 - c^2x^2\right)}{6c^3} + \frac{bx^2}{6c}$$

```
[Out] 1/6*b*x^2/c+1/3*x^3*(a+b*arctanh(c*x))+1/6*b*ln(-c^2*x^2+1)/c^3
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5916, 266, 43}

$$\frac{1}{3}x^3 \left(a + b\tanh^{-1}(cx)\right) + \frac{b\log\left(1 - c^2x^2\right)}{6c^3} + \frac{bx^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x^2)/(6*c) + (x^3*(a + b*ArcTanh[c*x]))/3 + (b*Log[1 - c^2*x^2])/(6*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(a + b\tanh^{-1}(cx)\right) dx &= \frac{1}{3}x^3 \left(a + b\tanh^{-1}(cx)\right) - \frac{1}{3}(bc)\int \frac{x^3}{1 - c^2x^2}\,dx \\
&= \frac{1}{3}x^3 \left(a + b\tanh^{-1}(cx)\right) - \frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{x}{1 - c^2x}\,dx, x, x^2\right) \\
&= \frac{1}{3}x^3 \left(a + b\tanh^{-1}(cx)\right) - \frac{1}{6}(bc)\,\text{Subst}\left(\int \left(-\frac{1}{c^2} - \frac{1}{c^2\left(-1 + c^2x\right)}\right) dx, x, x^2\right) \\
&= \frac{bx^2}{6c} + \frac{1}{3}x^3 \left(a + b\tanh^{-1}(cx)\right) + \frac{b\log\left(1 - c^2x^2\right)}{6c^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.11

$$\frac{ax^3}{3} + \frac{b\log\left(1 - c^2x^2\right)}{6c^3} + \frac{1}{3}bx^3\tanh^{-1}(cx) + \frac{bx^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x^2)/(6*c) + (a*x^3)/3 + (b*x^3*ArcTanh[c*x])/3 + (b*Log[1 - c^2*x^2])/(
6*c^3)
```

fricas [A] time = 0.58, size = 58, normalized size = 1.26

$$\frac{bc^3x^3\log\left(-\frac{cx+1}{cx-1}\right)+2\,ac^3x^3+bc^2x^2+b\log\left(c^2x^2-1\right)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(b*c^3*x^3*log(-(c*x + 1)/(c*x - 1)) + 2*a*c^3*x^3 + b*c^2*x^2 + b*log(
c^2*x^2 - 1))/c^3
```

giac [B] time = 0.23, size = 258, normalized size = 5.61

$$\frac{1}{3}c\left(\frac{\left(\frac{3\,(cx+1)^2b}{(cx-1)^2}+b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3c^4}{(cx-1)^3}-\frac{3\,(cx+1)^2c^4}{(cx-1)^2}+\frac{3\,(cx+1)c^4}{cx-1}-c^4}+\frac{2\left(\frac{3\,(cx+1)^2a}{(cx-1)^2}+a+\frac{(cx+1)^2b}{(cx-1)^2}-\frac{(cx+1)b}{cx-1}\right)}{\frac{(cx+1)^3c^4}{(cx-1)^3}-\frac{3\,(cx+1)^2c^4}{(cx-1)^2}+\frac{3\,(cx+1)c^4}{cx-1}-c^4}-\frac{b\log\left(-\frac{cx+1}{cx-1}+1\right)}{c^4}+\frac{b\log\left(-\right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*c*((3*(c*x + 1)^2*b/(c*x - 1)^2 + b)*log(-(c*x + 1)/(c*x - 1))/((c*x +
1)^3*c^4/(c*x - 1)^3 - 3*(c*x + 1)^2*c^4/(c*x - 1)^2 + 3*(c*x + 1)*c^4/(c*x
 - 1) - c^4) + 2*(3*(c*x + 1)^2*a/(c*x - 1)^2 + a + (c*x + 1)^2*b/(c*x - 1)
^2 - (c*x + 1)*b/(c*x - 1))/((c*x + 1)^3*c^4/(c*x - 1)^3 - 3*(c*x + 1)^2*c^
4/(c*x - 1)^2 + 3*(c*x + 1)*c^4/(c*x - 1) - c^4) - b*log(-(c*x + 1)/(c*x -
1) + 1)/c^4 + b*log(-(c*x + 1)/(c*x - 1))/c^4)
```

maple [A] time = 0.01, size = 51, normalized size = 1.11

$$\frac{x^3a}{3}+\frac{b\,x^3\,\mathrm{arctanh}\,(cx)}{3}+\frac{b\,x^2}{6c}+\frac{b\ln\,(cx-1)}{6c^3}+\frac{b\ln\,(cx+1)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c*x)+1/6*b*x^2/c+1/6/c^3*b*ln(c*x-1)+1/6/c^3*b*
ln(c*x+1)
```

maxima [A] time = 0.31, size = 44, normalized size = 0.96

$$\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\,\mathrm{artanh}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b
```

mupad [B] time = 0.74, size = 44, normalized size = 0.96

$$\frac{\frac{b\,\ln\left(c^2\,x^2-1\right)}{6}+\frac{b\,c^2\,x^2}{6}}{c^3}+\frac{a\,x^3}{3}+\frac{b\,x^3\,\mathrm{atanh}\,(c\,x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x)),x)
```

```
[Out] ((b*log(c^2*x^2 - 1))/6 + (b*c^2*x^2)/6)/c^3 + (a*x^3)/3 + (b*x^3*atanh(c*x
))/3
```

sympy [A]   time = 0.73, size = 58, normalized size = 1.26

$$\begin{cases} \frac{ax^3}{3} + \frac{bx^3 \operatorname{atanh}(cx)}{3} + \frac{bx^2}{6c} + \frac{b \log\left(x - \frac{1}{c}\right)}{3c^3} + \frac{b \operatorname{atanh}(cx)}{3c^3} & \text{for } c \neq 0 \\ \frac{ax^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*atanh(c*x)/3 + b*x**2/(6*c) + b*log(x - 1/c)/(
3*c**3) + b*atanh(c*x)/(3*c**3), Ne(c, 0)), (a*x**3/3, True))
```

3.5 $\int x\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{b\tanh^{-1}(cx)}{2c^2}+\frac{bx}{2c}$$

```
[Out] 1/2*b*x/c-1/2*b*arctanh(c*x)/c^2+1/2*x^2*(a+b*arctanh(c*x))
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5916, 321, 206}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{b\tanh^{-1}(cx)}{2c^2}+\frac{bx}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(2*c) - (b*ArcTanh[c*x])/(2*c^2) + (x^2*(a + b*ArcTanh[c*x]))/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tanh^{-1}(cx)\right)dx&=\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}(bc)\int\frac{x^2}{1-c^2x^2}\,dx\\&=\frac{bx}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{b\int\frac{1}{1-c^2x^2}\,dx}{2c}\\&=\frac{bx}{2c}-\frac{b\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.59

$$\frac{ax^2}{2}+\frac{b\log(1-cx)}{4c^2}-\frac{b\log(cx+1)}{4c^2}+\frac{1}{2}bx^2\tanh^{-1}(cx)+\frac{bx}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(2*c) + (a*x^2)/2 + (b*x^2*ArcTanh[c*x])/2 + (b*Log[1 - c*x])/(4*c^2)
 - (b*Log[1 + c*x])/(4*c^2)
```

fricas [A] time = 0.58, size = 48, normalized size = 1.30

$$\frac{2\,ac^2x^2 + 2\,bcx + \left(bc^2x^2 - b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*x^2 + 2*b*c*x + (b*c^2*x^2 - b)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

giac [B] time = 0.29, size = 148, normalized size = 4.00

$$c\left(\frac{(cx+1)b\log\left(-\frac{cx+1}{cx-1}\right)}{\left(\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2\,(cx+1)c^3}{cx-1} + c^3\right)(cx-1)} + \frac{\frac{2\,(cx+1)a}{cx-1} + \frac{(cx+1)b}{cx-1} - b}{\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2\,(cx+1)c^3}{cx-1} + c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] c*((c*x + 1)*b*log(-(c*x + 1)/(c*x - 1))/((((c*x + 1)^2*c^3/(c*x - 1)^2 - 2*
(c*x + 1)*c^3/(c*x - 1) + c^3)*(c*x - 1)) + (2*(c*x + 1)*a/(c*x - 1) + (c*x
 + 1)*b/(c*x - 1) - b)/((c*x + 1)^2*c^3/(c*x - 1)^2 - 2*(c*x + 1)*c^3/(c*x
- 1) + c^3))
```

maple [A] time = 0.01, size = 49, normalized size = 1.32

$$\frac{a\,x^2}{2} + \frac{b\,x^2\,\mathrm{arctanh}\,(cx)}{2} + \frac{bx}{2c} + \frac{b\ln(cx-1)}{4c^2} - \frac{b\ln(cx+1)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c*x)+1/2*b*x/c+1/4/c^2*b*ln(c*x-1)-1/4/c^2*b*ln
(c*x+1)
```

maxima [A] time = 0.30, size = 50, normalized size = 1.35

$$\frac{1}{2}\,ax^2 + \frac{1}{4}\left(2\,x^2\,\mathrm{artanh}\,(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c
*x - 1)/c^3))*b
```

mupad [B] time = 0.73, size = 35, normalized size = 0.95

$$\frac{a\,x^2}{2} - \frac{\frac{b\,\mathrm{atanh}(c\,x)}{2} - \frac{b\,c\,x}{2}}{c^2} + \frac{b\,x^2\,\mathrm{atanh}\,(c\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x)),x)
```

```
[Out] (a*x^2)/2 - ((b*atanh(c*x))/2 - (b*c*x)/2)/c^2 + (b*x^2*atanh(c*x))/2
```

sympy [A] time = 0.49, size = 42, normalized size = 1.14

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{atanh}(cx)}{2} + \frac{bx}{2c} - \frac{b \operatorname{atanh}(cx)}{2c^2} & \text{for } c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*atanh(c*x)/2 + b*x/(2*c) - b*atanh(c*x)/(2*c**
2), Ne(c, 0)), (a*x**2/2, True))
```

3.6 $\int \left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=30

$$ax+\frac{b\log\left(1-c^2x^2\right)}{2c}+bx\tanh^{-1}(cx)$$

```
[Out] a*x+b*x*arctanh(c*x)+1/2*b*ln(-c^2*x^2+1)/c
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5910, 260}

$$ax+\frac{b\log\left(1-c^2x^2\right)}{2c}+bx\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tanh^{-1}(cx)\right)\,dx &= ax+b\int \tanh^{-1}(cx)\,dx\\ &= ax+bx\tanh^{-1}(cx)-(bc)\int \frac{x}{1-c^2x^2}\,dx\\ &= ax+bx\tanh^{-1}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$ax+\frac{b\log\left(1-c^2x^2\right)}{2c}+bx\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c)
```

fricas [A] time = 0.56, size = 42, normalized size = 1.40

$$\frac{bcx\log\left(-\frac{cx+1}{cx-1}\right)+2\,acx+b\log\left(c^2x^2-1\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x*log(-(c*x + 1)/(c*x - 1)) + 2*a*c*x + b*log(c^2*x^2 - 1))/c
```

giac [B] time = 0.19, size = 156, normalized size = 5.20

$$bc\left(\frac{\log\left(\frac{|-cx-1|}{|cx-1|}\right)}{c^2} - \frac{\log\left(\left|-\frac{cx+1}{cx-1}+1\right|\right)}{c^2} + \frac{\log\left(-\frac{\frac{c\left(\frac{cx+1}{cx-1}+1\right)}{\frac{(cx+1)c}{cx-1}-c}+1}{\frac{c\left(\frac{cx+1}{cx-1}+1\right)}{\frac{(cx+1)c}{cx-1}-c}-1}\right)}{c^2\left(\frac{cx+1}{cx-1}-1\right)}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x),x, algorithm="giac")
```

```
[Out] b*c*(log(abs(-c*x - 1)/abs(c*x - 1))/c^2 - log(abs(-(c*x + 1)/(c*x - 1) + 1
))/c^2 + log(-(c*((c*x + 1)/(c*x - 1) + 1)/((c*x + 1)*c/(c*x - 1) - c) + 1)
/(c*((c*x + 1)/(c*x - 1) + 1)/((c*x + 1)*c/(c*x - 1) - c) - 1))/(c^2*((c*x
+ 1)/(c*x - 1) - 1))) + a*x
```

maple [A] time = 0.00, size = 29, normalized size = 0.97

$$ax + bx\,\mathrm{arctanh}\,(cx) + \frac{b\ln\left(-c^2x^2+1\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x),x)
```

```
[Out] a*x+b*x*arctanh(c*x)+1/2*b*ln(-c^2*x^2+1)/c
```

maxima [A] time = 0.33, size = 30, normalized size = 1.00

$$ax + \frac{\left(2\,cx\,\mathrm{artanh}\,(cx) + \log\left(-c^2x^2+1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b/c
```

mupad [B] time = 0.68, size = 27, normalized size = 0.90

$$a\,x + \frac{b\,\ln\left(c^2\,x^2-1\right)}{2\,c} + b\,x\,\mathrm{atanh}\,(c\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c*x),x)
```

```
[Out] a*x + (b*log(c^2*x^2 - 1))/(2*c) + b*x*atanh(c*x)
```

sympy [A] time = 0.33, size = 27, normalized size = 0.90

$$ax + b\left(\begin{cases} x\,\mathrm{atanh}\,(cx) + \frac{\log\,(cx+1)}{c} - \frac{\mathrm{atanh}\,(cx)}{c} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*atanh(c*x) + log(c*x + 1)/c - atanh(c*x)/c, Ne(c, 0)),
 (0, True))
```

3.7 $\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx$

Optimal. Leaf size=26

$$a\log(x)-\frac{1}{2}b\text{Li}_2(-cx)+\frac{1}{2}b\text{Li}_2(cx)$$

```
[Out] a*ln(x)-1/2*b*polylog(2,-c*x)+1/2*b*polylog(2,c*x)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5912}

$$-\frac{1}{2}b\text{PolyLog}(2,-cx)+\frac{1}{2}b\text{PolyLog}(2,cx)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x,x]
[Out] a*Log[x] - (b*PolyLog[2, -(c*x)])/2 + (b*PolyLog[2, c*x])/2
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx=a\log(x)-\frac{1}{2}b\text{Li}_2(-cx)+\frac{1}{2}b\text{Li}_2(cx)$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.92

$$a\log(x)+\frac{1}{2}b(\text{Li}_2(cx)-\text{Li}_2(-cx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x,x]
[Out] a*Log[x] + (b*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/2
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\,(cx)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x,x, algorithm="fricas")
[Out] integral((b*arctanh(c*x) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\,(cx)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/x, x)
```

maple [B] time = 0.01, size = 47, normalized size = 1.81

$$a \ln(cx) + b \ln(cx) \operatorname{arctanh}(cx) - \frac{b \operatorname{dilog}(cx)}{2} - \frac{b \operatorname{dilog}(cx+1)}{2} - \frac{b \ln(cx) \ln(cx+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x,x)
```

```
[Out] a*ln(c*x)+b*ln(c*x)*arctanh(c*x)-1/2*b*dilog(c*x)-1/2*b*dilog(c*x+1)-1/2*b*
ln(c*x)*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} b \int \frac{\log(cx+1) - \log(-cx+1)}{x} \, dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{atanh}(c\,x)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/x,x)
```

```
[Out] int((a + b*atanh(c*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}(cx)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x,x)
```

```
[Out] Integral((a + b*atanh(c*x))/x, x)
```

3.8 $\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx$

Optimal. Leaf size=36

$$-\frac{a+b\tanh^{-1}(cx)}{x}-\frac{1}{2}bc\log\left(1-c^2x^2\right)+bc\log(x)$$

```
[Out] (-a-b*arctanh(c*x))/x+b*c*ln(x)-1/2*b*c*ln(-c^2*x^2+1)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5916, 266, 36, 29, 31}

$$-\frac{a+b\tanh^{-1}(cx)}{x}-\frac{1}{2}bc\log\left(1-c^2x^2\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^2,x]
```

```
[Out] -((a + b*ArcTanh[c*x])/x) + b*c*Log[x] - (b*c*Log[1 - c^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx = -\frac{a+b\tanh^{-1}(cx)}{x} + (bc)\int \frac{1}{x\left(1-c^2x^2\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}(cx)}{x} + \frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1-c^2x\right)}\,dx, x, x^2\right)$$
$$= -\frac{a+b\tanh^{-1}(cx)}{x} + \frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) + \frac{1}{2}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x}\,dx, x, x^2\right)$$
$$= -\frac{a+b\tanh^{-1}(cx)}{x} + bc\log(x) - \frac{1}{2}bc\log\left(1-c^2x^2\right)$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.08

$$-\frac{a}{x} - \frac{1}{2}bc\log\left(1-c^2x^2\right) + bc\log(x) - \frac{b\tanh^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTanh[c*x])/x + b*c*Log[x] - (b*c*Log[1 - c^2*x^2])/2
```

fricas [A] time = 0.70, size = 47, normalized size = 1.31

$$-\frac{bcx\log\left(c^2x^2-1\right) - 2\,bcx\log(x) + b\log\left(-\frac{cx+1}{cx-1}\right) + 2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*x*log(c^2*x^2 - 1) - 2*b*c*x*log(x) + b*log(-(c*x + 1)/(c*x - 1))
 + 2*a)/x
```

giac [B] time = 0.13, size = 94, normalized size = 2.61

$$\left(b\log\left(-\frac{cx+1}{cx-1}-1\right) - b\log\left(-\frac{cx+1}{cx-1}\right) + \frac{b\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{cx+1}{cx-1}+1} + \frac{2\,a}{\frac{cx+1}{cx-1}+1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] (b*log(-(c*x + 1)/(c*x - 1) - 1) - b*log(-(c*x + 1)/(c*x - 1)) + b*log(-(c*
x + 1)/(c*x - 1))/((c*x + 1)/(c*x - 1) + 1) + 2*a/((c*x + 1)/(c*x - 1) + 1)
)*c
```

maple [A] time = 0.01, size = 45, normalized size = 1.25

$$-\frac{a}{x} - \frac{b\,\text{arctanh}\,(cx)}{x} + cb\ln(cx) - \frac{cb\ln(cx-1)}{2} - \frac{cb\ln(cx+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c*x)+c*b*ln(c*x)-1/2*c*b*ln(c*x-1)-1/2*c*b*ln(c*x+1)
```

maxima [A] time = 0.32, size = 39, normalized size = 1.08

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\operatorname{artanh}\left(cx\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b - a/x
```

mupad [B] time = 0.70, size = 33, normalized size = 0.92

$$b\,c\,\ln(x)-\frac{a+b\,\operatorname{atanh}\left(c\,x\right)}{x}-\frac{b\,c\,\ln\left(c^2\,x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/x^2,x)
```

```
[Out] b*c*log(x) - (a + b*atanh(c*x))/x - (b*c*log(c^2*x^2 - 1))/2
```

sympy [A] time = 0.73, size = 41, normalized size = 1.14

$$\begin{cases}-\frac{a}{x}+bc\log\left(x\right)-bc\log\left(x-\frac{1}{c}\right)-bc\,\operatorname{atanh}\left(cx\right)-\frac{b\,\operatorname{atanh}\left(cx\right)}{x} & \text{for } c\neq 0\\ -\frac{a}{x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*c*log(x) - b*c*log(x - 1/c) - b*c*atanh(c*x) - b*atanh(
c*x)/x, Ne(c, 0)), (-a/x, True))
```

3.9 $\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx$

Optimal. Leaf size=37

$$-\frac{a+b\tanh^{-1}(cx)}{2x^2}+\frac{1}{2}bc^2\tanh^{-1}(cx)-\frac{bc}{2x}$$

```
[Out] -1/2*b*c/x+1/2*b*c^2*arctanh(c*x)+1/2*(-a-b*arctanh(c*x))/x^2
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5916, 325, 206}

$$-\frac{a+b\tanh^{-1}(cx)}{2x^2}+\frac{1}{2}bc^2\tanh^{-1}(cx)-\frac{bc}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^3,x]
```

```
[Out] -(b*c)/(2*x) + (b*c^2*ArcTanh[c*x])/2 - (a + b*ArcTanh[c*x])/(2*x^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{2x^2}+\frac{1}{2}(bc)\int \frac{1}{x^2\left(1-c^2x^2\right)}\,dx\\ &= -\frac{bc}{2x}-\frac{a+b\tanh^{-1}(cx)}{2x^2}+\frac{1}{2}\left(bc^3\right)\int \frac{1}{1-c^2x^2}\,dx\\ &= -\frac{bc}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)-\frac{a+b\tanh^{-1}(cx)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.59

$$-\frac{a}{2x^2}-\frac{1}{4}bc^2\log(1-cx)+\frac{1}{4}bc^2\log(cx+1)-\frac{b\tanh^{-1}(cx)}{2x^2}-\frac{bc}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*c)/(2*x) - (b*ArcTanh[c*x])/(2*x^2) - (b*c^2*Log[1 - c*x])/
4 + (b*c^2*Log[1 + c*x])/4
```

fricas [A] time = 1.30, size = 43, normalized size = 1.16

$$-\frac{2\,bcx - \left(bc^2x^2 - b\right)\log\left(-\frac{cx+1}{cx-1}\right) + 2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*c*x - (b*c^2*x^2 - b)*log(-(c*x + 1)/(c*x - 1)) + 2*a)/x^2
```

giac [B] time = 0.15, size = 135, normalized size = 3.65

$$\left(\frac{(cx+1)bc\log\left(-\frac{cx+1}{cx-1}\right)}{(cx-1)\left(\frac{(cx+1)^2}{(cx-1)^2} + \frac{2\,(cx+1)}{cx-1} + 1\right)} + \frac{\frac{2\,(cx+1)ac}{cx-1} + \frac{(cx+1)bc}{cx-1} + bc}{\frac{(cx+1)^2}{(cx-1)^2} + \frac{2\,(cx+1)}{cx-1} + 1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] ((c*x + 1)*b*c*log(-(c*x + 1)/(c*x - 1))/((c*x - 1)*((c*x + 1)^2/(c*x - 1)^
2 + 2*(c*x + 1)/(c*x - 1) + 1)) + (2*(c*x + 1)*a*c/(c*x - 1) + (c*x + 1)*b*
c/(c*x - 1) + b*c)/((c*x + 1)^2/(c*x - 1)^2 + 2*(c*x + 1)/(c*x - 1) + 1))*c
```

maple [A] time = 0.01, size = 49, normalized size = 1.32

$$-\frac{a}{2x^2} - \frac{b\,\mathrm{arctanh}\,(cx)}{2x^2} - \frac{bc}{2x} - \frac{c^2b\ln(cx-1)}{4} + \frac{c^2b\ln(cx+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x)-1/2*b*c/x-1/4*c^2*b*ln(c*x-1)+1/4*c^2*b*l
n(c*x+1)
```

maxima [A] time = 0.32, size = 45, normalized size = 1.22

$$\frac{1}{4}\left(\left(c\log(cx+1) - c\log(cx-1) - \frac{2}{x}\right)c - \frac{2\,\mathrm{artanh}\,(cx)}{x^2}\right)b - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b - 1/
2*a/x^2
```

mupad [B] time = 0.73, size = 46, normalized size = 1.24

$$\frac{b\,c\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)\sqrt{-c^2}}{2} - \frac{\frac{a}{2} + \frac{b\,\mathrm{atanh}(c\,x)}{2} + \frac{b\,c\,x}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/x^3,x)
```

```
[Out] (b*c*atan((c^2*x)/(-c^2)^(1/2))*(-c^2)^(1/2))/2 - (a/2 + (b*atanh(c*x))/2 +
 (b*c*x)/2)/x^2
```

sympy [**A**] time = 0.58, size = 36, normalized size = 0.97

$$-\frac{a}{2x^2}+\frac{bc^2\operatorname{atanh}(cx)}{2}-\frac{bc}{2x}-\frac{b\operatorname{atanh}(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3,x)
```

```
[Out] -a/(2*x**2) + b*c**2*atanh(c*x)/2 - b*c/(2*x) - b*atanh(c*x)/(2*x**2)
```

3.10 $\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx$

Optimal. Leaf size=54

$$-\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{3}bc^3\log(x)-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)-\frac{bc}{6x^2}$$

```
[Out] -1/6*b*c/x^2+1/3*(-a-b*arctanh(c*x))/x^3+1/3*b*c^3*ln(x)-1/6*b*c^3*ln(-c^2*
x^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5916, 266, 44}

$$-\frac{a+b\tanh^{-1}(cx)}{3x^3}-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)+\frac{1}{3}bc^3\log(x)-\frac{bc}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^4,x]
```

```
[Out] -(b*c)/(6*x^2) - (a + b*ArcTanh[c*x])/(3*x^3) + (b*c^3*Log[x])/3 - (b*c^3*L
og[1 - c^2*x^2])/6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{3}(bc)\int \frac{1}{x^3\left(1-c^2x^2\right)}\,dx\\
&= -\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1-c^2x\right)}\,dx,x,x^2\right)\\
&= -\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{6}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{c^2}{x}-\frac{c^4}{-1+c^2x}\right)dx,x,x^2\right)\\
&= -\frac{bc}{6x^2}-\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{3}bc^3\log(x)-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.09

$$-\frac{a}{3x^3}+\frac{1}{3}bc^3\log(x)-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)-\frac{b\tanh^{-1}(cx)}{3x^3}-\frac{bc}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*c)/(6*x^2) - (b*ArcTanh[c*x])/(3*x^3) + (b*c^3*Log[x])/3 -
(b*c^3*Log[1 - c^2*x^2])/6
```

fricas [A] time = 0.59, size = 59, normalized size = 1.09

$$-\frac{bc^3x^3\log\left(c^2x^2-1\right)-2\,bc^3x^3\log(x)+bcx+b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c^3*x^3*log(c^2*x^2 - 1) - 2*b*c^3*x^3*log(x) + b*c*x + b*log(-(c*x
 + 1)/(c*x - 1)) + 2*a)/x^3
```

giac [B] time = 0.14, size = 251, normalized size = 4.65

$$\frac{1}{3}\left(bc^2\log\left(-\frac{cx+1}{cx-1}-1\right)-bc^2\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{3(cx+1)^2bc^2}{(cx-1)^2}+bc^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3(cx+1)^2}{(cx-1)^2}+\frac{3(cx+1)}{cx-1}+1}+\frac{2\left(\frac{3(cx+1)^2ac^2}{(cx-1)^2}+ac^2+\frac{(cx+1)^2bc^2}{(cx-1)^2}\right.}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3(cx+1)^2}{(cx-1)^2}+\frac{3(cx+1}{cx-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(b*c^2*log(-(c*x + 1)/(c*x - 1) - 1) - b*c^2*log(-(c*x + 1)/(c*x - 1))
+ (3*(c*x + 1)^2*b*c^2/(c*x - 1)^2 + b*c^2)*log(-(c*x + 1)/(c*x - 1))/((c*x
 + 1)^3/(c*x - 1)^3 + 3*(c*x + 1)^2/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1
) + 2*(3*(c*x + 1)^2*a*c^2/(c*x - 1)^2 + a*c^2 + (c*x + 1)^2*b*c^2/(c*x - 1
)^2 + (c*x + 1)*b*c^2/(c*x - 1))/((c*x + 1)^3/(c*x - 1)^3 + 3*(c*x + 1)^2/(
c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1))*c
```

maple [A] time = 0.01, size = 59, normalized size = 1.09

$$-\frac{a}{3x^3}-\frac{b\,\mathrm{arctanh}\,(cx)}{3x^3}-\frac{bc}{6x^2}+\frac{c^3b\ln(cx)}{3}-\frac{c^3b\ln(cx-1)}{6}-\frac{c^3b\ln(cx+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x)-1/6*b*c/x^2+1/3*c^3*b*ln(c*x)-1/6*c^3*b*l
n(c*x-1)-1/6*c^3*b*ln(c*x+1)
```

maxima [A] time = 0.32, size = 49, normalized size = 0.91

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}\,(cx)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b - 1/3*a/x^3
```

mupad [B] time = 0.73, size = 46, normalized size = 0.85

$$\frac{b\,c^3\,\ln(x)}{3}-\frac{b\,c^3\,\ln\left(c^2\,x^2-1\right)}{6}-\frac{\frac{a}{3}+\frac{b\,\mathrm{atanh}(c\,x)}{3}+\frac{b\,c\,x}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/x^4,x)
```

```
[Out] (b*c^3*log(x))/3 - (b*c^3*log(c^2*x^2 - 1))/6 - (a/3 + (b*atanh(c*x))/3 + (
b*c*x)/6)/x^3
```

sympy [A] time = 1.22, size = 70, normalized size = 1.30

$$\begin{cases} -\frac{a}{3x^3}+\frac{bc^3\log(x)}{3}-\frac{bc^3\log\left(x-\frac{1}{c}\right)}{3}-\frac{bc^3\operatorname{atanh}(cx)}{3}-\frac{bc}{6x^2}-\frac{b\operatorname{atanh}(cx)}{3x^3} & \text{for}\ c \neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) + b*c**3*log(x)/3 - b*c**3*log(x - 1/c)/3 - b*c**3*a
tanh(c*x)/3 - b*c/(6*x**2) - b*atanh(c*x)/(3*x**3), Ne(c, 0)), (-a/(3*x**3)
, True))
```

3.11 $\int \frac{a+b\tanh^{-1}(cx)}{x^5}\,dx$

Optimal. Leaf size=48

$$-\frac{a+b\tanh^{-1}(cx)}{4x^4}+\frac{1}{4}bc^4\tanh^{-1}(cx)-\frac{bc^3}{4x}-\frac{bc}{12x^3}$$

```
[Out] -1/12*b*c/x^3-1/4*b*c^3/x+1/4*b*c^4*arctanh(c*x)+1/4*(-a-b*arctanh(c*x))/x^
4
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5916, 325, 206}

$$-\frac{a+b\tanh^{-1}(cx)}{4x^4}-\frac{bc^3}{4x}+\frac{1}{4}bc^4\tanh^{-1}(cx)-\frac{bc}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^5,x]
```

```
[Out] -(b*c)/(12*x^3) - (b*c^3)/(4*x) + (b*c^4*ArcTanh[c*x])/4 - (a + b*ArcTanh[c
*x])/(4*x^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{x^5}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{4x^4}+\frac{1}{4}(bc)\int \frac{1}{x^4\left(1-c^2x^2\right)}\,dx\\
&= -\frac{bc}{12x^3}-\frac{a+b\tanh^{-1}(cx)}{4x^4}+\frac{1}{4}\left(bc^3\right)\int \frac{1}{x^2\left(1-c^2x^2\right)}\,dx\\
&= -\frac{bc}{12x^3}-\frac{bc^3}{4x}-\frac{a+b\tanh^{-1}(cx)}{4x^4}+\frac{1}{4}\left(bc^5\right)\int \frac{1}{1-c^2x^2}\,dx\\
&= -\frac{bc}{12x^3}-\frac{bc^3}{4x}+\frac{1}{4}bc^4\tanh^{-1}(cx)-\frac{a+b\tanh^{-1}(cx)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 70, normalized size = 1.46

$$-\frac{a}{4x^4}-\frac{1}{8}bc^4\log(1-cx)+\frac{1}{8}bc^4\log(cx+1)-\frac{bc^3}{4x}-\frac{b\tanh^{-1}(cx)}{4x^4}-\frac{bc}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (b*c)/(12*x^3) - (b*c^3)/(4*x) - (b*ArcTanh[c*x])/(4*x^4) - (b
*c^4*Log[1 - c*x])/8 + (b*c^4*Log[1 + c*x])/8
```

fricas [A] time = 1.21, size = 52, normalized size = 1.08

$$-\frac{6\,bc^3x^3+2\,bcx-3\left(bc^4x^4-b\right)\log\left(-\frac{cx+1}{cx-1}\right)+6\,a}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(6*b*c^3*x^3 + 2*b*c*x - 3*(b*c^4*x^4 - b)*log(-(c*x + 1)/(c*x - 1))
 + 6*a)/x^4
```

giac [B] time = 0.13, size = 292, normalized size = 6.08

$$\frac{1}{3}c\left(\frac{3\left(\frac{(cx+1)^3bc^3}{(cx-1)^3}+\frac{(cx+1)bc^3}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}+\frac{\frac{6\,(cx+1)^3ac^3}{(cx-1)^3}+\frac{6\,(cx+1)ac^3}{cx-1}+\frac{3\,(cx+1)^3bc^3}{(cx-1)^3}+\frac{6\,(cx+1)^2bc^3}{(cx-1)^2}+\frac{5\,(cx+1)bc^3}{cx-1}+}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/3*c*(3*((c*x + 1)^3*b*c^3/(c*x - 1)^3 + (c*x + 1)*b*c^3/(c*x - 1))*log(-(
c*x + 1)/(c*x - 1))/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 +
6*(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + (6*(c*x + 1)^3*a*c
^3/(c*x - 1)^3 + 6*(c*x + 1)*a*c^3/(c*x - 1) + 3*(c*x + 1)^3*b*c^3/(c*x - 1
)^3 + 6*(c*x + 1)^2*b*c^3/(c*x - 1)^2 + 5*(c*x + 1)*b*c^3/(c*x - 1) + 2*b*c
^3)/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c
*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1))
```

maple [A] time = 0.01, size = 58, normalized size = 1.21

$$-\frac{a}{4x^4}-\frac{b\,\mathrm{arctanh}\,(cx)}{4x^4}-\frac{bc}{12x^3}-\frac{b\,c^3}{4x}-\frac{c^4b\ln\,(cx-1)}{8}+\frac{c^4b\ln\,(cx+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctanh(c*x)-1/12*b*c/x^3-1/4*b*c^3/x-1/8*c^4*b*ln(c*x
-1)+1/8*c^4*b*ln(c*x+1)
```

maxima [A] time = 0.32, size = 60, normalized size = 1.25

$$\frac{1}{24}\left(\left(3\,c^3\log\,(cx+1)-3\,c^3\log\,(cx-1)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\mathrm{artanh}\,(cx)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
 6*arctanh(c*x)/x^4)*b - 1/4*a/x^4
```

mupad [B] time = 1.05, size = 59, normalized size = 1.23

$$\frac{b\,\ln\left(1-c\,x\right)}{8\,x^4}-\frac{b\,\ln\left(c\,x+1\right)}{8\,x^4}-\frac{b\,c^3\,x^3+\frac{b\,c\,x}{3}+a}{4\,x^4}-\frac{b\,c^4\,\mathrm{atan}\left(c\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/x^5,x)
```

```
[Out] (b*log(1 - c*x))/(8*x^4) - (b*c^4*atan(c*x*1i)*1i)/4 - (b*log(c*x + 1))/(8*
x^4) - (a + b*c^3*x^3 + (b*c*x)/3)/(4*x^4)
```

sympy [A] time = 0.90, size = 46, normalized size = 0.96

$$-\frac{a}{4x^4}+\frac{bc^4\,\mathrm{atanh}\,(cx)}{4}-\frac{bc^3}{4x}-\frac{bc}{12x^3}-\frac{b\,\mathrm{atanh}\,(cx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**5,x)
```

```
[Out] -a/(4*x**4) + b*c**4*atanh(c*x)/4 - b*c**3/(4*x) - b*c/(12*x**3) - b*atanh(
c*x)/(4*x**4)
```

3.12 $\int \frac{a+b\tanh^{-1}(cx)}{x^6}\,dx$

Optimal. Leaf size=65

$$-\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)-\frac{bc}{20x^4}$$

```
[Out] -1/20*b*c/x^4-1/10*b*c^3/x^2+1/5*(-a-b*arctanh(c*x))/x^5+1/5*b*c^5*ln(x)-1/
10*b*c^5*ln(-c^2*x^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5916, 266, 44}

$$-\frac{a+b\tanh^{-1}(cx)}{5x^5}-\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)+\frac{1}{5}bc^5\log(x)-\frac{bc}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^6,x]
```

```
[Out] -(b*c)/(20*x^4) - (b*c^3)/(10*x^2) - (a + b*ArcTanh[c*x])/(5*x^5) + (b*c^5*
Log[x])/5 - (b*c^5*Log[1 - c^2*x^2])/10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{x^6}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{5}(bc)\int \frac{1}{x^5\left(1-c^2x^2\right)}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{10}(bc)\,\text{Subst}\left(\int \frac{1}{x^3\left(1-c^2x\right)}\,dx,x,x^2\right)\\ &= -\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{10}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{c^2}{x^2}+\frac{c^4}{x}-\frac{c^6}{-1+c^2x}\right)dx,x,x^2\right)\\ &= -\frac{bc}{20x^4}-\frac{bc^3}{10x^2}-\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 70, normalized size = 1.08

$$-\frac{a}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)-\frac{b\tanh^{-1}(cx)}{5x^5}-\frac{bc}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (b*c)/(20*x^4) - (b*c^3)/(10*x^2) - (b*ArcTanh[c*x])/(5*x^5) +
 (b*c^5*Log[x])/5 - (b*c^5*Log[1 - c^2*x^2])/10
```

fricas [A] time = 0.63, size = 70, normalized size = 1.08

$$-\frac{2\,bc^5x^5\log\left(c^2x^2-1\right)-4\,bc^5x^5\log(x)+2\,bc^3x^3+bcx+2\,b\log\left(-\frac{cx+1}{cx-1}\right)+4\,a}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*b*c^5*x^5*log(c^2*x^2 - 1) - 4*b*c^5*x^5*log(x) + 2*b*c^3*x^3 + b*
c*x + 2*b*log(-(c*x + 1)/(c*x - 1)) + 4*a)/x^5
```

giac [B] time = 0.16, size = 397, normalized size = 6.11

$$\frac{1}{5}\left(bc^4\log\left(-\frac{cx+1}{cx-1}-1\right)-bc^4\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\left(\frac{5\,(cx+1)^4bc^4}{(cx-1)^4}+\frac{10\,(cx+1)^2bc^4}{(cx-1)^2}+bc^4\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5}{(cx-1)^5}+\frac{5\,(cx+1)^4}{(cx-1)^4}+\frac{10\,(cx+1)^3}{(cx-1)^3}+\frac{10\,(cx+1)^2}{(cx-1)^2}+\frac{5\,(cx+1)}{cx-1}+1}+\frac{2\left(\frac{5\,(cx+1)}{(cx-1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] 1/5*(b*c^4*log(-(c*x + 1)/(c*x - 1) - 1) - b*c^4*log(-(c*x + 1)/(c*x - 1))
+ (5*(c*x + 1)^4*b*c^4/(c*x - 1)^4 + 10*(c*x + 1)^2*b*c^4/(c*x - 1)^2 + b*c
^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x
 - 1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x
+ 1)/(c*x - 1) + 1) + 2*(5*(c*x + 1)^4*a*c^4/(c*x - 1)^4 + 10*(c*x + 1)^2*a
*c^4/(c*x - 1)^2 + a*c^4 + 2*(c*x + 1)^4*b*c^4/(c*x - 1)^4 + 4*(c*x + 1)^3*
b*c^4/(c*x - 1)^3 + 4*(c*x + 1)^2*b*c^4/(c*x - 1)^2 + 2*(c*x + 1)*b*c^4/(c*
x - 1))/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(c*x + 1)
^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1) + 1))*c
```

maple [A] time = 0.01, size = 68, normalized size = 1.05

$$-\frac{a}{5x^5}-\frac{b\,\mathrm{arctanh}\,(cx)}{5x^5}-\frac{bc}{20x^4}-\frac{b\,c^3}{10x^2}+\frac{c^5b\ln(cx)}{5}-\frac{c^5b\ln(cx-1)}{10}-\frac{c^5b\ln(cx+1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctanh(c*x)-1/20*b*c/x^4-1/10*b*c^3/x^2+1/5*c^5*b*ln(
c*x)-1/10*c^5*b*ln(c*x-1)-1/10*c^5*b*ln(c*x+1)
```

maxima [A] time = 0.33, size = 61, normalized size = 0.94

$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\,\mathrm{artanh}\,(cx)}{x^5}\right)b-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c +
4*arctanh(c*x)/x^5)*b - 1/5*a/x^5
```

mupad [B] time = 0.91, size = 71, normalized size = 1.09

$$\frac{b\,c^5\,\ln(x)}{5}-\frac{b\,c^5\,\ln\left(c^2\,x^2-1\right)}{10}-\frac{\frac{b\,c^3\,x^3}{2}+\frac{b\,c\,x}{4}+a}{5\,x^5}-\frac{b\,\ln\left(c\,x+1\right)}{10\,x^5}+\frac{b\,\ln\left(1-c\,x\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/x^6,x)
```

```
[Out] (b*c^5*log(x))/5 - (b*c^5*log(c^2*x^2 - 1))/10 - (a + (b*c^3*x^3)/2 + (b*c*
x)/4)/(5*x^5) - (b*log(c*x + 1))/(10*x^5) + (b*log(1 - c*x))/(10*x^5)
```

sympy [A] time = 1.91, size = 80, normalized size = 1.23

$$\begin{cases} -\frac{a}{5x^5}+\frac{bc^5\log(x)}{5}-\frac{bc^5\log\left(x-\frac{1}{c}\right)}{5}-\frac{bc^5\operatorname{atanh}(cx)}{5}-\frac{bc^3}{10x^2}-\frac{bc}{20x^4}-\frac{b\operatorname{atanh}(cx)}{5x^5} & \text{for } c\neq 0 \\ -\frac{a}{5x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + b*c**5*log(x)/5 - b*c**5*log(x - 1/c)/5 - b*c**5*a
tanh(c*x)/5 - b*c**3/(10*x**2) - b*c/(20*x**4) - b*atanh(c*x)/(5*x**5), Ne(
c, 0)), (-a/(5*x**5), True))
```

3.13 $\int x^5 \left(a+b\tanh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=145

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{6c^6}+\frac{abx}{3c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{b^2x\tanh}{3c^5}$$

```
[Out] 1/3*a*b*x/c^5+4/45*b^2*x^2/c^4+1/60*b^2*x^4/c^2+1/3*b^2*x*arctanh(c*x)/c^5+
1/9*b*x^3*(a+b*arctanh(c*x))/c^3+1/15*b*x^5*(a+b*arctanh(c*x))/c-1/6*(a+b*a
rctanh(c*x))^2/c^6+1/6*x^6*(a+b*arctanh(c*x))^2+23/90*b^2*ln(-c^2*x^2+1)/c^
6
```

Rubi [A] time = 0.33, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5916, 5980, 266, 43, 5910, 260, 5948}

$$\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{abx}{3c^5}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{6c^6}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{b^2x^4}{60c^2}+\frac{4b^2}{45c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*x)/(3*c^5) + (4*b^2*x^2)/(45*c^4) + (b^2*x^4)/(60*c^2) + (b^2*x*ArcTan
h[c*x])/(3*c^5) + (b*x^3*(a + b*ArcTanh[c*x]))/(9*c^3) + (b*x^5*(a + b*ArcT
anh[c*x]))/(15*c) - (a + b*ArcTanh[c*x])^2/(6*c^6) + (x^6*(a + b*ArcTanh[c*
x])^2)/6 + (23*b^2*Log[1 - c^2*x^2])/(90*c^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int x^5\left(a+b\tanh^{-1}(cx)\right)^2 dx = \frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{3}(bc)\int\frac{x^6\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx$$

$$=\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b\int x^4\left(a+b\tanh^{-1}(cx)\right)dx}{3c}-\frac{b\int\frac{x^4\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}d}{3c}$$

$$=\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{15}b^2\int\frac{x^5}{1-c^2x^2}dx+\frac{b\int}{}$$

$$=\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2-$$

$$=\frac{abx}{3c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{6c^6}$$

$$=\frac{abx}{3c^5}+\frac{b^2x^2}{30c^4}+\frac{b^2x^4}{60c^2}+\frac{b^2x\tanh^{-1}(cx)}{3c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+b\tanh\right.}{15}$$

$$=\frac{abx}{3c^5}+\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}+\frac{b^2x\tanh^{-1}(cx)}{3c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+b\right.}{1}$$

Mathematica [A] time = 0.07, size = 164, normalized size = 1.13

$$\frac{30a^2c^6x^6+12abc^5x^5+20abc^3x^3+4bcx\tanh^{-1}(cx)\left(15ac^5x^5+b\left(3c^4x^4+5c^2x^2+15\right)\right)+60abcx+2b(15a+}{180c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (60*a*b*c*x + 16*b^2*c^2*x^2 + 20*a*b*c^3*x^3 + 3*b^2*c^4*x^4 + 12*a*b*c^5*
x^5 + 30*a^2*c^6*x^6 + 4*b*c*x*(15*a*c^5*x^5 + b*(15 + 5*c^2*x^2 + 3*c^4*x^
4))*ArcTanh[c*x] + 30*b^2*(-1 + c^6*x^6)*ArcTanh[c*x]^2 + 2*b*(15*a + 23*b)
*Log[1 - c*x] - 30*a*b*Log[1 + c*x] + 46*b^2*Log[1 + c*x])/(180*c^6)
```

fricas [A] time = 1.18, size = 193, normalized size = 1.33

$$\frac{60a^2c^6x^6+24abc^5x^5+6b^2c^4x^4+40abc^3x^3+32b^2c^2x^2+120abcx+15\left(b^2c^6x^6-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2-4\left(15a\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/360*(60*a^2*c^6*x^6 + 24*a*b*c^5*x^5 + 6*b^2*c^4*x^4 + 40*a*b*c^3*x^3 + 3
2*b^2*c^2*x^2 + 120*a*b*c*x + 15*(b^2*c^6*x^6 - b^2)*log(-(c*x + 1)/(c*x -
1))^2 - 4*(15*a*b - 23*b^2)*log(c*x + 1) + 4*(15*a*b + 23*b^2)*log(c*x - 1)
 + 4*(15*a*b*c^6*x^6 + 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 + 15*b^2*c*x)*log(-(c*
x + 1)/(c*x - 1)))/c^6
```

giac [B] time = 0.18, size = 889, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/90*(15*(3*(c*x + 1)^5*b^2/(c*x - 1)^5 + 10*(c*x + 1)^3*b^2/(c*x - 1)^3 +
3*(c*x + 1)*b^2/(c*x - 1))*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^6*c^7/(c*
x - 1)^6 - 6*(c*x + 1)^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4 -
 20*(c*x + 1)^3*c^7/(c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x +
 1)*c^7/(c*x - 1) + c^7) + 2*(90*(c*x + 1)^5*a*b/(c*x - 1)^5 + 300*(c*x + 1
)^3*a*b/(c*x - 1)^3 + 90*(c*x + 1)*a*b/(c*x - 1) + 45*(c*x + 1)^5*b^2/(c*x
- 1)^5 - 135*(c*x + 1)^4*b^2/(c*x - 1)^4 + 230*(c*x + 1)^3*b^2/(c*x - 1)^3
- 210*(c*x + 1)^2*b^2/(c*x - 1)^2 + 93*(c*x + 1)*b^2/(c*x - 1) - 23*b^2)*lo
g(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6*c^7/(c*x - 1)^6 - 6*(c*x + 1)^5*c^7/(c
*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4 - 20*(c*x + 1)^3*c^7/(c*x - 1)^3
 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x + 1)*c^7/(c*x - 1) + c^7) + 4*(4
5*(c*x + 1)^5*a^2/(c*x - 1)^5 + 150*(c*x + 1)^3*a^2/(c*x - 1)^3 + 45*(c*x +
 1)*a^2/(c*x - 1) + 45*(c*x + 1)^5*a*b/(c*x - 1)^5 - 135*(c*x + 1)^4*a*b/(c
*x - 1)^4 + 230*(c*x + 1)^3*a*b/(c*x - 1)^3 - 210*(c*x + 1)^2*a*b/(c*x - 1)
^2 + 93*(c*x + 1)*a*b/(c*x - 1) - 23*a*b + 11*(c*x + 1)^5*b^2/(c*x - 1)^5 -
 38*(c*x + 1)^4*b^2/(c*x - 1)^4 + 54*(c*x + 1)^3*b^2/(c*x - 1)^3 - 38*(c*x
+ 1)^2*b^2/(c*x - 1)^2 + 11*(c*x + 1)*b^2/(c*x - 1))/((c*x + 1)^6*c^7/(c*x
- 1)^6 - 6*(c*x + 1)^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4 - 2
0*(c*x + 1)^3*c^7/(c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x + 1
)*c^7/(c*x - 1) + c^7) - 46*b^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^7 + 46*b^2*
log(-(c*x + 1)/(c*x - 1))/c^7)*c
```

maple [B] time = 0.03, size = 314, normalized size = 2.17

$$\frac{x^6a^2}{6}+\frac{b^2x^6\operatorname{arctanh}(cx)^2}{6}+\frac{b^2\operatorname{arctanh}(cx)\,x^5}{15c}+\frac{b^2\operatorname{arctanh}(cx)\,x^3}{9c^3}+\frac{b^2x\operatorname{arctanh}(cx)}{3c^5}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{6c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/6*x^6*a^2+1/6*b^2*x^6*arctanh(c*x)^2+1/15/c*b^2*arctanh(c*x)*x^5+1/9/c^3*
b^2*arctanh(c*x)*x^3+1/3*b^2*x*arctanh(c*x)/c^5+1/6/c^6*b^2*arctanh(c*x)*ln
(c*x-1)-1/6/c^6*b^2*arctanh(c*x)*ln(c*x+1)+1/24/c^6*b^2*ln(c*x-1)^2-1/12/c^
6*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/24/c^6*b^2*ln(c*x+1)^2-1/12/c^6*b^2*ln(-1
/2*c*x+1/2)*ln(c*x+1)+1/12/c^6*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/60*b^
2*x^4/c^2+4/45*b^2*x^2/c^4+23/90/c^6*b^2*ln(c*x-1)+23/90/c^6*b^2*ln(c*x+1)+
1/3*a*b*x^6*arctanh(c*x)+1/15/c*x^5*a*b+1/9*a*b*x^3/c^3+1/3*a*b*x/c^5+1/6/c
^6*a*b*ln(c*x-1)-1/6/c^6*a*b*ln(c*x+1)
```

maxima [A] time = 0.37, size = 215, normalized size = 1.48

$$\frac{1}{6}\,b^2x^6\operatorname{artanh}(cx)^2+\frac{1}{6}\,a^2x^6+\frac{1}{90}\left(30\,x^6\operatorname{artanh}(cx)+c\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log(cx+1)}{c^7}+\frac{15\,\log(cx}{c^7}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctanh(c*x)^2 + 1/6*a^2*x^6 + 1/90*(30*x^6*arctanh(c*x) + c*(2
*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)
/c^7))*a*b + 1/360*(4*c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x
+ 1)/c^7 + 15*log(c*x - 1)/c^7)*arctanh(c*x) + (6*c^4*x^4 + 32*c^2*x^2 - 2*
(15*log(c*x - 1) - 46)*log(c*x + 1) + 15*log(c*x + 1)^2 + 15*log(c*x - 1)^2
 + 92*log(c*x - 1))/c^6)*b^2
```

mupad [B] time = 1.04, size = 171, normalized size = 1.18

$$\frac{46\,b^2\,\ln\left(c^2\,x^2-1\right)-30\,b^2\,\mathrm{atanh}\left(c\,x\right)^2+30\,a^2\,c^6\,x^6+16\,b^2\,c^2\,x^2+3\,b^2\,c^4\,x^4-60\,a\,b\,\mathrm{atanh}\left(c\,x\right)+20\,b^2\,c^3\,x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x))^2,x)
```

```
[Out] (46*b^2*log(c^2*x^2 - 1) - 30*b^2*atanh(c*x)^2 + 30*a^2*c^6*x^6 + 16*b^2*c^
2*x^2 + 3*b^2*c^4*x^4 - 60*a*b*atanh(c*x) + 20*b^2*c^3*x^3*atanh(c*x) + 12*
b^2*c^5*x^5*atanh(c*x) + 60*b^2*c*x*atanh(c*x) + 30*b^2*c^6*x^6*atanh(c*x)^
2 + 20*a*b*c^3*x^3 + 12*a*b*c^5*x^5 + 60*a*b*c*x + 60*a*b*c^6*x^6*atanh(c*x
))/(180*c^6)
```

sympy [A] time = 2.74, size = 211, normalized size = 1.46

$$\begin{cases} \frac{a^2x^6}{6} + \frac{abx^6\,\mathrm{atanh}\,(cx)}{3} + \frac{abx^5}{15c} + \frac{abx^3}{9c^3} + \frac{abx}{3c^5} - \frac{ab\,\mathrm{atanh}\,(cx)}{3c^6} + \frac{b^2x^6\,\mathrm{atanh}^2\,(cx)}{6} + \frac{b^2x^5\,\mathrm{atanh}\,(cx)}{15c} + \frac{b^2x^4}{60c^2} + \frac{b^2x^3\,\mathrm{atanh}\,(cx)}{9c^3} + \frac{4b^2}{45c} \\ \frac{a^2x^6}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 + a*b*x**6*atanh(c*x)/3 + a*b*x**5/(15*c) + a*b*x**3
/(9*c**3) + a*b*x/(3*c**5) - a*b*atanh(c*x)/(3*c**6) + b**2*x**6*atanh(c*x)
**2/6 + b**2*x**5*atanh(c*x)/(15*c) + b**2*x**4/(60*c**2) + b**2*x**3*atanh
(c*x)/(9*c**3) + 4*b**2*x**2/(45*c**4) + b**2*x*atanh(c*x)/(3*c**5) + 23*b*
*2*log(x - 1/c)/(45*c**6) - b**2*atanh(c*x)**2/(6*c**6) + 23*b**2*atanh(c*x
)/(45*c**6), Ne(c, 0)), (a**2*x**6/6, True))
```

3.14 $\int x^4 \left(a+b \tanh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=162

$$\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{5c^5}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^5}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^4\left(a\right.}{}$$

```
[Out] 3/10*b^2*x/c^4+1/30*b^2*x^3/c^2-3/10*b^2*arctanh(c*x)/c^5+1/5*b*x^2*(a+b*ar
ctanh(c*x))/c^3+1/10*b*x^4*(a+b*arctanh(c*x))/c+1/5*(a+b*arctanh(c*x))^2/c^
5+1/5*x^5*(a+b*arctanh(c*x))^2-2/5*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^5-
1/5*b^2*polylog(2,1-2/(-c*x+1))/c^5
```

Rubi [A] time = 0.30, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5916, 5980, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^5}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{5c^5}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^5}+\frac{1}{5}x^5\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (3*b^2*x)/(10*c^4) + (b^2*x^3)/(30*c^2) - (3*b^2*ArcTanh[c*x])/(10*c^5) + (
b*x^2*(a + b*ArcTanh[c*x]))/(5*c^3) + (b*x^4*(a + b*ArcTanh[c*x]))/(10*c) +
 (a + b*ArcTanh[c*x])^2/(5*c^5) + (x^5*(a + b*ArcTanh[c*x])^2)/5 - (2*b*(a
+ b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^5) - (b^2*PolyLog[2, 1 - 2/(1 - c*
x)])/(5*c^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{5}(2bc)\int\frac{x^5\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx$$

$$=\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{(2b)\int x^3\left(a+b\tanh^{-1}(cx)\right)\,dx}{5c}-\frac{(2b)\int\frac{x^3\left(a+b\tanh^{-1}\right.}{1-c^2x^2}}{5c}$$

$$=\frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{10}b^2\int\frac{x^4}{1-c^2x^2}\,dx+\frac{(2b}{}$$

$$=\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{5c^5}+\frac{1}{5}x^5$$

$$=\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c}+\frac{\left(a+b\tanh\right.}{5c^5}$$

$$=\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}-\frac{3b^2\tanh^{-1}(cx)}{10c^5}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{bx^4\left(a+b\tanh^{-1}(\right.}{10c}$$

$$=\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}-\frac{3b^2\tanh^{-1}(cx)}{10c^5}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{bx^4\left(a+b\tanh^{-1}(\right.}{10c}$$

Mathematica [A] time = 0.50, size = 161, normalized size = 0.99

$$\frac{6a^2c^5x^5 + 3abc^4x^4 + 6abc^2x^2 + 6ab\log\left(c^2x^2-1\right) + 3b\tanh^{-1}(cx)\left(4ac^5x^5 + b\left(c^4x^4+2c^2x^2-3\right) - 4b\log\left(e^{-2\tanh}\right.\right.}{30c^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (-9*a*b + 9*b^2*c*x + 6*a*b*c^2*x^2 + b^2*c^3*x^3 + 3*a*b*c^4*x^4 + 6*a^2*c
^5*x^5 + 6*b^2*(-1 + c^5*x^5)*ArcTanh[c*x]^2 + 3*b*ArcTanh[c*x]*(4*a*c^5*x^
5 + b*(-3 + 2*c^2*x^2 + c^4*x^4) - 4*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 6*a*
b*Log[-1 + c^2*x^2] + 6*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(30*c^5)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^4\,\text{artanh}\,(cx)^2 + 2\,abx^4\,\text{artanh}\,(cx) + a^2x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^4*arctanh(c*x)^2 + 2*a*b*x^4*arctanh(c*x) + a^2*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{artanh}\,(cx) + a)^2 x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^4, x)
```

maple [B] time = 0.02, size = 306, normalized size = 1.89

$$\frac{x^5a^2}{5} + \frac{x^5b^2\,\text{arctanh}\,(cx)^2}{5} + \frac{b^2\,\text{arctanh}\,(cx)\,x^4}{10c} + \frac{b^2\,\text{arctanh}\,(cx)\,x^2}{5c^3} + \frac{b^2\,\text{arctanh}\,(cx)\ln(cx-1)}{5c^5} + \frac{b^2\,\text{arctanh}\,(cx)\ln}{5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/5*x^5*a^2+1/5*x^5*b^2*arctanh(c*x)^2+1/10/c*b^2*arctanh(c*x)*x^4+1/5/c^3*
b^2*arctanh(c*x)*x^2+1/5/c^5*b^2*arctanh(c*x)*ln(c*x-1)+1/5/c^5*b^2*arctanh
(c*x)*ln(c*x+1)+1/30*b^2*x^3/c^2+3/10*b^2*x/c^4+3/20/c^5*b^2*ln(c*x-1)-3/20
/c^5*b^2*ln(c*x+1)+1/20/c^5*b^2*ln(c*x-1)^2-1/5/c^5*b^2*dilog(1/2+1/2*c*x)-
1/10/c^5*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/20/c^5*b^2*ln(c*x+1)^2+1/10/c^5*b^
2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/10/c^5*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+
2/5*x^5*a*b*arctanh(c*x)+1/10/c*x^4*a*b+1/5*a*b*x^2/c^3+1/5/c^5*a*b*ln(c*x-
1)+1/5/c^5*a*b*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{5}a^2x^5 + \frac{1}{10}\left(4x^5\,\text{artanh}\,(cx) + c\left(\frac{c^2x^4+2x^2}{c^4} + \frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)ab - \frac{1}{36000}\left(24c^6\left(\frac{2\left(3c^4x^5+5c^2x^3+15x\right)}{c^{10}} - \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 + 1/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c
^2*x^2 - 1)/c^6))*a*b - 1/36000*(24*c^6*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c
^10 - 15*log(c*x + 1)/c^11 + 15*log(c*x - 1)/c^11) - 45*c^5*((c^2*x^4 + 2*x
^2)/c^8 + 2*log(c^2*x^2 - 1)/c^10) - 1080000*c^5*integrate(1/150*x^5*log(c*
x + 1)/(c^6*x^2 - c^4), x) + 50*c^4*(2*(c^2*x^3 + 3*x)/c^8 - 3*log(c*x + 1)
/c^9 + 3*log(c*x - 1)/c^9) - 300*c^3*(x^2/c^6 + log(c^2*x^2 - 1)/c^8) + 900
*c^2*(2*x/c^6 - log(c*x + 1)/c^7 + log(c*x - 1)/c^7) - 540000*c*integrate(1
/150*x*log(c*x + 1)/(c^6*x^2 - c^4), x) - 60*(30*c^5*x^5*log(c*x + 1)^2 + (
12*c^5*x^5 - 15*c^4*x^4 + 20*c^3*x^3 - 30*c^2*x^2 + 60*c*x - 60*(c^5*x^5 +
1)*log(c*x + 1))*log(-c*x + 1))/c^5 - (72*(c*x - 1)^5*(25*log(-c*x + 1)^2 -
 10*log(-c*x + 1) + 2) + 1125*(c*x - 1)^4*(8*log(-c*x + 1)^2 - 4*log(-c*x +
 1) + 1) + 2000*(c*x - 1)^3*(9*log(-c*x + 1)^2 - 6*log(-c*x + 1) + 2) + 900
0*(c*x - 1)^2*(2*log(-c*x + 1)^2 - 2*log(-c*x + 1) + 1) + 9000*(c*x - 1)*(l
og(-c*x + 1)^2 - 2*log(-c*x + 1) + 2))/c^5 + 1800*log(150*c^6*x^2 - 150*c^4
)/c^5 - 540000*integrate(1/150*log(c*x + 1)/(c^6*x^2 - c^4), x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,(a+b\,\mathrm{atanh}\,(c\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x))^2,x)
```

```
[Out] int(x^4*(a + b*atanh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\,(a+b\,\mathrm{atanh}\,(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c*x))**2, x)
```

3.15 $\int x^3 \left(a+b\tanh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=113

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4}+\frac{abx}{2c^3}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{b^2x\tanh^{-1}(cx)}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2\log\left(1-\right.}{3c^4}$$

```
[Out] 1/2*a*b*x/c^3+1/12*b^2*x^2/c^2+1/2*b^2*x*arctanh(c*x)/c^3+1/6*b*x^3*(a+b*ar
ctanh(c*x))/c-1/4*(a+b*arctanh(c*x))^2/c^4+1/4*x^4*(a+b*arctanh(c*x))^2+1/3
*b^2*ln(-c^2*x^2+1)/c^4
```

Rubi [A] time = 0.22, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5916, 5980, 266, 43, 5910, 260, 5948}

$$\frac{abx}{2c^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{b^2x^2}{12c^2}+\frac{b^2\log\left(1-c^2x^2\right)}{3c^4}+\frac{b^2x\tanh^{-}}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*x)/(2*c^3) + (b^2*x^2)/(12*c^2) + (b^2*x*ArcTanh[c*x])/(2*c^3) + (b*x^
3*(a + b*ArcTanh[c*x]))/(6*c) - (a + b*ArcTanh[c*x])^2/(4*c^4) + (x^4*(a +
b*ArcTanh[c*x])^2)/4 + (b^2*Log[1 - c^2*x^2])/(3*c^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\tanh^{-1}(cx)\right)^2dx &= \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{2}(bc)\int\frac{x^4\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx\\ &= \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b\int x^2\left(a+b\tanh^{-1}(cx)\right)dx}{2c}-\frac{b\int\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}d\end{aligned}$$

$$\begin{aligned} &= \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{6}b^2\int\frac{x^3}{1-c^2x^2}dx+\frac{b\int(\\ &= \frac{abx}{2c^3}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)\\ &= \frac{abx}{2c^3}+\frac{b^2x\tanh^{-1}(cx)}{2c^3}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4(\\ &= \frac{abx}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2x\tanh^{-1}(cx)}{2c^3}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 132, normalized size = 1.17

$$\frac{3a^2c^4x^4+2abc^3x^3+2bcx\tanh^{-1}(cx)\left(3ac^3x^3+b\left(c^2x^2+3\right)\right)+6abcx+b(3a+4b)\log(1-cx)-3ab\log(cx+}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (6*a*b*c*x + b^2*c^2*x^2 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + 2*b*c*x*(3*a*c^3
*x^3 + b*(3 + c^2*x^2))*ArcTanh[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTanh[c*x]^2
+ b*(3*a + 4*b)*Log[1 - c*x] - 3*a*b*Log[1 + c*x] + 4*b^2*Log[1 + c*x])/(12
*c^4)
```

fricas [A] time = 0.57, size = 160, normalized size = 1.42

$$\frac{12\,a^2c^4x^4+8\,abc^3x^3+4\,b^2c^2x^2+24\,abcx+3\left(b^2c^4x^4-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2-4\left(3\,ab-4\,b^2\right)\log\left(cx+1\right)+4\left(3\,a\right.}{48\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(12*a^2*c^4*x^4 + 8*a*b*c^3*x^3 + 4*b^2*c^2*x^2 + 24*a*b*c*x + 3*(b^2*
c^4*x^4 - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 4*(3*a*b - 4*b^2)*log(c*x + 1)
```

```
 + 4*(3*a*b + 4*b^2)*log(c*x - 1) + 4*(3*a*b*c^4*x^4 + b^2*c^3*x^3 + 3*b^2*
c*x)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

giac [B] time = 0.17, size = 603, normalized size = 5.34

$$\frac{1}{6}\left(\frac{3\left(\frac{(cx+1)^3b^2}{(cx-1)^3}+\frac{(cx+1)b^2}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)^2}{\frac{(cx+1)^4c^5}{(cx-1)^4}-\frac{4(cx+1)^3c^5}{(cx-1)^3}+\frac{6(cx+1)^2c^5}{(cx-1)^2}-\frac{4(cx+1)c^5}{cx-1}+c^5}+\frac{2\left(\frac{6(cx+1)^3ab}{(cx-1)^3}+\frac{6(cx+1)ab}{cx-1}+\frac{3(cx+1)^3b^2}{(cx-1)^3}-\frac{6(cx+1)^2b^2}{(cx-1)^2}+\frac{5(cx+1)b^2}{cx-1}\cdots\right.}{\frac{(cx+1)^4c^5}{(cx-1)^4}-\frac{4(cx+1)^3c^5}{(cx-1)^3}+\frac{6(cx+1)^2c^5}{(cx-1)^2}-\frac{4(cx+1)c^5}{cx-1}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*((c*x + 1)^3*b^2/(c*x - 1)^3 + (c*x + 1)*b^2/(c*x - 1))*log(-(c*x +
1)/(c*x - 1))^2/((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - 1)^
3 + 6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^5/(c*x - 1) + c^5) + 2*(6
*(c*x + 1)^3*a*b/(c*x - 1)^3 + 6*(c*x + 1)*a*b/(c*x - 1) + 3*(c*x + 1)^3*b^
2/(c*x - 1)^3 - 6*(c*x + 1)^2*b^2/(c*x - 1)^2 + 5*(c*x + 1)*b^2/(c*x - 1) -
 2*b^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(c*x + 1
)^3*c^5/(c*x - 1)^3 + 6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^5/(c*x
- 1) + c^5) + 2*(6*(c*x + 1)^3*a^2/(c*x - 1)^3 + 6*(c*x + 1)*a^2/(c*x - 1)
+ 6*(c*x + 1)^3*a*b/(c*x - 1)^3 - 12*(c*x + 1)^2*a*b/(c*x - 1)^2 + 10*(c*x
+ 1)*a*b/(c*x - 1) - 4*a*b + (c*x + 1)^3*b^2/(c*x - 1)^3 - 2*(c*x + 1)^2*b^
2/(c*x - 1)^2 + (c*x + 1)*b^2/(c*x - 1))/((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(
c*x + 1)^3*c^5/(c*x - 1)^3 + 6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^
5/(c*x - 1) + c^5) - 4*b^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 + 4*b^2*log(-(
c*x + 1)/(c*x - 1))/c^5)*c
```

maple [B] time = 0.02, size = 278, normalized size = 2.46

$$\frac{a^2x^4}{4}+\frac{b^2x^4\operatorname{arctanh}(cx)^2}{4}+\frac{b^2\operatorname{arctanh}(cx)x^3}{6c}+\frac{b^2x\operatorname{arctanh}(cx)}{2c^3}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{4c^4}-\frac{b^2\operatorname{arctanh}(cx)\ln(\cdots}{4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/4*a^2*x^4+1/4*b^2*x^4*arctanh(c*x)^2+1/6/c*b^2*arctanh(c*x)*x^3+1/2*b^2*x
*arctanh(c*x)/c^3+1/4/c^4*b^2*arctanh(c*x)*ln(c*x-1)-1/4/c^4*b^2*arctanh(c*
x)*ln(c*x+1)+1/16/c^4*b^2*ln(c*x-1)^2-1/8/c^4*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)
+1/16/c^4*b^2*ln(c*x+1)^2-1/8/c^4*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/8/c^4*b^
2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/12*b^2*x^2/c^2+1/3/c^4*b^2*ln(c*x-1)+1
/3/c^4*b^2*ln(c*x+1)+1/2*x^4*a*b*arctanh(c*x)+1/6*a*b*x^3/c+1/2*a*b*x/c^3+1
/4/c^4*a*b*ln(c*x-1)-1/4/c^4*a*b*ln(c*x+1)
```

maxima [A] time = 0.33, size = 189, normalized size = 1.67

$$\frac{1}{4}b^2x^4\operatorname{artanh}(cx)^2+\frac{1}{4}a^2x^4+\frac{1}{12}\left(6x^4\operatorname{artanh}(cx)+c\left(\frac{2\left(c^2x^3+3x\right)}{c^4}-\frac{3\log(cx+1)}{c^5}+\frac{3\log(cx-1)}{c^5}\right)\right)ab+\frac{1}{48}\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c*x)^2 + 1/4*a^2*x^4 + 1/12*(6*x^4*arctanh(c*x) + c*(2*
(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b + 1/48*
(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*arct
anh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(c*x + 1
)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*b^2
```

mupad [B] time = 0.92, size = 134, normalized size = 1.19

$$\frac{4\,b^2\,\ln\left(c^2\,x^2-1\right)-3\,b^2\,\mathrm{atanh}\left(c\,x\right)^2+3\,a^2\,c^4\,x^4+b^2\,c^2\,x^2-6\,a\,b\,\mathrm{atanh}\left(c\,x\right)+2\,b^2\,c^3\,x^3\,\mathrm{atanh}\left(c\,x\right)+6\,b^2\,c\,x}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))^2,x)
```

```
[Out] (4*b^2*log(c^2*x^2 - 1) - 3*b^2*atanh(c*x)^2 + 3*a^2*c^4*x^4 + b^2*c^2*x^2
- 6*a*b*atanh(c*x) + 2*b^2*c^3*x^3*atanh(c*x) + 6*b^2*c*x*atanh(c*x) + 3*b^
2*c^4*x^4*atanh(c*x)^2 + 2*a*b*c^3*x^3 + 6*a*b*c*x + 6*a*b*c^4*x^4*atanh(c*
x))/(12*c^4)
```

sympy [A] time = 1.68, size = 168, normalized size = 1.49

$$\begin{cases} \frac{a^2x^4}{4} + \frac{abx^4\operatorname{atanh}(cx)}{2} + \frac{abx^3}{6c} + \frac{abx}{2c^3} - \frac{ab\operatorname{atanh}(cx)}{2c^4} + \frac{b^2x^4\operatorname{atanh}^2(cx)}{4} + \frac{b^2x^3\operatorname{atanh}(cx)}{6c} + \frac{b^2x^2}{12c^2} + \frac{b^2x\operatorname{atanh}(cx)}{2c^3} + \frac{2b^2\log\left(x-\frac{1}{c}\right)}{3c^4} \\ \frac{a^2x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atanh(c*x)/2 + a*b*x**3/(6*c) + a*b*x/(2*
c**3) - a*b*atanh(c*x)/(2*c**4) + b**2*x**4*atanh(c*x)**2/4 + b**2*x**3*ata
nh(c*x)/(6*c) + b**2*x**2/(12*c**2) + b**2*x*atanh(c*x)/(2*c**3) + 2*b**2*l
og(x - 1/c)/(3*c**4) - b**2*atanh(c*x)**2/(4*c**4) + 2*b**2*atanh(c*x)/(3*c
**4), Ne(c, 0)), (a**2*x**4/4, True))
```

3.16 $\int x^2 \left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=130

$$\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c}-\frac{b^2\text{Li}_2\left(\right.}{3}$$

[Out] 1/3*b^2*x/c^2-1/3*b^2*arctanh(c*x)/c^3+1/3*b*x^2*(a+b*arctanh(c*x))/c+1/3*(a+b*arctanh(c*x))^2/c^3+1/3*x^3*(a+b*arctanh(c*x))^2-2/3*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^3-1/3*b^2*polylog(2,1-2/(-c*x+1))/c^3

Rubi [A] time = 0.20, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5916, 5980, 321, 206, 5984, 5918, 2402, 2315}

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^2}{}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*ArcTanh[c*x])^2,x]

[Out] (b^2*x)/(3*c^2) - (b^2*ArcTanh[c*x])/(3*c^3) + (b*x^2*(a + b*ArcTanh[c*x]))/(3*c) + (a + b*ArcTanh[c*x])^2/(3*c^3) + (x^3*(a + b*ArcTanh[c*x])^2)/3 - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^3) - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^3)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \left(a+b\tanh^{-1}(cx)\right)^2 dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{1}{3}(2bc)\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx \\
&= \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(2b)\int x\left(a+b\tanh^{-1}(cx)\right)dx}{3c} - \frac{(2b)\int \frac{x\left(a+b\tanh^{-1}(c\right.}{1-c^2x^2}}{3c} \\
&= \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3} + \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{1}{3}b^{\cdot} \\
&= \frac{b^2x}{3c^2} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3} + \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^{\cdot} \\
&= \frac{b^2x}{3c^2} - \frac{b^2\tanh^{-1}(cx)}{3c^3} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3} + \frac{1}{3}x^3\left(a\right. \\
&= \frac{b^2x}{3c^2} - \frac{b^2\tanh^{-1}(cx)}{3c^3} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3} + \frac{1}{3}x^3\left(a\right.
\end{aligned}
$$

Mathematica [A] time = 0.28, size = 122, normalized size = 0.94

$$\frac{a^2c^3x^3 + abc^2x^2 + ab\log\left(c^2x^2-1\right) + b\tanh^{-1}(cx)\left(2ac^3x^3 + bc^2x^2 - 2b\log\left(e^{-2\tanh^{-1}(cx)}+1\right) - b\right) + b^2\left(c^3x^3\right.}{3c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (b^2*c*x + a*b*c^2*x^2 + a^2*c^3*x^3 + b^2*(-1 + c^3*x^3)*ArcTanh[c*x]^2 +
b*ArcTanh[c*x]*(-b + b*c^2*x^2 + 2*a*c^3*x^3 - 2*b*Log[1 + E^(-2*ArcTanh[c*
```

```
x])]) + a*b*Log[-1 + c^2*x^2] + b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(3*c^
3)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\,\text{artanh}\,(cx)^2 + 2\,abx^2\,\text{artanh}\,(cx) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{artanh}\,(cx) + a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2, x)
```

maple [B] time = 0.01, size = 270, normalized size = 2.08

$$\frac{x^3a^2}{3}+\frac{b^2x^3\,\text{arctanh}\,(cx)^2}{3}+\frac{b^2\,\text{arctanh}\,(cx)\,x^2}{3c}+\frac{b^2\,\text{arctanh}\,(cx)\ln(cx-1)}{3c^3}+\frac{b^2\,\text{arctanh}\,(cx)\ln(cx+1)}{3c^3}+\frac{b^2x}{3c^2}+\frac{b^2\,1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arctanh(c*x)^2+1/3/c*b^2*arctanh(c*x)*x^2+1/3/c^3*b
^2*arctanh(c*x)*ln(c*x-1)+1/3/c^3*b^2*arctanh(c*x)*ln(c*x+1)+1/3*b^2*x/c^2+
1/6/c^3*b^2*ln(c*x-1)-1/6/c^3*b^2*ln(c*x+1)+1/12/c^3*b^2*ln(c*x-1)^2-1/3/c^
3*b^2*dilog(1/2+1/2*c*x)-1/6/c^3*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/12/c^3*b^2
*ln(c*x+1)^2-1/6/c^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/6/c^3*b^2*ln(-1
/2*c*x+1/2)*ln(c*x+1)+2/3*a*b*x^3*arctanh(c*x)+1/3*a*b*x^2/c+1/3/c^3*a*b*ln
(c*x-1)+1/3/c^3*a*b*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^2x^3+\frac{1}{3}\left(2\,x^3\,\text{artanh}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)ab-\frac{1}{216}\left(2\,c^4\left(\frac{2\left(c^2x^3+3\,x\right)}{c^6}-\frac{3\,\log(cx+1)}{c^7}+\frac{3\,\log(cx-}{c^7}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))
*a*b - 1/216*(2*c^4*(2*(c^2*x^3 + 3*x)/c^6 - 3*log(c*x + 1)/c^7 + 3*log(c*x
 - 1)/c^7) - 3*c^3*(x^2/c^4 + log(c^2*x^2 - 1)/c^6) - 648*c^3*integrate(1/9
*x^3*log(c*x + 1)/(c^4*x^2 - c^2), x) + 9*c^2*(2*x/c^4 - log(c*x + 1)/c^5 +
 log(c*x - 1)/c^5) - 324*c*integrate(1/9*x*log(c*x + 1)/(c^4*x^2 - c^2), x)
 - 6*(3*c^3*x^3*log(c*x + 1)^2 + (2*c^3*x^3 - 3*c^2*x^2 + 6*c*x - 6*(c^3*x^
3 + 1)*log(c*x + 1))*log(-c*x + 1))/c^3 - (2*(c*x - 1)^3*(9*log(-c*x + 1)^2
 - 6*log(-c*x + 1) + 2) + 27*(c*x - 1)^2*(2*log(-c*x + 1)^2 - 2*log(-c*x +
1) + 1) + 54*(c*x - 1)*(log(-c*x + 1)^2 - 2*log(-c*x + 1) + 2))/c^3 + 18*lo
g(9*c^4*x^2 - 9*c^2)/c^3 - 324*integrate(1/9*log(c*x + 1)/(c^4*x^2 - c^2),
x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,(a+b\,\mathrm{atanh}\,(c\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))^2,x)
```

```
[Out] int(x^2*(a + b*atanh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,(a+b\,\mathrm{atanh}\,(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x))**2, x)
```

3.17 $\int x \left(a + b \tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=75

$$-\frac{\left(a + b\tanh^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}x^2\left(a + b\tanh^{-1}(cx)\right)^2 + \frac{abx}{c} + \frac{b^2\log\left(1 - c^2x^2\right)}{2c^2} + \frac{b^2x\tanh^{-1}(cx)}{c}$$

```
[Out] a*b*x/c+b^2*x*arctanh(c*x)/c-1/2*(a+b*arctanh(c*x))^2/c^2+1/2*x^2*(a+b*arct
anh(c*x))^2+1/2*b^2*ln(-c^2*x^2+1)/c^2
```

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5916, 5980, 5910, 260, 5948}

$$-\frac{\left(a + b\tanh^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}x^2\left(a + b\tanh^{-1}(cx)\right)^2 + \frac{abx}{c} + \frac{b^2\log\left(1 - c^2x^2\right)}{2c^2} + \frac{b^2x\tanh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*x)/c + (b^2*x*ArcTanh[c*x])/c - (a + b*ArcTanh[c*x])^2/(2*c^2) + (x^2*
(a + b*ArcTanh[c*x])^2)/2 + (b^2*Log[1 - c^2*x^2])/(2*c^2)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}(cx)\right)^2 dx = \frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2-(bc)\int\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx$$
$$=\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b\int\left(a+b\tanh^{-1}(cx)\right)dx}{c}-\frac{b\int\frac{a+b\tanh^{-1}(cx)}{1-c^2x^2}dx}{c}$$
$$=\frac{abx}{c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b^2\int\tanh^{-1}(cx)\,dx}{c}$$
$$=\frac{abx}{c}+\frac{b^2x\tanh^{-1}(cx)}{c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2-b^2\int$$
$$=\frac{abx}{c}+\frac{b^2x\tanh^{-1}(cx)}{c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b^2\text{lo}}{}$$

Mathematica [A] time = 0.07, size = 90, normalized size = 1.20

$$\frac{a^2c^2x^2+2abcx+b(a+b)\log(1-cx)-ab\log(cx+1)+2bcx\tanh^{-1}(cx)(acx+b)+b^2\left(c^2x^2-1\right)\tanh^{-1}(cx)^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (2*a*b*c*x + a^2*c^2*x^2 + 2*b*c*x*(b + a*c*x)*ArcTanh[c*x] + b^2*(-1 + c^2
*x^2)*ArcTanh[c*x]^2 + b*(a + b)*Log[1 - c*x] - a*b*Log[1 + c*x] + b^2*Log[
1 + c*x])/(2*c^2)
```

fricas [A] time = 0.61, size = 122, normalized size = 1.63

$$\frac{4\,a^2c^2x^2+8\,abcx+\left(b^2c^2x^2-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2-4\left(ab-b^2\right)\log\left(cx+1\right)+4\left(ab+b^2\right)\log\left(cx-1\right)+4\left(abc^2x^2\right.}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^2*c^2*x^2 + 8*a*b*c*x + (b^2*c^2*x^2 - b^2)*log(-(c*x + 1)/(c*x -
1))^2 - 4*(a*b - b^2)*log(c*x + 1) + 4*(a*b + b^2)*log(c*x - 1) + 4*(a*b*c^
2*x^2 + b^2*c*x)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

giac [B] time = 0.16, size = 301, normalized size = 4.01

$$\frac{1}{2}\left(\frac{(cx+1)b^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{\left(\frac{(cx+1)^2c^3}{(cx-1)^2}-\frac{2(cx+1)c^3}{cx-1}+c^3\right)(cx-1)}+\frac{2\left(\frac{2(cx+1)ab}{cx-1}+\frac{(cx+1)b^2}{cx-1}-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^2c^3}{(cx-1)^2}-\frac{2(cx+1)c^3}{cx-1}+c^3}+\frac{4\left(\frac{(cx+1)a^2}{cx-1}+\frac{(cx+1)ab}{cx-1}-ab\right)}{\frac{(cx+1)^2c^3}{(cx-1)^2}-\frac{2(cx+1)c^3}{cx-1}+c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*((c*x + 1)*b^2*log(-(c*x + 1)/(c*x - 1))^2/(((c*x + 1)^2*c^3/(c*x - 1)^
2 - 2*(c*x + 1)*c^3/(c*x - 1) + c^3)*(c*x - 1)) + 2*(2*(c*x + 1)*a*b/(c*x -
 1) + (c*x + 1)*b^2/(c*x - 1) - b^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^2
*c^3/(c*x - 1)^2 - 2*(c*x + 1)*c^3/(c*x - 1) + c^3) + 4*((c*x + 1)*a^2/(c*x
 - 1) + (c*x + 1)*a*b/(c*x - 1) - a*b)/((c*x + 1)^2*c^3/(c*x - 1)^2 - 2*(c*
x + 1)*c^3/(c*x - 1) + c^3) - 2*b^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^3 + 2*b
^2*log(-(c*x + 1)/(c*x - 1))/c^3)*c
```

maple [B] time = 0.02, size = 239, normalized size = 3.19

$$\frac{a^2x^2}{2}+\frac{x^2b^2\operatorname{arctanh}(cx)^2}{2}+\frac{b^2x\operatorname{arctanh}(cx)}{c}+\frac{b^2\operatorname{arctanh}(cx)\ln(cx-1)}{2c^2}-\frac{b^2\operatorname{arctanh}(cx)\ln(cx+1)}{2c^2}+\frac{b^2\ln(cx-}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*x^2*b^2*arctanh(c*x)^2+b^2*x*arctanh(c*x)/c+1/2/c^2*b^2*arc
tanh(c*x)*ln(c*x-1)-1/2/c^2*b^2*arctanh(c*x)*ln(c*x+1)+1/8/c^2*b^2*ln(c*x-1
)^2-1/4/c^2*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/2/c^2*b^2*ln(c*x-1)+1/2/c^2*b^2
*ln(c*x+1)+1/8/c^2*b^2*ln(c*x+1)^2+1/4/c^2*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*
c*x)-1/4/c^2*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+a*b*x^2*arctanh(c*x)+a*b*x/c+1/
2/c^2*a*b*ln(c*x-1)-1/2/c^2*a*b*ln(c*x+1)
```

maxima [B] time = 0.33, size = 158, normalized size = 2.11

$$\frac{1}{2}b^2x^2\operatorname{artanh}(cx)^2+\frac{1}{2}a^2x^2+\frac{1}{2}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)ab+\frac{1}{8}\left(4c\left(\frac{2x}{c^2}-\frac{\log(cx}{c^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctanh(c*x)^2 + 1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x
/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b + 1/8*(4*c*(2*x/c^2 - log(
c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) - (2*(log(c*x - 1) - 2)*log(c
*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1))/c^2)*b^2
```

mupad [B] time = 0.78, size = 89, normalized size = 1.19

$$\frac{a^2x^2}{2}-\frac{\frac{b^2\operatorname{atanh}(cx)^2}{2}-\frac{b^2\ln(c^2x^2-1)}{2}-c\left(x\operatorname{atanh}(cx)\,b^2+a\,x\,b\right)+a\,b\operatorname{atanh}(cx)}{c^2}+\frac{b^2x^2\operatorname{atanh}(cx)^2}{2}+a\,b\,x^2\operatorname{atanh}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))^2,x)
```

```
[Out] (a^2*x^2)/2 - ((b^2*atanh(c*x)^2)/2 - (b^2*log(c^2*x^2 - 1))/2 - c*(b^2*x*a
tanh(c*x) + a*b*x) + a*b*atanh(c*x))/c^2 + (b^2*x^2*atanh(c*x)^2)/2 + a*b*x
^2*atanh(c*x)
```

sympy [A] time = 0.84, size = 114, normalized size = 1.52

$$\begin{cases}\frac{a^2x^2}{2}+abx^2\operatorname{atanh}(cx)+\frac{abx}{c}-\frac{ab\operatorname{atanh}(cx)}{c^2}+\frac{b^2x^2\operatorname{atanh}^2(cx)}{2}+\frac{b^2x\operatorname{atanh}(cx)}{c}+\frac{b^2\log\left(x-\frac{1}{c}\right)}{c^2}-\frac{b^2\operatorname{atanh}^2(cx)}{2c^2}+\frac{b^2\operatorname{atanh}(cx)}{c^2} \\ \frac{a^2x^2}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 + a*b*x**2*atanh(c*x) + a*b*x/c - a*b*atanh(c*x)/c**
2 + b**2*x**2*atanh(c*x)**2/2 + b**2*x*atanh(c*x)/c + b**2*log(x - 1/c)/c**
2 - b**2*atanh(c*x)**2/(2*c**2) + b**2*atanh(c*x)/c**2, Ne(c, 0)), (a**2*x*
*2/2, True))
```

3.18 $\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=74

$$x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{b^2\mathrm{Li}_2\left(1-\frac{2}{1-cx}\right)}{c}$$

```
[Out] (a+b*arctanh(c*x))^2/c+x*(a+b*arctanh(c*x))^2-2*b*(a+b*arctanh(c*x))*ln(2/(
-c*x+1))/c-b^2*polylog(2,1-2/(-c*x+1))/c
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5910, 5984, 5918, 2402, 2315}

$$-\frac{b^2\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a + b*ArcTanh[c*x])^2/c + x*(a + b*ArcTanh[c*x])^2 - (2*b*(a + b*ArcTanh[c
*x])*Log[2/(1 - c*x)])/c - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(cx)\right)^2 dx = x\left(a+b\tanh^{-1}(cx)\right)^2-(2bc)\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx$$
$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-(2b)\int \frac{a+b\tanh^{-1}(cx)}{1-cx}dx$$
$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c}+(2$$
$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c}-\frac{(2}{}$$
$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c}-\frac{b^2}{}$$

Mathematica [A] time = 0.15, size = 82, normalized size = 1.11

$$\frac{a\left(acx+b\log\left(1-c^2x^2\right)\right)+2b\tanh^{-1}(cx)\left(acx-b\log\left(e^{-2\tanh^{-1}(cx)}+1\right)\right)+b^2\text{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)+b^2(cx-1)\text{ta}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (b^2*(-1 + c*x)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c*x - b*Log[1 + E^(-2*
ArcTanh[c*x])]) + a*(a*c*x + b*Log[1 - c^2*x^2]) + b^2*PolyLog[2, -E^(-2*Ar
cTanh[c*x])])/c
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2\,\text{artanh}\,(cx)^2+2\,ab\,\text{artanh}\,(cx)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{artanh}\,(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2, x)
```

maple [A] time = 0.19, size = 123, normalized size = 1.66

$$x\,b^2\,\text{arctanh}\,(cx)^2+2xab\,\text{arctanh}\,(cx)+\frac{b^2\,\text{arctanh}\,(cx)^2}{c}-\frac{2\,\text{arctanh}\,(cx)\ln\left(1+\frac{(cx+1)^2}{-c^2x^2+1}\right)b^2}{c}+a^2x+\frac{ab\ln\left(-c^2x^2+\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2,x)
```

```
[Out] x*b^2*arctanh(c*x)^2+2*x*a*b*arctanh(c*x)+1/c*b^2*arctanh(c*x)^2-2/c*arctan
h(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))*b^2+a^2*x+1/c*a*b*ln(-c^2*x^2+1)-1/c*po
lylog(2,-(c*x+1)^2/(-c^2*x^2+1))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\left(c^2\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right) - 6c\int \frac{x\log(cx+1)}{c^2x^2-1}\,dx - \frac{(cx-1)\left(\log(-cx+1)^2 - 2\log(-cx+1\right.}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - 6*c*integrate(x
*log(c*x + 1)/(c^2*x^2 - 1), x) - (c*x - 1)*(log(-c*x + 1)^2 - 2*log(-c*x +
 1) + 2)/c - (c*x*log(c*x + 1)^2 + 2*(c*x - (c*x + 1)*log(c*x + 1))*log(-c*
x + 1))/c + log(c^2*x^2 - 1)/c - 2*integrate(log(c*x + 1)/(c^2*x^2 - 1), x)
)*b^2 + a^2*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{atanh}\,(c\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2,x)
```

```
[Out] int((a + b*atanh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{atanh}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2, x)
```

3.19 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=117

$$-b\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+b\text{Li}_2\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}\right.$$

```
[Out] -2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))-b*(a+b*arctanh(c*x))*polylog
(2,1-2/(-c*x+1))+b*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))+1/2*b^2*poly
log(3,1-2/(-c*x+1))-1/2*b^2*polylog(3,-1+2/(-c*x+1))
```

Rubi [A] time = 0.26, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5914, 6052, 5948, 6058, 6610}

$$-b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+b\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - b*(a + b*ArcTanh[c*x])*
PolyLog[2, 1 - 2/(1 - c*x)] + b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 -
 c*x)] + (b^2*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*PolyLog[3, -1 + 2/(1 -
c*x)])/2
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx = 2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-(4bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)\tanh^{-1}\left(1\right.}{1-c^2x^2}$$
$$= 2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)+(2bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right.}{1-c^2x^2}$$
$$= 2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-b\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2}{1-cx}\right)$$
$$= 2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-b\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2}{1-cx}\right)$$

Mathematica [A] time = 0.08, size = 120, normalized size = 1.03

$$\frac{1}{2}b\left(2\text{Li}_2\left(\frac{cx+1}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)-2\text{Li}_2\left(\frac{cx+1}{cx-1}\right)\left(a+b\tanh^{-1}(cx)\right)+b\left(\text{Li}_3\left(\frac{cx+1}{cx-1}\right)-\text{Li}_3\left(\frac{cx+1}{1-cx}\right)\right)\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^2*ArcTanh[(1 + c*x)/(-1 + c*x)] + (b*(2*(a + b*ArcTa
nh[c*x])*PolyLog[2, (1 + c*x)/(1 - c*x)] - 2*(a + b*ArcTanh[c*x])*PolyLog[2
, (1 + c*x)/(-1 + c*x)] + b*(-PolyLog[3, (1 + c*x)/(1 - c*x)] + PolyLog[3,
(1 + c*x)/(-1 + c*x)])))/2
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}(cx)^2+2\,ab\,\text{artanh}(cx)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{artanh}(cx)+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/x, x)
```

maple [C] time = 0.28, size = 701, normalized size = 5.99

$$a^2\ln(cx)+b^2\ln(cx)\,\text{arctanh}(cx)^2-b^2\,\text{arctanh}(cx)\,\text{polylog}\left(2,-\frac{(cx+1)^2}{-c^2x^2+1}\right)+\frac{b^2\,\text{polylog}\left(3,-\frac{(cx+1)^2}{-c^2x^2+1}\right)}{2}-b^2\,\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x,x)
```

```
[Out] a^2*ln(c*x)+b^2*ln(c*x)*arctanh(c*x)^2-b^2*arctanh(c*x)*polylog(2,-(c*x+1)^
2/(-c^2*x^2+1))+1/2*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-b^2*arctanh(c*x)
^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1
)^(1/2))+2*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2*pol
ylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^
2+1)^(1/2))+2*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2
*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*I*b^2*Pi*csgn(I/(1+(c*x+1)^2/(-
c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^
2*arctanh(c*x)^2+1/2*I*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+
(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c
^2*x^2+1)))*arctanh(c*x)^2-1/2*I*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*
csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x
)^2+1/2*I*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1
)))^3*arctanh(c*x)^2+2*a*b*ln(c*x)*arctanh(c*x)-a*b*ln(c*x)*ln(c*x+1)-a*b*d
ilog(c*x)-a*b*dilog(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\log(x) + \int \frac{b^2\left(\log(cx+1) - \log(-cx+1)\right)^2}{4x} + \frac{ab\left(\log(cx+1) - \log(-cx+1)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c*x + 1) - log(-c*x + 1))^2/x + a*b*(lo
g(c*x + 1) - log(-c*x + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}\,(c\,x))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/x,x)
```

```
[Out] int((a + b*atanh(c*x))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/x, x)
```

3.20 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=71

$$c\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2bc\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)+b^2(-c)\mathrm{Li}_2\left(\frac{2}{cx+1}-1\right)$$

```
[Out] c*(a+b*arctanh(c*x))^2-(a+b*arctanh(c*x))^2/x+2*b*c*(a+b*arctanh(c*x))*ln(2
-2/(c*x+1))-b^2*c*polylog(2,-1+2/(c*x+1))
```

Rubi [A] time = 0.15, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5916, 5988, 5932, 2447}

$$b^2(-c)\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+c\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2bc\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x^2,x]
```

```
[Out] c*(a + b*ArcTanh[c*x])^2 - (a + b*ArcTanh[c*x])^2/x + 2*b*c*(a + b*ArcTanh[
c*x])*Log[2 - 2/(1 + c*x)] - b^2*c*PolyLog[2, -1 + 2/(1 + c*x)]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} + (2bc)\int \frac{a+b\tanh^{-1}(cx)}{x\left(1-c^2x^2\right)}\,dx$$
$$= c\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} + (2bc)\int \frac{a+b\tanh^{-1}(cx)}{x(1+cx)}\,dx$$
$$= c\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right.$$
$$= c\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right.$$

Mathematica [A] time = 0.15, size = 94, normalized size = 1.32

$$\frac{-a\left(a+bcx\log\left(1-c^2x^2\right)-2bcx\log(cx)\right)+2b\tanh^{-1}(cx)\left(bcx\log\left(1-e^{-2\tanh^{-1}(cx)}\right)-a\right)-b^2cx\text{Li}_2\left(e^{-2\tanh^{-1}(cx)}\right.}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^2,x]
```

```
[Out] (b^2*(-1 + c*x)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(-a + b*c*x*Log[1 - E^(-2
*ArcTanh[c*x])]) - a*(a - 2*b*c*x*Log[c*x] + b*c*x*Log[1 - c^2*x^2]) - b^2*
c*x*PolyLog[2, E^(-2*ArcTanh[c*x])])/x
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\,(cx)^2+2\,ab\,\text{artanh}\,(cx)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/x^2, x)
```

maple [B] time = 0.02, size = 248, normalized size = 3.49

$$-\frac{a^2}{x}-\frac{b^2\,\text{arctanh}\,(cx)^2}{x}+2c\,b^2\ln(cx)\,\text{arctanh}\,(cx)-c\,b^2\,\text{arctanh}\,(cx)\ln(cx-1)-c\,b^2\,\text{arctanh}\,(cx)\ln(cx+1)-\frac{c\,b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2,x)
```

```
[Out] -a^2/x-b^2/x*arctanh(c*x)^2+2*c*b^2*ln(c*x)*arctanh(c*x)-c*b^2*arctanh(c*x)
*ln(c*x-1)-c*b^2*arctanh(c*x)*ln(c*x+1)-1/4*c*b^2*ln(c*x-1)^2+c*b^2*dilog(1
/2+1/2*c*x)+1/2*c*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/4*c*b^2*ln(c*x+1)^2-1/2*c
*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2*c*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-
c*b^2*dilog(c*x)-c*b^2*dilog(c*x+1)-c*b^2*ln(c*x)*ln(c*x+1)-2*a*b/x*arctanh
(c*x)+2*c*a*b*ln(c*x)-c*a*b*ln(c*x-1)-c*a*b*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\mathrm{artanh}\left(cx\right)}{x}\right)ab-\frac{1}{4}\,b^2\left(\frac{\log\left(-cx+1\right)^2}{x}+\int-\frac{\left(cx-1\right)\log\left(cx+1\right)^2+2\left(cx-\right.}{c\!:}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] -(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b - 1/4*b^2*(log(-c
*x + 1)^2/x + integrate(-((c*x - 1)*log(c*x + 1)^2 + 2*(c*x - (c*x - 1)*log
(c*x + 1))*log(-c*x + 1))/(c*x^3 - x^2), x)) - a^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/x^2,x)
```

```
[Out] int((a + b*atanh(c*x))^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(cx\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/x**2, x)
```

3.21 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=80

$$\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{1}{2}b^2c^2\log\left(1-c^2x^2\right)+b^2c^2\log(x)$$

```
[Out] -b*c*(a+b*arctanh(c*x))/x+1/2*c^2*(a+b*arctanh(c*x))^2-1/2*(a+b*arctanh(c*x
))^2/x^2+b^2*c^2*ln(x)-1/2*b^2*c^2*ln(-c^2*x^2+1)
```

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5916, 5982, 266, 36, 29, 31, 5948}

$$\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{1}{2}b^2c^2\log\left(1-c^2x^2\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x^3,x]
```

```
[Out] -((b*c*(a + b*ArcTanh[c*x]))/x) + (c^2*(a + b*ArcTanh[c*x])^2)/2 - (a + b*A
rcTanh[c*x])^2/(2*x^2) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} + (bc)\int \frac{a+b\tanh^{-1}(cx)}{x^2\left(1-c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} + (bc)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx + \left(bc^3\right)\int \frac{a+b\tanh^{-1}(cx)}{1-c^2x^2}\,c$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} + \left(b^2c^2\right)$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} + \frac{1}{2}\left(b^2c\right.$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} + \frac{1}{2}\left(b^2c\right.$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} + b^2c^2\,\mathrm{lo}$$

Mathematica [A] time = 0.07, size = 101, normalized size = 1.26

$$-\frac{a^2+bc^2x^2(a+b)\log(1-cx)-bc^2x^2(a-b)\log(cx+1)+2abcx+2b\tanh^{-1}(cx)(a+bcx)-2b^2c^2x^2\log(x)-}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^3,x]
```

```
[Out] -1/2*(a^2 + 2*a*b*c*x + 2*b*(a + b*c*x)*ArcTanh[c*x] - b^2*(-1 + c^2*x^2)*A
rcTanh[c*x]^2 - 2*b^2*c^2*x^2*Log[x] + b*(a + b)*c^2*x^2*Log[1 - c*x] - (a
- b)*b*c^2*x^2*Log[1 + c*x])/x^2
```

fricas [A] time = 0.71, size = 135, normalized size = 1.69

$$\frac{8\,b^2c^2x^2\log(x)+4\left(ab-b^2\right)c^2x^2\log\left(cx+1\right)-4\left(ab+b^2\right)c^2x^2\log\left(cx-1\right)-8\,abcx+\left(b^2c^2x^2-b^2\right)\log\left(-\frac{cx+}{cx-}\right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^2*c^2*x^2*log(x) + 4*(a*b - b^2)*c^2*x^2*log(c*x + 1) - 4*(a*b + b
^2)*c^2*x^2*log(c*x - 1) - 8*a*b*c*x + (b^2*c^2*x^2 - b^2)*log(-(c*x + 1)/(
c*x - 1))^2 - 4*a^2 - 4*(b^2*c*x + a*b)*log(-(c*x + 1)/(c*x - 1)))/x^2
```

giac [B] time = 0.16, size = 278, normalized size = 3.48

$$\frac{1}{2}\left(2\,b^2c\log\left(-\frac{cx+1}{cx-1}-1\right)-2\,b^2c\log\left(-\frac{cx+1}{cx-1}\right)+\frac{(cx+1)b^2c\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx-1)\left(\frac{(cx+1)^2}{(cx-1)^2}+\frac{2\,(cx+1)}{cx-1}+1\right)}+\frac{2\left(\frac{2\,(cx+1)abc}{cx-1}+\frac{(cx+1)b^2c}{cx-1}+\right.}{\frac{(cx+1)^2}{(cx-1)^2}+\frac{2\,(cx+}{cx-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*c*log(-(c*x + 1)/(c*x - 1) - 1) - 2*b^2*c*log(-(c*x + 1)/(c*x -
1)) + (c*x + 1)*b^2*c*log(-(c*x + 1)/(c*x - 1))^2/((c*x - 1)*((c*x + 1)^2/(
c*x - 1)^2 + 2*(c*x + 1)/(c*x - 1) + 1)) + 2*(2*(c*x + 1)*a*b*c/(c*x - 1) +
 (c*x + 1)*b^2*c/(c*x - 1) + b^2*c)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^2/
(c*x - 1)^2 + 2*(c*x + 1)/(c*x - 1) + 1) + 4*((c*x + 1)*a^2*c/(c*x - 1) + (
c*x + 1)*a*b*c/(c*x - 1) + a*b*c)/((c*x + 1)^2/(c*x - 1)^2 + 2*(c*x + 1)/(c
*x - 1) + 1))*c
```

maple [B] time = 0.02, size = 253, normalized size = 3.16

$$-\frac{a^2}{2x^2}-\frac{b^2\operatorname{arctanh}(cx)^2}{2x^2}-\frac{c\,b^2\operatorname{arctanh}(cx)}{x}-\frac{c^2b^2\operatorname{arctanh}(cx)\ln(cx-1)}{2}+\frac{c^2b^2\operatorname{arctanh}(cx)\ln(cx+1)}{2}-\frac{c^2b^2\ln(c}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2-1/2*b^2/x^2*arctanh(c*x)^2-c*b^2*arctanh(c*x)/x-1/2*c^2*b^2*ar
ctanh(c*x)*ln(c*x-1)+1/2*c^2*b^2*arctanh(c*x)*ln(c*x+1)-1/8*c^2*b^2*ln(c*x-
1)^2+1/4*c^2*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+c^2*b^2*ln(c*x)-1/2*c^2*b^2*ln(c
*x-1)-1/2*c^2*b^2*ln(c*x+1)-1/8*c^2*b^2*ln(c*x+1)^2+1/4*c^2*b^2*ln(-1/2*c*x
+1/2)*ln(c*x+1)-1/4*c^2*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-a*b/x^2*arctan
h(c*x)-a*b*c/x-1/2*c^2*a*b*ln(c*x-1)+1/2*c^2*a*b*ln(c*x+1)
```

maxima [B] time = 0.32, size = 151, normalized size = 1.89

$$\frac{1}{2}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\operatorname{artanh}(cx)}{x^2}\right)ab+\frac{1}{8}\left(\left(2\left(\log(cx-1)-2\right)\log(cx+1)-\log(cx+1)^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b +
1/8*((2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 -
 4*log(c*x - 1) + 8*log(x))*c^2 + 4*(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)
*c*arctanh(c*x))*b^2 - 1/2*b^2*arctanh(c*x)^2/x^2 - 1/2*a^2/x^2
```

mupad [B] time = 1.49, size = 246, normalized size = 3.08

$$\frac{b^2\,c^2\ln(c\,x+1)^2}{8}-\frac{a^2}{2\,x^2}+\frac{b^2\,c^2\ln(1-c\,x)^2}{8}-\frac{b^2\ln(c\,x+1)^2}{8\,x^2}-\frac{b^2\ln(1-c\,x)^2}{8\,x^2}+b^2\,c^2\,\ln(x)-\frac{b^2\,c^2\,\ln(c\,x-1)}{2}-\frac{b^2\,c^2\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/x^3,x)
```

```
[Out] (b^2*c^2*log(c*x + 1)^2)/8 - a^2/(2*x^2) + (b^2*c^2*log(1 - c*x)^2)/8 - (b^
2*log(c*x + 1)^2)/(8*x^2) - (b^2*log(1 - c*x)^2)/(8*x^2) + b^2*c^2*log(x) -
 (b^2*c^2*log(c*x - 1))/2 - (b^2*c^2*log(c*x + 1))/2 - (a*b*log(c*x + 1))/(
2*x^2) + (a*b*log(1 - c*x))/(2*x^2) + (b^2*log(c*x + 1)*log(1 - c*x))/(4*x^
2) - (a*b*c)/x - (b^2*c*log(c*x + 1))/(2*x) + (b^2*c*log(1 - c*x))/(2*x) -
(a*b*c^2*log(c*x - 1))/2 + (a*b*c^2*log(c*x + 1))/2 - (b^2*c^2*log(c*x + 1)
*log(1 - c*x))/4
```

sympy [A] time = 1.14, size = 126, normalized size = 1.58

$$\begin{cases} -\frac{a^2}{2x^2} + abc^2 \operatorname{atanh}(cx) - \frac{abc}{x} - \frac{ab \operatorname{atanh}(cx)}{x^2} + b^2c^2 \log(x) - b^2c^2 \log\left(x - \frac{1}{c}\right) + \frac{b^2c^2 \operatorname{atanh}^2(cx)}{2} - b^2c^2 \operatorname{atanh}(cx) - \\ -\frac{a^2}{2x^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) + a*b*c**2*atanh(c*x) - a*b*c/x - a*b*atanh(c*x)/
x**2 + b**2*c**2*log(x) - b**2*c**2*log(x - 1/c) + b**2*c**2*atanh(c*x)**2/
2 - b**2*c**2*atanh(c*x) - b**2*c*atanh(c*x)/x - b**2*atanh(c*x)**2/(2*x**2
), Ne(c, 0)), (-a**2/(2*x**2), True))
```

3.22 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=130

$$\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}bc^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-$$

```
[Out] -1/3*b^2*c^2/x+1/3*b^2*c^3*arctanh(c*x)-1/3*b*c*(a+b*arctanh(c*x))/x^2+1/3*
c^3*(a+b*arctanh(c*x))^2-1/3*(a+b*arctanh(c*x))^2/x^3+2/3*b*c^3*(a+b*arctan
h(c*x))*ln(2-2/(c*x+1))-1/3*b^2*c^3*polylog(2,-1+2/(c*x+1))
```

Rubi [A] time = 0.23, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5916, 5982, 325, 206, 5988, 5932, 2447}

$$-\frac{1}{3}b^2c^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}bc^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{bc\left(a+b\,\text{ta}\right.}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x^4,x]
```

```
[Out] -(b^2*c^2)/(3*x) + (b^2*c^3*ArcTanh[c*x])/3 - (b*c*(a + b*ArcTanh[c*x]))/(3
*x^2) + (c^3*(a + b*ArcTanh[c*x])^2)/3 - (a + b*ArcTanh[c*x])^2/(3*x^3) + (
2*b*c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/3 - (b^2*c^3*PolyLog[2,
-1 + 2/(1 + c*x)])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bc)\int \frac{a+b\tanh^{-1}(cx)}{x^3\left(1-c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bc)\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx+\frac{1}{3}\left(2bc^3\right)\int \frac{a+b\tanh^{-1}}{x\left(1-c^2x\right.}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(b^2c\right.$$

$$= -\frac{b^2c^2}{3x}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}$$

$$= -\frac{b^2c^2}{3x}+\frac{1}{3}b^2c^3\tanh^{-1}(cx)-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2-$$

Mathematica [A] time = 0.38, size = 145, normalized size = 1.12

$$-\frac{a^2-2abc^3x^3\log(cx)+b\tanh^{-1}(cx)\left(2a-bc^3x^3-2bc^3x^3\log\left(1-e^{-2\tanh^{-1}(cx)}\right)+bcx\right)+abc^3x^3\log\left(1-c^2x\right.}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^4,x]
```

```
[Out] -1/3*(a^2 + a*b*c*x + b^2*c^2*x^2 + b^2*(1 - c^3*x^3)*ArcTanh[c*x]^2 + b*Ar
cTanh[c*x]*(2*a + b*c*x - b*c^3*x^3 - 2*b*c^3*x^3*Log[1 - E^(-2*ArcTanh[c*x
])]) - 2*a*b*c^3*x^3*Log[c*x] + a*b*c^3*x^3*Log[1 - c^2*x^2] + b^2*c^3*x^3*
PolyLog[2, E^(-2*ArcTanh[c*x])])/x^3
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\,(cx)^2+2\,ab\,\text{artanh}\,(cx)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/x^4, x)
```

maple [B] time = 0.02, size = 339, normalized size = 2.61

$$-\frac{a^2}{3x^3} - \frac{b^2 \operatorname{arctanh}(cx)^2}{3x^3} - \frac{c\,b^2 \operatorname{arctanh}(cx)}{3x^2} + \frac{2c^3b^2 \ln(cx) \operatorname{arctanh}(cx)}{3} - \frac{c^3b^2 \operatorname{arctanh}(cx) \ln(cx-1)}{3} - \frac{c^3b^2 \operatorname{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^4,x)
```

```
[Out] -1/3*a^2/x^3-1/3*b^2/x^3*arctanh(c*x)^2-1/3*c*b^2*arctanh(c*x)/x^2+2/3*c^3*
b^2*ln(c*x)*arctanh(c*x)-1/3*c^3*b^2*arctanh(c*x)*ln(c*x-1)-1/3*c^3*b^2*arc
tanh(c*x)*ln(c*x+1)-1/3*b^2*c^2/x-1/6*c^3*b^2*ln(c*x-1)+1/6*c^3*b^2*ln(c*x+
1)-1/12*c^3*b^2*ln(c*x-1)^2+1/3*c^3*b^2*dilog(1/2+1/2*c*x)+1/6*c^3*b^2*ln(c
*x-1)*ln(1/2+1/2*c*x)+1/12*c^3*b^2*ln(c*x+1)^2+1/6*c^3*b^2*ln(-1/2*c*x+1/2)
*ln(1/2+1/2*c*x)-1/6*c^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/3*c^3*b^2*dilog(c
*x)-1/3*c^3*b^2*dilog(c*x+1)-1/3*c^3*b^2*ln(c*x)*ln(c*x+1)-2/3*a*b/x^3*arct
anh(c*x)-1/3*c*a*b/x^2+2/3*c^3*a*b*ln(c*x)-1/3*c^3*a*b*ln(c*x-1)-1/3*c^3*a*
b*ln(c*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(\left(c^2 \log\left(c^2x^2-1\right) - c^2 \log\left(x^2\right) + \frac{1}{x^2}\right)c + \frac{2 \operatorname{artanh}(cx)}{x^3}\right)ab - \frac{1}{12}b^2\left(\frac{\log(-cx+1)^2}{x^3} + 3\int -\frac{3(cx-1)\log(cx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*a*b - 1/12*b^2*(log(-c*x + 1)^2/x^3 + 3*integrate(-1/3*(3*(c*x - 1)*log(c*
x + 1)^2 + 2*(c*x - 3*(c*x - 1)*log(c*x + 1))*log(-c*x + 1))/(c*x^5 - x^4),
 x)) - 1/3*a^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atanh}(c\,x))^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/x^4,x)
```

```
[Out] int((a + b*atanh(c*x))^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/x**4, x)
```

3.23 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=117

$$\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}+\frac{2}{3}b^2c^4\log(x)-\frac{b^2c^2}{12x^2}$$

```
[Out] -1/12*b^2*c^2/x^2-1/6*b*c*(a+b*arctanh(c*x))/x^3-1/2*b*c^3*(a+b*arctanh(c*x
))/x+1/4*c^4*(a+b*arctanh(c*x))^2-1/4*(a+b*arctanh(c*x))^2/x^4+2/3*b^2*c^4*
ln(x)-1/3*b^2*c^4*ln(-c^2*x^2+1)
```

Rubi [A] time = 0.23, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5916, 5982, 266, 44, 36, 29, 31, 5948}

$$\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}-\frac{b^2c^2}{12x^2}-\frac{1}{3}b^2c^4\log(1$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x^5,x]
```

```
[Out] -(b^2*c^2)/(12*x^2) - (b*c*(a + b*ArcTanh[c*x]))/(6*x^3) - (b*c^3*(a + b*Ar
cTanh[c*x]))/(2*x) + (c^4*(a + b*ArcTanh[c*x])^2)/4 - (a + b*ArcTanh[c*x])^
2/(4*x^4) + (2*b^2*c^4*Log[x])/3 - (b^2*c^4*Log[1 - c^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
```

```
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^5}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}+\frac{1}{2}(bc)\int \frac{a+b\tanh^{-1}(cx)}{x^4\left(1-c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}+\frac{1}{2}(bc)\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx+\frac{1}{2}\left(bc^3\right)\int \frac{a+b\tanh^{-1}(c}{x^2\left(1-c^2x^2\right.}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}+\frac{1}{6}\left(b^2c^2\right)\int \frac{1}{x^3\left(1-c^2x^2\right)}\,dx+\frac{1}{2}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{(a}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{(a}{}$$

$$= -\frac{b^2c^2}{12x^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right.$$

$$= -\frac{b^2c^2}{12x^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right.$$

Mathematica [A] time = 0.08, size = 164, normalized size = 1.40

$$-\frac{3a^2+3abc^4x^4\log(1-cx)-3abc^4x^4\log(cx+1)+6abc^3x^3+2b\tanh^{-1}(cx)\left(3a+3bc^3x^3+bcx\right)+2abcx-8}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^5,x]
```

```
[Out] -1/12*(3*a^2 + 2*a*b*c*x + b^2*c^2*x^2 + 6*a*b*c^3*x^3 + 2*b*(3*a + b*c*x +
 3*b*c^3*x^3)*ArcTanh[c*x] - 3*b^2*(-1 + c^4*x^4)*ArcTanh[c*x]^2 - 8*b^2*c^
4*x^4*Log[x] + 3*a*b*c^4*x^4*Log[1 - c*x] + 4*b^2*c^4*x^4*Log[1 - c*x] - 3*
a*b*c^4*x^4*Log[1 + c*x] + 4*b^2*c^4*x^4*Log[1 + c*x])/x^4
```

fricas [A] time = 0.90, size = 173, normalized size = 1.48

$$\frac{32\,b^2c^4x^4\log(x)+4\left(3\,ab-4\,b^2\right)c^4x^4\log\left(cx+1\right)-4\left(3\,ab+4\,b^2\right)c^4x^4\log\left(cx-1\right)-24\,abc^3x^3-4\,b^2c^2x^2-8}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(32*b^2*c^4*x^4*log(x) + 4*(3*a*b - 4*b^2)*c^4*x^4*log(c*x + 1) - 4*(3
*a*b + 4*b^2)*c^4*x^4*log(c*x - 1) - 24*a*b*c^3*x^3 - 4*b^2*c^2*x^2 - 8*a*b
*c*x + 3*(b^2*c^4*x^4 - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 12*a^2 - 4*(3*b^
2*c^3*x^3 + b^2*c*x + 3*a*b)*log(-(c*x + 1)/(c*x - 1)))/x^4
```

giac [B] time = 0.15, size = 612, normalized size = 5.23

$$\frac{1}{6}\left(4\,b^2c^3\log\left(-\frac{cx+1}{cx-1}-1\right)-4\,b^2c^3\log\left(-\frac{cx+1}{cx-1}\right)+\frac{3\left(\frac{(cx+1)^3b^2c^3}{(cx-1)^3}+\frac{(cx+1)b^2c^3}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)^2}{\frac{(cx+1)^4}{(cx-1)^4}+\frac{4\,(cx+1)^3}{(cx-1)^3}+\frac{6\,(cx+1)^2}{(cx-1)^2}+\frac{4\,(cx+1)}{cx-1}+1}+\frac{2\left(\frac{6\,(cx+1)^3abc^3}{(cx-1)^3}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] 1/6*(4*b^2*c^3*log(-(c*x + 1)/(c*x - 1) - 1) - 4*b^2*c^3*log(-(c*x + 1)/(c*
x - 1)) + 3*((c*x + 1)^3*b^2*c^3/(c*x - 1)^3 + (c*x + 1)*b^2*c^3/(c*x - 1))
*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x
- 1)^3 + 6*(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + 2*(6*(c*x
 + 1)^3*a*b*c^3/(c*x - 1)^3 + 6*(c*x + 1)*a*b*c^3/(c*x - 1) + 3*(c*x + 1)^3
*b^2*c^3/(c*x - 1)^3 + 6*(c*x + 1)^2*b^2*c^3/(c*x - 1)^2 + 5*(c*x + 1)*b^2*
c^3/(c*x - 1) + 2*b^2*c^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^4/(c*x - 1)
^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c
*x - 1) + 1) + 2*(6*(c*x + 1)^3*a^2*c^3/(c*x - 1)^3 + 6*(c*x + 1)*a^2*c^3/(
c*x - 1) + 6*(c*x + 1)^3*a*b*c^3/(c*x - 1)^3 + 12*(c*x + 1)^2*a*b*c^3/(c*x
- 1)^2 + 10*(c*x + 1)*a*b*c^3/(c*x - 1) + 4*a*b*c^3 + (c*x + 1)^3*b^2*c^3/(
c*x - 1)^3 + 2*(c*x + 1)^2*b^2*c^3/(c*x - 1)^2 + (c*x + 1)*b^2*c^3/(c*x - 1
))/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c*
x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1))*c
```

maple [B] time = 0.03, size = 290, normalized size = 2.48

$$-\frac{a^2}{4x^4}-\frac{b^2\operatorname{arctanh}(cx)^2}{4x^4}-\frac{c\,b^2\operatorname{arctanh}(cx)}{6x^3}-\frac{c^3b^2\operatorname{arctanh}(cx)}{2x}-\frac{c^4b^2\operatorname{arctanh}(cx)\ln(cx-1)}{4}+\frac{c^4b^2\operatorname{arctanh}(cx)\ln}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^5,x)
```

```
[Out] -1/4*a^2/x^4-1/4*b^2/x^4*arctanh(c*x)^2-1/6*c*b^2*arctanh(c*x)/x^3-1/2*c^3*
b^2*arctanh(c*x)/x-1/4*c^4*b^2*arctanh(c*x)*ln(c*x-1)+1/4*c^4*b^2*arctanh(c
*x)*ln(c*x+1)-1/16*c^4*b^2*ln(c*x-1)^2+1/8*c^4*b^2*ln(c*x-1)*ln(1/2+1/2*c*x
)-1/16*c^4*b^2*ln(c*x+1)^2-1/8*c^4*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/8
*c^4*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/12*b^2*c^2/x^2+2/3*c^4*b^2*ln(c*x)-1/
3*c^4*b^2*ln(c*x-1)-1/3*c^4*b^2*ln(c*x+1)-1/2*a*b/x^4*arctanh(c*x)-1/6*a*b*
c/x^3-1/2*c^3*a*b/x-1/4*c^4*a*b*ln(c*x-1)+1/4*c^4*a*b*ln(c*x+1)
```

maxima [B] time = 0.34, size = 224, normalized size = 1.91

$$\frac{1}{12}\left(\left(3\,c^3\log(cx+1)-3\,c^3\log(cx-1)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\operatorname{artanh}(cx)}{x^4}\right)ab+\frac{1}{48}\left(\left(32\,c^2\log(x)-\frac{3\,c^2x^2\log(c}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
 6*arctanh(c*x)/x^4)*a*b + 1/48*((32*c^2*log(x) - (3*c^2*x^2*log(c*x + 1)^2
 + 3*c^2*x^2*log(c*x - 1)^2 + 16*c^2*x^2*log(c*x - 1) - 2*(3*c^2*x^2*log(c*
x - 1) - 8*c^2*x^2)*log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*log(c*x + 1) - 3*
c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*b^2 - 1/4*b^2*arc
tanh(c*x)^2/x^4 - 1/4*a^2/x^4
```

mupad [B] time = 1.90, size = 303, normalized size = 2.59

$$\frac{b^2\,c^4\ln\left(c\,x+1\right)^2}{16}-\frac{a^2}{4\,x^4}+\frac{b^2\,c^4\ln\left(1-c\,x\right)^2}{16}-\frac{b^2\ln\left(c\,x+1\right)^2}{16\,x^4}-\frac{b^2\ln\left(1-c\,x\right)^2}{16\,x^4}-\frac{b^2\,c^2}{12\,x^2}+\frac{2\,b^2\,c^4\,\ln(x)}{3}-\frac{b^2\,c^4\,\ln\left(c\,x\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2/x^5,x)
```

```
[Out] (b^2*c^4*log(c*x + 1)^2)/16 - a^2/(4*x^4) + (b^2*c^4*log(1 - c*x)^2)/16 - (
b^2*log(c*x + 1)^2)/(16*x^4) - (b^2*log(1 - c*x)^2)/(16*x^4) - (b^2*c^2)/(1
2*x^2) + (2*b^2*c^4*log(x))/3 - (b^2*c^4*log(c*x - 1))/3 - (b^2*c^4*log(c*x
 + 1))/3 - (a*b*log(c*x + 1))/(4*x^4) + (a*b*log(1 - c*x))/(4*x^4) + (b^2*l
og(c*x + 1)*log(1 - c*x))/(8*x^4) - (a*b*c)/(6*x^3) - (b^2*c*log(c*x + 1))/
(12*x^3) + (b^2*c*log(1 - c*x))/(12*x^3) - (a*b*c^3)/(2*x) - (b^2*c^3*log(c
*x + 1))/(4*x) + (b^2*c^3*log(1 - c*x))/(4*x) - (a*b*c^4*log(c*x - 1))/4 +
(a*b*c^4*log(c*x + 1))/4 - (b^2*c^4*log(c*x + 1)*log(1 - c*x))/8
```

sympy [A] time = 2.00, size = 184, normalized size = 1.57

$$\begin{cases} -\frac{a^2}{4x^4}+\frac{abc^4\operatorname{atanh}(cx)}{2}-\frac{abc^3}{2x}-\frac{abc}{6x^3}-\frac{ab\operatorname{atanh}(cx)}{2x^4}+\frac{2b^2c^4\log(x)}{3}-\frac{2b^2c^4\log\left(x-\frac{1}{c}\right)}{3}+\frac{b^2c^4\operatorname{atanh}^2(cx)}{4}-\frac{2b^2c^4\operatorname{atanh}(cx)}{3}-\frac{b^2c^3}{} \\ -\frac{a^2}{4x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) + a*b*c**4*atanh(c*x)/2 - a*b*c**3/(2*x) - a*b*c/
(6*x**3) - a*b*atanh(c*x)/(2*x**4) + 2*b**2*c**4*log(x)/3 - 2*b**2*c**4*log
(x - 1/c)/3 + b**2*c**4*atanh(c*x)**2/4 - 2*b**2*c**4*atanh(c*x)/3 - b**2*c
**3*atanh(c*x)/(2*x) - b**2*c**2/(12*x**2) - b**2*c*atanh(c*x)/(6*x**3) - b
**2*atanh(c*x)**2/(4*x**4), Ne(c, 0)), (-a**2/(4*x**4), True))
```

3.24 $\int x^5 \left(a+b\tanh^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=247

$$-\frac{23b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{15c^6}+\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{6c^6}+\cdots$$

```
[Out] 19/60*b^3*x/c^5+1/60*b^3*x^3/c^3-19/60*b^3*arctanh(c*x)/c^6+4/15*b^2*x^2*(a
+b*arctanh(c*x))/c^4+1/20*b^2*x^4*(a+b*arctanh(c*x))/c^2+23/30*b*(a+b*arcta
nh(c*x))^2/c^6+1/2*b*x*(a+b*arctanh(c*x))^2/c^5+1/6*b*x^3*(a+b*arctanh(c*x)
)^2/c^3+1/10*b*x^5*(a+b*arctanh(c*x))^2/c-1/6*(a+b*arctanh(c*x))^3/c^6+1/6*
x^6*(a+b*arctanh(c*x))^3-23/15*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^6-23
/30*b^3*polylog(2,1-2/(-c*x+1))/c^6
```

Rubi [A] time = 0.96, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5916, 5980, 302, 206, 321, 5984, 5918, 2402, 2315, 5910, 5948}

$$-\frac{23b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{30c^6}+\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}+\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}-\frac{23b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{15c^6}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (19*b^3*x)/(60*c^5) + (b^3*x^3)/(60*c^3) - (19*b^3*ArcTanh[c*x])/(60*c^6) +
 (4*b^2*x^2*(a + b*ArcTanh[c*x]))/(15*c^4) + (b^2*x^4*(a + b*ArcTanh[c*x]))
/(20*c^2) + (23*b*(a + b*ArcTanh[c*x])^2)/(30*c^6) + (b*x*(a + b*ArcTanh[c*
x])^2)/(2*c^5) + (b*x^3*(a + b*ArcTanh[c*x])^2)/(6*c^3) + (b*x^5*(a + b*Arc
Tanh[c*x])^2)/(10*c) - (a + b*ArcTanh[c*x])^3/(6*c^6) + (x^6*(a + b*ArcTanh
[c*x])^3)/6 - (23*b^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(15*c^6) - (23
*b^3*PolyLog[2, 1 - 2/(1 - c*x)])/(30*c^6)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^5 \left(a+b\tanh^{-1}(cx)\right)^3 dx = \frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{1}{2}(bc)\int \frac{x^6\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx$$
$$= \frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3 + \frac{b\int x^4\left(a+b\tanh^{-1}(cx)\right)^2 dx}{2c} - \frac{b\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx}{2c}$$
$$= \frac{bx^5\left(a+b\tanh^{-1}(cx)\right)^2}{10c} + \frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{1}{5}b^2\int \frac{x^5\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}$$
$$= \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{6c^3} + \frac{bx^5\left(a+b\tanh^{-1}(cx)\right)^2}{10c} + \frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3 + \frac{b}{}$$
$$= \frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2} + \frac{bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c^5} + \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{6c^3} + \frac{bx}{}$$
$$= \frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4} + \frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2} + \frac{23b\left(a+b\tanh^{-1}(cx)\right)^2}{30c^6} +$$
$$= \frac{19b^3x}{60c^5} + \frac{b^3x^3}{60c^3} + \frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4} + \frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2} + \frac{23b\left(a+b\right.}{}$$
$$= \frac{19b^3x}{60c^5} + \frac{b^3x^3}{60c^3} - \frac{19b^3\tanh^{-1}(cx)}{60c^6} + \frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4} + \frac{b^2x^4\left(a+b\tanh^{-1}\right.}{20c^2}$$
$$= \frac{19b^3x}{60c^5} + \frac{b^3x^3}{60c^3} - \frac{19b^3\tanh^{-1}(cx)}{60c^6} + \frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4} + \frac{b^2x^4\left(a+b\tanh^{-1}\right.}{20c^2}$$

Mathematica [A] time = 0.78, size = 305, normalized size = 1.23

$$\frac{10a^3c^6x^6 + b\tanh^{-1}(cx)\left(30a^2c^6x^6 + 4abcx\left(3c^4x^4+5c^2x^2+15\right) + b^2\left(3c^4x^4+16c^2x^2-19\right) - 92b^2\log\left(e^{-2\tanh^{-1}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (-19*a*b^2 + 30*a^2*b*c*x + 19*b^3*c*x + 16*a*b^2*c^2*x^2 + 10*a^2*b*c^3*x^
3 + b^3*c^3*x^3 + 3*a*b^2*c^4*x^4 + 6*a^2*b*c^5*x^5 + 10*a^3*c^6*x^6 + 2*b^
2*(b*(-23 + 15*c*x + 5*c^3*x^3 + 3*c^5*x^5) + 15*a*(-1 + c^6*x^6))*ArcTanh[
c*x]^2 + 10*b^3*(-1 + c^6*x^6)*ArcTanh[c*x]^3 + b*ArcTanh[c*x]*(30*a^2*c^6*
x^6 + 4*a*b*c*x*(15 + 5*c^2*x^2 + 3*c^4*x^4) + b^2*(-19 + 16*c^2*x^2 + 3*c^
4*x^4) - 92*b^2*Log[1 + E^(-2*ArcTanh[c*x])]) + 15*a^2*b*Log[1 - c*x] - 15*
a^2*b*Log[1 + c*x] + 46*a*b^2*Log[1 - c^2*x^2] + 46*b^3*PolyLog[2, -E^(-2*A
rcTanh[c*x])])/(60*c^6)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^5\,\text{artanh}(cx)^3 + 3\,ab^2x^5\,\text{artanh}(cx)^2 + 3\,a^2bx^5\,\text{artanh}(cx) + a^3x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^5*arctanh(c*x)^3 + 3*a*b^2*x^5*arctanh(c*x)^2 + 3*a^2*b*x^5*
arctanh(c*x) + a^3*x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{artanh}(cx) + a)^3 x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^5, x)
```

maple [C] time = 2.33, size = 1330, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x))^3,x)
```

```
[Out] -23/15/c^6*b^3*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-23/15/c^6*b^
3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2/c^6*b^3*arctanh(c*x)^
2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*a^2*b*x^6*arctanh(c*x)+1/2*a*b^2*x^6*a
rctanh(c*x)^2-1/8*I/c^6*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c
*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)
))*arctanh(c*x)^2+19/60*b^3*x/c^5+1/60*b^3*x^3/c^3-19/60*b^3*arctanh(c*x)/c
^6+1/8*I/c^6*b^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(
c^2*x^2-1))*arctanh(c*x)^2+1/4*I/c^6*b^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/
2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2+1/8*I/c^6*b^3*Pi*csgn(I/
(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2
*x^2+1)))^2*arctanh(c*x)^2-1/8*I/c^6*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*c
sgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/
4/c^6*b^3*arctanh(c*x)^2*ln(c*x+1)+1/6*b^3*x^6*arctanh(c*x)^3+23/30/c^6*b^3
*arctanh(c*x)^2-1/6/c^6*b^3*arctanh(c*x)^3-23/15/c^6*b^3*dilog(1+I*(c*x+1)/
(-c^2*x^2+1)^(1/2))-23/15/c^6*b^3*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/3
/c^6*b^3+1/6*x^6*a^3-1/4*I/c^6*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*
arctanh(c*x)^2+1/4*I/c^6*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctan
h(c*x)^2+1/8*I/c^6*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2+1/
8*I/c^6*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*a
rctanh(c*x)^2+1/2/c^6*a*b^2*arctanh(c*x)*ln(c*x-1)-1/2/c^6*a*b^2*arctanh(c*
x)*ln(c*x+1)-1/4/c^6*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/4/c^6*a*b^2*ln(-1/2*
c*x+1/2)*ln(c*x+1)+1/4/c^6*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/3/c^3*a
*b^2*arctanh(c*x)*x^3+1/c^5*a*b^2*arctanh(c*x)*x+1/5/c*a*b^2*x^5*arctanh(c*
x)-1/4*I/c^6*b^3*Pi*arctanh(c*x)^2+4/15/c^4*x^2*a*b^2+1/2/c^5*x*a^2*b+1/10/
c*x^5*a^2*b+1/20/c^2*a*b^2*x^4+1/6/c^3*a^2*b*x^3+1/10/c*b^3*arctanh(c*x)^2*
x^5+1/6/c^3*b^3*arctanh(c*x)^2*x^3+1/2/c^5*b^3*arctanh(c*x)^2*x+1/20/c^2*b^
3*arctanh(c*x)*x^4+4/15/c^4*b^3*arctanh(c*x)*x^2+1/8/c^6*a*b^2*ln(c*x-1)^2+
1/8/c^6*a*b^2*ln(c*x+1)^2+23/30/c^6*a*b^2*ln(c*x-1)+23/30/c^6*a*b^2*ln(c*x+
1)+1/4/c^6*a^2*b*ln(c*x-1)-1/4/c^6*a^2*b*ln(c*x+1)+1/4/c^6*b^3*arctanh(c*x)
^2*ln(c*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b^2*x^6*arctanh(c*x)^2 + 1/6*a^3*x^6 + 1/60*(30*x^6*arctanh(c*x) + c*
(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x -
1)/c^7))*a^2*b + 1/120*(4*c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(
c*x + 1)/c^7 + 15*log(c*x - 1)/c^7)*arctanh(c*x) + (6*c^4*x^4 + 32*c^2*x^2
```

```
- 2*(15*log(c*x - 1) - 46)*log(c*x + 1) + 15*log(c*x + 1)^2 + 15*log(c*x -
1)^2 + 92*log(c*x - 1))/c^6)*a*b^2 - 1/1728000*(500*c^7*((2*c^4*x^6 + 3*c^2
*x^4 + 6*x^2)/c^11 + 6*log(c^2*x^2 - 1)/c^13) + 728*c^6*(2*(3*c^4*x^5 + 5*c
^2*x^3 + 15*x)/c^11 - 15*log(c*x + 1)/c^12 + 15*log(c*x - 1)/c^12) + 1485*c
^5*((c^2*x^4 + 2*x^2)/c^9 + 2*log(c^2*x^2 - 1)/c^11) - 622080000*c^5*integr
ate(1/3600*x^5*log(c*x + 1)/(c^7*x^2 - c^5), x) + 9750*c^4*(2*(c^2*x^3 + 3*
x)/c^9 - 3*log(c*x + 1)/c^10 + 3*log(c*x - 1)/c^10) - 2700*c^3*(x^2/c^7 + l
og(c^2*x^2 - 1)/c^9) - 1036800000*c^3*integrate(1/3600*x^3*log(c*x + 1)/(c^
7*x^2 - c^5), x) + 227700*c^2*(2*x/c^7 - log(c*x + 1)/c^8 + log(c*x - 1)/c^
8) - 5495040000*c*integrate(1/3600*x*log(c*x + 1)/(c^7*x^2 - c^5), x) + (10
00*(36*log(-c*x + 1)^3 - 18*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 1)*(c*x - 1
)^6 + 1728*(125*log(-c*x + 1)^3 - 75*log(-c*x + 1)^2 + 30*log(-c*x + 1) - 6
)*(c*x - 1)^5 + 16875*(32*log(-c*x + 1)^3 - 24*log(-c*x + 1)^2 + 12*log(-c*
x + 1) - 3)*(c*x - 1)^4 + 80000*(9*log(-c*x + 1)^3 - 9*log(-c*x + 1)^2 + 6*
log(-c*x + 1) - 2)*(c*x - 1)^3 + 135000*(4*log(-c*x + 1)^3 - 6*log(-c*x + 1
)^2 + 6*log(-c*x + 1) - 3)*(c*x - 1)^2 + 216000*(log(-c*x + 1)^3 - 3*log(-c
*x + 1)^2 + 6*log(-c*x + 1) - 6)*(c*x - 1))/c^6 - 60*(600*(c^6*x^6 - 1)*log
(c*x + 1)^3 + 240*(3*c^5*x^5 + 5*c^3*x^3 + 15*c*x)*log(c*x + 1)^2 - 30*(10*
c^6*x^6 - 12*c^5*x^5 + 15*c^4*x^4 - 20*c^3*x^3 + 30*c^2*x^2 - 60*c*x - 60*(
c^6*x^6 - 1)*log(c*x + 1) + 37)*log(-c*x + 1)^2 + (100*c^6*x^6 + 264*c^5*x^
5 - 165*c^4*x^4 + 1140*c^3*x^3 - 1230*c^2*x^2 - 1800*(c^6*x^6 - 1)*log(c*x
+ 1)^2 + 8820*c*x - 480*(3*c^5*x^5 + 5*c^3*x^3 + 15*c*x + 23)*log(c*x + 1))
*log(-c*x + 1))/c^6 + 264600*log(3600*c^7*x^2 - 3600*c^5)/c^6 - 2384640000*
integrate(1/3600*log(c*x + 1)/(c^7*x^2 - c^5), x))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \left(a + b \operatorname{atanh}(c\,x)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x))^3,x)
[Out] int(x^5*(a + b*atanh(c*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a + b \operatorname{atanh}(cx)\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x))**3,x)
[Out] Integral(x**5*(a + b*atanh(c*x))**3, x)
```

3.25 $\int x^4 \left(a + b\tanh^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=262

$$-\frac{3b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^5}+\frac{9ab^2x}{10c^4}+\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{5c^5}-\frac{9b\left(a+b\tanh\right.}{20c^5}$$

```
[Out] 9/10*a*b^2*x/c^4+1/20*b^3*x^2/c^3+9/10*b^3*x*arctanh(c*x)/c^4+1/10*b^2*x^3*
(a+b*arctanh(c*x))/c^2-9/20*b*(a+b*arctanh(c*x))^2/c^5+3/10*b*x^2*(a+b*arct
anh(c*x))^2/c^3+3/20*b*x^4*(a+b*arctanh(c*x))^2/c+1/5*(a+b*arctanh(c*x))^3/
c^5+1/5*x^5*(a+b*arctanh(c*x))^3-3/5*b*(a+b*arctanh(c*x))^2*ln(2/(-c*x+1))/
c^5+1/2*b^3*ln(-c^2*x^2+1)/c^5-3/5*b^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c
*x+1))/c^5+3/10*b^3*polylog(3,1-2/(-c*x+1))/c^5
```

Rubi [A] time = 0.77, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5916, 5980, 266, 43, 5910, 260, 5948, 5984, 5918, 6058, 6610}

$$-\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^5}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{10c^5}+\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}+\frac{9ab^2x}{10c^4}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (9*a*b^2*x)/(10*c^4) + (b^3*x^2)/(20*c^3) + (9*b^3*x*ArcTanh[c*x])/(10*c^4)
 + (b^2*x^3*(a + b*ArcTanh[c*x]))/(10*c^2) - (9*b*(a + b*ArcTanh[c*x])^2)/(
20*c^5) + (3*b*x^2*(a + b*ArcTanh[c*x])^2)/(10*c^3) + (3*b*x^4*(a + b*ArcTa
nh[c*x])^2)/(20*c) + (a + b*ArcTanh[c*x])^3/(5*c^5) + (x^5*(a + b*ArcTanh[c
*x])^3)/5 - (3*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/(5*c^5) + (b^3*Lo
g[1 - c^2*x^2])/(2*c^5) - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 -
 c*x)])/(5*c^5) + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(10*c^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^4 \left(a+b\tanh^{-1}(cx)\right)^3 dx = \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{1}{5}(3bc)\int \frac{x^5\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx$$

$$= \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^3 + \frac{(3b)\int x^3\left(a+b\tanh^{-1}(cx)\right)^2 dx}{5c} - \frac{(3b)\int \frac{x^3(a+b\tanh}{1-c^2}}{5c}$$

$$= \frac{3bx^4\left(a+b\tanh^{-1}(cx)\right)^2}{20c} + \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{1}{10}\left(3b^2\right)\int \frac{x^4\left(a+b\tan\right.}{1-c}$$

$$= \frac{3bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{10c^3} + \frac{3bx^4\left(a+b\tanh^{-1}(cx)\right)^2}{20c} + \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{5c^5} +$$

$$= \frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2} + \frac{3bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{10c^3} + \frac{3bx^4\left(a+b\tanh^{-1}(cx)\right)}{20c}$$

$$= \frac{9ab^2x}{10c^4} + \frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2} - \frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{20c^5} + \frac{3bx^2\left(a+b\tanh\right.}{10c^3}$$

$$= \frac{9ab^2x}{10c^4} + \frac{9b^3x\tanh^{-1}(cx)}{10c^4} + \frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2} - \frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{20c^5}$$

$$= \frac{9ab^2x}{10c^4} + \frac{b^3x^2}{20c^3} + \frac{9b^3x\tanh^{-1}(cx)}{10c^4} + \frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2} - \frac{9b\left(a+b\tanh\right.}{20c^5}$$

Mathematica [A] time = 0.81, size = 383, normalized size = 1.46

$$4a^3c^5x^5 + 12a^2bc^5x^5\tanh^{-1}(cx) + 3a^2bc^4x^4 + 6a^2bc^2x^2 + 6a^2b\log\left(1-c^2x^2\right) + 12ab^2c^5x^5\tanh^{-1}(cx)^2 + 6ab^2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (-b^3 + 18*a*b^2*c*x + 6*a^2*b*c^2*x^2 + b^3*c^2*x^2 + 2*a*b^2*c^3*x^3 + 3*
a^2*b*c^4*x^4 + 4*a^3*c^5*x^5 - 18*a*b^2*ArcTanh[c*x] + 18*b^3*c*x*ArcTanh[
c*x] + 12*a*b^2*c^2*x^2*ArcTanh[c*x] + 2*b^3*c^3*x^3*ArcTanh[c*x] + 6*a*b^2
*c^4*x^4*ArcTanh[c*x] + 12*a^2*b*c^5*x^5*ArcTanh[c*x] - 12*a*b^2*ArcTanh[c*
x]^2 - 9*b^3*ArcTanh[c*x]^2 + 6*b^3*c^2*x^2*ArcTanh[c*x]^2 + 3*b^3*c^4*x^4*
ArcTanh[c*x]^2 + 12*a*b^2*c^5*x^5*ArcTanh[c*x]^2 - 4*b^3*ArcTanh[c*x]^3 + 4
*b^3*c^5*x^5*ArcTanh[c*x]^3 - 24*a*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c
*x])] - 12*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 6*a^2*b*Log[1
- c^2*x^2] + 10*b^3*Log[1 - c^2*x^2] + 12*b^2*(a + b*ArcTanh[c*x])*PolyLog[
2, -E^(-2*ArcTanh[c*x])] + 6*b^3*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(20*c^5)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^4\,\text{artanh}(cx)^3 + 3\,ab^2x^4\,\text{artanh}(cx)^2 + 3\,a^2bx^4\,\text{artanh}(cx) + a^3x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^4*arctanh(c*x)^3 + 3*a*b^2*x^4*arctanh(c*x)^2 + 3*a^2*b*x^4*
arctanh(c*x) + a^3*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{artanh}\,(cx) + a)^3 x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^4, x)
```

maple [C] time = 2.22, size = 1275, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))^3,x)
```

```
[Out] 9/10*a*b^2*x/c^4+9/10*b^3*x*arctanh(c*x)/c^4+3/20/c*x^4*a^2*b+3/10/c^3*a^2*
b*x^2+1/10/c^2*b^3*arctanh(c*x)*x^3+3/10/c^3*b^3*arctanh(c*x)^2*x^2+3/20/c*
b^3*arctanh(c*x)^2*x^4+3/10/c^5*b^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/c^
5*b^3*ln(1+(c*x+1)^2/(-c^2*x^2+1))+3/5/c^3*a*b^2*arctanh(c*x)*x^2+3/5/c^5*a
*b^2*arctanh(c*x)*ln(c*x-1)+3/5/c^5*a*b^2*arctanh(c*x)*ln(c*x+1)-3/10/c^5*a
*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+3/10/c^5*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/
10/c^5*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-3/10*I/c^5*b^3*arctanh(c*x)^2
*Pi+3/20/c^5*a*b^2*ln(c*x-1)^2-3/20/c^5*a*b^2*ln(c*x+1)^2+9/20/c^5*a*b^2*ln
(c*x-1)-9/20/c^5*a*b^2*ln(c*x+1)+3/10/c^5*a^2*b*ln(c*x-1)+3/10/c^5*a^2*b*ln
(c*x+1)-3/5/c^5*a*b^2*dilog(1/2+1/2*c*x)-3/5/c^5*b^3*arctanh(c*x)^2*ln(2)+3
/10/c^5*b^3*arctanh(c*x)^2*ln(c*x-1)+3/10/c^5*b^3*arctanh(c*x)^2*ln(c*x+1)-
3/5/c^5*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-3/5/c^5*b^3*arcta
nh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+3/5*x^5*a^2*b*arctanh(c*x)+3/5*x
^5*a*b^2*arctanh(c*x)^2-3/10*I/c^5*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2
*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*Pi-3/20*I/c^5*b^3*arctanh(c*x
)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi+3
/20*I/c^5*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2
/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi-3/20*I/c^5*b^3*arctanh(c*x)^2
*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)
^2/(-c^2*x^2+1)))^2*Pi+1/10*a*b^2*x^3/c^2+1/20*b^3*x^2/c^3-3/20*I/c^5*b^3*a
rctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi-3/10*I/c^5*b^3*arctanh(c*x
)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi-3/20*I/c^5*b^3*arctanh(c*x)^2*c
sgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi+3/10*I/c^5*b^3
*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi+1/5*x^5*a^3+3/20*I/
c^5*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-
c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi-1/
20/c^5*b^3+3/10/c*a*b^2*x^4*arctanh(c*x)+1/5*x^5*b^3*arctanh(c*x)^3-9/20/c^
5*b^3*arctanh(c*x)^2+1/5/c^5*b^3*arctanh(c*x)^3+1/c^5*b^3*arctanh(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{5}a^3x^5+\frac{3}{20}\left(4\,x^5\,\text{artanh}\,(cx)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)a^2b-\frac{2\left(b^3c^5x^5-b^3\right)\log\left(-cx+1\right)^3-3\left(4\,ab^2c^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/5*a^3*x^5 + 3/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c
^2*x^2 - 1)/c^6))*a^2*b - 1/80*(2*(b^3*c^5*x^5 - b^3)*log(-c*x + 1)^3 - 3*(
4*a*b^2*c^5*x^5 + b^3*c^4*x^4 + 2*b^3*c^2*x^2 + 2*(b^3*c^5*x^5 + b^3)*log(c
*x + 1))*log(-c*x + 1)^2)/c^5 - integrate(-1/40*(5*(b^3*c^5*x^5 - b^3*c^4*x
```

```
^4)*log(c*x + 1)^3 + 30*(a*b^2*c^5*x^5 - a*b^2*c^4*x^4)*log(c*x + 1)^2 - 3*
(4*a*b^2*c^5*x^5 + b^3*c^4*x^4 + 2*b^3*c^2*x^2 + 5*(b^3*c^5*x^5 - b^3*c^4*x
^4)*log(c*x + 1)^2 - 2*(10*a*b^2*c^4*x^4 - (10*a*b^2*c^5 + b^3*c^5)*x^5 - b
^3)*log(c*x + 1))*log(-c*x + 1))/(c^5*x - c^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,(a+b\,\mathrm{atanh}\,(c\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x))^3,x)
```

```
[Out] int(x^4*(a + b*atanh(c*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\,(a+b\,\mathrm{atanh}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c*x))**3, x)
```

3.26 $\int x^3 \left(a + b \tanh^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=185

$$-\frac{2b^2 \log\left(\frac{2}{1-cx}\right)\left(a + b\tanh^{-1}(cx)\right)}{c^4} + \frac{b^2x^2\left(a + b\tanh^{-1}(cx)\right)}{4c^2} - \frac{\left(a + b\tanh^{-1}(cx)\right)^3}{4c^4} + \frac{b\left(a + b\tanh^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a\right.}{}$$

```
[Out] 1/4*b^3*x/c^3-1/4*b^3*arctanh(c*x)/c^4+1/4*b^2*x^2*(a+b*arctanh(c*x))/c^2+b
*(a+b*arctanh(c*x))^2/c^4+3/4*b*x*(a+b*arctanh(c*x))^2/c^3+1/4*b*x^3*(a+b*a
rctanh(c*x))^2/c-1/4*(a+b*arctanh(c*x))^3/c^4+1/4*x^4*(a+b*arctanh(c*x))^3-
2*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4-b^3*polylog(2,1-2/(-c*x+1))/c^4
```

Rubi [A] time = 0.57, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 5910, 5948}

$$-\frac{b^3\text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{c^4} + \frac{b^2x^2\left(a + b\tanh^{-1}(cx)\right)}{4c^2} - \frac{2b^2 \log\left(\frac{2}{1-cx}\right)\left(a + b\tanh^{-1}(cx)\right)}{c^4} + \frac{3bx\left(a + b\tanh^{-1}(cx)\right)^2}{4c^3} -$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (b^3*x)/(4*c^3) - (b^3*ArcTanh[c*x])/(4*c^4) + (b^2*x^2*(a + b*ArcTanh[c*x]
))/(4*c^2) + (b*(a + b*ArcTanh[c*x])^2)/c^4 + (3*b*x*(a + b*ArcTanh[c*x])^2
)/(4*c^3) + (b*x^3*(a + b*ArcTanh[c*x])^2)/(4*c) - (a + b*ArcTanh[c*x])^3/(
4*c^4) + (x^4*(a + b*ArcTanh[c*x])^3)/4 - (2*b^2*(a + b*ArcTanh[c*x])*Log[2
/(1 - c*x)])/c^4 - (b^3*PolyLog[2, 1 - 2/(1 - c*x)])/c^4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
```

```
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}(cx)\right)^3 dx = \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{1}{4}(3bc)\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx$$

$$= \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^3 + \frac{(3b)\int x^2\left(a+b\tanh^{-1}(cx)\right)^2 dx}{4c} - \frac{(3b)\int \frac{x^2(a+b\tanh^{-1}(c}{1-c^2x^2}}{4c}$$

$$= \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{4c} + \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{1}{2}b^2\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}$$

$$= \frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{4c^3} + \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4c^4} + \frac{1}{4}x^4\left(\right.$$

$$= \frac{b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{4c^3} + \frac{bx^3}{}$$

$$= \frac{b^3x}{4c^3} + \frac{b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{4c^3}$$

$$= \frac{b^3x}{4c^3} - \frac{b^3\tanh^{-1}(cx)}{4c^4} + \frac{b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a\right.}{}$$

$$= \frac{b^3x}{4c^3} - \frac{b^3\tanh^{-1}(cx)}{4c^4} + \frac{b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a\right.}{}$$

Mathematica [A] time = 0.60, size = 245, normalized size = 1.32

$$\frac{2a^3c^4x^4 + 2b\tanh^{-1}(cx)\left(3a^2c^4x^4 + 2abcx\left(c^2x^2+3\right) + b^2\left(c^2x^2-1\right) - 8b^2\log\left(e^{-2\tanh^{-1}(cx)}+1\right)\right) + 2a^2bc^3x^3 + 6}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (-2*a*b^2 + 6*a^2*b*c*x + 2*b^3*c*x + 2*a*b^2*c^2*x^2 + 2*a^2*b*c^3*x^3 + 2
*a^3*c^4*x^4 + 2*b^2*(b*(-4 + 3*c*x + c^3*x^3) + 3*a*(-1 + c^4*x^4))*ArcTan
h[c*x]^2 + 2*b^3*(-1 + c^4*x^4)*ArcTanh[c*x]^3 + 2*b*ArcTanh[c*x]*(3*a^2*c^
4*x^4 + b^2*(-1 + c^2*x^2) + 2*a*b*c*x*(3 + c^2*x^2) - 8*b^2*Log[1 + E^(-2*
ArcTanh[c*x])]) + 3*a^2*b*Log[1 - c*x] - 3*a^2*b*Log[1 + c*x] + 8*a*b^2*Log
[1 - c^2*x^2] + 8*b^3*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(8*c^4)
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^3\,\text{artanh}(cx)^3 + 3\,ab^2x^3\,\text{artanh}(cx)^2 + 3\,a^2bx^3\,\text{artanh}(cx) + a^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctanh(c*x)^3 + 3*a*b^2*x^3*arctanh(c*x)^2 + 3*a^2*b*x^3*
arctanh(c*x) + a^3*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{artanh}(cx) + a)^3 x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^3, x)
```

maple [C] time = 1.41, size = 1245, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^3,x)
```

```
[Out] 3/16*I/c^4*b^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^
2*x^2-1))*arctanh(c*x)^2+3/16*I/c^4*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)
))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^
2-3/16*I/c^4*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+3/8*I/c^4*b^3*Pi*csgn(I*(c
*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-2/
c^4*b^3*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2/c^4*b^3*dilog(1-I*(c*x+1)/(
-c^2*x^2+1)^(1/2))+1/c^4*b^3*arctanh(c*x)^2-1/4/c^4*b^3*arctanh(c*x)^3+1/4*
x^4*b^3*arctanh(c*x)^3+3/4/c^4*a*b^2*arctanh(c*x)*ln(c*x-1)-3/4/c^4*a*b^2*a
rctanh(c*x)*ln(c*x+1)-3/8/c^4*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-3/8/c^4*a*b^2
*ln(-1/2*c*x+1/2)*ln(c*x+1)+3/8/c^4*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+
3/2/c^3*a*b^2*arctanh(c*x)*x+1/2/c*a*b^2*arctanh(c*x)*x^3-3/8*I/c^4*b^3*Pi*
arctanh(c*x)^2+1/4*b^3*x/c^3-1/4*b^3*arctanh(c*x)/c^4-3/16*I/c^4*b^3*Pi*csg
n(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1
)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2-1/4/c^4*b^3+1/4*
x^4*a^3+1/4/c^2*x^2*a*b^2+3/4/c^3*x*a^2*b+1/4/c*a^2*b*x^3+3/16/c^4*a*b^2*ln
(c*x-1)^2+3/16/c^4*a*b^2*ln(c*x+1)^2+1/c^4*a*b^2*ln(c*x-1)+1/c^4*a*b^2*ln(c
*x+1)+3/8/c^4*a^2*b*ln(c*x-1)-3/8/c^4*a^2*b*ln(c*x+1)+3/8/c^4*b^3*arctanh(c
*x)^2*ln(c*x-1)-3/8/c^4*b^3*arctanh(c*x)^2*ln(c*x+1)-2/c^4*b^3*arctanh(c*x)
*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2/c^4*b^3*arctanh(c*x)*ln(1-I*(c*x+1)/(
-c^2*x^2+1)^(1/2))+3/4/c^4*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2)
)+1/4/c*b^3*arctanh(c*x)^2*x^3+3/4/c^3*b^3*arctanh(c*x)^2*x+1/4/c^2*b^3*arc
tanh(c*x)*x^2+3/4*x^4*a^2*b*arctanh(c*x)+3/4*a*b^2*x^4*arctanh(c*x)^2+3/8*I
/c^4*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-3/8*I/c^4*b
^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2+3/16*I/c^4*b^3*Pi
*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2+3/16*I/c^4*b^3*Pi*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctanh(c*x)^2 + 1/4*a^3*x^4 + 1/8*(6*x^4*arctanh(c*x) + c*(2
*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a^2*b + 1/
16*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*a
rctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(c*x
+ 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*a*b^2 - 1/9216*(27*c^5*((
c^2*x^4 + 2*x^2)/c^7 + 2*log(c^2*x^2 - 1)/c^9) + 74*c^4*(2*(c^2*x^3 + 3*x)/
c^7 - 3*log(c*x + 1)/c^8 + 3*log(c*x - 1)/c^8) + 60*c^3*(x^2/c^5 + log(c^2*
x^2 - 1)/c^7) - 221184*c^3*integrate(1/96*x^3*log(c*x + 1)/(c^5*x^2 - c^3),
 x) + 1692*c^2*(2*x/c^5 - log(c*x + 1)/c^6 + log(c*x - 1)/c^6) - 1105920*c*
integrate(1/96*x*log(c*x + 1)/(c^5*x^2 - c^3), x) + (9*(32*log(-c*x + 1)^3
- 24*log(-c*x + 1)^2 + 12*log(-c*x + 1) - 3)*(c*x - 1)^4 + 128*(9*log(-c*x
+ 1)^3 - 9*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 2)*(c*x - 1)^3 + 432*(4*log(
-c*x + 1)^3 - 6*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 3)*(c*x - 1)^2 + 1152*(
log(-c*x + 1)^3 - 3*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 6)*(c*x - 1))/c^4 -
```

```
 12*(24*(c^4*x^4 - 1)*log(c*x + 1)^3 + 48*(c^3*x^3 + 3*c*x)*log(c*x + 1)^2
- 6*(3*c^4*x^4 - 4*c^3*x^3 + 6*c^2*x^2 - 12*c*x - 12*(c^4*x^4 - 1)*log(c*x
+ 1) + 7)*log(-c*x + 1)^2 + (9*c^4*x^4 + 28*c^3*x^3 - 18*c^2*x^2 - 72*(c^4*
x^4 - 1)*log(c*x + 1)^2 + 300*c*x - 96*(c^3*x^3 + 3*c*x + 4)*log(c*x + 1))*
log(-c*x + 1))/c^4 + 1800*log(96*c^5*x^2 - 96*c^3)/c^4 - 442368*integrate(1
/96*log(c*x + 1)/(c^5*x^2 - c^3), x))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,(a+b\,\mathrm{atanh}\,(c\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x))^3,x)
```

```
[Out] int(x^3*(a + b*atanh(c*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,(a+b\,\mathrm{atanh}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*x))**3, x)
```

3.27 $\int x^2 \left(a+b\tanh^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=197

$$-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3}+\frac{ab^2x}{c^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c^3}-\frac{b\log\left(\frac{2}{1-cx}\right)\left(a+b\text{t}\right.}{c^3}$$

```
[Out] a*b^2*x/c^2+b^3*x*arctanh(c*x)/c^2-1/2*b*(a+b*arctanh(c*x))^2/c^3+1/2*b*x^2
*(a+b*arctanh(c*x))^2/c+1/3*(a+b*arctanh(c*x))^3/c^3+1/3*x^3*(a+b*arctanh(c
*x))^3-b*(a+b*arctanh(c*x))^2*ln(2/(-c*x+1))/c^3+1/2*b^3*ln(-c^2*x^2+1)/c^3
-b^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/c^3+1/2*b^3*polylog(3,1-2/(
-c*x+1))/c^3
```

Rubi [A] time = 0.45, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5916, 5980, 5910, 260, 5948, 5984, 5918, 6058, 6610}

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3}+\frac{b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c^3}+\frac{ab^2x}{c^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3}+\left(a+b\text{t}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (a*b^2*x)/c^2 + (b^3*x*ArcTanh[c*x])/c^2 - (b*(a + b*ArcTanh[c*x])^2)/(2*c^
3) + (b*x^2*(a + b*ArcTanh[c*x])^2)/(2*c) + (a + b*ArcTanh[c*x])^3/(3*c^3)
+ (x^3*(a + b*ArcTanh[c*x])^3)/3 - (b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x
)])/c^3 + (b^3*Log[1 - c^2*x^2])/(2*c^3) - (b^2*(a + b*ArcTanh[c*x])*PolyLo
g[2, 1 - 2/(1 - c*x)])/c^3 + (b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c^3)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^2 \left(a+b\tanh^{-1}(cx)\right)^3 dx = \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^3-(bc)\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx$$

$$=\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^3+\frac{b\int x\left(a+b\tanh^{-1}(cx)\right)^2dx}{c}-\frac{b\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx}{c}$$

$$=\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^3-b^2\int$$

$$=\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^3-\frac{b\left(a\right.}{}$$

$$=\frac{ab^2x}{c^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c^3}+$$

$$=\frac{ab^2x}{c^2}+\frac{b^3x\tanh^{-1}(cx)}{c^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{\left(a+\right.}{}$$

$$=\frac{ab^2x}{c^2}+\frac{b^3x\tanh^{-1}(cx)}{c^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{\left(a+\right.}{}$$

Mathematica [A] time = 0.56, size = 250, normalized size = 1.27

$$\frac{2a^3c^3x^3 + 6a^2bc^3x^3\tanh^{-1}(cx) + 3a^2bc^2x^2 + 3a^2b\log\left(1-c^2x^2\right) + 6ab^2\left(\left(c^3x^3-1\right)\tanh^{-1}(cx)^2 + \tanh^{-1}(cx)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 + 6*a^2*b*c^3*x^3*ArcTanh[c*x] + 3*a^2*b*L
og[1 - c^2*x^2] + 6*a*b^2*(c*x + (-1 + c^3*x^3)*ArcTanh[c*x]^2 + ArcTanh[c*
x]*(-1 + c^2*x^2 - 2*Log[1 + E^(-2*ArcTanh[c*x])]) + PolyLog[2, -E^(-2*ArcT
anh[c*x])]) + b^3*(6*c*x*ArcTanh[c*x] - 3*ArcTanh[c*x]^2 + 3*c^2*x^2*ArcTan
h[c*x]^2 - 2*ArcTanh[c*x]^3 + 2*c^3*x^3*ArcTanh[c*x]^3 - 6*ArcTanh[c*x]^2*L
og[1 + E^(-2*ArcTanh[c*x])] + 3*Log[1 - c^2*x^2] + 6*ArcTanh[c*x]*PolyLog[2
, -E^(-2*ArcTanh[c*x])] + 3*PolyLog[3, -E^(-2*ArcTanh[c*x])]))/(6*c^3)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^2\operatorname{artanh}(cx)^3 + 3\,ab^2x^2\operatorname{artanh}(cx)^2 + 3\,a^2bx^2\operatorname{artanh}(cx) + a^3x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctanh(c*x)^3 + 3*a*b^2*x^2*arctanh(c*x)^2 + 3*a^2*b*x^2*
arctanh(c*x) + a^3*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{artanh}(cx) + a)^3x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^2, x)
```

maple [C] time = 1.41, size = 1177, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^3,x)
```

```
[Out] 1/3*x^3*a^3+a*b^2*x/c^2+b^3*x*arctanh(c*x)/c^2+1/4*I/c^3*b^3*arctanh(c*x)^2
*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2
/(-c^2*x^2+1)))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))+1/c*a*b^2*arctanh(c*x)*x
^2+1/c^3*a*b^2*arctanh(c*x)*ln(c*x-1)+1/c^3*a*b^2*arctanh(c*x)*ln(c*x+1)-1/
2/c^3*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/2/c^3*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x
+1)-1/2/c^3*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/2*I/c^3*b^3*arctanh(c*
x)^2*Pi+1/3*x^3*b^3*arctanh(c*x)^3-1/2/c^3*b^3*arctanh(c*x)^2+1/3/c^3*b^3*a
rctanh(c*x)^3+1/c^3*b^3*arctanh(c*x)+1/2/c^3*b^3*polylog(3,-(c*x+1)^2/(-c^2
*x^2+1))-1/c^3*b^3*ln(1+(c*x+1)^2/(-c^2*x^2+1))-1/4*I/c^3*b^3*arctanh(c*x)^
2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/4*I/c^3*b^3*arctanh(c*x)^2*Pi*csgn(I
*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3-1/2*I/c^3*b^3*arctanh(
c*x)^2*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3+1/2*I/c^3*b^3*arctanh(c*x)^2
*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2+1/2/c*a^2*b*x^2+a^2*b*x^3*arctanh(
c*x)+a*b^2*x^3*arctanh(c*x)^2+1/2/c*b^3*arctanh(c*x)^2*x^2-1/c^3*b^3*arctan
h(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-1/c^3*b^3*arctanh(c*x)^2*ln(2)-1/
```

```
c^3*a*b^2*dilog(1/2+1/2*c*x)+1/2/c^3*b^3*arctanh(c*x)^2*ln(c*x-1)+1/2/c^3*b
^3*arctanh(c*x)^2*ln(c*x+1)-1/c^3*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1
)^(1/2))+1/4/c^3*a*b^2*ln(c*x-1)^2-1/4/c^3*a*b^2*ln(c*x+1)^2+1/2/c^3*a*b^2*
ln(c*x-1)-1/2/c^3*a*b^2*ln(c*x+1)+1/2/c^3*a^2*b*ln(c*x-1)+1/2/c^3*a^2*b*ln(
c*x+1)-1/2*I/c^3*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn
(I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/4*I/c^3*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+
1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^
2-1/4*I/c^3*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x
+1)/(-c^2*x^2+1)^(1/2))^2-1/4*I/c^3*b^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/
(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1))
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^3x^3+\frac{1}{2}\left(2x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)a^2b-\frac{\left(b^3c^3x^3-b^3\right)\log(-cx+1)^3-3\left(2ab^2c^3x^3+b^3c^2x^2+\right.}{24c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 + 1/2*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))
*a^2*b - 1/24*((b^3*c^3*x^3 - b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c^3*x^3 + b
^3*c^2*x^2 + (b^3*c^3*x^3 + b^3)*log(c*x + 1))*log(-c*x + 1)^2)/c^3 - integ
rate(-1/8*((b^3*c^3*x^3 - b^3*c^2*x^2)*log(c*x + 1)^3 + 6*(a*b^2*c^3*x^3 -
a*b^2*c^2*x^2)*log(c*x + 1)^2 - (4*a*b^2*c^3*x^3 + 2*b^3*c^2*x^2 + 3*(b^3*c
^3*x^3 - b^3*c^2*x^2)*log(c*x + 1)^2 - 2*(6*a*b^2*c^2*x^2 - (6*a*b^2*c^3 +
b^3*c^3)*x^3 - b^3)*log(c*x + 1))*log(-c*x + 1))/(c^3*x - c^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x))^3,x)
```

```
[Out] int(x^2*(a + b*atanh(c*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(a+b\operatorname{atanh}{\left(cx\right)}\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x))**3, x)
```

3.28 $\int x\left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=123

$$-\frac{3b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^3+\frac{3}{}$$

```
[Out] 3/2*b*(a+b*arctanh(c*x))^2/c^2+3/2*b*x*(a+b*arctanh(c*x))^2/c-1/2*(a+b*arct
anh(c*x))^3/c^2+1/2*x^2*(a+b*arctanh(c*x))^3-3*b^2*(a+b*arctanh(c*x))*ln(2/
(-c*x+1))/c^2-3/2*b^3*polylog(2,1-2/(-c*x+1))/c^2
```

Rubi [A] time = 0.25, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5916, 5980, 5910, 5984, 5918, 2402, 2315, 5948}

$$-\frac{3b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c^2}-\frac{3b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (3*b*(a + b*ArcTanh[c*x])^2)/(2*c^2) + (3*b*x*(a + b*ArcTanh[c*x])^2)/(2*c)
 - (a + b*ArcTanh[c*x])^3/(2*c^2) + (x^2*(a + b*ArcTanh[c*x])^3)/2 - (3*b^2
*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c^2 - (3*b^3*PolyLog[2, 1 - 2/(1 -
c*x)])/(2*c^2)
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
```

```
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}(cx)\right)^3\,dx = \frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{2}(3bc)\int\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx$$

$$=\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^3+\frac{(3b)\int\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{2c}-\frac{(3b)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,d}{2c}$$

$$=\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^3-\left(3b^2\right)$$

$$=\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a\right.$$

$$=\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a\right.$$

$$=\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a\right.$$

$$=\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a\right.$$

Mathematica [A] time = 0.30, size = 161, normalized size = 1.31

$$\frac{a\left(2a^2c^2x^2+6abcx+3ab\log(1-cx)-3ab\log(cx+1)+6b^2\log\left(1-c^2x^2\right)\right)+6b^2(cx-1)\tanh^{-1}(cx)^2(acx+a+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (6*b^2*(-1 + c*x)*(a + b + a*c*x)*ArcTanh[c*x]^2 + 2*b^3*(-1 + c^2*x^2)*Arc
Tanh[c*x]^3 + 6*b*ArcTanh[c*x]*(a*c*x*(2*b + a*c*x) - 2*b^2*Log[1 + E^(-2*A
```

```
rcTanh[c*x])]) + a*(6*a*b*c*x + 2*a^2*c^2*x^2 + 3*a*b*Log[1 - c*x] - 3*a*b*
Log[1 + c*x] + 6*b^2*Log[1 - c^2*x^2]) + 6*b^3*PolyLog[2, -E^(-2*ArcTanh[c*
x])])/(4*c^2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3 x \operatorname{artanh}(cx)^3 + 3\,ab^2 x \operatorname{artanh}(cx)^2 + 3\,a^2 bx \operatorname{artanh}(cx) + a^3 x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctanh(c*x)^3 + 3*a*b^2*x*arctanh(c*x)^2 + 3*a^2*b*x*arctan
h(c*x) + a^3*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{artanh}(cx) + a)^3 x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x, x)
```

maple [C] time = 0.64, size = 6097, normalized size = 49.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2}\,ab^2 x^2 \operatorname{artanh}(cx)^2 + \frac{1}{2}\,a^3 x^2 + \frac{3}{4}\left(2\,x^2 \operatorname{artanh}(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)a^2 b + \frac{3}{8}\left(4\,c\left(\frac{2\,x}{c^2} - \frac{\text{lo}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arctanh(c*x)^2 + 1/2*a^3*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(2
*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a^2*b + 3/8*(4*c*(2*x/c^2 -
log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) - (2*(log(c*x - 1) - 2)*l
og(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1))/c^2)*a*b^2
- 1/64*(3*c^3*(x^2/c^3 + log(c^2*x^2 - 1)/c^5) + 21*c^2*(2*x/c^3 - log(c*x
+ 1)/c^4 + log(c*x - 1)/c^4) - 576*c*integrate(1/4*x*log(c*x + 1)/(c^3*x^2
- c), x) - 2*(12*c*x*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x + 1)^3 - 3*(c
^2*x^2 - 2*c*x - 2*(c^2*x^2 - 1)*log(c*x + 1) + 1)*log(-c*x + 1)^2 + 3*(c^2
*x^2 - 2*(c^2*x^2 - 1)*log(c*x + 1)^2 + 6*c*x - 8*(c*x + 1)*log(c*x + 1))*l
og(-c*x + 1))/c^2 + ((4*log(-c*x + 1)^3 - 6*log(-c*x + 1)^2 + 6*log(-c*x +
1) - 3)*(c*x - 1)^2 + 8*(log(-c*x + 1)^3 - 3*log(-c*x + 1)^2 + 6*log(-c*x +
 1) - 6)*(c*x - 1))/c^2 + 18*log(4*c^3*x^2 - 4*c)/c^2 - 192*integrate(1/4*l
og(c*x + 1)/(c^3*x^2 - c), x))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a + b\,\text{atanh}(c\,x))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x))^3,x)
```

```
[Out] int(x*(a + b*atanh(c*x))^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{atanh}\left(cx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x*(a + b*atanh(c*x))**3, x)
```

3.29 $\int \left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=108

$$-\frac{3b^2\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}-\frac{3b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}$$

```
[Out] (a+b*arctanh(c*x))^3/c+x*(a+b*arctanh(c*x))^3-3*b*(a+b*arctanh(c*x))^2*ln(2
/(-c*x+1))/c-3*b^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/c+3/2*b^3*pol
ylog(3,1-2/(-c*x+1))/c
```

Rubi [A] time = 0.22, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5910, 5984, 5918, 5948, 6058, 6610}

$$-\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c}+x\left(a+b\tanh^{-1}(cx)\right)^3+\frac{\left(a+b\tanh\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (a + b*ArcTanh[c*x])^3/c + x*(a + b*ArcTanh[c*x])^3 - (3*b*(a + b*ArcTanh[c
*x])^2*Log[2/(1 - c*x)])/c - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(
1 - c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c)
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
```

```
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(cx)\right)^3 dx = x\left(a+b\tanh^{-1}(cx)\right)^3-(3bc)\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx$$
$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-(3b)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1-cx}dx$$
$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right)}{c}+\left(\right.$$
$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right)}{c}-\frac{3}{}$$
$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right)}{c}-\frac{3}{}$$

Mathematica [A] time = 0.29, size = 161, normalized size = 1.49

$$\frac{2a^3cx+3a^2b\log\left(1-c^2x^2\right)+6a^2bcx\tanh^{-1}(cx)+6ab^2\left(\text{Li}_2\left(-e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\left((cx-1)\tanh^{-1}(cx)-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (2*a^3*c*x + 6*a^2*b*c*x*ArcTanh[c*x] + 3*a^2*b*Log[1 - c^2*x^2] + 6*a*b^2*
(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x])]) +
PolyLog[2, -E^(-2*ArcTanh[c*x])]) + b^3*(2*ArcTanh[c*x]^2*((-1 + c*x)*ArcTa
nh[c*x] - 3*Log[1 + E^(-2*ArcTanh[c*x])]) + 6*ArcTanh[c*x]*PolyLog[2, -E^(-
2*ArcTanh[c*x])] + 3*PolyLog[3, -E^(-2*ArcTanh[c*x])]))/(2*c)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3\,\text{artanh}\,(cx)^3+3\,ab^2\,\text{artanh}\,(cx)^2+3\,a^2b\,\text{artanh}\,(cx)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x)
 + a^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{artanh}\,(cx)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3, x)
```

maple [B] time = 0.24, size = 261, normalized size = 2.42

$$x\,a^3+b^3x\,\mathrm{arctanh}\,(cx)^3+\frac{b^3\,\mathrm{arctanh}\,(cx)^3}{c}-\frac{3b^3\,\mathrm{arctanh}\,(cx)^2\ln\left(1+\frac{(cx+1)^2}{-c^2x^2+1}\right)}{c}-\frac{3b^3\,\mathrm{arctanh}\,(cx)\,\mathrm{polylog}\left(2,-\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3,x)
```

```
[Out] x*a^3+b^3*x*arctanh(c*x)^3+1/c*b^3*arctanh(c*x)^3-3/c*b^3*arctanh(c*x)^2*ln
(1+(c*x+1)^2/(-c^2*x^2+1))-3/c*b^3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*
x^2+1))+3/2/c*b^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+3*x*a*b^2*arctanh(c*x)
^2+3/c*a*b^2*arctanh(c*x)^2-6/c*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))*a
*b^2-3/c*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))*a*b^2+3*x*a^2*b*arctanh(c*x)+3/
2/c*a^2*b*ln(-c^2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3x+\frac{3\left(2\,cx\,\mathrm{artanh}\,(cx)+\log\left(-c^2x^2+1\right)\right)a^2b}{2\,c}-\frac{\left(b^3cx-b^3\right)\log\left(-cx+1\right)^3-3\left(2\,ab^2cx+\left(b^3cx+b^3\right)\log\left(cx\right.\right.}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a^2*b/c - 1/8*((b^3*c*
x - b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c*x + (b^3*c*x + b^3)*log(c*x + 1))*l
og(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b
^2*c*x - a*b^2)*log(c*x + 1)^2 - 3*(4*a*b^2*c*x + (b^3*c*x - b^3)*log(c*x +
 1)^2 - 2*(2*a*b^2 - b^3 - (2*a*b^2*c + b^3*c)*x)*log(c*x + 1))*log(-c*x +
1))/(c*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{atanh}\,(c\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3,x)
```

```
[Out] int((a + b*atanh(c*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{atanh}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3, x)
```

3.30 $\int \frac{\left(a+b \tanh^{-1}(cx)\right)^3}{x} \, dx$

Optimal. Leaf size=184

$$\frac{3}{2}b^2\text{Li}_3\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{3}{2}b^2\text{Li}_3\left(\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{3}{2}b\text{Li}_2\left(1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}\right.$$

```
[Out] -2*(a+b*arctanh(c*x))^3*arctanh(-1+2/(-c*x+1))-3/2*b*(a+b*arctanh(c*x))^2*p
olylog(2,1-2/(-c*x+1))+3/2*b*(a+b*arctanh(c*x))^2*polylog(2,-1+2/(-c*x+1))+
3/2*b^2*(a+b*arctanh(c*x))*polylog(3,1-2/(-c*x+1))-3/2*b^2*(a+b*arctanh(c*x
))*polylog(3,-1+2/(-c*x+1))-3/4*b^3*polylog(4,1-2/(-c*x+1))+3/4*b^3*polylog
(4,-1+2/(-c*x+1))
```

Rubi [A] time = 0.45, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5914, 6052, 5948, 6058, 6062, 6610}

$$\frac{3}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{3}{2}b^2\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{3}{2}b\text{PolyLog}\left(2,\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x,x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^3*ArcTanh[1 - 2/(1 - c*x)] - (3*b*(a + b*ArcTanh[c*x
])^2*PolyLog[2, 1 - 2/(1 - c*x)])/2 + (3*b*(a + b*ArcTanh[c*x])^2*PolyLog[2
, -1 + 2/(1 - c*x)])/2 + (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, 1 - 2/(1 -
c*x)])/2 - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, -1 + 2/(1 - c*x)])/2 - (3
*b^3*PolyLog[4, 1 - 2/(1 - c*x)])/4 + (3*b^3*PolyLog[4, -1 + 2/(1 - c*x)])/
4
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx = 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-(6bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(\right.}{1-c^2x^2}$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)+(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-c}\right.}{1-c^2x^2}$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-\frac{3}{2}b\left(a+b\tanh^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1-c}\right.$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-\frac{3}{2}b\left(a+b\tanh^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1-c}\right.$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-\frac{3}{2}b\left(a+b\tanh^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1-c}\right.$$

Mathematica [A] time = 0.14, size = 178, normalized size = 0.97

$$\frac{3}{4}b\left(2\text{Li}_2\left(\frac{cx+1}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2-2\text{Li}_2\left(\frac{cx+1}{cx-1}\right)\left(a+b\tanh^{-1}(cx)\right)^2+b\left(-2\text{Li}_3\left(\frac{cx+1}{1-cx}\right)\left(a+b\tanh^{-1}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x,x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^3*ArcTanh[(1 + c*x)/(-1 + c*x)] + (3*b*(2*(a + b*Arc
Tanh[c*x])^2*PolyLog[2, (1 + c*x)/(1 - c*x)] - 2*(a + b*ArcTanh[c*x])^2*Pol
yLog[2, (1 + c*x)/(-1 + c*x)] + b*(-2*(a + b*ArcTanh[c*x])*PolyLog[3, (1 +
c*x)/(1 - c*x)] + 2*(a + b*ArcTanh[c*x])*PolyLog[3, (1 + c*x)/(-1 + c*x)] +
 b*(PolyLog[4, (1 + c*x)/(1 - c*x)] - PolyLog[4, (1 + c*x)/(-1 + c*x)]))))/
4
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{artanh}\,(cx)^3+3\,ab^2\,\text{artanh}\,(cx)^2+3\,a^2b\,\text{artanh}\,(cx)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x, x)
```

maple [C] time = 0.26, size = 1470, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x,x)
```

```
[Out] a^3*ln(c*x)-3/4*b^3*polylog(4,-(c*x+1)^2/(-c^2*x^2+1))+6*b^3*polylog(4,(c*x
+1)/(-c^2*x^2+1)^(1/2))+6*b^3*polylog(4,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*
b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arct
anh(c*x)^3+3/2*I*a*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x
+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x
^2+1)))*arctanh(c*x)^2-3/2*a^2*b*dilog(c*x)-3/2*a^2*b*dilog(c*x+1)+3/2*a*b^
2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-6*a*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)
^(1/2))-6*a*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+b^3*arctanh(c*x)^3*l
n(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*b^3*arctanh(c*x)^2*polylog(2,-(c*x+1)/(-c
^2*x^2+1)^(1/2))-6*b^3*arctanh(c*x)*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+
b^3*ln(c*x)*arctanh(c*x)^3-b^3*arctanh(c*x)^3*ln((c*x+1)^2/(-c^2*x^2+1)-1)-
3/2*b^3*arctanh(c*x)^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+3/2*b^3*arctanh(c
*x)*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+b^3*arctanh(c*x)^3*ln(1-(c*x+1)/(-c^
2*x^2+1)^(1/2))+3*b^3*arctanh(c*x)^2*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-
6*b^3*arctanh(c*x)*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*I*b^3*Pi*csgn(
I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^
2/(-c^2*x^2+1)))^2*arctanh(c*x)^3-1/2*I*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^
2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arct
anh(c*x)^3+3/2*I*a*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-
c^2*x^2+1)))^3*arctanh(c*x)^2-3*a*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2
+1)-1)+3*a*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*a*b^2*arct
anh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+3*a*b^2*arctanh(c*x)^2*ln(1+
(c*x+1)/(-c^2*x^2+1)^(1/2))+6*a*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x
^2+1)^(1/2))+3*a*b^2*ln(c*x)*arctanh(c*x)^2-3*a*b^2*arctanh(c*x)*polylog(2,
-(c*x+1)^2/(-c^2*x^2+1))+3*a^2*b*ln(c*x)*arctanh(c*x)-3/2*a^2*b*ln(c*x)*ln(
c*x+1)-3/2*I*a*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-3/2*I*a*b^2*Pi
*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c
*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+1/2*I*b^3*Pi*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2
+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*x)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \log(x) + \int \frac{b^3 \left(\log(cx+1) - \log(-cx+1)\right)^3}{8x} + \frac{3\, ab^2 \left(\log(cx+1) - \log(-cx+1)\right)^2}{4x} + \frac{3\, a^2 b \left(\log(cx+1) - \log(-\right.}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c*x + 1) - log(-c*x + 1))^3/x + 3/4*a*b
^2*(log(c*x + 1) - log(-c*x + 1))^2/x + 3/2*a^2*b*(log(c*x + 1) - log(-c*x
+ 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}\,(c\,x))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/x,x)
```

```
[Out] int((a + b*atanh(c*x))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x, x)
```

3.31 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^2}\,dx$

Optimal. Leaf size=102

$$-3b^2c\text{Li}_2\left(\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)+c\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}+3bc\log\left(2-\frac{2}{cx+1}\right)\left(a+\right.$$

```
[Out] c*(a+b*arctanh(c*x))^3-(a+b*arctanh(c*x))^3/x+3*b*c*(a+b*arctanh(c*x))^2*ln
(2-2/(c*x+1))-3*b^2*c*(a+b*arctanh(c*x))*polylog(2,-1+2/(c*x+1))-3/2*b^3*c*
polylog(3,-1+2/(c*x+1))
```

Rubi [A] time = 0.27, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5916, 5988, 5932, 5948, 6056, 6610}

$$-3b^2c\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{3}{2}b^3c\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)+c\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\,\text{ta}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^2,x]
```

```
[Out] c*(a + b*ArcTanh[c*x])^3 - (a + b*ArcTanh[c*x])^3/x + 3*b*c*(a + b*ArcTanh[
c*x])^2*Log[2 - 2/(1 + c*x)] - 3*b^2*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 +
 2/(1 + c*x)] - (3*b^3*c*PolyLog[3, -1 + 2/(1 + c*x)])/2
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
```

```
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x} + (3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x\left(1-c^2x^2\right)}\,dx$$

$$= c\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x} + (3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(1+cx)}\,dx$$

$$= c\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x} + 3bc\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{\quad}{1}\right.$$

$$= c\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x} + 3bc\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{\quad}{1}\right.$$

$$= c\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x} + 3bc\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{\quad}{1}\right.$$

Mathematica [C] time = 0.34, size = 196, normalized size = 1.92

$$-\frac{a^3}{x}-\frac{3}{2}a^2bc\log\left(1-c^2x^2\right)+3a^2bc\log(x)-\frac{3a^2b\tanh^{-1}(cx)}{x}+3ab^2c\left(\tanh^{-1}(cx)\left(-\frac{\tanh^{-1}(cx)}{cx}+\tanh^{-1}(cx)+2\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*ArcTanh[c*x])/x + 3*a^2*b*c*Log[x] - (3*a^2*b*c*Log[1 -
 c^2*x^2])/2 + 3*a*b^2*c*(ArcTanh[c*x]*(ArcTanh[c*x] - ArcTanh[c*x]/(c*x) +
 2*Log[1 - E^(-2*ArcTanh[c*x])]) - PolyLog[2, E^(-2*ArcTanh[c*x])]) + b^3*c
*((I/8)*Pi^3 - ArcTanh[c*x]^3 - ArcTanh[c*x]^3/(c*x) + 3*ArcTanh[c*x]^2*Log
[1 - E^(2*ArcTanh[c*x])] + 3*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] -
(3*PolyLog[3, E^(2*ArcTanh[c*x])])/2)
```

fricas [F] time = 1.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^3\,\mathrm{artanh}\,(cx)^3+3\,ab^2\,\mathrm{artanh}\,(cx)^2+3\,a^2b\,\mathrm{artanh}\,(cx)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{artanh}\,(cx)+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^2, x)
```

maple [C] time = 0.54, size = 1583, normalized size = 15.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x^2,x)
```

```
[Out] -6*c*b^3*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-6*c*b^3*polylog(3,-(c*x+1)/(
-c^2*x^2+1)^(1/2))-c*b^3*arctanh(c*x)^3-b^3/x*arctanh(c*x)^3+3/4*I*c*b^3*Pi
*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh
(c*x)^2-3/2*I*c*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-a^3/x+3/2*I*c
*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)
))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(c*
x)^2-3/4*I*c*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^
2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*arctanh(
c*x)^2-3*c*a*b^2*dilog(c*x+1)-3/2*c*b^3*arctanh(c*x)^2*ln(c*x-1)-3/2*c*b^3*
arctanh(c*x)^2*ln(c*x+1)+3*c*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/
2))+3*c*b^3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c*b^3*arctanh
(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*c*b^3*arctanh(c*x)*polylog(2,-(c
*x+1)/(-c^2*x^2+1)^(1/2))+3*c*b^3*ln(c*x)*arctanh(c*x)^2+6*c*b^3*arctanh(c*
x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-3*c*b^3*arctanh(c*x)^2*ln((c*x+1)^
2/(-c^2*x^2+1)-1)-3/4*c*a*b^2*ln(c*x-1)^2+3/4*c*a*b^2*ln(c*x+1)^2-3/2*c*a^2
*b*ln(c*x-1)-3/2*c*a^2*b*ln(c*x+1)+3*c*b^3*arctanh(c*x)^2*ln(2)+3*c*a*b^2*d
ilog(1/2+1/2*c*x)+3*c*a^2*b*ln(c*x)-3*c*a*b^2*dilog(c*x)-3*a^2*b/x*arctanh(
c*x)-3*a*b^2/x*arctanh(c*x)^2+3/4*I*c*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^
3*arctanh(c*x)^2-3/2*I*c*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctan
h(c*x)^2+3/4*I*c*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2
+1)))^3*arctanh(c*x)^2+3/2*I*c*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*
arctanh(c*x)^2+3/2*I*c*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^
2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-3/2*I*c*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*
x^2+1)))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*ar
ctanh(c*x)^2-3/4*I*c*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/
(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2+3/4*I*c*b^3*Pi*csg
n(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(
-c^2*x^2+1)))^2*arctanh(c*x)^2+3/2*I*c*b^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(
1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2+6*c*a*b^2*arctanh(c*x)
*ln(c*x)-3*c*a*b^2*ln(c*x)*ln(c*x+1)-3*c*a*b^2*arctanh(c*x)*ln(c*x-1)-3*c*a
*b^2*arctanh(c*x)*ln(c*x+1)+3/2*c*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-3/2*c*a*b
^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+3/2*c*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+
3/2*I*c*b^3*Pi*arctanh(c*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\operatorname{artanh}\left(cx\right)}{x}\right)a^2b-\frac{a^3}{x}-\frac{\left(b^3cx-b^3\right)\log\left(-cx+1\right)^3+3\left(2\,ab^2+\left(b^3cx+b^3\right)\mathrm{lo}\right.}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a^2*b - a^3/x - 1
/8*((b^3*c*x - b^3)*log(-c*x + 1)^3 + 3*(2*a*b^2 + (b^3*c*x + b^3)*log(c*x
+ 1))*log(-c*x + 1)^2)/x - integrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 +
```

```
 6*(a*b^2*c*x - a*b^2)*log(c*x + 1)^2 + 3*(4*a*b^2*c*x - (b^3*c*x - b^3)*lo
g(c*x + 1)^2 + 2*(b^3*c^2*x^2 + 2*a*b^2 - (2*a*b^2*c - b^3*c)*x)*log(c*x +
1))*log(-c*x + 1))/(c*x^3 - x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}\,(c\,x))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/x^2,x)
```

```
[Out] int((a + b*atanh(c*x))^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**2, x)
```

3.32 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^3}\,dx$

Optimal. Leaf size=123

$$3b^2c^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)}{2x^2}$$

```
[Out] 3/2*b*c^2*(a+b*arctanh(c*x))^2-3/2*b*c*(a+b*arctanh(c*x))^2/x+1/2*c^2*(a+b*
arctanh(c*x))^3-1/2*(a+b*arctanh(c*x))^3/x^2+3*b^2*c^2*(a+b*arctanh(c*x))*l
n(2-2/(c*x+1))-3/2*b^3*c^2*polylog(2,-1+2/(c*x+1))
```

Rubi [A] time = 0.29, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5916, 5982, 5988, 5932, 2447, 5948}

$$-\frac{3}{2}b^3c^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+3b^2c^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^3,x]
```

```
[Out] (3*b*c^2*(a + b*ArcTanh[c*x])^2)/2 - (3*b*c*(a + b*ArcTanh[c*x])^2)/(2*x) +
 (c^2*(a + b*ArcTanh[c*x])^3)/2 - (a + b*ArcTanh[c*x])^3/(2*x^2) + 3*b^2*c^
2*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (3*b^3*c^2*PolyLog[2, -1 + 2/
(1 + c*x)])/2
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^3}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2\left(1-c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx+\frac{1}{2}\left(3bc^3\right)\int \frac{\left(a+b\,\mathrm{ta}\right.}{1-}$$

$$= -\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2x^2}+\left(3b\right.$$

$$= \frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3-$$

$$= \frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3-$$

$$= \frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3-$$

Mathematica [A] time = 0.29, size = 192, normalized size = 1.56

$$\frac{a\left(-2a^2-3abc^2x^2\log(1-cx)+3abc^2x^2\log(cx+1)-6abcx+12b^2c^2x^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)\right)-6b\tanh^{-1}(cx)\left(a^2+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x^3,x]
```

```
[Out] (6*b^2*(-1 + c*x)*(a + a*c*x + b*c*x)*ArcTanh[c*x]^2 + 2*b^3*(-1 + c^2*x^2)
*ArcTanh[c*x]^3 - 6*b*ArcTanh[c*x]*(a^2 + 2*a*b*c*x - 2*b^2*c^2*x^2*Log[1 -
 E^(-2*ArcTanh[c*x])]) + a*(-2*a^2 - 6*a*b*c*x - 3*a*b*c^2*x^2*Log[1 - c*x]
 + 3*a*b*c^2*x^2*Log[1 + c*x] + 12*b^2*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]]
) - 6*b^3*c^2*x^2*PolyLog[2, E^(-2*ArcTanh[c*x])])/(4*x^2)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^3\,\mathrm{artanh}\,(cx)^3+3\,ab^2\,\mathrm{artanh}\,(cx)^2+3\,a^2b\,\mathrm{artanh}\,(cx)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{artanh}\,(cx) + a)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)^3/x^3, x)
```

maple [C] time = 0.71, size = 5098, normalized size = 41.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x^3,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\text{artanh}\,(cx)}{x^2}\right)a^2b+\frac{3}{8}\left(\left(2\left(\log\left(cx-1\right)-2\right)\log\left(cx+1\right)-\log\left(cx+1\right)^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="maxima")
[Out] 3/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a^2*b
+ 3/8*((2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2
 - 4*log(c*x - 1) + 8*log(x))*c^2 + 4*(c*log(c*x + 1) - c*log(c*x - 1) - 2/
x)*c*arctanh(c*x))*a*b^2 - 1/16*b^3*(((c^2*x^2 - 1)*log(-c*x + 1)^3 + 3*(2*
c*x - (c^2*x^2 - 1)*log(c*x + 1))*log(-c*x + 1)^2)/x^2 + 2*integrate(-((c*x
 - 1)*log(c*x + 1)^3 + 3*(2*c^2*x^2 - (c*x - 1)*log(c*x + 1)^2 - (c^3*x^3 -
 c*x)*log(c*x + 1))*log(-c*x + 1))/(c*x^4 - x^3), x)) - 3/2*a*b^2*arctanh(c
*x)^2/x^2 - 1/2*a^3/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\text{atanh}\,(c\,x))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/x^3,x)
[Out] int((a + b*atanh(c*x))^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{atanh}\,(cx))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**3,x)
[Out] Integral((a + b*atanh(c*x))**3/x**3, x)
```

3.33 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^4}\,dx$

Optimal. Leaf size=200

$$-b^2c^3\text{Li}_2\left(\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^3+\frac{1}{2}bc^3\left(a+b\tanh\right.$$

```
[Out] -b^2*c^2*(a+b*arctanh(c*x))/x+1/2*b*c^3*(a+b*arctanh(c*x))^2-1/2*b*c*(a+b*a
rctanh(c*x))^2/x^2+1/3*c^3*(a+b*arctanh(c*x))^3-1/3*(a+b*arctanh(c*x))^3/x^
3+b^3*c^3*ln(x)-1/2*b^3*c^3*ln(-c^2*x^2+1)+b*c^3*(a+b*arctanh(c*x))^2*ln(2-
2/(c*x+1))-b^2*c^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(c*x+1))-1/2*b^3*c^3*p
olylog(3,-1+2/(c*x+1))
```

Rubi [A] time = 0.50, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 6056, 6610}

$$-b^2c^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}b^3c^3\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{3}c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^4,x]
```

```
[Out] -((b^2*c^2*(a + b*ArcTanh[c*x]))/x) + (b*c^3*(a + b*ArcTanh[c*x])^2)/2 - (b
*c*(a + b*ArcTanh[c*x])^2)/(2*x^2) + (c^3*(a + b*ArcTanh[c*x])^3)/3 - (a +
b*ArcTanh[c*x])^3/(3*x^3) + b^3*c^3*Log[x] - (b^3*c^3*Log[1 - c^2*x^2])/2 +
 b*c^3*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)] - b^2*c^3*(a + b*ArcTanh
[c*x])*PolyLog[2, -1 + 2/(1 + c*x)] - (b^3*c^3*PolyLog[3, -1 + 2/(1 + c*x)]
)/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
```

```
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^4}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3x^3} + (bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3\left(1-c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3x^3} + (bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx + \left(bc^3\right)\int \frac{\left(a+b\tanh^{-1}(\right.}{x\left(1-c^2x^2\right.}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3x^3} + \left(b^2c^2\right.$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} + \frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3x^3} + bc^3\left(\right.$$

$$= -\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{1}{2}bc^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} +$$

$$= -\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{1}{2}bc^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} +$$

$$= -\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{1}{2}bc^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} +$$

$$= -\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{1}{2}bc^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} +$$

Mathematica [C] time = 0.94, size = 323, normalized size = 1.62

$$-\frac{8a^3 - 24a^2bc^3x^3\log(x) + 12a^2bc^3x^3\log\left(1-c^2x^2\right) + 12a^2bcx + 24a^2b\tanh^{-1}(cx) + 24ab^2\left(c^3x^3\mathrm{Li}_2\left(e^{-2\tanh^{-1}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x^4,x]
```

```
[Out] -1/24*(8*a^3 + 12*a^2*b*c*x + 24*a^2*b*ArcTanh[c*x] - 24*a^2*b*c^3*x^3*Log[
x] + 12*a^2*b*c^3*x^3*Log[1 - c^2*x^2] + 24*a*b^2*(c^2*x^2 + (1 - c^3*x^3)*
ArcTanh[c*x]^2 - c*x*ArcTanh[c*x]*(-1 + c^2*x^2 + 2*c^2*x^2*Log[1 - E^(-2*A
rcTanh[c*x])]) + c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x])]) + b^3*((-I)*c^3*P
i^3*x^3 + 24*c^2*x^2*ArcTanh[c*x] + 12*c*x*ArcTanh[c*x]^2 - 12*c^3*x^3*ArcT
anh[c*x]^2 + 8*ArcTanh[c*x]^3 + 8*c^3*x^3*ArcTanh[c*x]^3 - 24*c^3*x^3*ArcTa
nh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 24*c^3*x^3*Log[(c*x)/Sqrt[1 - c^2*x
^2]] - 24*c^3*x^3*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + 12*c^3*x^3*
PolyLog[3, E^(2*ArcTanh[c*x])]))/x^3
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^3\,\mathrm{artanh}\,(cx)^3 + 3\,ab^2\,\mathrm{artanh}\,(cx)^2 + 3\,a^2b\,\mathrm{artanh}\,(cx) + a^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{artanh}(cx) + a)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^4, x)
```

maple [C] time = 1.94, size = 1838, normalized size = 9.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x^4,x)
```

```
[Out] -1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*Pi+1/2*I
*c^3*b^3*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^
2*x^2+1)))^3*Pi+1/4*I*c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^
3*Pi+1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*Pi+1
/4*I*c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2
*x^2+1)))^3*Pi+1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-
1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I/(1
+(c*x+1)^2/(-c^2*x^2+1)))*Pi-1/4*I*c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/
(c^2*x^2-1))*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1+(c*x+1)^2/(-c^2*x^2+1)))*Pi-1/3*a^3/x^3-1/2*I*c^3*b^3*arctanh(c*x)^2*c
sgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(1+(c*x
+1)^2/(-c^2*x^2+1)))^2*Pi-1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-
c^2*x^2+1)-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1
)))*Pi+1/4*I*c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*cs
gn(I*(c*x+1)^2/(c^2*x^2-1))*Pi+1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)/
(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*Pi-1/4*I*c^3*b^3*arctan
h(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x
+1)^2/(-c^2*x^2+1)))^2*Pi+1/4*I*c^3*b^3*arctanh(c*x)^2*csgn(I/(1+(c*x+1)^2/
(-c^2*x^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*P
i+c^3*b^3*ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)+c^3*b^3*ln(1+(c*x+1)/(-c^2*x^2+1
)^(1/2))+1/2*c^3*b^3*arctanh(c*x)^2-1/3*c^3*b^3*arctanh(c*x)^3-c^3*b^3*arct
anh(c*x)-1/3*b^3/x^3*arctanh(c*x)^3-2*c^3*b^3*polylog(3,(c*x+1)/(-c^2*x^2+1
)^(1/2))-2*c^3*b^3*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-c^2*a*b^2/x-1/2*c
*a^2*b/x^2+1/2*c^3*a*b^2*ln(c*x+1)-1/2*c^3*a^2*b*ln(c*x-1)-1/2*c^3*a^2*b*ln
(c*x+1)+c^3*a^2*b*ln(c*x)-c^3*a*b^2*dilog(c*x)-c^3*a*b^2*dilog(c*x+1)+c^3*a
*b^2*dilog(1/2+1/2*c*x)-1/2*c*b^3*arctanh(c*x)^2/x^2-c^2*b^3*arctanh(c*x)/x
-1/4*c^3*a*b^2*ln(c*x-1)^2+1/4*c^3*a*b^2*ln(c*x+1)^2-1/2*c^3*a*b^2*ln(c*x-1
)+c^3*b^3*arctanh(c*x)^2*ln(2)-1/2*c^3*b^3*arctanh(c*x)^2*ln(c*x-1)-1/2*c^3
*b^3*arctanh(c*x)^2*ln(c*x+1)+c^3*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1
)^(1/2))+c^3*b^3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+c^3*b^3*ar
ctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^3*b^3*arctanh(c*x)*polylo
g(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+c^3*b^3*ln(c*x)*arctanh(c*x)^2+2*c^3*b^3*a
rctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-c^3*b^3*arctanh(c*x)^2*ln
((c*x+1)^2/(-c^2*x^2+1)-1)-a^2*b/x^3*arctanh(c*x)-a*b^2/x^3*arctanh(c*x)^2-
c^3*a*b^2*arctanh(c*x)*ln(c*x+1)+1/2*c^3*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/
2*c^3*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2*c^3*a*b^2*ln(-1/2*c*x+1/2)*ln(1/
2+1/2*c*x)+2*c^3*a*b^2*arctanh(c*x)*ln(c*x)-c^3*a*b^2*ln(c*x)*ln(c*x+1)-c^3
*a*b^2*arctanh(c*x)*ln(c*x-1)-c*a*b^2*arctanh(c*x)/x^2+1/2*I*c^3*b^3*arctan
h(c*x)^2*Pi
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\left(c^2 \log\left(c^2x^2-1\right) - c^2 \log\left(x^2\right) + \frac{1}{x^2}\right)c + \frac{2\operatorname{artanh}(cx)}{x^3}\right)a^2b - \frac{a^3}{3x^3} - \frac{\left(b^3c^3x^3 - b^3\right)\log(-cx+1)^3 + 3\left(b^3cx + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*a^2*b - 1/3*a^3/x^3 - 1/24*((b^3*c^3*x^3 - b^3)*log(-c*x + 1)^3 + 3*(b^3*c
*x + 2*a*b^2 + (b^3*c^3*x^3 + b^3)*log(c*x + 1))*log(-c*x + 1)^2)/x^3 - int
egrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c*x - a*b^2)*log(c*x
 + 1)^2 + (2*b^3*c^2*x^2 + 4*a*b^2*c*x - 3*(b^3*c*x - b^3)*log(c*x + 1)^2 +
 2*(b^3*c^4*x^4 + 6*a*b^2 - (6*a*b^2*c - b^3*c)*x)*log(c*x + 1))*log(-c*x +
 1))/(c*x^5 - x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atanh}(c\,x))^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/x^4,x)
```

```
[Out] int((a + b*atanh(c*x))^3/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**4, x)
```

3.34 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^5}\,dx$

Optimal. Leaf size=187

$$2b^2c^4\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^3+bc^4\left(a+b\tanh^{-1}(c\right.$$

```
[Out] -1/4*b^3*c^3/x+1/4*b^3*c^4*arctanh(c*x)-1/4*b^2*c^2*(a+b*arctanh(c*x))/x^2+
b*c^4*(a+b*arctanh(c*x))^2-1/4*b*c*(a+b*arctanh(c*x))^2/x^3-3/4*b*c^3*(a+b*
arctanh(c*x))^2/x+1/4*c^4*(a+b*arctanh(c*x))^3-1/4*(a+b*arctanh(c*x))^3/x^4
+2*b^2*c^4*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b^3*c^4*polylog(2,-1+2/(c*x+1
))
```

Rubi [A] time = 0.63, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5916, 5982, 325, 206, 5988, 5932, 2447, 5948}

$$-b^3c^4\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+2b^2c^4\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^4\left(a+b\,\text{ta}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^5,x]
```

```
[Out] -(b^3*c^3)/(4*x) + (b^3*c^4*ArcTanh[c*x])/4 - (b^2*c^2*(a + b*ArcTanh[c*x])
)/(4*x^2) + b*c^4*(a + b*ArcTanh[c*x])^2 - (b*c*(a + b*ArcTanh[c*x])^2)/(4*
x^3) - (3*b*c^3*(a + b*ArcTanh[c*x])^2)/(4*x) + (c^4*(a + b*ArcTanh[c*x])^3
)/4 - (a + b*ArcTanh[c*x])^3/(4*x^4) + 2*b^2*c^4*(a + b*ArcTanh[c*x])*Log[2
 - 2/(1 + c*x)] - b^3*c^4*PolyLog[2, -1 + 2/(1 + c*x)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^5}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4x^4}+\frac{1}{4}(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4\left(1-c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4x^4}+\frac{1}{4}(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx+\frac{1}{4}\left(3bc^3\right)\int \frac{\left(a+b\,\mathrm{ta}\right.}{x^2\left(1\right.}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{4x^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4x^4}+\frac{1}{2}\left(b^2c^2\right)\int \frac{a+b\tanh^{-1}(cx)}{x^3\left(1-c^2x^2\right)}\,dx$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{4x^3}-\frac{3bc^3\left(a+b\tanh^{-1}(cx)\right)^2}{4x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^3-$$

$$= -\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+bc^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{4x^3}-\frac{3b}{}$$

$$= -\frac{b^3c^3}{4x}-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+bc^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\tanh^{-1}(cx\right.}{4x^3}$$

$$= -\frac{b^3c^3}{4x}+\frac{1}{4}b^3c^4\tanh^{-1}(cx)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+bc^4\left(a+b\tanh^{-1}(cx)\right)^2-$$

Mathematica [A] time = 0.70, size = 295, normalized size = 1.58

$$-\frac{2a^3+2b\tanh^{-1}(cx)\left(3a^2+2abcx\left(3c^2x^2+1\right)-8b^2c^4x^4\log\left(1-e^{-2\tanh^{-1}(cx)}\right)+b^2c^2x^2\left(1-c^2x^2\right)\right)+3a^2bc^4}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x^5,x]
```

```
[Out] -1/8*(2*a^3 + 2*a^2*b*c*x + 2*a*b^2*c^2*x^2 + 6*a^2*b*c^3*x^3 + 2*b^3*c^3*x
^3 - 2*a*b^2*c^4*x^4 + 2*b^2*(b*c*x*(1 + 3*c^2*x^2 - 4*c^3*x^3) + a*(3 - 3*
c^4*x^4))*ArcTanh[c*x]^2 - 2*b^3*(-1 + c^4*x^4)*ArcTanh[c*x]^3 + 2*b*ArcTan
h[c*x]*(3*a^2 + b^2*c^2*x^2*(1 - c^2*x^2) + 2*a*b*c*x*(1 + 3*c^2*x^2) - 8*b
^2*c^4*x^4*Log[1 - E^(-2*ArcTanh[c*x])]) + 3*a^2*b*c^4*x^4*Log[1 - c*x] - 3
*a^2*b*c^4*x^4*Log[1 + c*x] - 16*a*b^2*c^4*x^4*Log[(c*x)/Sqrt[1 - c^2*x^2]]
 + 8*b^3*c^4*x^4*PolyLog[2, E^(-2*ArcTanh[c*x])])/x^4
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{artanh}\,(cx)^3 + 3\,ab^2\,\text{artanh}\,(cx)^2 + 3\,a^2b\,\text{artanh}\,(cx) + a^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{artanh}\,(cx) + a)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^5, x)
```

maple [C] time = 1.71, size = 1281, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x^5,x)
```

```
[Out] 3/8*c^4*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-3/8*c^4*a*b^2*ln(-1/2*c*x+1/2)*ln(1
/2+1/2*c*x)-1/2*c*a*b^2*arctanh(c*x)/x^3-3/2*c^3*a*b^2*arctanh(c*x)/x+3/8*I
*c^4*b^3*Pi*arctanh(c*x)^2+3/8*c^4*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/4*c^4
*a*b^2*arctanh(c*x)*ln(c*x-1)+3/16*I*c^4*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x
^2+1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+
1)^2/(-c^2*x^2+1)))*arctanh(c*x)^2+1/4*b^3*c^4*arctanh(c*x)+3/8*I*c^4*b^3*P
i*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-3/16*I*c^4*b^3*Pi*csg
n(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-3/16*I*c^4*b^3*Pi*csgn(I*(c*x+1
)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^3*arctanh(c*x)^2-3/8*I*c^4*b^3*
Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-1/4*a^3/x^4-1/4*c*a^
2*b/x^3-1/4*c^2*a*b^2/x^2-3/4*c^3*a^2*b/x-1/4*c^2*b^3*arctanh(c*x)/x^2-1/4*
c*b^3*arctanh(c*x)^2/x^3-3/4*c^3*b^3*arctanh(c*x)^2/x-3/16*c^4*a*b^2*ln(c*x
-1)^2-3/16*c^4*a*b^2*ln(c*x+1)^2-c^4*a*b^2*ln(c*x-1)-c^4*a*b^2*ln(c*x+1)-3/
8*c^4*a^2*b*ln(c*x-1)+3/8*c^4*a^2*b*ln(c*x+1)+2*c^4*a*b^2*ln(c*x)-3/8*c^4*b
^3*arctanh(c*x)^2*ln(c*x-1)+3/8*c^4*b^3*arctanh(c*x)^2*ln(c*x+1)-3/4*c^4*b^
3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-1/4*c^4*b^3/(c*x+1-(-c^2*x^
2+1)^(1/2))*(-c^2*x^2+1)^(1/2)+1/4*c^4*b^3/((-c^2*x^2+1)^(1/2)+c*x+1)*(-c^2
*x^2+1)^(1/2)+2*c^4*b^3*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*a
^2*b/x^4*arctanh(c*x)-3/4*a*b^2/x^4*arctanh(c*x)^2+3/4*c^4*a*b^2*arctanh(c*
```

```
x)*ln(c*x+1)-3/8*I*c^4*b^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*
x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2+3/16*I*c^4*b^3*Pi*csgn(I*(c*x+1)^2/(c^
2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctan
h(c*x)^2-3/16*I*c^4*b^3*Pi*csgn(I/(1+(c*x+1)^2/(-c^2*x^2+1)))*csgn(I*(c*x+1
)^2/(c^2*x^2-1)/(1+(c*x+1)^2/(-c^2*x^2+1)))^2*arctanh(c*x)^2-3/16*I*c^4*b^3
*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arct
anh(c*x)^2-1/4*b^3/x^4*arctanh(c*x)^3-c^4*b^3*arctanh(c*x)^2+1/4*c^4*b^3*ar
ctanh(c*x)^3+2*c^4*b^3*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^4*b^3*dilog(
(c*x+1)/(-c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\operatorname{artanh}\left(cx\right)}{x^4}\right)a^2b+\frac{1}{16}\left(\left(32\,c^2\log(x)-\frac{3\,c^2x^2\log}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
6*arctanh(c*x)/x^4)*a^2*b + 1/16*((32*c^2*log(x) - (3*c^2*x^2*log(c*x + 1)^
2 + 3*c^2*x^2*log(c*x - 1)^2 + 16*c^2*x^2*log(c*x - 1) - 2*(3*c^2*x^2*log(c
*x - 1) - 8*c^2*x^2)*log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*log(c*x + 1) - 3
*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*a*b^2 - 1/32*b^3
*(((c^4*x^4 - 1)*log(-c*x + 1)^3 + (6*c^3*x^3 + 2*c*x - 3*(c^4*x^4 - 1)*log
(c*x + 1))*log(-c*x + 1)^2)/x^4 + 4*integrate(-1/2*(2*(c*x - 1)*log(c*x + 1
)^3 + (6*c^4*x^4 + 2*c^2*x^2 - 6*(c*x - 1)*log(c*x + 1)^2 - 3*(c^5*x^5 - c*
x)*log(c*x + 1))*log(-c*x + 1))/(c*x^6 - x^5), x)) - 3/4*a*b^2*arctanh(c*x)
^2/x^4 - 1/4*a^3/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3/x^5,x)
```

```
[Out] int((a + b*atanh(c*x))^3/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{atanh}\left(cx\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**5, x)
```

3.35 $\int (dx)^{5/2} \left(a + b \tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=124

$$\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}-\frac{2bd^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}+\frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}$$

```
[Out] 4/35*b*(d*x)^(5/2)/c-2/7*b*d^(5/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))/c^(7
/2)+2/7*(d*x)^(7/2)*(a+b*arctanh(c*x))/d-2/7*b*d^(5/2)*arctanh(c^(1/2)*(d*x
)^(1/2)/d^(1/2))/c^(7/2)+4/7*b*d^2*(d*x)^(1/2)/c^3
```

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = $\{5916, 321, 329, 212, 208, 205\}$

$$\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}+\frac{4bd^2\sqrt{dx}}{7c^3}-\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}-\frac{2bd^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}+\frac{4b(dx)^{5/2}}{35c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (4*b*d^2*Sqrt[d*x])/(7*c^3) + (4*b*(d*x)^(5/2))/(35*c) - (2*b*d^(5/2)*ArcTa
n[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/2)) + (2*(d*x)^(7/2)*(a + b*ArcTanh
[c*x]))/(7*d) - (2*b*d^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/
2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)dx &= \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{(2bc)\int\frac{(dx)^{7/2}}{1-c^2x^2}dx}{7d}\\ &= \frac{4b(dx)^{5/2}}{35c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{(2bd)\int\frac{(dx)^{3/2}}{1-c^2x^2}dx}{7c}\\ &= \frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{\left(2bd^3\right)\int\frac{1}{\sqrt{dx}\left(1-c^2x^2\right)}}{7c^3}\\ &= \frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{\left(4bd^2\right)\text{Subst}\left(\int\frac{1}{1-c^2\ldots}\right.}{7c^3}\\ &= \frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{\left(2bd^3\right)\text{Subst}\left(\int\frac{1}{d-c\ldots}\right.}{7c^3}\\ &= \frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}-\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx\ldots\right.}{7d}\end{aligned}$$

Mathematica [A] time = 0.08, size = 128, normalized size = 1.03

$$\frac{(dx)^{5/2}\left(10ac^{7/2}x^{7/2}+4bc^{5/2}x^{5/2}+10bc^{7/2}x^{7/2}\tanh^{-1}(cx)+20b\sqrt{c}\sqrt{x}+5b\log\left(1-\sqrt{c}\sqrt{x}\right)-5b\log\left(\sqrt{c}\sqrt{x}\ldots\right.}{35c^{7/2}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] ((d*x)^(5/2)*(20*b*Sqrt[c]*Sqrt[x] + 4*b*c^(5/2)*x^(5/2) + 10*a*c^(7/2)*x^(
7/2) - 10*b*ArcTan[Sqrt[c]*Sqrt[x]] + 10*b*c^(7/2)*x^(7/2)*ArcTanh[c*x] + 5
*b*Log[1 - Sqrt[c]*Sqrt[x]] - 5*b*Log[1 + Sqrt[c]*Sqrt[x]]))/(35*c^(7/2)*x^
(5/2))
```

fricas [A] time = 0.58, size = 296, normalized size = 2.39

$$\left[-\frac{10\,bd^2\sqrt{\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}\,c\sqrt{\frac{d}{c}}}{d}\right)-5\,bd^2\sqrt{\frac{d}{c}}\log\left(\frac{cdx-2\,\sqrt{dx}\,c\sqrt{\frac{d}{c}}+d}{cx-1}\right)-\left(5\,bc^3d^2x^3\log\left(-\frac{cx+1}{cx-1}\right)+10\,ac^3d^2x^3+4\,bc^2\ldots\right.}{35\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] [-1/35*(10*b*d^2*sqrt(d/c)*arctan(sqrt(d*x)*c*sqrt(d/c)/d) - 5*b*d^2*sqrt(d
/c)*log((c*d*x - 2*sqrt(d*x)*c*sqrt(d/c) + d)/(c*x - 1)) - (5*b*c^3*d^2*x^3
*log(-(c*x + 1)/(c*x - 1)) + 10*a*c^3*d^2*x^3 + 4*b*c^2*d^2*x^2 + 20*b*d^2)
```

```
*sqrt(d*x))/c^3, 1/35*(10*b*d^2*sqrt(-d/c)*arctan(sqrt(d*x)*c*sqrt(-d/c)/d)
 + 5*b*d^2*sqrt(-d/c)*log((c*d*x - 2*sqrt(d*x)*c*sqrt(-d/c) - d)/(c*x + 1))
 + (5*b*c^3*d^2*x^3*log(-(c*x + 1)/(c*x - 1)) + 10*a*c^3*d^2*x^3 + 4*b*c^2*
d^2*x^2 + 20*b*d^2)*sqrt(d*x))/c^3]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{5}{2}} (b \operatorname{artanh}(cx) + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*(b*arctanh(c*x) + a), x)
```

maple [A] time = 0.04, size = 107, normalized size = 0.86

$$\frac{2 (dx)^{\frac{7}{2}} a}{7d}+\frac{2b (dx)^{\frac{7}{2}} \operatorname{arctanh}(cx)}{7d}+\frac{4b (dx)^{\frac{5}{2}}}{35c}+\frac{4b\, d^2 \sqrt{dx}}{7c^3}-\frac{2d^3 b \arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7c^3\sqrt{cd}}-\frac{2d^3 b \operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7c^3\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a+b*arctanh(c*x)),x)
```

```
[Out] 2/7/d*(d*x)^(7/2)*a+2/7/d*b*(d*x)^(7/2)*arctanh(c*x)+4/35*b*(d*x)^(5/2)/c+4
/7*b*d^2*(d*x)^(1/2)/c^3-2/7*d^3*b/c^3/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*
d)^(1/2))-2/7*d^3*b/c^3/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

maxima [A] time = 0.42, size = 134, normalized size = 1.08

$$\frac{10 (dx)^{\frac{7}{2}} a + \left(10 (dx)^{\frac{7}{2}} \operatorname{artanh}(cx) - \frac{\left(\frac{10\, d^5 \arctan\left(\frac{\sqrt{dx}\, c}{\sqrt{cd}}\right)}{\sqrt{cd}\, c^4} - \frac{5\, d^5 \log\left(\frac{\sqrt{dx}\, c - \sqrt{cd}}{\sqrt{dx}\, c + \sqrt{cd}}\right)}{\sqrt{cd}\, c^4} - \frac{4\left((dx)^{\frac{5}{2}} c^2 d^2 + 5\sqrt{dx}\, d^4\right)}{c^4}\right) c}{d}\right) b}{35\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/35*(10*(d*x)^(7/2)*a + (10*(d*x)^(7/2)*arctanh(c*x) - (10*d^5*arctan(sqrt
(d*x)*c/sqrt(c*d))/(sqrt(c*d)*c^4) - 5*d^5*log((sqrt(d*x)*c - sqrt(c*d))/(s
qrt(d*x)*c + sqrt(c*d)))/(sqrt(c*d)*c^4) - 4*((d*x)^(5/2)*c^2*d^2 + 5*sqrt(
d*x)*d^4)/c^4)*c/d)*b)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\, \operatorname{atanh}(c\, x))\, (d\, x)^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d*x)^(5/2),x)
```

```
[Out] int((a + b*atanh(c*x))*(d*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{5}{2}} (a + b \operatorname{atanh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(a+b*atanh(c*x)),x)
```

```
[Out] Integral((d*x)**(5/2)*(a + b*atanh(c*x)), x)
```

3.36 $\int (dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)dx$

Optimal. Leaf size=106

$$\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}+\frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}-\frac{2bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}+\frac{4b(dx)^{3/2}}{15c}$$

```
[Out] 4/15*b*(d*x)^(3/2)/c+2/5*b*d^(3/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))/c^(5
/2)+2/5*(d*x)^(5/2)*(a+b*arctanh(c*x))/d-2/5*b*d^(3/2)*arctanh(c^(1/2)*(d*x
)^(1/2)/d^(1/2))/c^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5916, 321, 329, 298, 205, 208}

$$\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}+\frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}-\frac{2bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}+\frac{4b(dx)^{3/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (4*b*(d*x)^(3/2))/(15*c) + (2*b*d^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]
)/(5*c^(5/2)) + (2*(d*x)^(5/2)*(a + b*ArcTanh[c*x]))/(5*d) - (2*b*d^(3/2)*A
rcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)dx=\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{(2bc)\int\frac{(dx)^{5/2}}{1-c^2x^2}\,dx}{5d}$$

$$=\frac{4b(dx)^{3/2}}{15c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{(2bd)\int\frac{\sqrt{dx}}{1-c^2x^2}\,dx}{5c}$$

$$=\frac{4b(dx)^{3/2}}{15c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{(4b)\operatorname{Subst}\left(\int\frac{x^2}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{5c}$$

$$=\frac{4b(dx)^{3/2}}{15c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{\left(2bd^2\right)\operatorname{Subst}\left(\int\frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right.}{5c^2}$$

$$=\frac{4b(dx)^{3/2}}{15c}+\frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{2bd^{3/2}\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.06, size = 115, normalized size = 1.08

$$\frac{(dx)^{3/2}\left(6ac^{5/2}x^{5/2}+4bc^{3/2}x^{3/2}+6bc^{5/2}x^{5/2}\tanh^{-1}(cx)+3b\log\left(1-\sqrt{c}\sqrt{x}\right)-3b\log\left(\sqrt{c}\sqrt{x}+1\right)+6b\tan^{-1}\right.}{15c^{5/2}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] ((d*x)^(3/2)*(4*b*c^(3/2)*x^(3/2) + 6*a*c^(5/2)*x^(5/2) + 6*b*ArcTan[Sqrt[c
]*Sqrt[x]] + 6*b*c^(5/2)*x^(5/2)*ArcTanh[c*x] + 3*b*Log[1 - Sqrt[c]*Sqrt[x]
] - 3*b*Log[1 + Sqrt[c]*Sqrt[x]]))/(15*c^(5/2)*x^(3/2))
```

fricas [A] time = 0.66, size = 255, normalized size = 2.41

$$\left[\frac{6\,bd\sqrt{\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}\,c\sqrt{\frac{d}{c}}}{d}\right)+3\,bd\sqrt{\frac{d}{c}}\log\left(\frac{cdx-2\,\sqrt{dx}\,c\sqrt{\frac{d}{c}}+d}{cx-1}\right)+\left(3\,bc^2dx^2\log\left(-\frac{cx+1}{cx-1}\right)+6\,ac^2dx^2+4\,bcdx\right)\sqrt{dx}}{15\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] [1/15*(6*b*d*sqrt(d/c)*arctan(sqrt(d*x)*c*sqrt(d/c)/d) + 3*b*d*sqrt(d/c)*lo
g((c*d*x - 2*sqrt(d*x)*c*sqrt(d/c) + d)/(c*x - 1)) + (3*b*c^2*d*x^2*log(-(c
*x + 1)/(c*x - 1)) + 6*a*c^2*d*x^2 + 4*b*c*d*x)*sqrt(d*x))/c^2, 1/15*(6*b*d
*sqrt(-d/c)*arctan(sqrt(d*x)*c*sqrt(-d/c)/d) + 3*b*d*sqrt(-d/c)*log((c*d*x
+ 2*sqrt(d*x)*c*sqrt(-d/c) - d)/(c*x + 1)) + (3*b*c^2*d*x^2*log(-(c*x + 1)/
(c*x - 1)) + 6*a*c^2*d*x^2 + 4*b*c*d*x)*sqrt(d*x))/c^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\left(b\,\mathrm{artanh}\left(cx\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b*arctanh(c*x) + a), x)
```

maple [A] time = 0.04, size = 93, normalized size = 0.88

$$\frac{2\,(dx)^{\frac{5}{2}}\,a}{5d}+\frac{2b\,(dx)^{\frac{5}{2}}\,\mathrm{arctanh}\,(cx)}{5d}+\frac{4b\,(dx)^{\frac{3}{2}}}{15c}+\frac{2d^2b\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5c^2\sqrt{cd}}-\frac{2d^2b\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5c^2\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a+b*arctanh(c*x)),x)
```

```
[Out] 2/5/d*(d*x)^(5/2)*a+2/5/d*b*(d*x)^(5/2)*arctanh(c*x)+4/15*b*(d*x)^(3/2)/c+2
/5*d^2*b/c^2/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2))-2/5*d^2*b/c^2/(c
*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

maxima [A] time = 0.43, size = 118, normalized size = 1.11

$$\frac{6\,(dx)^{\frac{5}{2}}\,a+\left(6\,(dx)^{\frac{5}{2}}\,\mathrm{artanh}\,(cx)+\frac{\left(\frac{4\,(dx)^{\frac{3}{2}}\,d^2}{c^2}+\frac{6\,d^4\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^3}+\frac{3\,d^4\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}\,c^3}\right)c}{d}\right)b}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/15*(6*(d*x)^(5/2)*a + (6*(d*x)^(5/2)*arctanh(c*x) + (4*(d*x)^(3/2)*d^2/c^
2 + 6*d^4*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*c^3) + 3*d^4*log((sqrt(d
*x)*c - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/(sqrt(c*d)*c^3))*c/d)*b)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{atanh}\,(c\,x))\,\,(d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d*x)^(3/2),x)
```

```
[Out] int((a + b*atanh(c*x))*(d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\,(a+b\,\mathrm{atanh}\,(cx))\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(a+b*atanh(c*x)),x)
```

```
[Out] Integral((d*x)**(3/2)*(a + b*atanh(c*x)), x)
```

3.37 $\int \sqrt{dx}\left(a+b\tanh^{-1}(cx)\right)dx$

Optimal. Leaf size=106

$$\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}-\frac{2b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}+\frac{4b\sqrt{dx}}{3c}$$

```
[Out] 2/3*(d*x)^(3/2)*(a+b*arctanh(c*x))/d-2/3*b*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/
2))*d^(1/2)/c^(3/2)-2/3*b*arctanh(c^(1/2)*(d*x)^(1/2)/d^(1/2))*d^(1/2)/c^(3
/2)+4/3*b*(d*x)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5916, 321, 329, 212, 208, 205}

$$\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}-\frac{2b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}+\frac{4b\sqrt{dx}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (4*b*Sqrt[d*x])/(3*c) - (2*b*Sqrt[d]*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(
3*c^(3/2)) + (2*(d*x)^(3/2)*(a + b*ArcTanh[c*x]))/(3*d) - (2*b*Sqrt[d]*ArcT
anh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{dx}\left(a+b\tanh^{-1}(cx)\right)dx = \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d} - \frac{(2bc)\int \frac{(dx)^{3/2}}{1-c^2x^2}\,dx}{3d}$$
$$= \frac{4b\sqrt{dx}}{3c} + \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d} - \frac{(2bd)\int \frac{1}{\sqrt{dx}\,(1-c^2x^2)}\,dx}{3c}$$
$$= \frac{4b\sqrt{dx}}{3c} + \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d} - \frac{(4b)\operatorname{Subst}\left(\int \frac{1}{1-\frac{c^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{3c}$$
$$= \frac{4b\sqrt{dx}}{3c} + \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d} - \frac{(2bd)\operatorname{Subst}\left(\int \frac{1}{d-cx^2}\,dx, x, \sqrt{dx}\right)}{3c} - (2b$$
$$= \frac{4b\sqrt{dx}}{3c} - \frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}} + \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d} - \frac{2b\sqrt{d}\tanh^{-1}(}{3c^{3/2}}$$

Mathematica [A] time = 0.05, size = 114, normalized size = 1.08

$$\frac{\sqrt{dx}\left(2ac^{3/2}x^{3/2}+2bc^{3/2}x^{3/2}\tanh^{-1}(cx)+4b\sqrt{c}\sqrt{x}+b\log\left(1-\sqrt{c}\sqrt{x}\right)-b\log\left(\sqrt{c}\sqrt{x}+1\right)-2b\tan^{-1}\left(\sqrt{c}\sqrt{x}\right.\right.}{3c^{3/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (Sqrt[d*x]*(4*b*Sqrt[c]*Sqrt[x] + 2*a*c^(3/2)*x^(3/2) - 2*b*ArcTan[Sqrt[c]*
Sqrt[x]] + 2*b*c^(3/2)*x^(3/2)*ArcTanh[c*x] + b*Log[1 - Sqrt[c]*Sqrt[x]] -
b*Log[1 + Sqrt[c]*Sqrt[x]]))/(3*c^(3/2)*Sqrt[x])
```

fricas [A] time = 0.75, size = 223, normalized size = 2.10

$$\left[-\frac{2b\sqrt{\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}\,c\sqrt{\frac{d}{c}}}{d}\right)-b\sqrt{\frac{d}{c}}\log\left(\frac{cdx-2\sqrt{dx}\,c\sqrt{\frac{d}{c}}+d}{cx-1}\right)-\left(bcx\log\left(-\frac{cx+1}{cx-1}\right)+2\,acx+4\,b\right)\sqrt{dx}}{3\,c}, \frac{2\,b\sqrt{-\frac{d}{c}}\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*b*sqrt(d/c)*arctan(sqrt(d*x)*c*sqrt(d/c)/d) - b*sqrt(d/c)*log((c*d
*x - 2*sqrt(d*x)*c*sqrt(d/c) + d)/(c*x - 1)) - (b*c*x*log(-(c*x + 1)/(c*x -
 1)) + 2*a*c*x + 4*b)*sqrt(d*x))/c, 1/3*(2*b*sqrt(-d/c)*arctan(sqrt(d*x)*c*
sqrt(-d/c)/d) + b*sqrt(-d/c)*log((c*d*x - 2*sqrt(d*x)*c*sqrt(-d/c) - d)/(c*
x + 1)) + (b*c*x*log(-(c*x + 1)/(c*x - 1)) + 2*a*c*x + 4*b)*sqrt(d*x))/c]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,(b\,\mathrm{artanh}\,(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b*arctanh(c*x) + a), x)
```

maple [A] time = 0.03, size = 89, normalized size = 0.84

$$\frac{2\,(dx)^{\frac{3}{2}}\,a}{3d}+\frac{2b\,(dx)^{\frac{3}{2}}\operatorname{arctanh}(cx)}{3d}+\frac{4b\sqrt{dx}}{3c}-\frac{2db\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3c\sqrt{cd}}-\frac{2db\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3c\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a+b*arctanh(c*x)),x)
```

```
[Out] 2/3/d*(d*x)^(3/2)*a+2/3/d*b*(d*x)^(3/2)*arctanh(c*x)+4/3*b*(d*x)^(1/2)/c-2/
3*d*b/c/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2))-2/3*d*b/c/(c*d)^(1/2)
*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

maxima [A] time = 0.43, size = 119, normalized size = 1.12

$$\frac{2\,(dx)^{\frac{3}{2}}\,a+\left(2\,(dx)^{\frac{3}{2}}\operatorname{artanh}(cx)-\frac{\left(\frac{2\,d^{3}\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^{2}}-\frac{d^{3}\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}\,c^{2}}-\frac{4\,\sqrt{dx}\,d^{2}}{c^{2}}\right)c}{d}\right)b}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*(2*(d*x)^(3/2)*a + (2*(d*x)^(3/2)*arctanh(c*x) - (2*d^3*arctan(sqrt(d*x
)*c/sqrt(c*d))/(sqrt(c*d)*c^2) - d^3*log((sqrt(d*x)*c - sqrt(c*d))/(sqrt(d*
x)*c + sqrt(c*d)))/(sqrt(c*d)*c^2) - 4*sqrt(d*x)*d^2/c^2)*c/d)*b)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{atanh}(c\,x))\,\sqrt{d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d*x)^(1/2),x)
```

```
[Out] int((a + b*atanh(c*x))*(d*x)^(1/2), x)
```

sympy [C] time = 11.73, size = 685, normalized size = 6.46

$$\frac{2a\,(dx)^{\frac{3}{2}}}{3d}+\frac{2b\left(\begin{cases}\frac{4c^{2}\sqrt{d}\,(dx)^{\frac{3}{2}}\sqrt{\frac{1}{c}}\operatorname{atanh}(cx)}{12c^{2}\sqrt{d}\sqrt{\frac{1}{c}}+12ic^{2}\sqrt{d}\sqrt{\frac{1}{c}}}+\frac{4ic^{2}\sqrt{d}\,(dx)^{\frac{3}{2}}\sqrt{\frac{1}{c}}\operatorname{atanh}(cx)}{12c^{2}\sqrt{d}\sqrt{\frac{1}{c}}+12ic^{2}\sqrt{d}\sqrt{\frac{1}{c}}}+\frac{2ic^{2}d^{2}\log\left(i\sqrt{d}\sqrt{\frac{1}{c}}+\sqrt{dx}\right)}{12c^{4}\sqrt{d}\sqrt{\frac{1}{c}}+12ic^{4}\sqrt{d}\sqrt{\frac{1}{c}}}+\frac{8cd^{\frac{3}{2}}\sqrt{dx}\sqrt{\frac{1}{c}}}{12c^{2}\sqrt{d}\sqrt{\frac{1}{c}}+12ic^{2}\sqrt{d}\sqrt{\frac{1}{c}}} \\ 0\end{cases}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(a+b*atanh(c*x)),x)
```

```
[Out] 2*a*(d*x)**(3/2)/(3*d) + 2*b*Piecewise((4*c**2*sqrt(d)*(d*x)**(3/2)*sqrt(1/
c)*atanh(c*x)/(12*c**2*sqrt(d)*sqrt(1/c) + 12*I*c**2*sqrt(d)*sqrt(1/c)) + 4
```

```
*I*c**2*sqrt(d)*(d*x)**(3/2)*sqrt(1/c)*atanh(c*x)/(12*c**2*sqrt(d)*sqrt(1/c
) + 12*I*c**2*sqrt(d)*sqrt(1/c)) + 2*I*c**2*d**2*log(I*sqrt(d)*sqrt(1/c) +
sqrt(d*x))/(12*c**4*sqrt(d)*sqrt(1/c) + 12*I*c**4*sqrt(d)*sqrt(1/c)) + 8*c*
d**(3/2)*sqrt(d*x)*sqrt(1/c)/(12*c**2*sqrt(d)*sqrt(1/c) + 12*I*c**2*sqrt(d)
*sqrt(1/c)) + 8*I*c*d**(3/2)*sqrt(d*x)*sqrt(1/c)/(12*c**2*sqrt(d)*sqrt(1/c)
 + 12*I*c**2*sqrt(d)*sqrt(1/c)) + 4*I*c*d**2*log(-sqrt(d)*sqrt(1/c) + sqrt(
d*x))/(12*c**3*sqrt(d)*sqrt(1/c) + 12*I*c**3*sqrt(d)*sqrt(1/c)) - 6*I*c*d**
2*log(I*sqrt(d)*sqrt(1/c) + sqrt(d*x))/(12*c**3*sqrt(d)*sqrt(1/c) + 12*I*c*
*3*sqrt(d)*sqrt(1/c)) + 4*I*c*d**2*atanh(c*x)/(12*c**3*sqrt(d)*sqrt(1/c) +
12*I*c**3*sqrt(d)*sqrt(1/c)) + 4*d**2*log(-sqrt(d)*sqrt(1/c) + sqrt(d*x))/(
12*c**2*sqrt(d)*sqrt(1/c) + 12*I*c**2*sqrt(d)*sqrt(1/c)) - 4*d**2*log(-I*sq
rt(d)*sqrt(1/c) + sqrt(d*x))/(12*c**2*sqrt(d)*sqrt(1/c) + 12*I*c**2*sqrt(d)
*sqrt(1/c)) + 4*d**2*atanh(c*x)/(12*c**2*sqrt(d)*sqrt(1/c) + 12*I*c**2*sqrt
(d)*sqrt(1/c)), Ne(c, 0)), (0, True))/d
```

3.38 $\int \frac{a+b\tanh^{-1}(cx)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{dx}\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{2b\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}$$

```
[Out] 2*b*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))/c^(1/2)/d^(1/2)-2*b*arctanh(c^(1/2)
*(d*x)^(1/2)/d^(1/2))/c^(1/2)/d^(1/2)+2*(a+b*arctanh(c*x))*(d*x)^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5916, 329, 298, 205, 208}

$$\frac{2\sqrt{dx}\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{2b\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/Sqrt[d*x],x]
```

```
[Out] (2*b*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[c]*Sqrt[d]) + (2*Sqrt[d*x]*
(a + b*ArcTanh[c*x]))/d - (2*b*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[
c]*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}\,\left(a+b\tanh^{-1}(cx)\right)}{d} - \frac{(2bc)\int \frac{\sqrt{dx}}{1-c^2x^2}\,dx}{d}$$

$$= \frac{2\sqrt{dx}\,\left(a+b\tanh^{-1}(cx)\right)}{d} - \frac{(4bc)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d^2}$$

$$= \frac{2\sqrt{dx}\,\left(a+b\tanh^{-1}(cx)\right)}{d} - (2b)\,\text{Subst}\left(\int \frac{1}{d-cx^2}\,dx, x, \sqrt{dx}\right) + (2b)\,\text{Subst}\left(\int \frac{1}{d+cx}\right.$$

$$= \frac{2b\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}} + \frac{2\sqrt{dx}\,\left(a+b\tanh^{-1}(cx)\right)}{d} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}$$

Mathematica [A] time = 0.03, size = 98, normalized size = 1.15

$$\frac{\sqrt{x}\left(2a\sqrt{c}\sqrt{x}+b\log\left(1-\sqrt{c}\sqrt{x}\right)-b\log\left(\sqrt{c}\sqrt{x}+1\right)+2b\tan^{-1}\left(\sqrt{c}\sqrt{x}\right)+2b\sqrt{c}\sqrt{x}\tanh^{-1}(cx)\right)}{\sqrt{c}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/Sqrt[d*x],x]
```

```
[Out] (Sqrt[x]*(2*a*Sqrt[c]*Sqrt[x] + 2*b*ArcTan[Sqrt[c]*Sqrt[x]] + 2*b*Sqrt[c]*S
qrt[x]*ArcTanh[c*x] + b*Log[1 - Sqrt[c]*Sqrt[x]] - b*Log[1 + Sqrt[c]*Sqrt[x
]]))/(Sqrt[c]*Sqrt[d*x])
```

fricas [A] time = 0.55, size = 211, normalized size = 2.48

$$\left[-\frac{2\sqrt{cd}\,b\arctan\left(\frac{\sqrt{cd}\sqrt{dx}}{cdx}\right)-\sqrt{cd}\,b\log\left(\frac{cdx-2\sqrt{cd}\sqrt{dx}+d}{cx-1}\right)-\left(bc\log\left(-\frac{cx+1}{cx-1}\right)+2\,ac\right)\sqrt{dx}}{cd}, \frac{2\sqrt{-cd}\,b\arctan\left(\frac{\sqrt{-cd}\sqrt{}}{cdx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-(2*sqrt(c*d)*b*arctan(sqrt(c*d)*sqrt(d*x)/(c*d*x)) - sqrt(c*d)*b*log((c*d
*x - 2*sqrt(c*d)*sqrt(d*x) + d)/(c*x - 1)) - (b*c*log(-(c*x + 1)/(c*x - 1))
 + 2*a*c)*sqrt(d*x))/(c*d), (2*sqrt(-c*d)*b*arctan(sqrt(-c*d)*sqrt(d*x)/(c*
d*x)) - sqrt(-c*d)*b*log((c*d*x - 2*sqrt(-c*d)*sqrt(d*x) - d)/(c*x + 1)) +
(b*c*log(-(c*x + 1)/(c*x - 1)) + 2*a*c)*sqrt(d*x))/(c*d)]
```

giac [A] time = 0.15, size = 88, normalized size = 1.04

$$\frac{\left(2\,cd\left(\frac{\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}+\frac{\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{-cd}}\right)}{\sqrt{-cd}\,c}\right)+\sqrt{dx}\,\log\left(-\frac{cx+1}{cx-1}\right)\right)b+2\sqrt{dx}\,a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] ((2*c*d*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*c) + arctan(sqrt(d*x)*c/s
qrt(-c*d))/(sqrt(-c*d)*c)) + sqrt(d*x)*log(-(c*x + 1)/(c*x - 1)))*b + 2*sqr
t(d*x)*a)/d
```

maple [A] time = 0.04, size = 70, normalized size = 0.82

$$\frac{2a\sqrt{dx}}{d}+\frac{2b\sqrt{dx}\ \operatorname{arctanh}(cx)}{d}+\frac{2b\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{\sqrt{cd}}-\frac{2b\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(1/2),x)
```

```
[Out] 2/d*a*(d*x)^(1/2)+2/d*b*(d*x)^(1/2)*arctanh(c*x)+2*b/(c*d)^(1/2)*arctan(c*(
d*x)^(1/2)/(c*d)^(1/2))-2*b/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

maxima [A] time = 0.43, size = 103, normalized size = 1.21

$$\frac{\left(2\sqrt{dx}\ \operatorname{artanh}(cx)+\frac{\left(\frac{2d^2\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}c}+\frac{d^2\log\left(\frac{\sqrt{dx}c-\sqrt{cd}}{\sqrt{dx}c+\sqrt{cd}}\right)}{\sqrt{cd}c}\right)c}{d}\right)b+2\sqrt{dx}a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] ((2*sqrt(d*x)*arctanh(c*x) + (2*d^2*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d
)*c) + d^2*log((sqrt(d*x)*c - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/(sqrt(c
*d)*c))*c/d)*b + 2*sqrt(d*x)*a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{atanh}(c\,x)}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d*x)^(1/2),x)
```

```
[Out] int((a + b*atanh(c*x))/(d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{atanh}(cx)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(1/2),x)
```

```
[Out] Integral((a + b*atanh(c*x))/sqrt(d*x), x)
```

3.39 $\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{2b\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{2b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

```
[Out] 2*b*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))*c^(1/2)/d^(3/2)+2*b*arctanh(c^(1/2)
*(d*x)^(1/2)/d^(1/2))*c^(1/2)/d^(3/2)-2*(a+b*arctanh(c*x))/d/(d*x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5916, 329, 212, 208, 205}

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{2b\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{2b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d*x)^(3/2),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*(a + b*ArcTa
nh[c*x]))/(d*Sqrt[d*x]) + (2*b*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]
)/d^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{3/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}} + \frac{(2bc)\int \frac{1}{\sqrt{dx}\left(1-c^2x^2\right)}\,dx}{d}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}} + \frac{(4bc)\,\text{Subst}\left(\int \frac{1}{1-\frac{c^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d^2}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}} + \frac{(2bc)\,\text{Subst}\left(\int \frac{1}{d-cx^2}\,dx, x, \sqrt{dx}\right)}{d} + \frac{(2bc)\,\text{Subst}\left(\int \frac{1}{d+cx^2}\,dx\right.}{d}$$

$$= \frac{2b\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}} + \frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

Mathematica [A] time = 0.05, size = 99, normalized size = 1.16

$$\frac{x\left(-2a-b\sqrt{c}\sqrt{x}\log\left(1-\sqrt{c}\sqrt{x}\right)+b\sqrt{c}\sqrt{x}\log\left(\sqrt{c}\sqrt{x}+1\right)+2b\sqrt{c}\sqrt{x}\tan^{-1}\left(\sqrt{c}\sqrt{x}\right)-2b\tanh^{-1}(cx)\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d*x)^(3/2),x]
```

```
[Out] (x*(-2*a + 2*b*Sqrt[c]*Sqrt[x]*ArcTan[Sqrt[c]*Sqrt[x]] - 2*b*ArcTanh[c*x] -
 b*Sqrt[c]*Sqrt[x]*Log[1 - Sqrt[c]*Sqrt[x]] + b*Sqrt[c]*Sqrt[x]*Log[1 + Sqr
t[c]*Sqrt[x]]))/(d*x)^(3/2)
```

fricas [A] time = 0.65, size = 221, normalized size = 2.60

$$\left[-\frac{2\,bdx\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right) - bdx\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right) + \sqrt{dx}\left(b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a\right)}{d^2x}, -\frac{2\,bdx\sqrt{-\frac{c}{d}}\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(2*b*d*x*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) - b*d*x*sqrt(c/d)*lo
g((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) + sqrt(d*x)*(b*log(-(c*x + 1
)/(c*x - 1)) + 2*a))/(d^2*x), -(2*b*d*x*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c
/d)/(c*x)) - b*d*x*sqrt(-c/d)*log((c*x + 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x +
 1)) + sqrt(d*x)*(b*log(-(c*x + 1)/(c*x - 1)) + 2*a))/(d^2*x)]
```

giac [A] time = 0.17, size = 93, normalized size = 1.09

$$\frac{2\,bcd\left(\frac{\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,d} - \frac{\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{-cd}}\right)}{\sqrt{-cd}\,d}\right) - \frac{b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}} - \frac{2\,a}{\sqrt{dx}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] (2*b*c*d*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d) - arctan(sqrt(d*x)*c/
sqrt(-c*d))/(sqrt(-c*d)*d)) - b*log(-(c*d*x + d)/(c*d*x - d))/sqrt(d*x) - 2
*a/sqrt(d*x))/d
```

maple [A] time = 0.03, size = 78, normalized size = 0.92

$$-\frac{2a}{d\sqrt{dx}}-\frac{2b\operatorname{arctanh}(cx)}{d\sqrt{dx}}+\frac{2bc\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{d\sqrt{cd}}+\frac{2bc\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{d\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(3/2),x)
```

```
[Out] -2/d*a/(d*x)^(1/2)-2/d*b/(d*x)^(1/2)*arctanh(c*x)+2/d*b*c/(c*d)^(1/2)*arcta
n(c*(d*x)^(1/2)/(c*d)^(1/2))+2/d*b*c/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d
)^(1/2))
```

maxima [A] time = 0.43, size = 94, normalized size = 1.11

$$\frac{b\left(\frac{\left(\frac{2\,d\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}}-\frac{d\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}}\right)c}{d}-\frac{2\operatorname{artanh}(cx)}{\sqrt{dx}}\right)-\frac{2\,a}{\sqrt{dx}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] (b*((2*d*arctan(sqrt(d*x)*c/sqrt(c*d))/sqrt(c*d) - d*log((sqrt(d*x)*c - sqr
t(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/sqrt(c*d))*c/d - 2*arctanh(c*x)/sqrt(d*x
)) - 2*a/sqrt(d*x))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{atanh}(c\,x)}{(d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d*x)^(3/2),x)
```

```
[Out] int((a + b*atanh(c*x))/(d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{atanh}(cx)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(3/2),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d*x)**(3/2), x)
```

3.40 $\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}-\frac{2bc^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4bc}{3d^2\sqrt{dx}}$$

```
[Out] -2/3*b*c^(3/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(5/2)-2/3*(a+b*arctanh
(c*x))/d/(d*x)^(3/2)+2/3*b*c^(3/2)*arctanh(c^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(
5/2)-4/3*b*c/d^2/(d*x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5916, 325, 329, 298, 205, 208}

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}-\frac{2bc^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4bc}{3d^2\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d*x)^(5/2),x]
```

```
[Out] (-4*b*c)/(3*d^2*Sqrt[d*x]) - (2*b*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d
]])/(3*d^(5/2)) - (2*(a + b*ArcTanh[c*x]))/(3*d*(d*x)^(3/2)) + (2*b*c^(3/2)
*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{5/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{(2bc)\int \frac{1}{(dx)^{3/2}\left(1-c^2x^2\right)}\,dx}{3d}$$

$$= -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{\left(2bc^3\right)\int \frac{\sqrt{dx}}{1-c^2x^2}\,dx}{3d^3}$$

$$= -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{\left(4bc^3\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{3d^4}$$

$$= -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{\left(2bc^2\right)\text{Subst}\left(\int \frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)}{3d^2}-\frac{\left(2bc^2\right)\text{Subst}}{}$$

$$= -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2bc^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{2bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}$$

Mathematica [A] time = 0.07, size = 107, normalized size = 1.00

$$-\frac{x\left(2a+bc^{3/2}x^{3/2}\log\left(1-\sqrt{c}\sqrt{x}\right)-bc^{3/2}x^{3/2}\log\left(\sqrt{c}\sqrt{x}+1\right)+2bc^{3/2}x^{3/2}\tan^{-1}\left(\sqrt{c}\sqrt{x}\right)+4bcx+2b\tanh^{-1}(c\right.}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d*x)^(5/2),x]
```

```
[Out] -1/3*(x*(2*a + 4*b*c*x + 2*b*c^(3/2)*x^(3/2)*ArcTan[Sqrt[c]*Sqrt[x]] + 2*b*
ArcTanh[c*x] + b*c^(3/2)*x^(3/2)*Log[1 - Sqrt[c]*Sqrt[x]] - b*c^(3/2)*x^(3/
2)*Log[1 + Sqrt[c]*Sqrt[x]]))/(d*x)^(5/2)
```

fricas [A] time = 1.27, size = 243, normalized size = 2.27

$$\left[\frac{2\,bcdx^2\sqrt{\frac{c}{d}}\,\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)+bcdx^2\sqrt{\frac{c}{d}}\,\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)-\left(4\,bcx+b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a\right)\sqrt{dx}}{3\,d^3x^2},-\frac{2\,bcdx^2\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(2*b*c*d*x^2*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) + b*c*d*x^2*s
qrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) - (4*b*c*x + b*lo
g(-(c*x + 1)/(c*x - 1)) + 2*a)*sqrt(d*x))/(d^3*x^2), -1/3*(2*b*c*d*x^2*sqrt
(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - b*c*d*x^2*sqrt(-c/d)*log((c*x -
 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) + (4*b*c*x + b*log(-(c*x + 1)/(c*x
- 1)) + 2*a)*sqrt(d*x))/(d^3*x^2)]
```

giac [A] time = 0.19, size = 117, normalized size = 1.09

$$-\frac{\frac{2\,bc^2\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,d}+\frac{2\,bc^2\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{-cd}}\right)}{\sqrt{-cd}\,d}+\frac{b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}\,dx}+\frac{2\,(2\,bcdx+ad)}{\sqrt{dx}\,d^2x}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(2*b*c^2*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d) + 2*b*c^2*arctan(
sqrt(d*x)*c/sqrt(-c*d))/(sqrt(-c*d)*d) + b*log(-(c*d*x + d)/(c*d*x - d))/(s
qrt(d*x)*d*x) + 2*(2*b*c*d*x + a*d)/(sqrt(d*x)*d^2*x))/d
```

maple [A] time = 0.04, size = 94, normalized size = 0.88

$$-\frac{2a}{3d\,(dx)^{\frac{3}{2}}}-\frac{2b\operatorname{arctanh}(cx)}{3d\,(dx)^{\frac{3}{2}}}-\frac{4bc}{3d^2\sqrt{dx}}-\frac{2b\,c^2\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3d^2\sqrt{cd}}+\frac{2b\,c^2\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3d^2\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(5/2),x)
```

```
[Out] -2/3/d*a/(d*x)^(3/2)-2/3/d*b/(d*x)^(3/2)*arctanh(c*x)-4/3*b*c/d^2/(d*x)^(1/
2)-2/3/d^2*b*c^2/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2))+2/3/d^2*b*c^
2/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

maxima [A] time = 0.43, size = 101, normalized size = 0.94

$$-\frac{b\left(\frac{\left(\frac{2\,c\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{c\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{4}{\sqrt{dx}}\right)c}{d}+\frac{2\operatorname{artanh}(cx)}{(dx)^{\frac{3}{2}}}\right)+\frac{2\,a}{(dx)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(b*((2*c*arctan(sqrt(d*x)*c/sqrt(c*d))/sqrt(c*d) + c*log((sqrt(d*x)*c
- sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/sqrt(c*d) + 4/sqrt(d*x))*c/d + 2*ar
ctanh(c*x)/(d*x)^(3/2)) + 2*a/(d*x)^(3/2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{atanh}(c\,x)}{(d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d*x)^(5/2),x)
```

```
[Out] int((a + b*atanh(c*x))/(d*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{atanh}(cx)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(5/2),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d*x)**(5/2), x)
```

3.41 $\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{2bc^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{2bc^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{4bc}{15d^2(dx)^{3/2}}$$

```
[Out] -4/15*b*c/d^2/(d*x)^(3/2)+2/5*b*c^(5/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))
/d^(7/2)-2/5*(a+b*arctanh(c*x))/d/(d*x)^(5/2)+2/5*b*c^(5/2)*arctanh(c^(1/2)
*(d*x)^(1/2)/d^(1/2))/d^(7/2)
```

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5916, 325, 329, 212, 208, 205}

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{2bc^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{2bc^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{4bc}{15d^2(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d*x)^(7/2),x]
```

```
[Out] (-4*b*c)/(15*d^2*(d*x)^(3/2)) + (2*b*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqr
t[d]])/(5*d^(7/2)) - (2*(a + b*ArcTanh[c*x]))/(5*d*(d*x)^(5/2)) + (2*b*c^(5
/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*d^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{7/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{(2bc)\int \frac{1}{(dx)^{5/2}\left(1-c^2x^2\right)}\,dx}{5d}$$

$$= -\frac{4bc}{15d^2(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{\left(2bc^3\right)\int \frac{1}{\sqrt{dx}\left(1-c^2x^2\right)}\,dx}{5d^3}$$

$$= -\frac{4bc}{15d^2(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{\left(4bc^3\right)\text{Subst}\left(\int \frac{1}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{5d^4}$$

$$= -\frac{4bc}{15d^2(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{\left(2bc^3\right)\text{Subst}\left(\int \frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)}{5d^3}+\frac{\left(2bc^3\right)\text{Sub}}{}$$

$$= -\frac{4bc}{15d^2(dx)^{3/2}}+\frac{2bc^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{2bc^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}$$

Mathematica [A] time = 0.05, size = 108, normalized size = 1.01

$$\frac{x\left(-6a-3bc^{5/2}x^{5/2}\log\left(1-\sqrt{c}\sqrt{x}\right)+3bc^{5/2}x^{5/2}\log\left(\sqrt{c}\sqrt{x}+1\right)+6bc^{5/2}x^{5/2}\tan^{-1}\left(\sqrt{c}\sqrt{x}\right)-4bcx-6b\tanh^{-1}\right.}{15(dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d*x)^(7/2),x]
```

```
[Out] (x*(-6*a - 4*b*c*x + 6*b*c^(5/2)*x^(5/2)*ArcTan[Sqrt[c]*Sqrt[x]] - 6*b*ArcT
anh[c*x] - 3*b*c^(5/2)*x^(5/2)*Log[1 - Sqrt[c]*Sqrt[x]] + 3*b*c^(5/2)*x^(5/
2)*Log[1 + Sqrt[c]*Sqrt[x]]))/(15*(d*x)^(7/2))
```

fricas [A] time = 0.65, size = 253, normalized size = 2.36

$$\left[-\frac{6\,bc^2dx^3\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)-3\,bc^2dx^3\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)+\left(4\,bcx+3\,b\log\left(-\frac{cx+1}{cx-1}\right)+6\,a\right)\sqrt{dx}}{15\,d^4x^3},-\frac{6\,bc^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(6*b*c^2*d*x^3*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) - 3*b*c^2
*d*x^3*sqrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) + (4*b*c*
x + 3*b*log(-(c*x + 1)/(c*x - 1)) + 6*a)*sqrt(d*x))/(d^4*x^3), -1/15*(6*b*c
^2*d*x^3*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - 3*b*c^2*d*x^3*sqrt
(-c/d)*log((c*x + 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) + (4*b*c*x + 3*b*l
og(-(c*x + 1)/(c*x - 1)) + 6*a)*sqrt(d*x))/(d^4*x^3)]
```

giac [A] time = 0.20, size = 117, normalized size = 1.09

$$\frac{6\,bc^3\left(\frac{\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^2}-\frac{\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{-cd}}\right)}{\sqrt{-cd}\,d^2}\right)-\frac{3\,b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}\,d^2x^2}-\frac{2\,(2\,bcdx+3\,ad)}{\sqrt{dx}\,d^3x^2}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(6*b*c^3*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d^2) - arctan(sqrt(
d*x)*c/sqrt(-c*d))/(sqrt(-c*d)*d^2)) - 3*b*log(-(c*d*x + d)/(c*d*x - d))/(s
qrt(d*x)*d^2*x^2) - 2*(2*b*c*d*x + 3*a*d)/(sqrt(d*x)*d^3*x^2))/d
```

maple [A] time = 0.03, size = 94, normalized size = 0.88

$$-\frac{2a}{5d\,(dx)^{\frac{5}{2}}}-\frac{2b\arctanh\,(cx)}{5d\,(dx)^{\frac{5}{2}}}-\frac{4bc}{15d^2\,(dx)^{\frac{3}{2}}}+\frac{2b\,c^3\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5d^3\sqrt{cd}}+\frac{2b\,c^3\arctanh\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5d^3\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(7/2),x)
```

```
[Out] -2/5/d*a/(d*x)^(5/2)-2/5/d*b/(d*x)^(5/2)*arctanh(c*x)-4/15*b*c/d^2/(d*x)^(3
/2)+2/5/d^3*b*c^3/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2))+2/5/d^3*b*c
^3/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

maxima [A] time = 0.43, size = 112, normalized size = 1.05

$$\frac{b\left(\frac{\left(\frac{6\,c^2\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,d}-\frac{3\,c^2\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}\,d}-\frac{4}{(dx)^{\frac{3}{2}}}\right)c}{d}-\frac{6\;\mathrm{artanh}(cx)}{(dx)^{\frac{5}{2}}}\right)-\frac{6\,a}{(dx)^{\frac{5}{2}}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(b*((6*c^2*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d) - 3*c^2*log((sq
rt(d*x)*c - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/(sqrt(c*d)*d) - 4/(d*x)^(
3/2))*c/d - 6*arctanh(c*x)/(d*x)^(5/2)) - 6*a/(d*x)^(5/2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x\right)}{{\left(d\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d*x)^(7/2),x)
```

```
[Out] int((a + b*atanh(c*x))/(d*x)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{atanh}\,(cx)}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(7/2),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d*x)**(7/2), x)
```

3.42 $\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{9/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}-\frac{2bc^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}+\frac{2bc^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{4bc}{35d^2(dx)^{5/2}}$$

```
[Out] -4/35*b*c/d^2/(d*x)^(5/2)-2/7*b*c^(7/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))
/d^(9/2)-2/7*(a+b*arctanh(c*x))/d/(d*x)^(7/2)+2/7*b*c^(7/2)*arctanh(c^(1/2)
*(d*x)^(1/2)/d^(1/2))/d^(9/2)-4/7*b*c^3/d^4/(d*x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5916, 325, 329, 298, 205, 208}

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{2bc^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}+\frac{2bc^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{4bc}{35d^2(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d*x)^(9/2),x]
```

```
[Out] (-4*b*c)/(35*d^2*(d*x)^(5/2)) - (4*b*c^3)/(7*d^4*Sqrt[d*x]) - (2*b*c^(7/2)*
ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) - (2*(a + b*ArcTanh[c*x]))
/(7*d*(d*x)^(7/2)) + (2*b*c^(7/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*
d^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{9/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}} + \frac{(2bc)\int \frac{1}{(dx)^{7/2}\left(1-c^2x^2\right)}\,dx}{7d}$$

$$= -\frac{4bc}{35d^2(dx)^{5/2}} - \frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}} + \frac{\left(2bc^3\right)\int \frac{1}{(dx)^{3/2}\left(1-c^2x^2\right)}\,dx}{7d^3}$$

$$= -\frac{4bc}{35d^2(dx)^{5/2}} - \frac{4bc^3}{7d^4\sqrt{dx}} - \frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}} + \frac{\left(2bc^5\right)\int \frac{\sqrt{dx}}{1-c^2x^2}\,dx}{7d^5}$$

$$= -\frac{4bc}{35d^2(dx)^{5/2}} - \frac{4bc^3}{7d^4\sqrt{dx}} - \frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}} + \frac{\left(4bc^5\right)\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{7d^6}$$

$$= -\frac{4bc}{35d^2(dx)^{5/2}} - \frac{4bc^3}{7d^4\sqrt{dx}} - \frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}} + \frac{\left(2bc^4\right)\mathrm{Subst}\left(\int \frac{1}{d-cx^2}\,dx, x, \sqrt{dx}\right)}{7d^4} -$$

$$= -\frac{4bc}{35d^2(dx)^{5/2}} - \frac{4bc^3}{7d^4\sqrt{dx}} - \frac{2bc^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} - \frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}} + \frac{2bc^{7/2}\tanh}{7d}$$

Mathematica [A] time = 0.07, size = 122, normalized size = 0.98

$$-\frac{\sqrt{dx}\left(10a+5bc^{7/2}x^{7/2}\log\left(1-\sqrt{c}\sqrt{x}\right)-5bc^{7/2}x^{7/2}\log\left(\sqrt{c}\sqrt{x}+1\right)+10bc^{7/2}x^{7/2}\tan^{-1}\left(\sqrt{c}\sqrt{x}\right)+20bc^3x^3+4\right.}{35d^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d*x)^(9/2),x]
```

```
[Out] -1/35*(Sqrt[d*x]*(10*a + 4*b*c*x + 20*b*c^3*x^3 + 10*b*c^(7/2)*x^(7/2)*ArcT
an[Sqrt[c]*Sqrt[x]] + 10*b*ArcTanh[c*x] + 5*b*c^(7/2)*x^(7/2)*Log[1 - Sqrt[
c]*Sqrt[x]] - 5*b*c^(7/2)*x^(7/2)*Log[1 + Sqrt[c]*Sqrt[x]]))/(d^5*x^4)
```

fricas [A] time = 1.48, size = 272, normalized size = 2.18

$$\left[\frac{10\,bc^3dx^4\sqrt{\frac{c}{d}}\,\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)+5\,bc^3dx^4\sqrt{\frac{c}{d}}\,\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)-\left(20\,bc^3x^3+4\,bcx+5\,b\log\left(-\frac{cx+1}{cx-1}\right)+10\,a\right)}{35\,d^5x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/35*(10*b*c^3*d*x^4*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) + 5*b*c^3
*d*x^4*sqrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) - (20*b*c
^3*x^3 + 4*b*c*x + 5*b*log(-(c*x + 1)/(c*x - 1)) + 10*a)*sqrt(d*x))/(d^5*x^
4), -1/35*(10*b*c^3*d*x^4*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - 5
*b*c^3*d*x^4*sqrt(-c/d)*log((c*x - 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) +
 (20*b*c^3*x^3 + 4*b*c*x + 5*b*log(-(c*x + 1)/(c*x - 1)) + 10*a)*sqrt(d*x))
/(d^5*x^4)]
```

giac [A] time = 0.20, size = 135, normalized size = 1.08

$$-\frac{\frac{10\,bc^4\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^3}+\frac{10\,bc^4\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{-cd}}\right)}{\sqrt{-cd}\,d^3}+\frac{5\,b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}\,d^3x^3}+\frac{2\left(10\,bc^3d^3x^3+2\,bcd^3x+5\,ad^3\right)}{\sqrt{dx}\,d^6x^3}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(10*b*c^4*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d^3) + 10*b*c^4*ar
ctan(sqrt(d*x)*c/sqrt(-c*d))/(sqrt(-c*d)*d^3) + 5*b*log(-(c*d*x + d)/(c*d*x
 - d))/(sqrt(d*x)*d^3*x^3) + 2*(10*b*c^3*d^3*x^3 + 2*b*c*d^3*x + 5*a*d^3)/(
sqrt(d*x)*d^6*x^3))/d
```

maple [A] time = 0.04, size = 108, normalized size = 0.86

$$-\frac{2a}{7d\,(dx)^{\frac{7}{2}}}-\frac{2b\,\mathrm{arctanh}\,(cx)}{7d\,(dx)^{\frac{7}{2}}}-\frac{4bc}{35d^2\,(dx)^{\frac{5}{2}}}-\frac{4b\,c^3}{7d^4\sqrt{dx}}-\frac{2b\,c^4\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7d^4\sqrt{cd}}+\frac{2b\,c^4\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7d^4\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(9/2),x)
```

```
[Out] -2/7/d*a/(d*x)^(7/2)-2/7/d*b/(d*x)^(7/2)*arctanh(c*x)-4/35*b*c/d^2/(d*x)^(5
/2)-4/7*b*c^3/d^4/(d*x)^(1/2)-2/7/d^4*b*c^4/(c*d)^(1/2)*arctan(c*(d*x)^(1/2
)/(c*d)^(1/2))+2/7/d^4*b*c^4/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

maxima [A] time = 0.42, size = 130, normalized size = 1.04

$$-\frac{b\left(\frac{\left(\frac{10\,c^3\arctan\left(\frac{\sqrt{dx}\,c}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^2}+\frac{5\,c^3\log\left(\frac{\sqrt{dx}\,c-\sqrt{cd}}{\sqrt{dx}\,c+\sqrt{cd}}\right)}{\sqrt{cd}\,d^2}+\frac{4\left(5\,c^2d^2x^2+d^2\right)}{(dx)^{\frac{5}{2}}\,d^2}\right)c}{d}+\frac{10\,\mathrm{artanh}(cx)}{(dx)^{\frac{7}{2}}}\right)+\frac{10\,a}{(dx)^{\frac{7}{2}}}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(b*((10*c^3*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d^2) + 5*c^3*log
((sqrt(d*x)*c - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/(sqrt(c*d)*d^2) + 4*(
5*c^2*d^2*x^2 + d^2)/((d*x)^(5/2)*d^2))*c/d + 10*arctanh(c*x)/(d*x)^(7/2))
+ 10*a/(d*x)^(7/2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x\right)}{{\left(d\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))/(d*x)^(9/2),x)
```

```
[Out] int((a + b*atanh(c*x))/(d*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(9/2),x)
```

```
[Out] Timed out
```

3.43 $\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left((dx)^m \left(a+b\tanh^{-1}(cx)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x))^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx=\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx$$

Mathematica [A] time = 3.83, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^3, x]
```

fricas [A] time = 1.08, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^3\,\mathrm{artanh}\,(cx)^3+3\,ab^2\,\mathrm{artanh}\,(cx)^2+3\,a^2b\,\mathrm{artanh}\,(cx)+a^3\right)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\mathrm{artanh}\,(cx)+a)^3(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)^3*(d*x)^m, x)
```

maple [A] time = 1.76, size = 0, normalized size = 0.00

$$\int (dx)^m\,(a+b\,\mathrm{arctanh}\,(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3 d^m x x^m \log\left(-cx+1\right)^3}{8\left(m+1\right)}+\frac{(dx)^{m+1}\,a^3}{d(m+1)}+\int\frac{\left(b^3cd^m(m+1)x-b^3d^m(m+1)\right)x^m\log\left(cx+1\right)^3+6\left(ab^2cd^m(m+1)x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*d^m*x*x^m*log(-c*x + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1))
+ integrate(1/8*((b^3*c*d^m*(m + 1)*x - b^3*d^m*(m + 1))*x^m*log(c*x + 1)^3
 + 6*(a*b^2*c*d^m*(m + 1)*x - a*b^2*d^m*(m + 1))*x^m*log(c*x + 1)^2 + 12*(a
^2*b*c*d^m*(m + 1)*x - a^2*b*d^m*(m + 1))*x^m*log(c*x + 1) + 3*((b^3*c*d^m*
(m + 1)*x - b^3*d^m*(m + 1))*x^m*log(c*x + 1) - (2*a*b^2*d^m*(m + 1) - (2*a
*b^2*c*d^m*(m + 1) + b^3*c*d^m)*x)*x^m)*log(-c*x + 1)^2 - 3*((b^3*c*d^m*(m
+ 1)*x - b^3*d^m*(m + 1))*x^m*log(c*x + 1)^2 + 4*(a*b^2*c*d^m*(m + 1)*x - a
*b^2*d^m*(m + 1))*x^m*log(c*x + 1) + 4*(a^2*b*c*d^m*(m + 1)*x - a^2*b*d^m*(
m + 1))*x^m)*log(-c*x + 1))/(c*(m + 1)*x - m - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int {\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^3\,{\left(d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^3*(d*x)^m,x)
```

```
[Out] int((a + b*atanh(c*x))^3*(d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx\right)^{m} \left(a + b \operatorname{atanh}{\left(cx \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x))**3, x)
```

3.44 $\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}(cx)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x))^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$$

Mathematica [A] time = 2.52, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^2, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{artanh}(cx) + a)^2 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*(d*x)^m, x)
```

maple [A] time = 1.41, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{arctanh}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2 d^m x x^m \log(-cx+1)^2}{4(m+1)} + \frac{(dx)^{m+1} a^2}{d(m+1)} - \int -\frac{\left(b^2 c d^m (m+1)x - b^2 d^m (m+1)\right) x^m \log(cx+1)^2 + 4\left(abcd^m (m+1)x - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d^m*x*x^m*log(-c*x + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1)) -
 integrate(-1/4*((b^2*c*d^m*(m + 1)*x - b^2*d^m*(m + 1))*x^m*log(c*x + 1)^2
 + 4*(a*b*c*d^m*(m + 1)*x - a*b*d^m*(m + 1))*x^m*log(c*x + 1) - 2*((b^2*c*d
^m*(m + 1)*x - b^2*d^m*(m + 1))*x^m*log(c*x + 1) - (2*a*b*d^m*(m + 1) - (2*
a*b*c*d^m*(m + 1) + b^2*c*d^m)*x)*x^m)*log(-c*x + 1))/(c*(m + 1)*x - m - 1)
, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (a + b\,\mathrm{atanh}\,(c\,x))^2\,(d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^2*(d*x)^m,x)
```

```
[Out] int((a + b*atanh(c*x))^2*(d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b\,\mathrm{atanh}\,(cx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x))**2, x)
```

3.45 $\int (dx)^m \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=72

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}(cx)\right)}{d(m+1)}-\frac{bc(dx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{d^2(m+1)(m+2)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctanh(c*x))/d/(1+m)-b*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*
m],[2+1/2*m],c^2*x^2)/d^2/(1+m)/(2+m)
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5916, 364}

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}(cx)\right)}{d(m+1)}-\frac{bc(dx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{d^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d^2*(1 + m)*(2 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+b\tanh^{-1}(cx)\right)dx&=\frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx)\right)}{d(1+m)}-\frac{(bc)\int\frac{(dx)^{1+m}}{1-c^2x^2}\,dx}{d(1+m)}\\&=\frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx)\right)}{d(1+m)}-\frac{bc(dx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};c^2x^2\right)}{d^2(1+m)(2+m)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.82

$$-\frac{x(dx)^m\left(bcx\,{}_2F_1\left(1,\frac{m}{2}+1;\frac{m}{2}+2;c^2x^2\right)-(m+2)\left(a+b\tanh^{-1}(cx)\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x]),x]
```

```
[Out] -((x*(d*x)^m*(-((2 + m)*(a + b*ArcTanh[c*x])) + b*c*x*Hypergeometric2F1[1,
1 + m/2, 2 + m/2, c^2*x^2]))/((1 + m)*(2 + m)))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left((b\operatorname{artanh}(cx) + a)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{artanh}(cx) + a)(dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(d*x)^m, x)
```

maple [F] time = 1.47, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b\operatorname{arctanh}(cx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(2\,cd^m\int \frac{xx^m}{c^2(m+1)x^2 - m - 1}\,dx + \frac{d^m xx^m\log(cx+1) - d^m xx^m\log(-cx+1)}{m+1}\right)b + \frac{(dx)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*d^m*integrate(x*x^m/(c^2*(m + 1)*x^2 - m - 1), x) + (d^m*x*x^m*log
(c*x + 1) - d^m*x*x^m*log(-c*x + 1))/(m + 1))*b + (d*x)^(m + 1)*a/(d*(m + 1
))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\operatorname{atanh}(c\,x))\,(d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))*(d*x)^m,x)
```

```
[Out] int((a + b*atanh(c*x))*(d*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b\operatorname{atanh}(cx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x)), x)
```

3.46 $\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{(dx)^m}{a+b\tanh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x]), x]
```

fricas [A] time = 1.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b\,\text{artanh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arctanh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\,\text{artanh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x) + a), x)
```

maple [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b \operatorname{arctanh}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x)),x)
[Out] int((d*x)^m/(a+b*arctanh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctanh(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b \operatorname{atanh}(c\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x)),x)
[Out] int((d*x)^m/(a + b*atanh(c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b \operatorname{atanh}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x)),x)
[Out] Integral((d*x)**m/(a + b*atanh(c*x)), x)
```

3.47 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x])^2,x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x])^2, x]
```

fricas [A] time = 1.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b^2\,\text{artanh}(cx)^2+2\,ab\,\text{artanh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b\,\text{artanh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x) + a)^2, x)
```

maple [A] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b\operatorname{arctanh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(c^2d^mx^2 - d^m\right)x^m}{b^2c\log(cx+1) - b^2c\log(-cx+1) + 2\,abc} + \int -\frac{2\left(c^2d^m(m+2)x^2 - d^m m\right)x^m}{b^2cx\log(cx+1) - b^2cx\log(-cx+1) + 2\,abcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 2*(c^2*d^m*x^2 - d^m)*x^m/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) + 2*a*b
*c) + integrate(-2*(c^2*d^m*(m + 2)*x^2 - d^m*m)*x^m/(b^2*c*x*log(c*x + 1)
- b^2*c*x*log(-c*x + 1) + 2*a*b*c*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{(a + b\,\operatorname{atanh}(c\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x))^2,x)
```

```
[Out] int((d*x)^m/(a + b*atanh(c*x))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b\operatorname{atanh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral((d*x)**m/(a + b*atanh(c*x))**2, x)
```

3.48 $\int \left(a+b\tanh^{-1}(cx)\right)^p\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\left(a+b\tanh^{-1}(cx)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*arctanh(c*x))^p,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\tanh^{-1}(cx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x])^p,x]
[Out] Defer[Int][(a + b*ArcTanh[c*x])^p, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(cx)\right)^p\,dx=\int \left(a+b\tanh^{-1}(cx)\right)^p\,dx$$

Mathematica [A] time = 0.48, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^{-1}(cx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x])^p,x]
[Out] Integrate[(a + b*ArcTanh[c*x])^p, x]
```

fricas [A] time = 2.14, size = 0, normalized size = 0.00

$$\text{integral}\left((b\,\text{artanh}\,(cx)+a)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^p,x, algorithm="fricas")
[Out] integral((b*arctanh(c*x) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{artanh}\,(cx)+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^p,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x) + a)^p, x)
```

maple [A] time = 0.30, size = 0, normalized size = 0.00

$$\int (a+b\,\text{arctanh}\,(cx))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^p,x)
```

```
[Out] int((a+b*arctanh(c*x))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{artanh}(cx) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*arctanh(c*x) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int (a + b \operatorname{atanh}(c\,x))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^p,x)
```

```
[Out] int((a + b*atanh(c*x))^p, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atanh}(cx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**p,x)
```

```
[Out] Integral((a + b*atanh(c*x))**p, x)
```

3.49 $\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p\,dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}(cx)\right)^p,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x))^p,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x])^p,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p\,dx = \int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p\,dx$$

Mathematica [A] time = 0.36, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^p,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

fricas [A] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left((dx)^m (b\,\text{artanh}(cx)+a)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="fricas")
[Out] integral((d*x)^m*(b*arctanh(c*x) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (b\,\text{artanh}(cx)+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="giac")
[Out] integrate((d*x)^m*(b*arctanh(c*x) + a)^p, x)
```

maple [A] time = 0.56, size = 0, normalized size = 0.00

$$\int (dx)^m (a+b\,\text{arctanh}(cx))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x))^p,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (b\operatorname{artanh}(cx) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m*(b*arctanh(c*x) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (a + b\operatorname{atanh}(c\,x))^p \, (d\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x))^p*(d*x)^m,x)
```

```
[Out] int((a + b*atanh(c*x))^p*(d*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x))**p,x)
```

```
[Out] Timed out
```

3.50 $\int x^7 \left(a + b \tanh^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=54

$$\frac{1}{8}x^8\left(a + b\tanh^{-1}\left(cx^2\right)\right) - \frac{b\tanh^{-1}\left(cx^2\right)}{8c^4} + \frac{bx^2}{8c^3} + \frac{bx^6}{24c}$$

```
[Out] 1/8*b*x^2/c^3+1/24*b*x^6/c-1/8*b*arctanh(c*x^2)/c^4+1/8*x^8*(a+b*arctanh(c*
x^2))
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 275, 302, 206}

$$\frac{1}{8}x^8\left(a + b\tanh^{-1}\left(cx^2\right)\right) + \frac{bx^2}{8c^3} - \frac{b\tanh^{-1}\left(cx^2\right)}{8c^4} + \frac{bx^6}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) + (b*x^6)/(24*c) - (b*ArcTanh[c*x^2])/(8*c^4) + (x^8*(a + b
*ArcTanh[c*x^2]))/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^7\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx=\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{4}(bc)\int\frac{x^9}{1-c^2x^4}dx$$
$$=\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{8}(bc)\,\text{Subst}\left(\int\frac{x^4}{1-c^2x^2}dx,x,x^2\right)$$
$$=\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{8}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^4}-\frac{x^2}{c^2}+\frac{1}{c^4\left(1-c^2x^2\right)}\right)dx,x,x^2\right)$$
$$=\frac{bx^2}{8c^3}+\frac{bx^6}{24c}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1-c^2x^2}dx,x,x^2\right)}{8c^3}$$
$$=\frac{bx^2}{8c^3}+\frac{bx^6}{24c}-\frac{b\tanh^{-1}\left(cx^2\right)}{8c^4}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)$$

Mathematica [A] time = 0.02, size = 78, normalized size = 1.44

$$\frac{ax^8}{8}+\frac{b\log\left(1-cx^2\right)}{16c^4}-\frac{b\log\left(cx^2+1\right)}{16c^4}+\frac{bx^2}{8c^3}+\frac{bx^6}{24c}+\frac{1}{8}bx^8\tanh^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) + (b*x^6)/(24*c) + (a*x^8)/8 + (b*x^8*ArcTanh[c*x^2])/8 + (
b*Log[1 - c*x^2])/(16*c^4) - (b*Log[1 + c*x^2])/(16*c^4)
```

fricas [A] time = 0.71, size = 64, normalized size = 1.19

$$\frac{6\,ac^4x^8+2\,bc^3x^6+6\,bcx^2+3\left(bc^4x^8-b\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{48\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/48*(6*a*c^4*x^8 + 2*b*c^3*x^6 + 6*b*c*x^2 + 3*(b*c^4*x^8 - b)*log(-(c*x^2
 + 1)/(c*x^2 - 1)))/c^4
```

giac [A] time = 0.16, size = 78, normalized size = 1.44

$$\frac{1}{16}\,bx^8\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{8}\,ax^8+\frac{bx^6}{24\,c}+\frac{bx^2}{8\,c^3}-\frac{b\log\left(cx^2+1\right)}{16\,c^4}+\frac{b\log\left(cx^2-1\right)}{16\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/16*b*x^8*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/8*a*x^8 + 1/24*b*x^6/c + 1/8*b
*x^2/c^3 - 1/16*b*log(c*x^2 + 1)/c^4 + 1/16*b*log(c*x^2 - 1)/c^4
```

maple [A] time = 0.03, size = 66, normalized size = 1.22

$$\frac{x^8a}{8}+\frac{b\,x^8\,\text{arctanh}\left(c\,x^2\right)}{8}+\frac{b\,x^6}{24c}+\frac{b\,x^2}{8c^3}-\frac{b\ln\left(c\,x^2+1\right)}{16c^4}+\frac{b\ln\left(c\,x^2-1\right)}{16c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/8*x^8*a+1/8*b*x^8*arctanh(c*x^2)+1/24*b*x^6/c+1/8*b*x^2/c^3-1/16*b/c^4*ln
(c*x^2+1)+1/16*b/c^4*ln(c*x^2-1)
```

maxima [A] time = 0.31, size = 69, normalized size = 1.28

$$\frac{1}{8}ax^8+\frac{1}{48}\left(6x^8\operatorname{artanh}\left(cx^2\right)+c\left(\frac{2\left(c^2x^6+3x^2\right)}{c^4}-\frac{3\log\left(cx^2+1\right)}{c^5}+\frac{3\log\left(cx^2-1\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/48*(6*x^8*arctanh(c*x^2) + c*(2*(c^2*x^6 + 3*x^2)/c^4 - 3*log
(c*x^2 + 1)/c^5 + 3*log(c*x^2 - 1)/c^5))*b
```

mupad [B] time = 1.06, size = 69, normalized size = 1.28

$$\frac{a\,x^8}{8}+\frac{b\,x^2}{8\,c^3}+\frac{b\,x^6}{24\,c}+\frac{b\,x^8\,\ln\left(c\,x^2+1\right)}{16}-\frac{b\,x^8\,\ln\left(1-c\,x^2\right)}{16}+\frac{b\,\mathrm{atan}\left(c\,x^2\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atanh(c*x^2)),x)
```

```
[Out] (a*x^8)/8 + (b*x^2)/(8*c^3) + (b*x^6)/(24*c) + (b*atan(c*x^2*1i)*1i)/(8*c^4
) + (b*x^8*log(c*x^2 + 1))/16 - (b*x^8*log(1 - c*x^2))/16
```

sympy [A] time = 17.37, size = 58, normalized size = 1.07

$$\begin{cases} \frac{ax^8}{8}+\frac{bx^8\operatorname{atanh}\left(cx^2\right)}{8}+\frac{bx^6}{24c}+\frac{bx^2}{8c^3}-\frac{b\operatorname{atanh}\left(cx^2\right)}{8c^4} & \text{for}\ c\neq 0\\ \frac{ax^8}{8} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**8/8 + b*x**8*atanh(c*x**2)/8 + b*x**6/(24*c) + b*x**2/(8*c*
*3) - b*atanh(c*x**2)/(8*c**4), Ne(c, 0)), (a*x**8/8, True))
```

3.51 $\int x^5 \left(a + b \tanh^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=48

$$\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{12c^3}+\frac{bx^4}{12c}$$

```
[Out] 1/12*b*x^4/c+1/6*x^6*(a+b*arctanh(c*x^2))+1/12*b*ln(-c^2*x^4+1)/c^3
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6097, 266, 43}

$$\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{12c^3}+\frac{bx^4}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^4)/(12*c) + (x^6*(a + b*ArcTanh[c*x^2]))/6 + (b*Log[1 - c^2*x^4])/(12*
c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx &= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{3}(bc)\int\frac{x^7}{1-c^2x^4}\,dx\\
&= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{12}(bc)\,\text{Subst}\left(\int\frac{x}{1-c^2x}\,dx,x,x^4\right)\\
&= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{12}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^2}-\frac{1}{c^2\left(-1+c^2x\right)}\right)dx,x,x^4\right)\\
&= \frac{bx^4}{12c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{12c^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.10

$$\frac{ax^6}{6}+\frac{b\log\left(1-c^2x^4\right)}{12c^3}+\frac{bx^4}{12c}+\frac{1}{6}bx^6\tanh^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^4)/(12*c) + (a*x^6)/6 + (b*x^6*ArcTanh[c*x^2])/6 + (b*Log[1 - c^2*x^4]
)/(12*c^3)
```

fricas [A] time = 1.07, size = 62, normalized size = 1.29

$$\frac{bc^3x^6\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,ac^3x^6+bc^2x^4+b\log\left(c^2x^4-1\right)}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/12*(b*c^3*x^6*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^3*x^6 + b*c^2*x^4 + b
*log(c^2*x^4 - 1))/c^3
```

giac [A] time = 0.13, size = 57, normalized size = 1.19

$$\frac{1}{12}\,bx^6\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{6}\,ax^6+\frac{bx^4}{12\,c}+\frac{b\log\left(c^2x^4-1\right)}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/12*b*x^6*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/6*a*x^6 + 1/12*b*x^4/c + 1/12*
b*log(c^2*x^4 - 1)/c^3
```

maple [A] time = 0.02, size = 45, normalized size = 0.94

$$\frac{x^6a}{6}+\frac{b\,x^6\,\mathrm{arctanh}\left(c\,x^2\right)}{6}+\frac{b\,x^4}{12c}+\frac{b\ln\left(c^2x^4-1\right)}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctanh(c*x^2)+1/12*b*x^4/c+1/12*b/c^3*ln(c^2*x^4-1)
```

maxima [A] time = 0.32, size = 46, normalized size = 0.96

$$\frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\,\mathrm{artanh}\left(cx^2\right)+\left(\frac{x^4}{c^2}+\frac{\log\left(c^2x^4-1\right)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctanh(c*x^2) + (x^4/c^2 + log(c^2*x^4 - 1)/c^4)*c
)*b
```

mupad [B] time = 0.79, size = 61, normalized size = 1.27

$$\frac{a\,x^6}{6}+\frac{b\,\ln\left(c^2\,x^4-1\right)}{12\,c^3}+\frac{b\,x^4}{12\,c}+\frac{b\,x^6\,\ln\left(c\,x^2+1\right)}{12}-\frac{b\,x^6\,\ln\left(1-c\,x^2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x^2)),x)
```

```
[Out] (a*x^6)/6 + (b*log(c^2*x^4 - 1))/(12*c^3) + (b*x^4)/(12*c) + (b*x^6*log(c*x
^2 + 1))/12 - (b*x^6*log(1 - c*x^2))/12
```

sympy [A] time = 12.46, size = 85, normalized size = 1.77

$$\begin{cases} \frac{ax^6}{6} + \frac{bx^6 \operatorname{atanh}(cx^2)}{6} + \frac{bx^4}{12c} + \frac{b \log\left(x - i\sqrt{\frac{1}{c}}\right)}{6c^3} + \frac{b \log\left(x + i\sqrt{\frac{1}{c}}\right)}{6c^3} - \frac{b \operatorname{atanh}(cx^2)}{6c^3} & \text{for } c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atanh(c*x**2)/6 + b*x**4/(12*c) + b*log(x - I*
sqrt(1/c))/(6*c**3) + b*log(x + I*sqrt(1/c))/(6*c**3) - b*atanh(c*x**2)/(6*
c**3), Ne(c, 0)), (a*x**6/6, True))
```

3.52 $\int x^3 \left(a + b \tanh^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=43

$$\frac{1}{4}x^4\left(a + b\tanh^{-1}\left(cx^2\right)\right) - \frac{b\tanh^{-1}\left(cx^2\right)}{4c^2} + \frac{bx^2}{4c}$$

```
[Out] 1/4*b*x^2/c-1/4*b*arctanh(c*x^2)/c^2+1/4*x^4*(a+b*arctanh(c*x^2))
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6097, 275, 321, 206}

$$\frac{1}{4}x^4\left(a + b\tanh^{-1}\left(cx^2\right)\right) - \frac{b\tanh^{-1}\left(cx^2\right)}{4c^2} + \frac{bx^2}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^2)/(4*c) - (b*ArcTanh[c*x^2])/(4*c^2) + (x^4*(a + b*ArcTanh[c*x^2]))/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \left(a+b\tanh^{-1}\left(cx^2\right)\right)dx = \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{2}(bc)\int\frac{x^5}{1-c^2x^4}dx$$
$$=\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{4}(bc)\,\text{Subst}\left(\int\frac{x^2}{1-c^2x^2}dx,x,x^2\right)$$
$$=\frac{bx^2}{4c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1-c^2x^2}dx,x,x^2\right)}{4c}$$
$$=\frac{bx^2}{4c}-\frac{b\tanh^{-1}\left(cx^2\right)}{4c^2}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)$$

Mathematica [A] time = 0.01, size = 67, normalized size = 1.56

$$\frac{ax^4}{4}+\frac{b\log\left(1-cx^2\right)}{8c^2}-\frac{b\log\left(cx^2+1\right)}{8c^2}+\frac{bx^2}{4c}+\frac{1}{4}bx^4\tanh^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^2)/(4*c) + (a*x^4)/4 + (b*x^4*ArcTanh[c*x^2])/4 + (b*Log[1 - c*x^2])/(
8*c^2) - (b*Log[1 + c*x^2])/(8*c^2)
```

fricas [A] time = 1.47, size = 54, normalized size = 1.26

$$\frac{2\,ac^2x^4+2\,bcx^2+\left(bc^2x^4-b\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*c^2*x^4 + 2*b*c*x^2 + (b*c^2*x^4 - b)*log(-(c*x^2 + 1)/(c*x^2 - 1)
))/c^2
```

giac [B] time = 0.16, size = 181, normalized size = 4.21

$$\frac{1}{2}c\left(\frac{\left(cx^2+1\right)b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{\left(\frac{\left(cx^2+1\right)^2c^3}{\left(cx^2-1\right)^2}-\frac{2\left(cx^2+1\right)c^3}{cx^2-1}+c^3\right)\left(cx^2-1\right)}+\frac{\frac{2\left(cx^2+1\right)a}{cx^2-1}+\frac{\left(cx^2+1\right)b}{cx^2-1}-b}{\frac{\left(cx^2+1\right)^2c^3}{\left(cx^2-1\right)^2}-\frac{2\left(cx^2+1\right)c^3}{cx^2-1}+c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/2*c*((c*x^2 + 1)*b*log(-(c*x^2 + 1)/(c*x^2 - 1))/(((c*x^2 + 1)^2*c^3/(c*x
^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2 - 1) + c^3)*(c*x^2 - 1)) + (2*(c*x^2 +
 1)*a/(c*x^2 - 1) + (c*x^2 + 1)*b/(c*x^2 - 1) - b)/((c*x^2 + 1)^2*c^3/(c*x^
2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2 - 1) + c^3))
```

maple [A] time = 0.03, size = 57, normalized size = 1.33

$$\frac{x^4a}{4}+\frac{b\,x^4\,\text{arctanh}\left(c\,x^2\right)}{4}+\frac{b\,x^2}{4c}-\frac{b\ln\left(c\,x^2+1\right)}{8c^2}+\frac{b\ln\left(c\,x^2-1\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x^2)+1/4*b*x^2/c-1/8*b/c^2*ln(c*x^2+1)+1/8*b/
c^2*ln(c*x^2-1)
```

maxima [A] time = 0.34, size = 58, normalized size = 1.35

$$\frac{1}{4}\,ax^4 + \frac{1}{8}\left(2\,x^4\,\mathrm{artanh}\left(cx^2\right) + c\left(\frac{2\,x^2}{c^2} - \frac{\log\left(cx^2+1\right)}{c^3} + \frac{\log\left(cx^2-1\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/8*(2*x^4*arctanh(c*x^2) + c*(2*x^2/c^2 - log(c*x^2 + 1)/c^3 +
 log(c*x^2 - 1)/c^3))*b
```

mupad [B] time = 0.94, size = 60, normalized size = 1.40

$$\frac{a\,x^4}{4} + \frac{b\,x^2}{4\,c} + \frac{b\,x^4\,\ln\left(c\,x^2+1\right)}{8} - \frac{b\,x^4\,\ln\left(1-c\,x^2\right)}{8} + \frac{b\,\mathrm{atan}\left(c\,x^2\,1\mathrm{i}\right)\,1\mathrm{i}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^2)),x)
```

```
[Out] (a*x^4)/4 + (b*x^2)/(4*c) + (b*atan(c*x^2*1i)*1i)/(4*c^2) + (b*x^4*log(c*x^
2 + 1))/8 - (b*x^4*log(1 - c*x^2))/8
```

sympy [A] time = 8.41, size = 48, normalized size = 1.12

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4\,\mathrm{atanh}\left(cx^2\right)}{4} + \frac{bx^2}{4c} - \frac{b\,\mathrm{atanh}\left(cx^2\right)}{4c^2} & \text{for}\ c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atanh(c*x**2)/4 + b*x**2/(4*c) - b*atanh(c*x**
2)/(4*c**2), Ne(c, 0)), (a*x**4/4, True))
```

3.53 $\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{4c}$$

```
[Out] 1/2*x^2*(a+b*arctanh(c*x^2))+1/4*b*ln(-c^2*x^4+1)/c
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6097, 260}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (x^2*(a + b*ArcTanh[c*x^2]))/2 + (b*Log[1 - c^2*x^4])/(4*c)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx&=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)-(bc)\int\frac{x^3}{1-c^2x^4}\,dx\\&=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{4c}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.14

$$\frac{ax^2}{2}+\frac{b\log\left(1-c^2x^4\right)}{4c}+\frac{1}{2}bx^2\tanh^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcTanh[c*x^2])/2 + (b*Log[1 - c^2*x^4])/(4*c)
```

fricas [A] time = 0.84, size = 50, normalized size = 1.35

$$\frac{bcx^2\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,acx^2+b\log\left(c^2x^4-1\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*(b*c*x^2*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x^2 + b*log(c^2*x^4 - 1)
)/c
```

giac [B] time = 0.15, size = 188, normalized size = 5.08

$$\frac{1}{2}\,ax^2+\frac{1}{2}\,bc\left(\frac{\log\left(\frac{|-cx^2-1|}{|cx^2-1|}\right)}{c^2}-\frac{\log\left(\left|-\frac{cx^2+1}{cx^2-1}+1\right|\right)}{c^2}+\frac{\log\left(-\frac{\frac{c\left(\frac{cx^2+1}{cx^2-1}+1\right)}{\frac{(cx^2+1)c}{cx^2-1}-c}+1}{\frac{c\left(\frac{cx^2+1}{cx^2-1}+1\right)}{\frac{(cx^2+1)c}{cx^2-1}-c}-1}\right)}{c^2\left(\frac{cx^2+1}{cx^2-1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2 + 1/2*b*c*(log(abs(-c*x^2 - 1)/abs(c*x^2 - 1))/c^2 - log(abs(-(c*
x^2 + 1)/(c*x^2 - 1) + 1))/c^2 + log(-(c*((c*x^2 + 1)/(c*x^2 - 1) + 1)/((c*
x^2 + 1)*c/(c*x^2 - 1) - c) + 1)/(c*((c*x^2 + 1)/(c*x^2 - 1) + 1)/((c*x^2 +
 1)*c/(c*x^2 - 1) - c) - 1))/(c^2*((c*x^2 + 1)/(c*x^2 - 1) - 1)))
```

maple [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{arctanh}\left(c\,x^2\right)}{2}+\frac{b\ln\left(-c^2x^4+1\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c*x^2)+1/4*b*ln(-c^2*x^4+1)/c
```

maxima [A] time = 0.31, size = 37, normalized size = 1.00

$$\frac{1}{2}\,ax^2+\frac{\left(2\,cx^2\,\mathrm{artanh}\left(cx^2\right)+\log\left(-c^2x^4+1\right)\right)b}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*c*x^2*arctanh(c*x^2) + log(-c^2*x^4 + 1))*b/c
```

mupad [B] time = 0.77, size = 52, normalized size = 1.41

$$\frac{a\,x^2}{2}+\frac{b\,\ln\left(c^2\,x^4-1\right)}{4\,c}+\frac{b\,x^2\,\ln\left(c\,x^2+1\right)}{4}-\frac{b\,x^2\,\ln\left(1-c\,x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^2)),x)
```

```
[Out] (a*x^2)/2 + (b*log(c^2*x^4 - 1))/(4*c) + (b*x^2*log(c*x^2 + 1))/4 - (b*x^2*
log(1 - c*x^2))/4
```

sympy [A] time = 6.94, size = 71, normalized size = 1.92

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{atanh}\left(cx^2\right)}{2} + \frac{b\log\left(x - i\sqrt{\frac{1}{c}}\right)}{2c} + \frac{b\log\left(x + i\sqrt{\frac{1}{c}}\right)}{2c} - \frac{b\operatorname{atanh}\left(cx^2\right)}{2c} & \text{for } c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*atanh(c*x**2)/2 + b*log(x - I*sqrt(1/c))/(2*c)
 + b*log(x + I*sqrt(1/c))/(2*c) - b*atanh(c*x**2)/(2*c), Ne(c, 0)), (a*x**2
/2, True))
```

3.54 $\int \frac{a+b\tanh^{-1}(cx^2)}{x}\,dx$

Optimal. Leaf size=30

$$a\log(x) - \frac{1}{4}b\text{Li}_2\left(-cx^2\right) + \frac{1}{4}b\text{Li}_2\left(cx^2\right)$$

```
[Out] a*ln(x)-1/4*b*polylog(2,-c*x^2)+1/4*b*polylog(2,c*x^2)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6095, 5912}

$$-\frac{1}{4}b\text{PolyLog}\left(2,-cx^2\right) + \frac{1}{4}b\text{PolyLog}\left(2,cx^2\right) + a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x,x]
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^2)])/4 + (b*PolyLog[2, c*x^2])/4
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx, x, x^2\right)\\ &= a\log(x) - \frac{1}{4}b\text{Li}_2\left(-cx^2\right) + \frac{1}{4}b\text{Li}_2\left(cx^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.93

$$a\log(x) + \frac{1}{4}b\left(\text{Li}_2\left(cx^2\right) - \text{Li}_2\left(-cx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x,x]
[Out] a*Log[x] + (b*(-PolyLog[2, -(c*x^2)] + PolyLog[2, c*x^2]))/4
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx^2\right) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^2) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{artanh}\left(cx^2\right) + a}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)/x, x)
```

maple [B] time = 0.06, size = 124, normalized size = 4.13

$$a \ln(x)+b \ln(x) \operatorname{arctanh}\left(c\, x^2\right)-\frac{b \ln(x) \ln\left(1+x\sqrt{-c}\right)}{2}-\frac{b \ln(x) \ln\left(1-x\sqrt{-c}\right)}{2}-\frac{b \operatorname{dilog}\left(1+x\sqrt{-c}\right)}{2}-\frac{b \operatorname{dilog}\left(1-\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x,x)
```

```
[Out] a*ln(x)+b*ln(x)*arctanh(c*x^2)-1/2*b*ln(x)*ln(1+x*(-c)^(1/2))-1/2*b*ln(x)*l
n(1-x*(-c)^(1/2))-1/2*b*dilog(1+x*(-c)^(1/2))-1/2*b*dilog(1-x*(-c)^(1/2))+1
/2*b*ln(x)*ln(1-x*c^(1/2))+1/2*b*ln(x)*ln(1+x*c^(1/2))+1/2*b*dilog(1-x*c^(1
/2))+1/2*b*dilog(1+x*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\, b \int \frac{\log\left(cx^2+1\right) - \log\left(-cx^2+1\right)}{x}\, dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c*x^2 + 1) - log(-c*x^2 + 1))/x, x) + a*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{atanh}\left(c\, x^2\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x,x)
```

```
[Out] int((a + b*atanh(c*x^2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}\left(cx^2\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))/x, x)
```

3.55 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^3}\,dx$

Optimal. Leaf size=40

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2}-\frac{1}{4}bc\log\left(1-c^2x^4\right)+bc\log(x)$$

```
[Out] 1/2*(-a-b*arctanh(c*x^2))/x^2+b*c*ln(x)-1/4*b*c*ln(-c^2*x^4+1)
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 266, 36, 29, 31}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2}-\frac{1}{4}bc\log\left(1-c^2x^4\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^3,x]
```

```
[Out] -(a + b*ArcTanh[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 - c^2*x^4])/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^3}\,dx = -\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2} + (bc)\int \frac{1}{x\left(1-c^2x^4\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2} + \frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1-c^2x\right)}\,dx, x, x^4\right)$$
$$= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2} + \frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right) + \frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x}\,dx, x,\right.$$
$$= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2} + bc\log(x) - \frac{1}{4}bc\log\left(1-c^2x^4\right)$$

Mathematica [A] time = 0.01, size = 45, normalized size = 1.12

$$-\frac{a}{2x^2} - \frac{1}{4}bc\log\left(1-c^2x^4\right) - \frac{b\tanh^{-1}\left(cx^2\right)}{2x^2} + bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*ArcTanh[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 - c^2*x^4
])/4
```

fricas [A] time = 0.70, size = 55, normalized size = 1.38

$$-\frac{bcx^2\log\left(c^2x^4-1\right) - 4\,bcx^2\log(x) + b\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*x^2*log(c^2*x^4 - 1) - 4*b*c*x^2*log(x) + b*log(-(c*x^2 + 1)/(c*x
^2 - 1)) + 2*a)/x^2
```

giac [A] time = 0.15, size = 51, normalized size = 1.28

$$-\frac{1}{4}\,bc\log\left(c^2x^4-1\right) + bc\log(x) - \frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{4\,x^2} - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*b*c*log(c^2*x^4 - 1) + b*c*log(x) - 1/4*b*log(-(c*x^2 + 1)/(c*x^2 - 1)
)/x^2 - 1/2*a/x^2
```

maple [A] time = 0.03, size = 49, normalized size = 1.22

$$-\frac{a}{2x^2} - \frac{b\,\text{arctanh}\left(c\,x^2\right)}{2x^2} + bc\ln(x) - \frac{bc\ln\left(c\,x^2+1\right)}{4} - \frac{bc\ln\left(c\,x^2-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x^2)+b*c*ln(x)-1/4*b*c*ln(c*x^2+1)-1/4*b*c*l
n(c*x^2-1)
```

maxima [A] time = 0.31, size = 41, normalized size = 1.02

$$-\frac{1}{4}\left(c\left(\log\left(c^{2}x^{4}-1\right)-\log\left(x^{4}\right)\right)+\frac{2\,\operatorname{artanh}\left(cx^{2}\right)}{x^{2}}\right)b-\frac{a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*b - 1/2*a/x^2
```

mupad [B] time = 0.85, size = 55, normalized size = 1.38

$$b\,c\,\ln(x)-\frac{a}{2\,x^{2}}-\frac{b\,c\,\ln\left(c^{2}\,x^{4}-1\right)}{4}-\frac{b\,\ln\left(c\,x^{2}+1\right)}{4\,x^{2}}+\frac{b\,\ln\left(1-c\,x^{2}\right)}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x^3,x)
```

```
[Out] b*c*log(x) - a/(2*x^2) - (b*c*log(c^2*x^4 - 1))/4 - (b*log(c*x^2 + 1))/(4*x
^2) + (b*log(1 - c*x^2))/(4*x^2)
```

sympy [A] time = 11.51, size = 80, normalized size = 2.00

$$\begin{cases} -\frac{a}{2x^{2}}+bc\log\left(x\right)-\frac{bc\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2}-\frac{bc\log\left(x+i\sqrt{\frac{1}{c}}\right)}{2}+\frac{bc\operatorname{atanh}\left(cx^{2}\right)}{2}-\frac{b\operatorname{atanh}\left(cx^{2}\right)}{2x^{2}} & \text{for}\: c\neq 0 \\ -\frac{a}{2x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) + b*c*log(x) - b*c*log(x - I*sqrt(1/c))/2 - b*c*log(
x + I*sqrt(1/c))/2 + b*c*atanh(c*x**2)/2 - b*atanh(c*x**2)/(2*x**2), Ne(c,
0)), (-a/(2*x**2), True))
```

3.56 $\int \frac{a+b \tanh^{-1}(cx^2)}{x^5} \, dx$

Optimal. Leaf size=41

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{4}bc^2\tanh^{-1}\left(cx^2\right)-\frac{bc}{4x^2}$$

```
[Out] -1/4*b*c/x^2+1/4*b*c^2*arctanh(c*x^2)+1/4*(-a-b*arctanh(c*x^2))/x^4
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 275, 325, 206}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{4}bc^2\tanh^{-1}\left(cx^2\right)-\frac{bc}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^5,x]
```

```
[Out] -(b*c)/(4*x^2) + (b*c^2*ArcTanh[c*x^2])/4 - (a + b*ArcTanh[c*x^2])/(4*x^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^5}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{2}(bc)\int \frac{1}{x^3\left(1-c^2x^4\right)}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1-c^2x^2\right)}\,dx,x,x^2\right)\\ &= -\frac{bc}{4x^2}-\frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,x^2\right)\\ &= -\frac{bc}{4x^2}+\frac{1}{4}bc^2\tanh^{-1}\left(cx^2\right)-\frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 1.59

$$-\frac{a}{4x^4}-\frac{1}{8}bc^2\log\left(1-cx^2\right)+\frac{1}{8}bc^2\log\left(cx^2+1\right)-\frac{bc}{4x^2}-\frac{b\tanh^{-1}\left(cx^2\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (b*c)/(4*x^2) - (b*ArcTanh[c*x^2])/(4*x^4) - (b*c^2*Log[1 - c*
x^2])/8 + (b*c^2*Log[1 + c*x^2])/8
```

fricas [A] time = 1.10, size = 49, normalized size = 1.20

$$-\frac{2\,bcx^2-\left(bc^2x^4-b\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*c*x^2 - (b*c^2*x^4 - b)*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x^4
```

giac [A] time = 0.14, size = 67, normalized size = 1.63

$$\frac{1}{8}\,bc^2\log\left(cx^2+1\right)-\frac{1}{8}\,bc^2\log\left(cx^2-1\right)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{8\,x^4}-\frac{bcx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b*c^2*log(c*x^2 + 1) - 1/8*b*c^2*log(c*x^2 - 1) - 1/8*b*log(-(c*x^2 + 1
)/(c*x^2 - 1))/x^4 - 1/4*(b*c*x^2 + a)/x^4
```

maple [A] time = 0.03, size = 55, normalized size = 1.34

$$-\frac{a}{4x^4}-\frac{b\,\mathrm{arctanh}\left(c\,x^2\right)}{4x^4}-\frac{bc}{4x^2}+\frac{b\,c^2\ln\left(c\,x^2+1\right)}{8}-\frac{b\,c^2\ln\left(c\,x^2-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctanh(c*x^2)-1/4*b*c/x^2+1/8*b*c^2*ln(c*x^2+1)-1/8*b
*c^2*ln(c*x^2-1)
```

maxima [A] time = 0.32, size = 51, normalized size = 1.24

$$\frac{1}{8}\left(\left(c\log\left(cx^2+1\right)-c\log\left(cx^2-1\right)-\frac{2}{x^2}\right)c-\frac{2\ \mathrm{artanh}\left(cx^2\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^4
)*b - 1/4*a/x^4
```

mupad [B] time = 1.00, size = 52, normalized size = 1.27

$$\frac{b\,c^2\,\mathrm{atanh}\left(c\,x^2\right)}{4}-\frac{\frac{a}{4}+\frac{b\,\ln\left(c\,x^2+1\right)}{8}-\frac{b\,\ln\left(1-c\,x^2\right)}{8}+\frac{b\,c\,x^2}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x^5,x)
```

```
[Out] (b*c^2*atanh(c*x^2))/4 - (a/4 + (b*log(c*x^2 + 1))/8 - (b*log(1 - c*x^2))/8
 + (b*c*x^2)/4)/x^4
```

sympy [A] time = 8.07, size = 41, normalized size = 1.00

$$-\frac{a}{4x^4}+\frac{bc^2\,\mathrm{atanh}\left(cx^2\right)}{4}-\frac{bc}{4x^2}-\frac{b\,\mathrm{atanh}\left(cx^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**5,x)
```

```
[Out] -a/(4*x**4) + b*c**2*atanh(c*x**2)/4 - b*c/(4*x**2) - b*atanh(c*x**2)/(4*x*
*4)
```

3.57 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^7}\,dx$

Optimal. Leaf size=56

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{3}bc^3\log(x)-\frac{1}{12}bc^3\log\left(1-c^2x^4\right)-\frac{bc}{12x^4}$$

```
[Out] -1/12*b*c/x^4+1/6*(-a-b*arctanh(c*x^2))/x^6+1/3*b*c^3*ln(x)-1/12*b*c^3*ln(-
c^2*x^4+1)
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6097, 266, 44}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{6x^6}-\frac{1}{12}bc^3\log\left(1-c^2x^4\right)+\frac{1}{3}bc^3\log(x)-\frac{bc}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^7,x]
```

```
[Out] -(b*c)/(12*x^4) - (a + b*ArcTanh[c*x^2])/(6*x^6) + (b*c^3*Log[x])/3 - (b*c^
3*Log[1 - c^2*x^4])/12
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^7}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{3}(bc)\int \frac{1}{x^5\left(1-c^2x^4\right)}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{12}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1-c^2x\right)}\,dx,x,x^4\right)\\
&= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{12}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{c^2}{x}-\frac{c^4}{-1+c^2x}\right)dx,x,x^4\right)\\
&= -\frac{bc}{12x^4}-\frac{a+b\tanh^{-1}\left(cx^2\right)}{6x^6}+\frac{1}{3}bc^3\log(x)-\frac{1}{12}bc^3\log\left(1-c^2x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 1.09

$$-\frac{a}{6x^6}+\frac{1}{3}bc^3\log(x)-\frac{1}{12}bc^3\log\left(1-c^2x^4\right)-\frac{bc}{12x^4}-\frac{b\tanh^{-1}\left(cx^2\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^7,x]
```

```
[Out] -1/6*a/x^6 - (b*c)/(12*x^4) - (b*ArcTanh[c*x^2])/(6*x^6) + (b*c^3*Log[x])/3
 - (b*c^3*Log[1 - c^2*x^4])/12
```

fricas [A] time = 0.89, size = 65, normalized size = 1.16

$$-\frac{bc^3x^6\log\left(c^2x^4-1\right)-4\,bc^3x^6\log(x)+bcx^2+b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(b*c^3*x^6*log(c^2*x^4 - 1) - 4*b*c^3*x^6*log(x) + b*c*x^2 + b*log(-(
c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x^6
```

giac [A] time = 0.15, size = 65, normalized size = 1.16

$$-\frac{1}{12}\,bc^3\log\left(c^2x^4-1\right)+\frac{1}{3}\,bc^3\log(x)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{12\,x^6}-\frac{bcx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="giac")
```

```
[Out] -1/12*b*c^3*log(c^2*x^4 - 1) + 1/3*b*c^3*log(x) - 1/12*b*log(-(c*x^2 + 1)/(
c*x^2 - 1))/x^6 - 1/12*(b*c*x^2 + 2*a)/x^6
```

maple [A] time = 0.03, size = 63, normalized size = 1.12

$$-\frac{a}{6x^6}-\frac{b\,\mathrm{arctanh}\left(c\,x^2\right)}{6x^6}-\frac{bc}{12x^4}+\frac{b\,c^3\ln(x)}{3}-\frac{b\,c^3\ln\left(c\,x^2+1\right)}{12}-\frac{b\,c^3\ln\left(c\,x^2-1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^7,x)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctanh(c*x^2)-1/12*b*c/x^4+1/3*b*c^3*ln(x)-1/12*b*c^3
*ln(c*x^2+1)-1/12*b*c^3*ln(c*x^2-1)
```

maxima [A] time = 0.31, size = 51, normalized size = 0.91

$$-\frac{1}{12}\left(\left(c^2\log\left(c^2x^4-1\right)-c^2\log\left(x^4\right)+\frac{1}{x^4}\right)c+\frac{2\,\mathrm{artanh}\left(cx^2\right)}{x^6}\right)b-\frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*((c^2*log(c^2*x^4 - 1) - c^2*log(x^4) + 1/x^4)*c + 2*arctanh(c*x^2)/x
^6)*b - 1/6*a/x^6
```

mupad [B] time = 0.88, size = 67, normalized size = 1.20

$$\frac{b\,c^3\,\ln(x)}{3}-\frac{b\,c^3\,\ln\left(c^2\,x^4-1\right)}{12}-\frac{a}{6\,x^6}-\frac{b\,c}{12\,x^4}-\frac{b\,\ln\left(c\,x^2+1\right)}{12\,x^6}+\frac{b\,\ln\left(1-c\,x^2\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x^7,x)
```

```
[Out] (b*c^3*log(x))/3 - (b*c^3*log(c^2*x^4 - 1))/12 - a/(6*x^6) - (b*c)/(12*x^4)
 - (b*log(c*x^2 + 1))/(12*x^6) + (b*log(1 - c*x^2))/(12*x^6)
```

sympy [A] time = 22.25, size = 97, normalized size = 1.73

$$\begin{cases} -\frac{a}{6x^6}+\frac{bc^3\log(x)}{3}-\frac{bc^3\log\left(x-i\sqrt{\frac{1}{c}}\right)}{6}-\frac{bc^3\log\left(x+i\sqrt{\frac{1}{c}}\right)}{6}+\frac{bc^3\operatorname{atanh}\left(cx^2\right)}{6}-\frac{bc}{12x^4}-\frac{b\operatorname{atanh}\left(cx^2\right)}{6x^6} & \text{for}\ c\neq 0 \\ -\frac{a}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**7,x)
```

```
[Out] Piecewise((-a/(6*x**6) + b*c**3*log(x)/3 - b*c**3*log(x - I*sqrt(1/c))/6 -
b*c**3*log(x + I*sqrt(1/c))/6 + b*c**3*atanh(c*x**2)/6 - b*c/(12*x**4) - b*
atanh(c*x**2)/(6*x**6), Ne(c, 0)), (-a/(6*x**6), True))
```

3.58 $\int x^4 \left(a + b \tanh^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=65

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}+\frac{2bx^3}{15c}$$

```
[Out] 2/15*b*x^3/c+1/5*b*arctan(x*c^(1/2))/c^(5/2)+1/5*x^5*(a+b*arctanh(c*x^2))-1
/5*b*arctanh(x*c^(1/2))/c^(5/2)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 321, 298, 203, 206}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}+\frac{2bx^3}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*x^3)/(15*c) + (b*ArcTan[Sqrt[c]*x])/(5*c^(5/2)) - (b*ArcTanh[Sqrt[c]*x
])/(5*c^(5/2)) + (x^5*(a + b*ArcTanh[c*x^2]))/5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx=\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{5}(2bc)\int\frac{x^6}{1-c^2x^4}dx$$
$$=\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{(2b)\int\frac{x^2}{1-c^2x^4}dx}{5c}$$
$$=\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\int\frac{1}{1-cx^2}dx}{5c^2}+\frac{b\int\frac{1}{1+cx^2}dx}{5c^2}$$
$$=\frac{2bx^3}{15c}+\frac{b\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)$$

Mathematica [A] time = 0.03, size = 93, normalized size = 1.43

$$\frac{ax^5}{5}+\frac{b\log\left(1-\sqrt{c}\,x\right)}{10c^{5/2}}-\frac{b\log\left(\sqrt{c}\,x+1\right)}{10c^{5/2}}+\frac{b\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}+\frac{2bx^3}{15c}+\frac{1}{5}bx^5\tanh^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*x^3)/(15*c) + (a*x^5)/5 + (b*ArcTan[Sqrt[c]*x])/(5*c^(5/2)) + (b*x^5*A
rcTanh[c*x^2])/5 + (b*Log[1 - Sqrt[c]*x])/(10*c^(5/2)) - (b*Log[1 + Sqrt[c]
*x])/(10*c^(5/2))
```

fricas [A] time = 0.88, size = 197, normalized size = 3.03

$$\left[\frac{3\,bc^3x^5\log\left(-\frac{cx^2+1}{cx^2-1}\right)+6\,ac^3x^5+4\,bc^2x^3+6\,b\sqrt{c}\,\arctan\left(\sqrt{c}\,x\right)+3\,b\sqrt{c}\,\log\left(\frac{cx^2-2\,\sqrt{c}x+1}{cx^2-1}\right)}{30\,c^3},\frac{3\,bc^3x^5\log\left(-\frac{cx^2+}{cx^2-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] [1/30*(3*b*c^3*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a*c^3*x^5 + 4*b*c^2*x^
3 + 6*b*sqrt(c)*arctan(sqrt(c)*x) + 3*b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x +
1)/(c*x^2 - 1)))/c^3, 1/30*(3*b*c^3*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a
*c^3*x^5 + 4*b*c^2*x^3 + 6*b*sqrt(-c)*arctan(sqrt(-c)*x) - 3*b*sqrt(-c)*log
((c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)))/c^3]
```

giac [A] time = 0.39, size = 73, normalized size = 1.12

$$\frac{1}{10}\,bx^5\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{5}\,ax^5+\frac{2\,bx^3}{15\,c}+\frac{b\arctan\left(\sqrt{c}\,x\right)}{5\,c^{\frac{5}{2}}}+\frac{b\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{5\,\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/10*b*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/5*a*x^5 + 2/15*b*x^3/c + 1/5*b
*arctan(sqrt(c)*x)/c^(5/2) + 1/5*b*arctan(c*x/sqrt(-c))/(sqrt(-c)*c^2)
```

maple [A] time = 0.03, size = 53, normalized size = 0.82

$$\frac{a\,x^5}{5}+\frac{b\,x^5\operatorname{arctanh}\left(c\,x^2\right)}{5}+\frac{2b\,x^3}{15c}+\frac{b\arctan\left(x\sqrt{c}\right)}{5c^{\frac{5}{2}}}-\frac{b\operatorname{arctanh}\left(x\sqrt{c}\right)}{5c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/5*a*x^5+1/5*b*x^5*arctanh(c*x^2)+2/15*b*x^3/c+1/5*b*arctan(x*c^(1/2))/c^(
5/2)-1/5*b*arctanh(x*c^(1/2))/c^(5/2)
```

maxima [A] time = 0.41, size = 69, normalized size = 1.06

$$\frac{1}{5}\,ax^5+\frac{1}{30}\left(6\,x^5\,\mathrm{artanh}\left(cx^2\right)+c\left(\frac{4\,x^3}{c^2}+\frac{6\,\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{7}{2}}}+\frac{3\,\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{7}{2}}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/30*(6*x^5*arctanh(c*x^2) + c*(4*x^3/c^2 + 6*arctan(sqrt(c)*x)
/c^(7/2) + 3*log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(7/2)))*b
```

mupad [B] time = 0.98, size = 72, normalized size = 1.11

$$\frac{a\,x^5}{5}+\frac{2\,b\,x^3}{15\,c}+\frac{b\,\mathrm{atan}\left(\sqrt{c}\,x\right)}{5\,c^{5/2}}+\frac{b\,x^5\,\ln\left(c\,x^2+1\right)}{10}-\frac{b\,x^5\,\ln\left(1-c\,x^2\right)}{10}+\frac{b\,\mathrm{atan}\left(\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{5\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x^2)),x)
```

```
[Out] (a*x^5)/5 + (2*b*x^3)/(15*c) + (b*atan(c^(1/2)*x))/(5*c^(5/2)) + (b*atan(c^
(1/2)*x*1i)*1i)/(5*c^(5/2)) + (b*x^5*log(c*x^2 + 1))/10 - (b*x^5*log(1 - c*
x^2))/10
```

sympy [A] time = 11.71, size = 238, normalized size = 3.66

$$\begin{cases}\frac{ax^5}{5}+\frac{bx^5\,\mathrm{atanh}\left(cx^2\right)}{5}+\frac{2bx^3}{15c}+\frac{b\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(x+i\sqrt{\frac{1}{c}}\right)}{20c}-\frac{ib\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(x+i\sqrt{\frac{1}{c}}\right)}{20c}-\frac{b\sqrt{\frac{1}{c}}\log\left(x-i\sqrt{\frac{1}{c}}\right)}{10c^2}-\frac{ib\sqrt{\frac{1}{c}}\log\left(x-i\sqrt{\frac{1}{c}}\right)}{10c^2}-\frac{3b\sqrt{\frac{1}{c}}\log\left(x\right.}{20c^2} \\ \frac{ax^5}{5}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atanh(c*x**2)/5 + 2*b*x**3/(15*c) + b*(1/c)**(
3/2)*log(x + I*sqrt(1/c))/(20*c) - I*b*(1/c)**(3/2)*log(x + I*sqrt(1/c))/(2
0*c) - b*sqrt(1/c)*log(x - I*sqrt(1/c))/(10*c**2) - I*b*sqrt(1/c)*log(x - I
*sqrt(1/c))/(10*c**2) - 3*b*sqrt(1/c)*log(x + I*sqrt(1/c))/(20*c**2) + 3*I*
b*sqrt(1/c)*log(x + I*sqrt(1/c))/(20*c**2) + b*sqrt(1/c)*log(x - sqrt(1/c))
/(5*c**2) + b*sqrt(1/c)*atanh(c*x**2)/(5*c**2), Ne(c, 0)), (a*x**5/5, True)
)
```

3.59 $\int x^2 \left(a+b\tanh^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=63

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{2bx}{3c}$$

```
[Out] 2/3*b*x/c-1/3*b*arctan(x*c^(1/2))/c^(3/2)+1/3*x^3*(a+b*arctanh(c*x^2))-1/3*
b*arctanh(x*c^(1/2))/c^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 321, 212, 206, 203}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{2bx}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*x)/(3*c) - (b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) - (b*ArcTanh[Sqrt[c]*x])/
(3*c^(3/2)) + (x^3*(a + b*ArcTanh[c*x^2]))/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx=\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{3}(2bc)\int\frac{x^4}{1-c^2x^4}dx$$
$$=\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{(2b)\int\frac{1}{1-c^2x^4}dx}{3c}$$
$$=\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\int\frac{1}{1-cx^2}dx}{3c}-\frac{b\int\frac{1}{1+cx^2}dx}{3c}$$
$$=\frac{2bx}{3c}-\frac{b\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)$$

Mathematica [A] time = 0.02, size = 91, normalized size = 1.44

$$\frac{ax^3}{3}+\frac{b\log\left(1-\sqrt{c}\,x\right)}{6c^{3/2}}-\frac{b\log\left(\sqrt{c}\,x+1\right)}{6c^{3/2}}-\frac{b\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{1}{3}bx^3\tanh^{-1}\left(cx^2\right)+\frac{2bx}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*x)/(3*c) + (a*x^3)/3 - (b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) + (b*x^3*ArcT
anh[c*x^2])/3 + (b*Log[1 - Sqrt[c]*x])/(6*c^(3/2)) - (b*Log[1 + Sqrt[c]*x])
/(6*c^(3/2))
```

fricas [A] time = 0.92, size = 186, normalized size = 2.95

$$\left[\frac{bc^2x^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,ac^2x^3+4\,bcx-2\,b\sqrt{c}\,\arctan\left(\sqrt{c}\,x\right)+b\sqrt{c}\,\log\left(\frac{cx^2-2\,\sqrt{c}\,x+1}{cx^2-1}\right)}{6\,c^2},\frac{bc^2x^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,ac^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] [1/6*(b*c^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^2*x^3 + 4*b*c*x - 2*b
*sqrt(c)*arctan(sqrt(c)*x) + b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2
 - 1)))/c^2, 1/6*(b*c^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^2*x^3 + 4
*b*c*x + 2*b*sqrt(-c)*arctan(sqrt(-c)*x) - b*sqrt(-c)*log((c*x^2 + 2*sqrt(-
c)*x - 1)/(c*x^2 + 1)))/c^2]
```

giac [A] time = 0.30, size = 75, normalized size = 1.19

$$-\frac{1}{3}\,bc^5\left(\frac{\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{13}{2}}}-\frac{\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^6}\right)+\frac{1}{6}\,bx^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{3}\,ax^3+\frac{2\,bx}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] -1/3*b*c^5*(arctan(sqrt(c)*x)/c^(13/2) - arctan(c*x/sqrt(-c))/(sqrt(-c)*c^6
)) + 1/6*b*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/3*a*x^3 + 2/3*b*x/c
```

maple [A] time = 0.03, size = 51, normalized size = 0.81

$$\frac{x^3a}{3}+\frac{b\,x^3\,\mathrm{arctanh}\left(c\,x^2\right)}{3}+\frac{2bx}{3c}-\frac{b\arctan\left(x\sqrt{c}\right)}{3c^{\frac{3}{2}}}-\frac{b\,\mathrm{arctanh}\left(x\sqrt{c}\right)}{3c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c*x^2)+2/3*b*x/c-1/3*b*arctan(x*c^(1/2))/c^(3/2
)-1/3*b*arctanh(x*c^(1/2))/c^(3/2)
```

maxima [A] time = 0.41, size = 66, normalized size = 1.05

$$\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\operatorname{artanh}\left(cx^2\right)+c\left(\frac{4\,x}{c^2}-\frac{2\,\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{5}{2}}}+\frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c*x^2) + c*(4*x/c^2 - 2*arctan(sqrt(c)*x)/c^
(5/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(5/2)))*b
```

mupad [B] time = 0.85, size = 70, normalized size = 1.11

$$\frac{a\,x^3}{3}-\frac{b\,\mathrm{atan}\left(\sqrt{c}\,x\right)}{3\,c^{3/2}}+\frac{2\,b\,x}{3\,c}+\frac{b\,x^3\,\ln\left(c\,x^2+1\right)}{6}-\frac{b\,x^3\,\ln\left(1-c\,x^2\right)}{6}+\frac{b\,\mathrm{atan}\left(\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{3\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^2)),x)
```

```
[Out] (a*x^3)/3 - (b*atan(c^(1/2)*x))/(3*c^(3/2)) + (b*atan(c^(1/2)*x*1i)*1i)/(3*
c^(3/2)) + (2*b*x)/(3*c) + (b*x^3*log(c*x^2 + 1))/6 - (b*x^3*log(1 - c*x^2)
)/6
```

sympy [A] time = 8.54, size = 559, normalized size = 8.87

$$\begin{cases} \frac{4ac^2x^3\sqrt{\frac{1}{c}}}{12c^2\sqrt{\frac{1}{c}}+12ic^2\sqrt{\frac{1}{c}}}+\frac{4iac^2x^3\sqrt{\frac{1}{c}}}{12c^2\sqrt{\frac{1}{c}}+12ic^2\sqrt{\frac{1}{c}}}+\frac{4bc^2x^3\sqrt{\frac{1}{c}}\operatorname{atanh}\left(cx^2\right)}{12c^2\sqrt{\frac{1}{c}}+12ic^2\sqrt{\frac{1}{c}}}+\frac{4ibc^2x^3\sqrt{\frac{1}{c}}\operatorname{atanh}\left(cx^2\right)}{12c^2\sqrt{\frac{1}{c}}+12ic^2\sqrt{\frac{1}{c}}}+\frac{2ibc^2\log\left(x+i\sqrt{\frac{1}{c}}\right)}{12c^4\sqrt{\frac{1}{c}}+12ic^4\sqrt{\frac{1}{c}}}+\frac{8bcx\sqrt{\frac{1}{c}}}{12c^2\sqrt{\frac{1}{c}}+12ic^2} \\ \frac{ax^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((4*a*c**2*x**3*sqrt(1/c)/(12*c**2*sqrt(1/c) + 12*I*c**2*sqrt(1/c)
) + 4*I*a*c**2*x**3*sqrt(1/c)/(12*c**2*sqrt(1/c) + 12*I*c**2*sqrt(1/c)) + 4
*b*c**2*x**3*sqrt(1/c)*atanh(c*x**2)/(12*c**2*sqrt(1/c) + 12*I*c**2*sqrt(1/
c)) + 4*I*b*c**2*x**3*sqrt(1/c)*atanh(c*x**2)/(12*c**2*sqrt(1/c) + 12*I*c**
2*sqrt(1/c)) + 2*I*b*c**2*log(x + I*sqrt(1/c))/(12*c**4*sqrt(1/c) + 12*I*c*
*4*sqrt(1/c)) + 8*b*c*x*sqrt(1/c)/(12*c**2*sqrt(1/c) + 12*I*c**2*sqrt(1/c))
 + 8*I*b*c*x*sqrt(1/c)/(12*c**2*sqrt(1/c) + 12*I*c**2*sqrt(1/c)) - 6*I*b*c*
log(x + I*sqrt(1/c))/(12*c**3*sqrt(1/c) + 12*I*c**3*sqrt(1/c)) + 4*I*b*c*lo
g(x - sqrt(1/c))/(12*c**3*sqrt(1/c) + 12*I*c**3*sqrt(1/c)) + 4*I*b*c*atanh(
c*x**2)/(12*c**3*sqrt(1/c) + 12*I*c**3*sqrt(1/c)) - 4*b*log(x - I*sqrt(1/c)
)/(12*c**2*sqrt(1/c) + 12*I*c**2*sqrt(1/c)) + 4*b*log(x - sqrt(1/c))/(12*c*
*2*sqrt(1/c) + 12*I*c**2*sqrt(1/c)) + 4*b*atanh(c*x**2)/(12*c**2*sqrt(1/c)
+ 12*I*c**2*sqrt(1/c)), Ne(c, 0)), (a*x**3/3, True))
```

3.60 $\int \left(a+b \tanh^{-1}\left(c x^2\right)\right) \, dx$

Optimal. Leaf size=44

$$a x+b x \tanh^{-1}\left(c x^2\right)+\frac{b \tan^{-1}\left(\sqrt{c} x\right)}{\sqrt{c}}-\frac{b \tanh^{-1}\left(\sqrt{c} x\right)}{\sqrt{c}}$$

```
[Out] a*x+b*x*arctanh(c*x^2)+b*arctan(x*c^(1/2))/c^(1/2)-b*arctanh(x*c^(1/2))/c^(
1/2)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6091, 298, 203, 206}

$$a x+b x \tanh^{-1}\left(c x^2\right)+\frac{b \tan^{-1}\left(\sqrt{c} x\right)}{\sqrt{c}}-\frac{b \tanh^{-1}\left(\sqrt{c} x\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x^2],x]
```

```
[Out] a*x + (b*ArcTan[Sqrt[c]*x])/Sqrt[c] - (b*ArcTanh[Sqrt[c]*x])/Sqrt[c] + b*x*
ArcTanh[c*x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b \tanh^{-1}\left(c x^2\right)\right) \, dx &= a x+b \int \tanh^{-1}\left(c x^2\right) \, dx\\
&= a x+b x \tanh^{-1}\left(c x^2\right)-(2 b c) \int \frac{x^2}{1-c^2 x^4} \, dx\\
&= a x+b x \tanh^{-1}\left(c x^2\right)-b \int \frac{1}{1-c x^2} \, dx+b \int \frac{1}{1+c x^2} \, dx\\
&= a x+\frac{b \tan^{-1}\left(\sqrt{c} x\right)}{\sqrt{c}}-\frac{b \tanh^{-1}\left(\sqrt{c} x\right)}{\sqrt{c}}+b x \tanh^{-1}\left(c x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.30

$$ax + bx\tanh^{-1}\left(cx^2\right) + \frac{b\left(\log\left(1-\sqrt{c}\,x\right) - \log\left(\sqrt{c}\,x+1\right) + 2\tan^{-1}\left(\sqrt{c}\,x\right)\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x^2],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^2] + (b*(2*ArcTan[Sqrt[c]*x] + Log[1 - Sqrt[c]*x] - L
og[1 + Sqrt[c]*x]))/(2*Sqrt[c])
```

fricas [B] time = 0.88, size = 160, normalized size = 3.64

$$\left[\frac{bcx\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,acx + 2\,b\sqrt{c}\,\arctan\left(\sqrt{c}\,x\right) + b\sqrt{c}\,\log\left(\frac{cx^2-2\,\sqrt{c}\,x+1}{cx^2-1}\right)}{2\,c}, \frac{bcx\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,acx + 2\,b\sqrt{-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(b*c*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x + 2*b*sqrt(c)*arctan(sq
rt(c)*x) + b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2 - 1)))/c, 1/2*(b*
c*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x + 2*b*sqrt(-c)*arctan(sqrt(-c)*
x) - b*sqrt(-c)*log((c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)))/c]
```

giac [B] time = 0.13, size = 83, normalized size = 1.89

$$\frac{1}{2}\left(c\left(\frac{2\,\sqrt{|c|}\,\arctan\left(x\sqrt{|c|}\right)}{c^2} - \frac{\sqrt{|c|}\,\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)}{c^2} + \frac{\sqrt{|c|}\,\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{c^2}\right) + x\log\left(-\frac{cx^2+1}{cx^2-1}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(c*(2*sqrt(abs(c))*arctan(x*sqrt(abs(c)))/c^2 - sqrt(abs(c))*log(abs(x
+ 1/sqrt(abs(c))))/c^2 + sqrt(abs(c))*log(abs(x - 1/sqrt(abs(c))))/c^2) + x
*log(-(c*x^2 + 1)/(c*x^2 - 1)))*b + a*x
```

maple [A] time = 0.03, size = 37, normalized size = 0.84

$$ax + bx\,\mathrm{arctanh}\left(c\,x^2\right) + \frac{b\arctan\left(x\sqrt{c}\right)}{\sqrt{c}} - \frac{b\,\mathrm{arctanh}\left(x\sqrt{c}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^2),x)
```

```
[Out] a*x+b*x*arctanh(c*x^2)+b*arctan(x*c^(1/2))/c^(1/2)-b*arctanh(x*c^(1/2))/c^(
1/2)
```

maxima [A] time = 0.40, size = 55, normalized size = 1.25

$$\frac{1}{2}\left(c\left(\frac{2\,\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{3}{2}}} + \frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) + 2\,x\,\mathrm{artanh}\left(cx^2\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*arctan(sqrt(c)*x)/c^(3/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/
c^(3/2)) + 2*x*arctanh(c*x^2))*b + a*x
```

mupad [B] time = 0.79, size = 55, normalized size = 1.25

$$a\,x+\frac{b\,\mathrm{atan}\left(\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{b\,x\,\ln\left(c\,x^2+1\right)}{2}-\frac{b\,x\,\ln\left(1-c\,x^2\right)}{2}+\frac{b\,\mathrm{atan}\left(\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c*x^2),x)
```

```
[Out] a*x + (b*atan(c^(1/2)*x))/c^(1/2) + (b*atan(c^(1/2)*x*1i)*1i)/c^(1/2) + (b*
x*log(c*x^2 + 1))/2 - (b*x*log(1 - c*x^2))/2
```

sympy [A] time = 5.61, size = 178, normalized size = 4.05

$$ax+b\left(\begin{cases}\frac{c\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(x+i\sqrt{\frac{1}{c}}\right)}{4}-\frac{ic\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(x+i\sqrt{\frac{1}{c}}\right)}{4}+x\,\mathrm{atanh}\left(cx^2\right)-\frac{\sqrt{\frac{1}{c}}\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2}-\frac{i\sqrt{\frac{1}{c}}\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2}-\frac{3\sqrt{\frac{1}{c}}\log\left(x+i\sqrt{\frac{1}{c}}\right)}{4} \\ 0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**2),x)
```

```
[Out] a*x + b*Piecewise((c*(1/c)**(3/2)*log(x + I*sqrt(1/c))/4 - I*c*(1/c)**(3/2)
*log(x + I*sqrt(1/c))/4 + x*atanh(c*x**2) - sqrt(1/c)*log(x - I*sqrt(1/c))/
2 - I*sqrt(1/c)*log(x - I*sqrt(1/c))/2 - 3*sqrt(1/c)*log(x + I*sqrt(1/c))/4
 + 3*I*sqrt(1/c)*log(x + I*sqrt(1/c))/4 + sqrt(1/c)*log(x - sqrt(1/c)) + sq
rt(1/c)*atanh(c*x**2), Ne(c, 0)), (0, True))
```

3.61 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{x}+b\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)+b\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)$$

```
[Out] (-a-b*arctanh(c*x^2))/x+b*arctan(x*c^(1/2))*c^(1/2)+b*arctanh(x*c^(1/2))*c^
(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 212, 206, 203}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{x}+b\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)+b\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^2,x]
```

```
[Out] b*Sqrt[c]*ArcTan[Sqrt[c]*x] + b*Sqrt[c]*ArcTanh[Sqrt[c]*x] - (a + b*ArcTanh
[c*x^2])/x
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^2}\,dx = -\frac{a+b\tanh^{-1}\left(cx^2\right)}{x} + (2bc)\int \frac{1}{1-c^2x^4}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{x} + (bc)\int \frac{1}{1-cx^2}\,dx + (bc)\int \frac{1}{1+cx^2}\,dx$$
$$= b\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right) + b\sqrt{c}\,\tanh^{-1}\left(\sqrt{c}\,x\right) - \frac{a+b\tanh^{-1}\left(cx^2\right)}{x}$$

Mathematica [A] time = 0.02, size = 75, normalized size = 1.63

$$-\frac{a}{x} - \frac{b\tanh^{-1}\left(cx^2\right)}{x} - \frac{1}{2}b\sqrt{c}\,\log\left(1-\sqrt{c}\,x\right) + \frac{1}{2}b\sqrt{c}\,\log\left(\sqrt{c}\,x+1\right) + b\sqrt{c}\,\tan^{-1}\left(\sqrt{c}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^2,x]
```

```
[Out] -(a/x) + b*Sqrt[c]*ArcTan[Sqrt[c]*x] - (b*ArcTanh[c*x^2])/x - (b*Sqrt[c]*Lo
g[1 - Sqrt[c]*x])/2 + (b*Sqrt[c]*Log[1 + Sqrt[c]*x])/2
```

fricas [A] time = 1.16, size = 157, normalized size = 3.41

$$\left[\frac{2\,b\sqrt{c}\,x\arctan\left(\sqrt{c}\,x\right) + b\sqrt{c}\,x\log\left(\frac{cx^2+2\,\sqrt{c}\,x+1}{cx^2-1}\right) - b\log\left(-\frac{cx^2+1}{cx^2-1}\right) - 2\,a}{2\,x}, -\frac{2\,b\sqrt{-c}\,x\arctan\left(\sqrt{-c}\,x\right) - b\sqrt{-c}\,x\log\right.}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*sqrt(c)*x*arctan(sqrt(c)*x) + b*sqrt(c)*x*log((c*x^2 + 2*sqrt(c)*
x + 1)/(c*x^2 - 1)) - b*log(-(c*x^2 + 1)/(c*x^2 - 1)) - 2*a)/x, -1/2*(2*b*s
qrt(-c)*x*arctan(sqrt(-c)*x) - b*sqrt(-c)*x*log((c*x^2 + 2*sqrt(-c)*x - 1)/
(c*x^2 + 1)) + b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x]
```

giac [B] time = 0.17, size = 79, normalized size = 1.72

$$\frac{1}{2}\,bc\left(\frac{2\,\arctan\left(x\sqrt{|c|}\right)}{\sqrt{|c|}} + \frac{\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}} - \frac{\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}\right) - \frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{2\,x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c*(2*arctan(x*sqrt(abs(c)))/sqrt(abs(c)) + log(abs(x + 1/sqrt(abs(c))
))/sqrt(abs(c)) - log(abs(x - 1/sqrt(abs(c))))/sqrt(abs(c))) - 1/2*b*log(-(
c*x^2 + 1)/(c*x^2 - 1))/x - a/x
```

maple [A] time = 0.03, size = 42, normalized size = 0.91

$$-\frac{a}{x} - \frac{b\,\mathrm{arctanh}\left(c\,x^2\right)}{x} + b\arctan\left(x\sqrt{c}\right)\sqrt{c} + b\,\mathrm{arctanh}\left(x\sqrt{c}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c*x^2)+b*arctan(x*c^(1/2))*c^(1/2)+b*arctanh(x*c^(1/2))*c^
(1/2)
```

maxima [A] time = 0.41, size = 61, normalized size = 1.33

$$\frac{1}{2}\left(c\left(\frac{2\,\arctan\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{2\,\operatorname{artanh}\left(cx^2\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*arctan(sqrt(c)*x)/sqrt(c) - log((c*x - sqrt(c))/(c*x + sqrt(c)))/
sqrt(c)) - 2*arctanh(c*x^2)/x)*b - a/x
```

mupad [B] time = 0.92, size = 62, normalized size = 1.35

$$b\,\sqrt{c}\,\mathrm{atan}\left(\sqrt{c}\,x\right)-\frac{a}{x}-\frac{b\,\ln\left(c\,x^2+1\right)}{2\,x}+\frac{b\,\ln\left(1-c\,x^2\right)}{2\,x}-b\,\sqrt{c}\,\mathrm{atan}\left(\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x^2,x)
```

```
[Out] b*c^(1/2)*atan(c^(1/2)*x) - a/x - b*c^(1/2)*atan(c^(1/2)*x*1i)*1i - (b*log(
c*x^2 + 1))/(2*x) + (b*log(1 - c*x^2))/(2*x)
```

sympy [A] time = 10.15, size = 1292, normalized size = 28.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**2,x)
```

```
[Out] Piecewise((-a/x, Eq(c, 0)), (-(a - oo*b)/x, Eq(c, -1/x**2)), (-(a + oo*b)/x
, Eq(c, x**(-2))), (-a*c*x**4*sqrt(1/c)/(c*x**5*sqrt(1/c) + I*c*x**5*sqrt(1
/c) - x*sqrt(1/c)/c - I*x*sqrt(1/c)/c) - I*a*c*x**4*sqrt(1/c)/(c*x**5*sqrt(
1/c) + I*c*x**5*sqrt(1/c) - x*sqrt(1/c)/c - I*x*sqrt(1/c)/c) + a*sqrt(1/c)/
(c**2*x**5*sqrt(1/c) + I*c**2*x**5*sqrt(1/c) - x*sqrt(1/c) - I*x*sqrt(1/c))
 + I*a*sqrt(1/c)/(c**2*x**5*sqrt(1/c) + I*c**2*x**5*sqrt(1/c) - x*sqrt(1/c)
 - I*x*sqrt(1/c)) + I*b*c**2*x**5*log(x + I*sqrt(1/c))/(c**2*x**5*sqrt(1/c)
 + I*c**2*x**5*sqrt(1/c) - x*sqrt(1/c) - I*x*sqrt(1/c)) - I*b*c**2*x**5*log
(x - sqrt(1/c))/(c**2*x**5*sqrt(1/c) + I*c**2*x**5*sqrt(1/c) - x*sqrt(1/c)
- I*x*sqrt(1/c)) - I*b*c**2*x**5*atanh(c*x**2)/(c**2*x**5*sqrt(1/c) + I*c**
2*x**5*sqrt(1/c) - x*sqrt(1/c) - I*x*sqrt(1/c)) + b*c*x**5*log(x - I*sqrt(1
/c))/(c*x**5*sqrt(1/c) + I*c*x**5*sqrt(1/c) - x*sqrt(1/c)/c - I*x*sqrt(1/c)
/c) - b*c*x**5*log(x - sqrt(1/c))/(c*x**5*sqrt(1/c) + I*c*x**5*sqrt(1/c) -
x*sqrt(1/c)/c - I*x*sqrt(1/c)/c) - b*c*x**5*atanh(c*x**2)/(c*x**5*sqrt(1/c)
 + I*c*x**5*sqrt(1/c) - x*sqrt(1/c)/c - I*x*sqrt(1/c)/c) - b*c*x**4*sqrt(1/
c)*atanh(c*x**2)/(c*x**5*sqrt(1/c) + I*c*x**5*sqrt(1/c) - x*sqrt(1/c)/c - I
*x*sqrt(1/c)/c) - I*b*c*x**4*sqrt(1/c)*atanh(c*x**2)/(c*x**5*sqrt(1/c) + I*
c*x**5*sqrt(1/c) - x*sqrt(1/c)/c - I*x*sqrt(1/c)/c) - b*x*log(x - I*sqrt(1/
c))/(c**2*x**5*sqrt(1/c) + I*c**2*x**5*sqrt(1/c) - x*sqrt(1/c) - I*x*sqrt(1
/c)) - I*b*x*log(x + I*sqrt(1/c))/(c**2*x**5*sqrt(1/c) + I*c**2*x**5*sqrt(1
/c) - x*sqrt(1/c) - I*x*sqrt(1/c)) + b*x*log(x - sqrt(1/c))/(c**2*x**5*sqrt
(1/c) + I*c**2*x**5*sqrt(1/c) - x*sqrt(1/c) - I*x*sqrt(1/c)) + I*b*x*log(x
- sqrt(1/c))/(c**2*x**5*sqrt(1/c) + I*c**2*x**5*sqrt(1/c) - x*sqrt(1/c) - I
*x*sqrt(1/c)) + b*x*atanh(c*x**2)/(c**2*x**5*sqrt(1/c) + I*c**2*x**5*sqrt(1
/c) - x*sqrt(1/c) - I*x*sqrt(1/c)) + I*b*x*atanh(c*x**2)/(c**2*x**5*sqrt(1/
```

```
c) + I*c**2*x**5*sqrt(1/c) - x*sqrt(1/c) - I*x*sqrt(1/c)) + b*sqrt(1/c)*ata
nh(c*x**2)/(c**2*x**5*sqrt(1/c) + I*c**2*x**5*sqrt(1/c) - x*sqrt(1/c) - I*x
*sqrt(1/c)) + I*b*sqrt(1/c)*atanh(c*x**2)/(c**2*x**5*sqrt(1/c) + I*c**2*x**
5*sqrt(1/c) - x*sqrt(1/c) - I*x*sqrt(1/c)), True))
```

3.62 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^4}\,dx$

Optimal. Leaf size=63

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{2bc}{3x}$$

```
[Out] -2/3*b*c/x-1/3*b*c^(3/2)*arctan(x*c^(1/2))+1/3*(-a-b*arctanh(c*x^2))/x^3+1/
3*b*c^(3/2)*arctanh(x*c^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 325, 298, 203, 206}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{2bc}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^4,x]
```

```
[Out] (-2*b*c)/(3*x) - (b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 + (b*c^(3/2)*ArcTanh[Sqrt[
c]*x])/3 - (a + b*ArcTanh[c*x^2])/(3*x^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^4}\,dx = -\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3} + \frac{1}{3}(2bc)\int \frac{1}{x^2\left(1-c^2x^4\right)}\,dx$$
$$= -\frac{2bc}{3x} - \frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3} + \frac{1}{3}\left(2bc^3\right)\int \frac{x^2}{1-c^2x^4}\,dx$$
$$= -\frac{2bc}{3x} - \frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3} + \frac{1}{3}\left(bc^2\right)\int \frac{1}{1-cx^2}\,dx - \frac{1}{3}\left(bc^2\right)\int \frac{1}{1+cx^2}\,dx$$
$$= -\frac{2bc}{3x} - \frac{1}{3}bc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{1}{3}bc^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right) - \frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.44

$$-\frac{a}{3x^3}-\frac{1}{6}bc^{3/2}\log\left(1-\sqrt{c}\,x\right)+\frac{1}{6}bc^{3/2}\log\left(\sqrt{c}\,x+1\right)-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)-\frac{b\tanh^{-1}\left(cx^2\right)}{3x^3}-\frac{2bc}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (2*b*c)/(3*x) - (b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 - (b*ArcTanh[c
*x^2])/(3*x^3) - (b*c^(3/2)*Log[1 - Sqrt[c]*x])/6 + (b*c^(3/2)*Log[1 + Sqrt
[c]*x])/6
```

fricas [A] time = 0.98, size = 181, normalized size = 2.87

$$\left[-\frac{2\,bc^{\frac{3}{2}}x^3\arctan\left(\sqrt{c}\,x\right)-bc^{\frac{3}{2}}x^3\log\left(\frac{cx^2+2\,\sqrt{c}\,x+1}{cx^2-1}\right)+4\,bcx^2+b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a}{6\,x^3},-\frac{2\,b\sqrt{-c}\,cx^3\arctan\left(\sqrt{-c}\,x\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*b*c^(3/2)*x^3*arctan(sqrt(c)*x) - b*c^(3/2)*x^3*log((c*x^2 + 2*sqr
t(c)*x + 1)/(c*x^2 - 1)) + 4*b*c*x^2 + b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*
a)/x^3, -1/6*(2*b*sqrt(-c)*c*x^3*arctan(sqrt(-c)*x) - b*sqrt(-c)*c*x^3*log(
(c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) + 4*b*c*x^2 + b*log(-(c*x^2 + 1)/(c
*x^2 - 1)) + 2*a)/x^3]
```

giac [A] time = 0.18, size = 93, normalized size = 1.48

$$-\frac{bc^3\arctan\left(x\sqrt{|c|}\right)}{3\,|c|^{\frac{3}{2}}}+\frac{1}{6}\,bc\sqrt{|c|}\,\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)-\frac{bc^3\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{6\,|c|^{\frac{3}{2}}}-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{6\,x^3}-\frac{2\,bcx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*b*c^3*arctan(x*sqrt(abs(c)))/abs(c)^(3/2) + 1/6*b*c*sqrt(abs(c))*log(a
bs(x + 1/sqrt(abs(c)))) - 1/6*b*c^3*log(abs(x - 1/sqrt(abs(c))))/abs(c)^(3/
2) - 1/6*b*log(-(c*x^2 + 1)/(c*x^2 - 1))/x^3 - 1/3*(2*b*c*x^2 + a)/x^3
```

maple [A] time = 0.03, size = 51, normalized size = 0.81

$$-\frac{a}{3x^3}-\frac{b\,\mathrm{arctanh}\left(c\,x^2\right)}{3x^3}-\frac{2bc}{3x}-\frac{b\,c^{\frac{3}{2}}\arctan\left(x\sqrt{c}\right)}{3}+\frac{b\,c^{\frac{3}{2}}\,\mathrm{arctanh}\left(x\sqrt{c}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x^2)-2/3*b*c/x-1/3*b*c^(3/2)*arctan(x*c^(1/2
))+1/3*b*c^(3/2)*arctanh(x*c^(1/2))
```

maxima [A] time = 0.41, size = 65, normalized size = 1.03

$$-\frac{1}{6}\left(\left(2\sqrt{c}\arctan\left(\sqrt{c}x\right)+\sqrt{c}\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)+\frac{4}{x}\right)c+\frac{2\operatorname{artanh}\left(cx^2\right)}{x^3}\right)b-\frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((2*sqrt(c)*arctan(sqrt(c)*x) + sqrt(c)*log((c*x - sqrt(c))/(c*x + sqr
t(c))) + 4/x)*c + 2*arctanh(c*x^2)/x^3)*b - 1/3*a/x^3
```

mupad [B] time = 0.99, size = 71, normalized size = 1.13

$$\frac{b\ln\left(1-cx^2\right)}{6x^3}-\frac{b\,c^{3/2}\operatorname{atan}\left(\sqrt{c}\,x\right)}{3}-\frac{b\ln\left(cx^2+1\right)}{6x^3}-\frac{2bcx^2+a}{3x^3}-\frac{b\,c^{3/2}\operatorname{atan}\left(\sqrt{c}\,x\,1\mathrm{i}\right)1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x^4,x)
```

```
[Out] (b*log(1 - c*x^2))/(6*x^3) - (b*c^(3/2)*atan(c^(1/2)*x))/3 - (b*c^(3/2)*ata
n(c^(1/2)*x*1i)*1i)/3 - (b*log(c*x^2 + 1))/(6*x^3) - (a + 2*b*c*x^2)/(3*x^3
)
```

sympy [A] time = 14.52, size = 1822, normalized size = 28.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3), Eq(c, 0)), (-(a - oo*b)/(3*x**3), Eq(c, -1/x**2)),
(-(a + oo*b)/(3*x**3), Eq(c, x**(-2))), (a*c*x**4*sqrt(1/c)/(-3*c*x**7*sqrt
(1/c) + 3*I*c*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c)/c - 3*I*x**3*sqrt(1/c)/c) -
 I*a*c*x**4*sqrt(1/c)/(-3*c*x**7*sqrt(1/c) + 3*I*c*x**7*sqrt(1/c) + 3*x**3*
sqrt(1/c)/c - 3*I*x**3*sqrt(1/c)/c) - a*sqrt(1/c)/(-3*c**2*x**7*sqrt(1/c) +
 3*I*c**2*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c) - 3*I*x**3*sqrt(1/c)) + I*a*sqr
t(1/c)/(-3*c**2*x**7*sqrt(1/c) + 3*I*c**2*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c)
 - 3*I*x**3*sqrt(1/c)) + I*b*c**3*x**7*log(x + I*sqrt(1/c))/(-3*c**2*x**7*s
qrt(1/c) + 3*I*c**2*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c) - 3*I*x**3*sqrt(1/c))
 - I*b*c**3*x**7*log(x - sqrt(1/c))/(-3*c**2*x**7*sqrt(1/c) + 3*I*c**2*x**7
*sqrt(1/c) + 3*x**3*sqrt(1/c) - 3*I*x**3*sqrt(1/c)) - I*b*c**3*x**7*atanh(c
*x**2)/(-3*c**2*x**7*sqrt(1/c) + 3*I*c**2*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c)
 - 3*I*x**3*sqrt(1/c)) - b*c**2*x**7*log(x - I*sqrt(1/c))/(-3*c*x**7*sqrt(1
/c) + 3*I*c*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c)/c - 3*I*x**3*sqrt(1/c)/c) + b
*c**2*x**7*log(x - sqrt(1/c))/(-3*c*x**7*sqrt(1/c) + 3*I*c*x**7*sqrt(1/c) +
 3*x**3*sqrt(1/c)/c - 3*I*x**3*sqrt(1/c)/c) + b*c**2*x**7*atanh(c*x**2)/(-3
*c*x**7*sqrt(1/c) + 3*I*c*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c)/c - 3*I*x**3*sq
rt(1/c)/c) + 2*b*c**2*x**6*sqrt(1/c)/(-3*c*x**7*sqrt(1/c) + 3*I*c*x**7*sqrt
(1/c) + 3*x**3*sqrt(1/c)/c - 3*I*x**3*sqrt(1/c)/c) - 2*I*b*c**2*x**6*sqrt(1
/c)/(-3*c*x**7*sqrt(1/c) + 3*I*c*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c)/c - 3*I*
x**3*sqrt(1/c)/c) + b*c*x**4*sqrt(1/c)*atanh(c*x**2)/(-3*c*x**7*sqrt(1/c) +
 3*I*c*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c)/c - 3*I*x**3*sqrt(1/c)/c) - I*b*c*
```

```
x**4*sqrt(1/c)*atanh(c*x**2)/(-3*c*x**7*sqrt(1/c) + 3*I*c*x**7*sqrt(1/c) +
3*x**3*sqrt(1/c)/c - 3*I*x**3*sqrt(1/c)/c) - I*b*c*x**3*log(x + I*sqrt(1/c)
)/(-3*c**2*x**7*sqrt(1/c) + 3*I*c**2*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c) - 3*
I*x**3*sqrt(1/c)) + I*b*c*x**3*log(x - sqrt(1/c))/(-3*c**2*x**7*sqrt(1/c) +
 3*I*c**2*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c) - 3*I*x**3*sqrt(1/c)) + I*b*c*x
**3*atanh(c*x**2)/(-3*c**2*x**7*sqrt(1/c) + 3*I*c**2*x**7*sqrt(1/c) + 3*x**
3*sqrt(1/c) - 3*I*x**3*sqrt(1/c)) + b*x**3*log(x - I*sqrt(1/c))/(-3*c*x**7*
sqrt(1/c) + 3*I*c*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c)/c - 3*I*x**3*sqrt(1/c)/
c) - b*x**3*log(x - sqrt(1/c))/(-3*c*x**7*sqrt(1/c) + 3*I*c*x**7*sqrt(1/c)
+ 3*x**3*sqrt(1/c)/c - 3*I*x**3*sqrt(1/c)/c) - b*x**3*atanh(c*x**2)/(-3*c*x
**7*sqrt(1/c) + 3*I*c*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c)/c - 3*I*x**3*sqrt(1
/c)/c) - 2*b*x**2*sqrt(1/c)/(-3*c*x**7*sqrt(1/c) + 3*I*c*x**7*sqrt(1/c) + 3
*x**3*sqrt(1/c)/c - 3*I*x**3*sqrt(1/c)/c) + 2*I*b*x**2*sqrt(1/c)/(-3*c*x**7
*sqrt(1/c) + 3*I*c*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c)/c - 3*I*x**3*sqrt(1/c)
/c) - b*sqrt(1/c)*atanh(c*x**2)/(-3*c**2*x**7*sqrt(1/c) + 3*I*c**2*x**7*sqr
t(1/c) + 3*x**3*sqrt(1/c) - 3*I*x**3*sqrt(1/c)) + I*b*sqrt(1/c)*atanh(c*x**
2)/(-3*c**2*x**7*sqrt(1/c) + 3*I*c**2*x**7*sqrt(1/c) + 3*x**3*sqrt(1/c) - 3
*I*x**3*sqrt(1/c)), True))
```

3.63 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^6}\,dx$

Optimal. Leaf size=63

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{2bc}{15x^3}$$

```
[Out] -2/15*b*c/x^3+1/5*b*c^(5/2)*arctan(x*c^(1/2))+1/5*(-a-b*arctanh(c*x^2))/x^5
+1/5*b*c^(5/2)*arctanh(x*c^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 325, 212, 206, 203}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{2bc}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^6,x]
```

```
[Out] (-2*b*c)/(15*x^3) + (b*c^(5/2)*ArcTan[Sqrt[c]*x])/5 + (b*c^(5/2)*ArcTanh[Sq
rt[c]*x])/5 - (a + b*ArcTanh[c*x^2])/(5*x^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^6}\,dx = -\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}(2bc)\int \frac{1}{x^4\left(1-c^2x^4\right)}\,dx$$
$$= -\frac{2bc}{15x^3}-\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}\left(2bc^3\right)\int \frac{1}{1-c^2x^4}\,dx$$
$$= -\frac{2bc}{15x^3}-\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}\left(bc^3\right)\int \frac{1}{1-cx^2}\,dx+\frac{1}{5}\left(bc^3\right)\int \frac{1}{1+cx^2}\,dx$$
$$= -\frac{2bc}{15x^3}+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.44

$$-\frac{a}{5x^5}-\frac{1}{10}bc^{5/2}\log\left(1-\sqrt{c}\,x\right)+\frac{1}{10}bc^{5/2}\log\left(\sqrt{c}\,x+1\right)+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)-\frac{2bc}{15x^3}-\frac{b\tanh^{-1}\left(cx^2\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (2*b*c)/(15*x^3) + (b*c^(5/2)*ArcTan[Sqrt[c]*x])/5 - (b*ArcTan
h[c*x^2])/(5*x^5) - (b*c^(5/2)*Log[1 - Sqrt[c]*x])/10 + (b*c^(5/2)*Log[1 +
Sqrt[c]*x])/10
```

fricas [A] time = 1.02, size = 187, normalized size = 2.97

$$\left[\frac{6\,bc^{\frac{5}{2}}x^5\arctan\left(\sqrt{c}\,x\right)+3\,bc^{\frac{5}{2}}x^5\log\left(\frac{cx^2+2\,\sqrt{c}\,x+1}{cx^2-1}\right)-4\,bcx^2-3\,b\log\left(-\frac{cx^2+1}{cx^2-1}\right)-6\,a}{30\,x^5},-\frac{6\,b\sqrt{-c}\,c^2x^5\arctan\left(\sqrt{-c}\,x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(6*b*c^(5/2)*x^5*arctan(sqrt(c)*x) + 3*b*c^(5/2)*x^5*log((c*x^2 + 2*s
qrt(c)*x + 1)/(c*x^2 - 1)) - 4*b*c*x^2 - 3*b*log(-(c*x^2 + 1)/(c*x^2 - 1))
- 6*a)/x^5, -1/30*(6*b*sqrt(-c)*c^2*x^5*arctan(sqrt(-c)*x) - 3*b*sqrt(-c)*c
^2*x^5*log((c*x^2 + 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) + 4*b*c*x^2 + 3*b*log(-(
c*x^2 + 1)/(c*x^2 - 1)) + 6*a)/x^5]
```

giac [A] time = 0.22, size = 91, normalized size = 1.44

$$\frac{1}{10}\,bc^3\left(\frac{2\,\arctan\left(x\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}-\frac{\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}\right)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{10\,x^5}-\frac{2\,bcx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="giac")
```

```
[Out] 1/10*b*c^3*(2*arctan(x*sqrt(abs(c)))/sqrt(abs(c)) + log(abs(x + 1/sqrt(abs(
c))))/sqrt(abs(c)) - log(abs(x - 1/sqrt(abs(c))))/sqrt(abs(c))) - 1/10*b*lo
g(-(c*x^2 + 1)/(c*x^2 - 1))/x^5 - 1/15*(2*b*c*x^2 + 3*a)/x^5
```

maple [A] time = 0.03, size = 51, normalized size = 0.81

$$-\frac{a}{5x^5}-\frac{b\,\mathrm{arctanh}\left(c\,x^2\right)}{5x^5}-\frac{2bc}{15x^3}+\frac{b\,c^{\frac{5}{2}}\arctan\left(x\sqrt{c}\right)}{5}+\frac{b\,c^{\frac{5}{2}}\,\mathrm{arctanh}\left(x\sqrt{c}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctanh(c*x^2)-2/15*b*c/x^3+1/5*b*c^(5/2)*arctan(x*c^(
1/2))+1/5*b*c^(5/2)*arctanh(x*c^(1/2))
```

maxima [A] time = 0.40, size = 66, normalized size = 1.05

$$\frac{1}{30}\left(\left(6\,c^{\frac{3}{2}}\arctan\left(\sqrt{c}\,x\right)-3\,c^{\frac{3}{2}}\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)-\frac{4}{x^3}\right)c-\frac{6\,\mathrm{artanh}\left(cx^2\right)}{x^5}\right)b-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="maxima")
```

```
[Out] 1/30*((6*c^(3/2)*arctan(sqrt(c)*x) - 3*c^(3/2)*log((c*x - sqrt(c))/(c*x + s
qrt(c))) - 4/x^3)*c - 6*arctanh(c*x^2)/x^5)*b - 1/5*a/x^5
```

mupad [B] time = 1.03, size = 71, normalized size = 1.13

$$\frac{b\,c^{5/2}\,\mathrm{atan}\left(\sqrt{c}\,x\right)}{5}-\frac{\frac{2\,b\,c\,x^2}{3}+a}{5\,x^5}-\frac{b\,\ln\left(c\,x^2+1\right)}{10\,x^5}+\frac{b\,\ln\left(1-c\,x^2\right)}{10\,x^5}-\frac{b\,c^{5/2}\,\mathrm{atan}\left(\sqrt{c}\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/x^6,x)
```

```
[Out] (b*c^(5/2)*atan(c^(1/2)*x))/5 - (a + (2*b*c*x^2)/3)/(5*x^5) - (b*c^(5/2)*at
an(c^(1/2)*x*1i)*1i)/5 - (b*log(c*x^2 + 1))/(10*x^5) + (b*log(1 - c*x^2))/(
10*x^5)
```

sympy [A] time = 20.47, size = 1867, normalized size = 29.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**6,x)
```

```
[Out] Piecewise((-(a - oo*b)/(5*x**5), Eq(c, -1/x**2)), (-(a + oo*b)/(5*x**5), Eq
(c, x**(-2))), (-a/(5*x**5), Eq(c, 0)), (-3*a*c*x**4*sqrt(1/c)/(15*c*x**9*s
qrt(1/c) + 15*I*c*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c)/c - 15*I*x**5*sqrt(1/c
)/c) - 3*I*a*c*x**4*sqrt(1/c)/(15*c*x**9*sqrt(1/c) + 15*I*c*x**9*sqrt(1/c)
- 15*x**5*sqrt(1/c)/c - 15*I*x**5*sqrt(1/c)/c) + 3*a*sqrt(1/c)/(15*c**2*x**
9*sqrt(1/c) + 15*I*c**2*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c) - 15*I*x**5*sqrt
(1/c)) + 3*I*a*sqrt(1/c)/(15*c**2*x**9*sqrt(1/c) + 15*I*c**2*x**9*sqrt(1/c)
 - 15*x**5*sqrt(1/c) - 15*I*x**5*sqrt(1/c)) + 3*I*b*c**4*x**9*log(x + I*sqr
t(1/c))/(15*c**2*x**9*sqrt(1/c) + 15*I*c**2*x**9*sqrt(1/c) - 15*x**5*sqrt(1
/c) - 15*I*x**5*sqrt(1/c)) - 3*I*b*c**4*x**9*log(x - sqrt(1/c))/(15*c**2*x*
*9*sqrt(1/c) + 15*I*c**2*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c) - 15*I*x**5*sqr
t(1/c)) - 3*I*b*c**4*x**9*atanh(c*x**2)/(15*c**2*x**9*sqrt(1/c) + 15*I*c**2
*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c) - 15*I*x**5*sqrt(1/c)) + 3*b*c**3*x**9*
log(x - I*sqrt(1/c))/(15*c*x**9*sqrt(1/c) + 15*I*c*x**9*sqrt(1/c) - 15*x**5
*sqrt(1/c)/c - 15*I*x**5*sqrt(1/c)/c) - 3*b*c**3*x**9*log(x - sqrt(1/c))/(1
5*c*x**9*sqrt(1/c) + 15*I*c*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c)/c - 15*I*x**
5*sqrt(1/c)/c) - 3*b*c**3*x**9*atanh(c*x**2)/(15*c*x**9*sqrt(1/c) + 15*I*c*
x**9*sqrt(1/c) - 15*x**5*sqrt(1/c)/c - 15*I*x**5*sqrt(1/c)/c) - 2*b*c**2*x*
*6*sqrt(1/c)/(15*c*x**9*sqrt(1/c) + 15*I*c*x**9*sqrt(1/c) - 15*x**5*sqrt(1/
c)/c - 15*I*x**5*sqrt(1/c)/c) - 2*I*b*c**2*x**6*sqrt(1/c)/(15*c*x**9*sqrt(1
/c) + 15*I*c*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c)/c - 15*I*x**5*sqrt(1/c)/c)
- 3*I*b*c**2*x**5*log(x + I*sqrt(1/c))/(15*c**2*x**9*sqrt(1/c) + 15*I*c**2*
```

```
x**9*sqrt(1/c) - 15*x**5*sqrt(1/c) - 15*I*x**5*sqrt(1/c)) + 3*I*b*c**2*x**5
*log(x - sqrt(1/c))/(15*c**2*x**9*sqrt(1/c) + 15*I*c**2*x**9*sqrt(1/c) - 15
*x**5*sqrt(1/c) - 15*I*x**5*sqrt(1/c)) + 3*I*b*c**2*x**5*atanh(c*x**2)/(15*
c**2*x**9*sqrt(1/c) + 15*I*c**2*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c) - 15*I*x
**5*sqrt(1/c)) - 3*b*c*x**5*log(x - I*sqrt(1/c))/(15*c*x**9*sqrt(1/c) + 15*
I*c*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c)/c - 15*I*x**5*sqrt(1/c)/c) + 3*b*c*x
**5*log(x - sqrt(1/c))/(15*c*x**9*sqrt(1/c) + 15*I*c*x**9*sqrt(1/c) - 15*x*
*5*sqrt(1/c)/c - 15*I*x**5*sqrt(1/c)/c) + 3*b*c*x**5*atanh(c*x**2)/(15*c*x*
*9*sqrt(1/c) + 15*I*c*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c)/c - 15*I*x**5*sqrt
(1/c)/c) - 3*b*c*x**4*sqrt(1/c)*atanh(c*x**2)/(15*c*x**9*sqrt(1/c) + 15*I*c
*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c)/c - 15*I*x**5*sqrt(1/c)/c) - 3*I*b*c*x*
*4*sqrt(1/c)*atanh(c*x**2)/(15*c*x**9*sqrt(1/c) + 15*I*c*x**9*sqrt(1/c) - 1
5*x**5*sqrt(1/c)/c - 15*I*x**5*sqrt(1/c)/c) + 2*b*x**2*sqrt(1/c)/(15*c*x**9
*sqrt(1/c) + 15*I*c*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c)/c - 15*I*x**5*sqrt(1
/c)/c) + 2*I*b*x**2*sqrt(1/c)/(15*c*x**9*sqrt(1/c) + 15*I*c*x**9*sqrt(1/c)
- 15*x**5*sqrt(1/c)/c - 15*I*x**5*sqrt(1/c)/c) + 3*b*sqrt(1/c)*atanh(c*x**2
)/(15*c**2*x**9*sqrt(1/c) + 15*I*c**2*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c) -
15*I*x**5*sqrt(1/c)) + 3*I*b*sqrt(1/c)*atanh(c*x**2)/(15*c**2*x**9*sqrt(1/c
) + 15*I*c**2*x**9*sqrt(1/c) - 15*x**5*sqrt(1/c) - 15*I*x**5*sqrt(1/c)), Tr
ue))
```

3.64 $\int x^7 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=125

$$-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{8c^4}+\frac{abx^2}{4c^3}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{bx^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{12c}+\frac{b^2x^2\tanh^{-1}\left(cx^2\right)}{4c^3}+\frac{b^2x^4}{24c^2}$$

```
[Out] 1/4*a*b*x^2/c^3+1/24*b^2*x^4/c^2+1/4*b^2*x^2*arctanh(c*x^2)/c^3+1/12*b*x^6*
(a+b*arctanh(c*x^2))/c-1/8*(a+b*arctanh(c*x^2))^2/c^4+1/8*x^8*(a+b*arctanh(
c*x^2))^2+1/6*b^2*ln(-c^2*x^4+1)/c^4
```

Rubi [C] time = 1.55, antiderivative size = 636, normalized size of antiderivative = 5.09, number of steps used = 62, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {6099, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)}{16c^4}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)}{16c^4}+\frac{abx^2}{8c^3}-\frac{bx^4\left(2a-b\log\left(1-cx^2\right)\right)}{32c^2}-\frac{1}{192}b\left(-\frac{3\left(1-cx^2\right.}{c^4}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^7*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (a*b*x^2)/(8*c^3) + (23*b^2*x^2)/(192*c^3) + (b^2*x^4)/(128*c^2) - (7*b^2*x
^6)/(576*c) - (b^2*x^8)/256 + (3*b^2*(1 - c*x^2)^2)/(32*c^4) - (b^2*(1 - c*
x^2)^3)/(36*c^4) + (b^2*(1 - c*x^2)^4)/(256*c^4) - (5*b^2*Log[1 - c*x^2])/(
192*c^4) + (b^2*(1 - c*x^2)*Log[1 - c*x^2])/(16*c^4) + (b^2*Log[1 - c*x^2]^
2)/(32*c^4) - (b*x^4*(2*a - b*Log[1 - c*x^2]))/(32*c^2) + (b*x^6*(2*a - b*L
og[1 - c*x^2]))/(48*c) - (b*x^8*(2*a - b*Log[1 - c*x^2]))/64 + (x^8*(2*a -
b*Log[1 - c*x^2])^2)/32 - (b*(2*a - b*Log[1 - c*x^2])*((48*(1 - c*x^2))/c^4
 - (36*(1 - c*x^2)^2)/c^4 + (16*(1 - c*x^2)^3)/c^4 - (3*(1 - c*x^2)^4)/c^4
- (12*Log[1 - c*x^2])/c^4))/192 - (b*(2*a - b*Log[1 - c*x^2])*Log[(1 + c*x^
2)/2])/(16*c^4) + (b^2*Log[1 + c*x^2])/(24*c^4) + (b^2*x^6*Log[1 + c*x^2])/
(24*c) + (b^2*(1 + c*x^2)*Log[1 + c*x^2])/(8*c^4) + (b^2*Log[(1 - c*x^2)/2]
*Log[1 + c*x^2])/(16*c^4) + (b*x^8*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2])
/16 - (b^2*Log[1 + c*x^2]^2)/(32*c^4) + (b^2*x^8*Log[1 + c*x^2]^2)/32 + (b^
2*PolyLog[2, (1 - c*x^2)/2])/(16*c^4) + (b^2*PolyLog[2, (1 + c*x^2)/2])/(16
*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x^7 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x^7\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{1}{2}bx^7\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{4}\right.$$
$$= \frac{1}{4}\int x^7\left(2a-b\log\left(1-cx^2\right)\right)^2 dx - \frac{1}{2}b\int x^7\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx\right.$$
$$= \frac{1}{8}\text{Subst}\left(\int x^3(2a-b\log(1-cx))^2 dx, x, x^2\right) - \frac{1}{4}b\,\text{Subst}\left(\int x^3(-2a+b\log(1-\right.$$
$$= \frac{1}{32}x^8\left(2a-b\log\left(1-cx^2\right)\right)^2 + \frac{1}{16}bx^8\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right) + \frac{1}{32}b$$
$$= \frac{1}{32}x^8\left(2a-b\log\left(1-cx^2\right)\right)^2 + \frac{1}{16}bx^8\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right) + \frac{1}{32}b$$
$$= \frac{1}{32}x^8\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{1}{192}b\left(2a-b\log\left(1-cx^2\right)\right)\left(\frac{48\left(1-cx^2\right)}{c^4} - \frac{36\left(1\right.}{}\right.$$
$$= \frac{abx^2}{8c^3} - \frac{bx^4\left(2a-b\log\left(1-cx^2\right)\right)}{32c^2} + \frac{bx^6\left(2a-b\log\left(1-cx^2\right)\right)}{48c} - \frac{1}{64}bx^8\left(2a-b\right.$$
$$= \frac{abx^2}{8c^3} - \frac{bx^4\left(2a-b\log\left(1-cx^2\right)\right)}{32c^2} + \frac{bx^6\left(2a-b\log\left(1-cx^2\right)\right)}{48c} - \frac{1}{64}bx^8\left(2a-b\right.$$
$$= \frac{abx^2}{8c^3} + \frac{55b^2x^2}{192c^3} - \frac{5b^2x^4}{384c^2} + \frac{b^2x^6}{576c} - \frac{b^2x^8}{256} + \frac{3b^2\left(1-cx^2\right)^2}{32c^4} - \frac{b^2\left(1-cx^2\right)^3}{36c^4} + \frac{b^2\left(\right.}{2}$$
$$= \frac{abx^2}{8c^3} + \frac{55b^2x^2}{192c^3} - \frac{5b^2x^4}{384c^2} + \frac{b^2x^6}{576c} - \frac{b^2x^8}{256} + \frac{3b^2\left(1-cx^2\right)^2}{32c^4} - \frac{b^2\left(1-cx^2\right)^3}{36c^4} + \frac{b^2\left(\right.}{2}$$

Mathematica [A] time = 0.08, size = 146, normalized size = 1.17

$$\frac{3a^2c^4x^8 + 2abc^3x^6 + 2bcx^2\tanh^{-1}\left(cx^2\right)\left(3ac^3x^6 + b\left(c^2x^4+3\right)\right) + 6abcx^2 + b(3a+4b)\log\left(1-cx^2\right) - 3ab\log\left(c\right.}{24c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (6*a*b*c*x^2 + b^2*c^2*x^4 + 2*a*b*c^3*x^6 + 3*a^2*c^4*x^8 + 2*b*c*x^2*(3*a
*c^3*x^6 + b*(3 + c^2*x^4))*ArcTanh[c*x^2] + 3*b^2*(-1 + c^4*x^8)*ArcTanh[c
*x^2]^2 + b*(3*a + 4*b)*Log[1 - c*x^2] - 3*a*b*Log[1 + c*x^2] + 4*b^2*Log[1
 + c*x^2])/(24*c^4)
```

fricas [A] time = 0.86, size = 176, normalized size = 1.41

$$\frac{12\,a^2c^4x^8 + 8\,abc^3x^6 + 4\,b^2c^2x^4 + 24\,abcx^2 + 3\left(b^2c^4x^8 - b^2\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2 - 4\left(3\,ab - 4\,b^2\right)\log\left(cx^2+1\right) + 4\left(3\,a\right.}{96\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/96*(12*a^2*c^4*x^8 + 8*a*b*c^3*x^6 + 4*b^2*c^2*x^4 + 24*a*b*c*x^2 + 3*(b^
2*c^4*x^8 - b^2)*log(-(c*x^2 + 1)/(c*x^2 - 1))^2 - 4*(3*a*b - 4*b^2)*log(c*
x^2 + 1) + 4*(3*a*b + 4*b^2)*log(c*x^2 - 1) + 4*(3*a*b*c^4*x^8 + b^2*c^3*x^
6 + 3*b^2*c*x^2)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^4
```

giac [A] time = 0.28, size = 175, normalized size = 1.40

$$\frac{1}{8}a^2x^8+\frac{abx^6}{12c}+\frac{b^2x^4}{24c^2}+\frac{1}{32}\left(b^2x^8-\frac{b^2}{c^4}\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2+\frac{1}{24}\left(3abx^8+\frac{b^2x^6}{c}+\frac{3b^2x^2}{c^3}\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{abx^2}{4c^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*x^8 + 1/12*a*b*x^6/c + 1/24*b^2*x^4/c^2 + 1/32*(b^2*x^8 - b^2/c^4)*
log(-(c*x^2 + 1)/(c*x^2 - 1))^2 + 1/24*(3*a*b*x^8 + b^2*x^6/c + 3*b^2*x^2/c
^3)*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/4*a*b*x^2/c^3 - 1/24*(3*a*b - 4*b^2)*
log(c*x^2 + 1)/c^4 + 1/24*(3*a*b + 4*b^2)*log(c*x^2 - 1)/c^4
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^7\left(a+b\,\mathrm{arctanh}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int(x^7*(a+b*arctanh(c*x^2))^2,x)
```

maxima [A] time = 0.33, size = 217, normalized size = 1.74

$$\frac{1}{8}b^2x^8\,\mathrm{artanh}\left(cx^2\right)^2+\frac{1}{8}a^2x^8+\frac{1}{24}\left(6\,x^8\,\mathrm{artanh}\left(cx^2\right)+c\left(\frac{2\left(c^2x^6+3\,x^2\right)}{c^4}-\frac{3\,\log\left(cx^2+1\right)}{c^5}+\frac{3\,\log\left(cx^2-1\right)}{c^5}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8*arctanh(c*x^2)^2 + 1/8*a^2*x^8 + 1/24*(6*x^8*arctanh(c*x^2) + c
*(2*(c^2*x^6 + 3*x^2)/c^4 - 3*log(c*x^2 + 1)/c^5 + 3*log(c*x^2 - 1)/c^5))*a
*b + 1/96*(4*c*(2*(c^2*x^6 + 3*x^2)/c^4 - 3*log(c*x^2 + 1)/c^5 + 3*log(c*x^
2 - 1)/c^5)*arctanh(c*x^2) + (4*c^2*x^4 - 2*(3*log(c*x^2 - 1) - 8)*log(c*x^
2 + 1) + 3*log(c*x^2 + 1)^2 + 3*log(c*x^2 - 1)^2 + 16*log(c*x^2 - 1))/c^4)*
b^2
```

mupad [B] time = 1.73, size = 335, normalized size = 2.68

$$\frac{a^2\,x^8}{8}+\frac{b^2\,\ln\left(c\,x^2-1\right)}{6\,c^4}+\frac{b^2\,\ln\left(c\,x^2+1\right)}{6\,c^4}-\frac{b^2\ln\left(c\,x^2+1\right)^2}{32\,c^4}-\frac{b^2\ln\left(1-c\,x^2\right)^2}{32\,c^4}+\frac{b^2\,x^4}{24\,c^2}+\frac{b^2\,x^8\ln\left(c\,x^2+1\right)^2}{32}+\frac{b^2\,x^8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atanh(c*x^2))^2,x)
```

```
[Out] (a^2*x^8)/8 + (b^2*log(c*x^2 - 1))/(6*c^4) + (b^2*log(c*x^2 + 1))/(6*c^4) -
 (b^2*log(c*x^2 + 1)^2)/(32*c^4) - (b^2*log(1 - c*x^2)^2)/(32*c^4) + (b^2*x
^4)/(24*c^2) + (b^2*x^8*log(c*x^2 + 1)^2)/32 + (b^2*x^8*log(1 - c*x^2)^2)/3
2 + (b^2*x^2*log(c*x^2 + 1))/(8*c^3) - (b^2*x^2*log(1 - c*x^2))/(8*c^3) + (
b^2*x^6*log(c*x^2 + 1))/(24*c) - (b^2*x^6*log(1 - c*x^2))/(24*c) + (a*b*log
(c*x^2 - 1))/(8*c^4) - (a*b*log(c*x^2 + 1))/(8*c^4) + (a*b*x^8*log(c*x^2 +
```

```
1))/8 - (a*b*x^8*log(1 - c*x^2))/8 + (b^2*log(c*x^2 + 1)*log(1 - c*x^2))/(1
6*c^4) + (a*b*x^2)/(4*c^3) + (a*b*x^6)/(12*c) - (b^2*x^8*log(c*x^2 + 1)*log
(1 - c*x^2))/16
```

sympy [A] time = 23.15, size = 206, normalized size = 1.65

$$\begin{cases} \frac{a^2x^8}{8} + \frac{abx^8\operatorname{atanh}\left(cx^2\right)}{4} + \frac{abx^6}{12c} + \frac{abx^2}{4c^3} - \frac{ab\operatorname{atanh}\left(cx^2\right)}{4c^4} + \frac{b^2x^8\operatorname{atanh}^2\left(cx^2\right)}{8} + \frac{b^2x^6\operatorname{atanh}\left(cx^2\right)}{12c} + \frac{b^2x^4}{24c^2} + \frac{b^2x^2\operatorname{atanh}\left(cx^2\right)}{4c^3} + \frac{b^2\log\left(x\right.}{3c} \\ \frac{a^2x^8}{8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**8/8 + a*b*x**8*atanh(c*x**2)/4 + a*b*x**6/(12*c) + a*b*x
**2/(4*c**3) - a*b*atanh(c*x**2)/(4*c**4) + b**2*x**8*atanh(c*x**2)**2/8 +
b**2*x**6*atanh(c*x**2)/(12*c) + b**2*x**4/(24*c**2) + b**2*x**2*atanh(c*x*
*2)/(4*c**3) + b**2*log(x - I*sqrt(1/c))/(3*c**4) + b**2*log(x + I*sqrt(1/c
))/(3*c**4) - b**2*atanh(c*x**2)**2/(8*c**4) - b**2*atanh(c*x**2)/(3*c**4),
 Ne(c, 0)), (a**2*x**8/8, True))
```

3.65 $\int x^5 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=146

$$\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{6c^3}-\frac{b\log\left(\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3c^3}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{bx^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{6c}$$

```
[Out] 1/6*b^2*x^2/c^2-1/6*b^2*arctanh(c*x^2)/c^3+1/6*b*x^4*(a+b*arctanh(c*x^2))/c
+1/6*(a+b*arctanh(c*x^2))^2/c^3+1/6*x^6*(a+b*arctanh(c*x^2))^2-1/3*b*(a+b*a
rctanh(c*x^2))*ln(2/(-c*x^2+1))/c^3-1/6*b^2*polylog(2,1-2/(-c*x^2+1))/c^3
```

Rubi [B] time = 1.30, antiderivative size = 536, normalized size of antiderivative = 3.67, number of steps used = 53, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {6099, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)}{12c^3}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)}{12c^3}-\frac{abx^2}{6c^2}-\frac{1}{72}b\left(\frac{2\left(1-cx^2\right)^3}{c^3}-\frac{9\left(1-cx^2\right)^2}{c^3}+\frac{18\left(1-c\right.}{c^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] -(a*b*x^2)/(6*c^2) + (19*b^2*x^2)/(72*c^2) - (5*b^2*x^4)/(144*c) - (b^2*x^6
)/108 + (b^2*(1 - c*x^2)^2)/(16*c^3) - (b^2*(1 - c*x^2)^3)/(108*c^3) + (b^2
*Log[1 - c*x^2])/(72*c^3) - (b^2*(1 - c*x^2)*Log[1 - c*x^2])/(12*c^3) + (b^
2*Log[1 - c*x^2]^2)/(24*c^3) + (b*x^4*(2*a - b*Log[1 - c*x^2]))/(24*c) - (b
*x^6*(2*a - b*Log[1 - c*x^2]))/36 + (x^6*(2*a - b*Log[1 - c*x^2])^2)/24 - (
b*(2*a - b*Log[1 - c*x^2])*((18*(1 - c*x^2))/c^3 - (9*(1 - c*x^2)^2)/c^3 +
(2*(1 - c*x^2)^3)/c^3 - (6*Log[1 - c*x^2])/c^3))/72 + (b*(2*a - b*Log[1 - c
*x^2])*Log[(1 + c*x^2)/2])/(12*c^3) - (b^2*Log[1 + c*x^2])/(12*c^3) + (b^2*
x^4*Log[1 + c*x^2])/(12*c) + (b^2*Log[(1 - c*x^2)/2]*Log[1 + c*x^2])/(12*c^
3) + (b*x^6*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2])/12 + (b^2*Log[1 + c*x^
2]^2)/(24*c^3) + (b^2*x^6*Log[1 + c*x^2]^2)/24 - (b^2*PolyLog[2, (1 - c*x^2
)/2])/(12*c^3) + (b^2*PolyLog[2, (1 + c*x^2)/2])/(12*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
```

```
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x^5 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x^5\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{1}{2}bx^5\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{4}\right.$$
$$= \frac{1}{4}\int x^5\left(2a-b\log\left(1-cx^2\right)\right)^2 dx - \frac{1}{2}b\int x^5\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx\right.$$
$$= \frac{1}{8}\text{Subst}\left(\int x^2(2a-b\log(1-cx))^2\,dx, x, x^2\right) - \frac{1}{4}b\,\text{Subst}\left(\int x^2(-2a+b\log(1-\right.$$
$$= \frac{1}{24}x^6\left(2a-b\log\left(1-cx^2\right)\right)^2 + \frac{1}{12}bx^6\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{24}b$$
$$= \frac{1}{24}x^6\left(2a-b\log\left(1-cx^2\right)\right)^2 + \frac{1}{12}bx^6\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{24}b$$
$$= \frac{1}{24}x^6\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{1}{72}b\left(2a-b\log\left(1-cx^2\right)\right)\left(\frac{18\left(1-cx^2\right)}{c^3} - \frac{9\left(1-\right.}{c}\right.$$
$$= -\frac{abx^2}{6c^2} + \frac{bx^4\left(2a-b\log\left(1-cx^2\right)\right)}{24c} - \frac{1}{36}bx^6\left(2a-b\log\left(1-cx^2\right)\right)+\frac{1}{24}x^6\left(2a-\right.$$
$$= -\frac{abx^2}{6c^2} + \frac{bx^4\left(2a-b\log\left(1-cx^2\right)\right)}{24c} - \frac{1}{36}bx^6\left(2a-b\log\left(1-cx^2\right)\right)+\frac{1}{24}x^6\left(2a-\right.$$
$$= -\frac{abx^2}{6c^2} + \frac{13b^2x^2}{72c^2} + \frac{b^2x^4}{144c} - \frac{b^2x^6}{108} + \frac{b^2\left(1-cx^2\right)^2}{16c^3} - \frac{b^2\left(1-cx^2\right)^3}{108c^3} + \frac{b^2\log\left(1-cx\right.}{72c^3}$$
$$= -\frac{abx^2}{6c^2} + \frac{13b^2x^2}{72c^2} + \frac{b^2x^4}{144c} - \frac{b^2x^6}{108} + \frac{b^2\left(1-cx^2\right)^2}{16c^3} - \frac{b^2\left(1-cx^2\right)^3}{108c^3} + \frac{b^2\log\left(1-cx\right.}{72c^3}$$

Mathematica [A] time = 0.30, size = 132, normalized size = 0.90

$$\frac{a^2c^3x^6+abc^2x^4+ab\log\left(c^2x^4-1\right)+b\tanh^{-1}\left(cx^2\right)\left(2ac^3x^6+bc^2x^4-2b\log\left(e^{-2\tanh^{-1}\left(cx^2\right)}+1\right)-b\right)+b^2\left(c^3x^6\right.}{6c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (b^2*c*x^2 + a*b*c^2*x^4 + a^2*c^3*x^6 + b^2*(-1 + c^3*x^6)*ArcTanh[c*x^2]^
2 + b*ArcTanh[c*x^2]*(-b + b*c^2*x^4 + 2*a*c^3*x^6 - 2*b*Log[1 + E^(-2*ArcT
anh[c*x^2])]) + a*b*Log[-1 + c^2*x^4] + b^2*PolyLog[2, -E^(-2*ArcTanh[c*x^2
])])/(6*c^3)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^5\,\text{artanh}\left(cx^2\right)^2 + 2\,abx^5\,\text{artanh}\left(cx^2\right) + a^2x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^5*arctanh(c*x^2)^2 + 2*a*b*x^5*arctanh(c*x^2) + a^2*x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\operatorname{artanh}\left(cx^2\right)+a\right)^2 x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*x^5, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^5\left(a+b\operatorname{arctanh}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int(x^5*(a+b*arctanh(c*x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}a^2x^6+\frac{1}{6}\left(2\,x^6\operatorname{artanh}\left(cx^2\right)+\left(\frac{x^4}{c^2}+\frac{\log\left(c^2x^4-1\right)}{c^4}\right)c\right)ab+\frac{1}{432}\left(18\,x^6\log\left(-cx^2+1\right)^2-2\,c^4\left(\frac{2\left(c^2x^6+3\,x^2\right)}{c^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*x^6 + 1/6*(2*x^6*arctanh(c*x^2) + (x^4/c^2 + log(c^2*x^4 - 1)/c^4)*
c)*a*b + 1/432*(18*x^6*log(-c*x^2 + 1)^2 - 2*c^4*(2*(c^2*x^6 + 3*x^2)/c^6 -
 3*log(c*x^2 + 1)/c^7 + 3*log(c*x^2 - 1)/c^7) + 3*c^3*(x^4/c^4 + log(c^2*x^
4 - 1)/c^6) + 1296*c^3*integrate(1/9*x^7*log(c*x^2 + 1)/(c^4*x^4 - c^2), x)
 - 9*c^2*(2*x^2/c^4 - log(c*x^2 + 1)/c^5 + log(c*x^2 - 1)/c^5) - 6*c*((2*c^
2*x^6 + 3*c*x^4 + 6*x^2)/c^3 + 6*log(c*x^2 - 1)/c^4)*log(-c*x^2 + 1) + 648*
c*integrate(1/9*x^3*log(c*x^2 + 1)/(c^4*x^4 - c^2), x) + 6*(3*c^3*x^6*log(c
*x^2 + 1)^2 + (2*c^3*x^6 - 3*c^2*x^4 + 6*c*x^2 - 6*(c^3*x^6 + 1)*log(c*x^2
+ 1))*log(-c*x^2 + 1))/c^3 + (4*c^3*x^6 + 15*c^2*x^4 + 66*c*x^2 + 18*log(c*
x^2 - 1)^2 + 66*log(c*x^2 - 1))/c^3 - 18*log(9*c^4*x^4 - 9*c^2)/c^3 + 648*i
ntegrate(1/9*x*log(c*x^2 + 1)/(c^4*x^4 - c^2), x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(a+b\operatorname{atanh}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x^2))^2,x)
```

```
[Out] int(x^5*(a + b*atanh(c*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(a+b\operatorname{atanh}\left(cx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral(x**5*(a + b*atanh(c*x**2))**2, x)
```

3.66 $\int x^3 \left(a+b \tanh^{-1}\left(cx^2\right)\right)^2 \, dx$

Optimal. Leaf size=91

$$-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4c^2}+\frac{abx^2}{2c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{b^2\log\left(1-c^2x^4\right)}{4c^2}+\frac{b^2x^2\tanh^{-1}\left(cx^2\right)}{2c}$$

```
[Out] 1/2*a*b*x^2/c+1/2*b^2*x^2*arctanh(c*x^2)/c-1/4*(a+b*arctanh(c*x^2))^2/c^2+1
/4*x^4*(a+b*arctanh(c*x^2))^2+1/4*b^2*ln(-c^2*x^4+1)/c^2
```

Rubi [C] time = 0.97, antiderivative size = 524, normalized size of antiderivative = 5.76, number of steps used = 44, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 2439, 2416, 2394, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)}{8c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)}{8c^2}+\frac{\left(1-cx^2\right)^2\left(2a-b\log\left(1-cx^2\right)\right)^2}{16c^2}-\frac{\left(1-cx^2\right)\left(2a-b\log\right.}{8c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (3*a*b*x^2)/(4*c) - (b^2*x^4)/16 + (b^2*(1 - c*x^2)^2)/(32*c^2) + (b^2*(1 +
 c*x^2)^2)/(32*c^2) - (b^2*Log[1 - c*x^2])/(16*c^2) + (3*b^2*(1 - c*x^2)*Lo
g[1 - c*x^2])/(8*c^2) - (b*x^4*(2*a - b*Log[1 - c*x^2]))/16 + (b*(1 - c*x^2
)^2*(2*a - b*Log[1 - c*x^2]))/(16*c^2) - ((1 - c*x^2)*(2*a - b*Log[1 - c*x^
2])^2)/(8*c^2) + ((1 - c*x^2)^2*(2*a - b*Log[1 - c*x^2])^2)/(16*c^2) - (b*(
2*a - b*Log[1 - c*x^2])*Log[(1 + c*x^2)/2])/(8*c^2) - (b^2*Log[1 + c*x^2])/
(16*c^2) + (b^2*x^4*Log[1 + c*x^2])/16 + (3*b^2*(1 + c*x^2)*Log[1 + c*x^2])
/(8*c^2) - (b^2*(1 + c*x^2)^2*Log[1 + c*x^2])/(16*c^2) + (b^2*Log[(1 - c*x^
2)/2]*Log[1 + c*x^2])/(8*c^2) + (b*x^4*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x
^2])/8 - (b^2*(1 + c*x^2)*Log[1 + c*x^2]^2)/(8*c^2) + (b^2*(1 + c*x^2)^2*Lo
g[1 + c*x^2]^2)/(16*c^2) + (b^2*PolyLog[2, (1 - c*x^2)/2])/(8*c^2) + (b^2*P
olyLog[2, (1 + c*x^2)/2])/(8*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx=\int\left(\frac{1}{4}x^3\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx^3\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{4}\right.$$

$$=\frac{1}{4}\int x^3\left(2a-b\log\left(1-cx^2\right)\right)^2\,dx-\frac{1}{2}b\int x^3\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx\right.$$

$$=\frac{1}{8}\text{Subst}\left(\int x(2a-b\log(1-cx))^2\,dx,x,x^2\right)-\frac{1}{4}b\,\text{Subst}\left(\int x(-2a+b\log(1-c\right.$$

$$=\frac{1}{8}bx^4\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{8}\text{Subst}\left(\int\left(\frac{(2a-b\log(1-cx))^2}{c}-\right.\right.$$

$$=\frac{1}{8}bx^4\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{\text{Subst}\left(\int(2a-b\log(1-cx))^2\,dx,x,\right.}{8c}$$

$$=\frac{1}{8}bx^4\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{8}b\,\text{Subst}\left(\int x(-2a+b\log(1-cx))\right.$$

$$=\frac{abx^2}{4c}-\frac{1}{16}bx^4\left(2a-b\log\left(1-cx^2\right)\right)-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c^2}+\frac{\left(1-c\right.}{}$$

$$=\frac{3abx^2}{4c}-\frac{3b^2x^2}{8c}+\frac{b^2\left(1-cx^2\right)^2}{32c^2}+\frac{b^2\left(1+cx^2\right)^2}{32c^2}-\frac{1}{16}bx^4\left(2a-b\log\left(1-cx^2\right)\right)+$$

$$=\frac{3abx^2}{4c}-\frac{b^2x^4}{16}+\frac{b^2\left(1-cx^2\right)^2}{32c^2}+\frac{b^2\left(1+cx^2\right)^2}{32c^2}-\frac{b^2\log\left(1-cx^2\right)}{16c^2}+\frac{3b^2\left(1-cx^2\right.}{8}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 1.16

$$\frac{a^2c^2x^4 + 2abcx^2 + b(a+b)\log\left(1-cx^2\right) - ab\log\left(cx^2+1\right) + 2bcx^2\tanh^{-1}\left(cx^2\right)\left(acx^2+b\right) + b^2\left(c^2x^4-1\right)\tan}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (2*a*b*c*x^2 + a^2*c^2*x^4 + 2*b*c*x^2*(b + a*c*x^2)*ArcTanh[c*x^2] + b^2*(
-1 + c^2*x^4)*ArcTanh[c*x^2]^2 + b*(a + b)*Log[1 - c*x^2] - a*b*Log[1 + c*x
^2] + b^2*Log[1 + c*x^2])/(4*c^2)
```

fricas [A] time = 0.80, size = 138, normalized size = 1.52

$$\frac{4\,a^2c^2x^4 + 8\,abcx^2 + \left(b^2c^2x^4 - b^2\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2 - 4\left(ab-b^2\right)\log\left(cx^2+1\right) + 4\left(ab+b^2\right)\log\left(cx^2-1\right) + 4\left(ab\right.}{16\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^2*c^2*x^4 + 8*a*b*c*x^2 + (b^2*c^2*x^4 - b^2)*log(-(c*x^2 + 1)/(c
*x^2 - 1))^2 - 4*(a*b - b^2)*log(c*x^2 + 1) + 4*(a*b + b^2)*log(c*x^2 - 1)
+ 4*(a*b*c^2*x^4 + b^2*c*x^2)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^2
```

giac [B] time = 0.19, size = 361, normalized size = 3.97

$$\frac{1}{4}\left(\frac{\left(cx^2+1\right)b^2\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2}{\left(\frac{\left(cx^2+1\right)^2c^3}{\left(cx^2-1\right)^2} - \frac{2\left(cx^2+1\right)c^3}{cx^2-1} + c^3\right)\left(cx^2-1\right)} + \frac{2\left(\frac{2\left(cx^2+1\right)ab}{cx^2-1} + \frac{\left(cx^2+1\right)b^2}{cx^2-1} - b^2\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{\frac{\left(cx^2+1\right)^2c^3}{\left(cx^2-1\right)^2} - \frac{2\left(cx^2+1\right)c^3}{cx^2-1} + c^3} + \frac{4\left(\frac{\left(cx^2+1\right)a^2}{cx^2-1} + \frac{\left(cx^2+1\right)a}{cx^2-1}\right.}{\frac{\left(cx^2+1\right)^2c^3}{\left(cx^2-1\right)^2} - \frac{2\left(cx^2+1\right)c}{cx^2-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/4*((c*x^2 + 1)*b^2*log(-(c*x^2 + 1)/(c*x^2 - 1))^2/(((c*x^2 + 1)^2*c^3/(c
*x^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2 - 1) + c^3)*(c*x^2 - 1)) + 2*(2*(c*x
^2 + 1)*a*b/(c*x^2 - 1) + (c*x^2 + 1)*b^2/(c*x^2 - 1) - b^2)*log(-(c*x^2 +
1)/(c*x^2 - 1))/((c*x^2 + 1)^2*c^3/(c*x^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2
 - 1) + c^3) + 4*((c*x^2 + 1)*a^2/(c*x^2 - 1) + (c*x^2 + 1)*a*b/(c*x^2 - 1)
 - a*b)/((c*x^2 + 1)^2*c^3/(c*x^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2 - 1) +
c^3) - 2*b^2*log(-(c*x^2 + 1)/(c*x^2 - 1) + 1)/c^3 + 2*b^2*log(-(c*x^2 + 1)
/(c*x^2 - 1))/c^3)*c
```

maple [B] time = 0.26, size = 247, normalized size = 2.71

$$\frac{b^2\left(c^2x^4-1\right)\ln\left(c\,x^2+1\right)^2}{16c^2} + \frac{b\left(-x^4b\ln\left(-c\,x^2+1\right)c^2 + 2a\,c^2x^4 + 2bc\,x^2 + b\ln\left(-c\,x^2+1\right)\right)\ln\left(c\,x^2+1\right)}{8c^2} + \frac{b^2x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] 1/16*b^2*(c^2*x^4-1)/c^2*ln(c*x^2+1)^2+1/8*b*(-x^4*b*ln(-c*x^2+1)*c^2+2*a*c
^2*x^4+2*b*c*x^2+b*ln(-c*x^2+1))/c^2*ln(c*x^2+1)+1/16*b^2*x^4*ln(-c*x^2+1)^
2-1/4*a*b*x^4*ln(-c*x^2+1)+1/4*a^2*x^4-1/4/c*b^2*x^2*ln(-c*x^2+1)+1/2*a*b*x
^2/c-1/16/c^2*b^2*ln(-c*x^2+1)^2-1/4/c^2*b*ln(-c*x^2-1)*a+1/4/c^2*b^2*ln(-c
*x^2-1)+1/4/c^2*b*ln(-c*x^2+1)*a+1/4/c^2*b^2*ln(-c*x^2+1)
```

maxima [B] time = 0.33, size = 186, normalized size = 2.04

$$\frac{1}{4}b^2x^4\operatorname{artanh}\left(cx^2\right)^2+\frac{1}{4}a^2x^4+\frac{1}{4}\left(2x^4\operatorname{artanh}\left(cx^2\right)+c\left(\frac{2x^2}{c^2}-\frac{\log\left(cx^2+1\right)}{c^3}+\frac{\log\left(cx^2-1\right)}{c^3}\right)\right)ab+\frac{1}{16}\left(4c\left(\frac{2x^2}{c^2}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c*x^2)^2 + 1/4*a^2*x^4 + 1/4*(2*x^4*arctanh(c*x^2) + c*
(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3))*a*b + 1/16*(4*c*(2*x
^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3)*arctanh(c*x^2) - (2*(log(
c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 - log(c*x^2 - 1)^2 - 4*lo
g(c*x^2 - 1))/c^2)*b^2
```

mupad [B] time = 1.26, size = 275, normalized size = 3.02

$$\frac{a^2x^4}{4}+\frac{b^2\ln\left(cx^2-1\right)}{4c^2}+\frac{b^2\ln\left(cx^2+1\right)}{4c^2}-\frac{b^2\ln\left(cx^2+1\right)^2}{16c^2}-\frac{b^2\ln\left(1-cx^2\right)^2}{16c^2}+\frac{b^2x^4\ln\left(cx^2+1\right)^2}{16}+\frac{b^2x^4\ln\left(1-cx\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^2))^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*log(c*x^2 - 1))/(4*c^2) + (b^2*log(c*x^2 + 1))/(4*c^2) -
 (b^2*log(c*x^2 + 1)^2)/(16*c^2) - (b^2*log(1 - c*x^2)^2)/(16*c^2) + (b^2*x
^4*log(c*x^2 + 1)^2)/16 + (b^2*x^4*log(1 - c*x^2)^2)/16 + (b^2*x^2*log(c*x^
2 + 1))/(4*c) - (b^2*x^2*log(1 - c*x^2))/(4*c) + (a*b*log(c*x^2 - 1))/(4*c^
2) - (a*b*log(c*x^2 + 1))/(4*c^2) + (a*b*x^4*log(c*x^2 + 1))/4 - (a*b*x^4*l
og(1 - c*x^2))/4 + (b^2*log(c*x^2 + 1)*log(1 - c*x^2))/(8*c^2) + (a*b*x^2)/
(2*c) - (b^2*x^4*log(c*x^2 + 1)*log(1 - c*x^2))/8
```

sympy [A] time = 11.53, size = 163, normalized size = 1.79

$$\begin{cases}\frac{a^2x^4}{4}+\frac{abx^4\operatorname{atanh}\left(cx^2\right)}{2}+\frac{abx^2}{2c}-\frac{ab\operatorname{atanh}\left(cx^2\right)}{2c^2}+\frac{b^2x^4\operatorname{atanh}^2\left(cx^2\right)}{4}+\frac{b^2x^2\operatorname{atanh}\left(cx^2\right)}{2c}+\frac{b^2\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2c^2}+\frac{b^2\log\left(x+i\sqrt{\frac{1}{c}}\right)}{2c^2}-\frac{b^2\operatorname{atar}}{} \\ \frac{a^2x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atanh(c*x**2)/2 + a*b*x**2/(2*c) - a*b*at
anh(c*x**2)/(2*c**2) + b**2*x**4*atanh(c*x**2)**2/4 + b**2*x**2*atanh(c*x**
2)/(2*c) + b**2*log(x - I*sqrt(1/c))/(2*c**2) + b**2*log(x + I*sqrt(1/c))/(
2*c**2) - b**2*atanh(c*x**2)**2/(4*c**2) - b**2*atanh(c*x**2)/(2*c**2), Ne(
c, 0)), (a**2*x**4/4, True))
```

3.67 $\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{2c}-\frac{b\log\left(\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{c}-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx^2}\right)}{2c}$$

```
[Out] 1/2*(a+b*arctanh(c*x^2))^2/c+1/2*x^2*(a+b*arctanh(c*x^2))^2-b*(a+b*arctanh(
c*x^2))*ln(2/(-c*x^2+1))/c-1/2*b^2*polylog(2,1-2/(-c*x^2+1))/c
```

Rubi [B] time = 0.51, antiderivative size = 207, normalized size of antiderivative = 2.20, number of steps used = 28, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)}{4c}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)}{4c}+\frac{b\log\left(\frac{1}{2}\left(cx^2+1\right)\right)\left(2a-b\log\left(1-cx^2\right)\right)}{4c}+\frac{1}{4}bx^2\text{lo}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] -((1 - c*x^2)*(2*a - b*Log[1 - c*x^2])^2)/(8*c) + (b*(2*a - b*Log[1 - c*x^2
])*Log[(1 + c*x^2)/2])/(4*c) + (b^2*Log[(1 - c*x^2)/2]*Log[1 + c*x^2])/(4*c
) + (b*x^2*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2])/4 + (b^2*(1 + c*x^2)*Lo
g[1 + c*x^2]^2)/(8*c) - (b^2*PolyLog[2, (1 - c*x^2)/2])/(4*c) + (b^2*PolyLo
g[2, (1 + c*x^2)/2])/(4*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{4}\right.$$
$$=\frac{1}{4}\int x\left(2a-b\log\left(1-cx^2\right)\right)^2 dx-\frac{1}{2}b\int x\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx\right.$$
$$=\frac{1}{8}\operatorname{Subst}\left(\int (2a-b\log(1-cx))^2 dx, x, x^2\right)-\frac{1}{4}b\operatorname{Subst}\left(\int(-2a+b\log(1-cx\right.$$
$$=\frac{1}{4}bx^2\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)-\frac{\operatorname{Subst}\left(\int(2a-b\log(x))^2 dx, x, 1-\right.}{8c}$$
$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}+\frac{1}{4}bx^2\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)$$
$$=\frac{1}{2}abx^2+\frac{b^2x^2}{4}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}-\frac{b^2\left(1+cx^2\right)\log\left(1+cx^2\right)}{4c}$$
$$=\frac{b^2x^2}{2}+\frac{b^2\left(1-cx^2\right)\log\left(1-cx^2\right)}{4c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}+\frac{b\left(2a-\right.}{}$$
$$=\frac{b^2x^2}{4}+\frac{b^2\left(1-cx^2\right)\log\left(1-cx^2\right)}{4c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}+\frac{b\left(2a-\right.}{}$$
$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}+\frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(\frac{1}{2}\left(1+cx^2\right)\right)}{4c}$$

Mathematica [A] time = 0.16, size = 99, normalized size = 1.05

$$\frac{a\left(acx^2+b\log\left(1-c^2x^4\right)\right)+2b\tanh^{-1}\left(cx^2\right)\left(acx^2-b\log\left(e^{-2\tanh^{-1}\left(cx^2\right)}+1\right)\right)+b^2\operatorname{Li}_2\left(-e^{-2\tanh^{-1}\left(cx^2\right)}\right)+b^2\left(\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (b^2*(-1 + c*x^2)*ArcTanh[c*x^2]^2 + 2*b*ArcTanh[c*x^2]*(a*c*x^2 - b*Log[1
+ E^(-2*ArcTanh[c*x^2])]) + a*(a*c*x^2 + b*Log[1 - c^2*x^4]) + b^2*PolyLog[
2, -E^(-2*ArcTanh[c*x^2])])/(2*c)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^2x\operatorname{artanh}\left(cx^2\right)^2+2\,abx\operatorname{artanh}\left(cx^2\right)+a^2x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*arctanh(c*x^2)^2 + 2*a*b*x*arctanh(c*x^2) + a^2*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*x, x)
```

maple [A] time = 0.18, size = 144, normalized size = 1.53

$$\frac{x^2b^2\,\mathrm{arctanh}\left(c\,x^2\right)^2}{2}+x^2ab\,\mathrm{arctanh}\left(c\,x^2\right)+\frac{a^2x^2}{2}+\frac{b^2\,\mathrm{arctanh}\left(c\,x^2\right)^2}{2c}-\frac{\mathrm{arctanh}\left(c\,x^2\right)\ln\left(1+\frac{\left(c\,x^2+1\right)^2}{-c^2x^4+1}\right)b^2}{c}+\frac{ab\ln\left(\cdot\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] 1/2*x^2*b^2*arctanh(c*x^2)^2+x^2*a*b*arctanh(c*x^2)+1/2*a^2*x^2+1/2/c*b^2*a
rctanh(c*x^2)^2-1/c*arctanh(c*x^2)*ln(1+(c*x^2+1)^2/(-c^2*x^4+1))*b^2+1/2/c
*a*b*ln(-c^2*x^4+1)-1/2/c*polylog(2,-(c*x^2+1)^2/(-c^2*x^4+1))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2x^2+\frac{1}{8}\left(x^2\log\left(-cx^2+1\right)^2-c^2\left(\frac{2\,x^2}{c^2}-\frac{\log\left(cx^2+1\right)}{c^3}+\frac{\log\left(cx^2-1\right)}{c^3}\right)-2\,c\left(\frac{x^2}{c}+\frac{\log\left(cx^2-1\right)}{c^2}\right)\log\left(-cx^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/8*(x^2*log(-c*x^2 + 1)^2 - c^2*(2*x^2/c^2 - log(c*x^2 + 1)/
c^3 + log(c*x^2 - 1)/c^3) - 2*c*(x^2/c + log(c*x^2 - 1)/c^2)*log(-c*x^2 + 1
) + 12*c*integrate(x^3*log(c*x^2 + 1)/(c^2*x^4 - 1), x) + (c*x^2*log(c*x^2
+ 1)^2 + 2*(c*x^2 - (c*x^2 + 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/c + (2*c*x
^2 + log(c*x^2 - 1)^2 + 2*log(c*x^2 - 1))/c - log(c^2*x^4 - 1)/c + 4*integr
ate(x*log(c*x^2 + 1)/(c^2*x^4 - 1), x))*b^2 + 1/2*(2*c*x^2*arctanh(c*x^2) +
 log(-c^2*x^4 + 1))*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^2))^2,x)
```

```
[Out] int(x*(a + b*atanh(c*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{atanh}\left(cx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral(x*(a + b*atanh(c*x**2))**2, x)
```

3.68 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x}\,dx$

Optimal. Leaf size=137

$$-\frac{1}{2}b\text{Li}_2\left(1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{1}{2}b\text{Li}_2\left(\frac{2}{1-cx^2}-1\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)(a$$

```
[Out] -(a+b*arctanh(c*x^2))^2*arctanh(-1+2/(-c*x^2+1))-1/2*b*(a+b*arctanh(c*x^2))
*polylog(2,1-2/(-c*x^2+1))+1/2*b*(a+b*arctanh(c*x^2))*polylog(2,-1+2/(-c*x^
2+1))+1/4*b^2*polylog(3,1-2/(-c*x^2+1))-1/4*b^2*polylog(3,-1+2/(-c*x^2+1))
```

Rubi [A] time = 0.34, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-\frac{1}{2}b\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{1}{2}b\text{PolyLog}\left(2,\frac{2}{1-cx^2}-1\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{1}{4}b^2\text{Pol}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x,x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^2*ArcTanh[1 - 2/(1 - c*x^2)] - (b*(a + b*ArcTanh[c*x
^2])*PolyLog[2, 1 - 2/(1 - c*x^2)])/2 + (b*(a + b*ArcTanh[c*x^2])*PolyLog[2
, -1 + 2/(1 - c*x^2)])/2 + (b^2*PolyLog[3, 1 - 2/(1 - c*x^2)])/4 - (b^2*Pol
yLog[3, -1 + 2/(1 - c*x^2)])/4
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
```

```
n}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, x^2\right)$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-(2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\text{ta}}{1-c^2x}\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)+(bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\text{log}}{1-c^2x^2}\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-\frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)\text{Li}_2\left(1-\frac{2}{1-c}\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-\frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)\text{Li}_2\left(1-\frac{2}{1-c}\right.$$

Mathematica [A] time = 0.08, size = 183, normalized size = 1.34

$$\frac{1}{2}\left(2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{\text{Li}_2\left(\frac{-cx^2-1}{cx^2-1}\right)\left(-a-b\tanh^{-1}\left(cx^2\right)\right)}{2c}+\frac{b\text{Li}_3\left(\frac{-cx^2-1}{cx^2-1}\right)}{4c}\right)+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^2])^2*ArcTanh[1 - 2/(1 - c*x^2)] - 4*b*c*((((-a - b*A
rcTanh[c*x^2])*PolyLog[2, (-1 - c*x^2)/(-1 + c*x^2)])/(2*c) + (b*PolyLog[3,
 (-1 - c*x^2)/(-1 + c*x^2)])/(4*c))/2 + (-1/2*((-a - b*ArcTanh[c*x^2])*Poly
Log[2, (1 + c*x^2)/(-1 + c*x^2)])/c - (b*PolyLog[3, (1 + c*x^2)/(-1 + c*x^2
)])/(4*c))/2))/2
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(cx^2\right)^2+2\,ab\,\text{artanh}\left(cx^2\right)+a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{arctanh}\left(c\,x^{2}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\log(x) + \int \frac{b^{2}\left(\log\left(cx^{2}+1\right) - \log\left(-cx^{2}+1\right)\right)^{2}}{4\,x} + \frac{ab\left(\log\left(cx^{2}+1\right) - \log\left(-cx^{2}+1\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c*x^2 + 1) - log(-c*x^2 + 1))^2/x + a*b
*(log(c*x^2 + 1) - log(-c*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{atanh}\left(c\,x^{2}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/x,x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(cx^{2}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x, x)
```

3.69 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=87

$$\frac{1}{2}c\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{2x^2}+bc\log\left(2-\frac{2}{cx^2+1}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{2}b^2c\text{Li}_2\left(\frac{2}{cx^2+1}-1\right.$$

```
[Out] 1/2*c*(a+b*arctanh(c*x^2))^2-1/2*(a+b*arctanh(c*x^2))^2/x^2+b*c*(a+b*arctan
h(c*x^2))*ln(2-2/(c*x^2+1))-1/2*b^2*c*polylog(2,-1+2/(c*x^2+1))
```

Rubi [B] time = 0.63, antiderivative size = 237, normalized size of antiderivative = 2.72, number of steps used = 24, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {6099, 2454, 2397, 2392, 2391, 2395, 36, 29, 31, 2439, 2416, 2394, 2393}

$$-\frac{1}{2}b^2c\text{PolyLog}\left(2,-cx^2\right)+\frac{1}{2}b^2c\text{PolyLog}\left(2,cx^2\right)+\frac{1}{4}b^2c\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)-\frac{1}{4}b^2c\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)-$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^3,x]
```

```
[Out] 2*a*b*c*Log[x] - ((1 - c*x^2)*(2*a - b*Log[1 - c*x^2])^2)/(8*x^2) - (b*c*(2
*a - b*Log[1 - c*x^2])*Log[(1 + c*x^2)/2])/4 - (b^2*c*Log[(1 - c*x^2)/2]*Lo
g[1 + c*x^2])/4 - (b*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2])/(4*x^2) - (b^
2*(1 + c*x^2)*Log[1 + c*x^2]^2)/(8*x^2) - (b^2*c*PolyLog[2, -(c*x^2)])/2 +
(b^2*c*PolyLog[2, c*x^2])/2 + (b^2*c*PolyLog[2, (1 - c*x^2)/2])/4 - (b^2*c*
PolyLog[2, (1 + c*x^2)/2])/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^3}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^3} - \frac{b\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{2x^3} + \frac{b^2\log^2\left(1-\right.}{4x^3}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^3}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{x^3}\,dx +$$

$$= \frac{1}{8}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^2}\,dx, x, x^2\right) - \frac{1}{4}b\,\text{Subst}\left(\int \frac{(-2a+b\log(1-cx))\,\mathrm{l}}{x^2}\right.$$

$$= -\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{4x^2} - \frac{b^2\left(1-\right.}{}$$

$$= abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx\right.}{4x^2}$$

$$= abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx\right.}{4x^2}$$

$$= 2abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{1}{4}bc\left(2a-b\log\left(1-cx^2\right)\right)\log\left(\frac{1}{2}\right.$$

$$= 2abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{1}{4}bc\left(2a-b\log\left(1-cx^2\right)\right)\log\left(\frac{1}{2}\right.$$

$$= 2abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{1}{4}bc\left(2a-b\log\left(1-cx^2\right)\right)\log\left(\frac{1}{2}\right.$$

Mathematica [A] time = 0.16, size = 119, normalized size = 1.37

$$-\frac{a^2}{2x^2}+abc\left(-\frac{1}{2}\log\left(1-c^2x^4\right)+\log\left(cx^2\right)-\frac{\tanh^{-1}\left(cx^2\right)}{cx^2}\right)+\frac{1}{2}b^2c\left(\tanh^{-1}\left(cx^2\right)\left(-\frac{\tanh^{-1}\left(cx^2\right)}{cx^2}+\tanh^{-1}\left(cx^2\right)+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 + a*b*c*(-(ArcTanh[c*x^2]/(c*x^2)) + Log[c*x^2] - Log[1 - c^2*
x^4]/2) + (b^2*c*(ArcTanh[c*x^2]*(ArcTanh[c*x^2] - ArcTanh[c*x^2]/(c*x^2) +
 2*Log[1 - E^(-2*ArcTanh[c*x^2])]) - PolyLog[2, E^(-2*ArcTanh[c*x^2])]))/2
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(cx^2\right)^2+2\,ab\,\text{artanh}\left(cx^2\right)+a^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^3, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\,x^{2}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(c\left(\log\left(c^{2}x^{4}-1\right)-\log\left(x^{4}\right)\right)+\frac{2\,\mathrm{artanh}\left(cx^{2}\right)}{x^{2}}\right)ab-\frac{1}{8}\,b^{2}\left(\frac{\log\left(-cx^{2}+1\right)^{2}}{x^{2}}+2\int-\frac{\left(cx^{2}-1\right)\log\left(cx^{2}+1\right)^{2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*a*b - 1/8*b^2
*(log(-c*x^2 + 1)^2/x^2 + 2*integrate(-((c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(c
*x^2 - (c*x^2 - 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^5 - x^3), x)) - 1/
2*a^2/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x^{2}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/x^3,x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(cx^{2}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x**3, x)
```

3.70 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=88

$$\frac{1}{4}c^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-\frac{bc\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{2x^2}-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4x^4}-\frac{1}{4}b^2c^2\log\left(1-c^2x^4\right)+b^2c^2\log(x)$$

```
[Out] -1/2*b*c*(a+b*arctanh(c*x^2))/x^2+1/4*c^2*(a+b*arctanh(c*x^2))^2-1/4*(a+b*a
rctanh(c*x^2))^2/x^4+b^2*c^2*ln(x)-1/4*b^2*c^2*ln(-c^2*x^4+1)
```

Rubi [C] time = 1.06, antiderivative size = 360, normalized size of antiderivative = 4.09, number of steps used = 46, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2395, 44, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)+\frac{1}{8}bc^2\log\left(\frac{1}{2}\left(cx^2+1\right)\right)\left(2a-b\log\left(1-cx^2\right)\right)+$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^5,x]
```

```
[Out] b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c*x^2])/8 - (b*c*(2*a - b*Log[1 - c*x^2])
)/(8*x^2) - (b*c*(1 - c*x^2)*(2*a - b*Log[1 - c*x^2]))/(8*x^2) + (c^2*(2*a
- b*Log[1 - c*x^2])^2)/16 - (2*a - b*Log[1 - c*x^2])^2/(16*x^4) + (b*c^2*(2
*a - b*Log[1 - c*x^2])*Log[(1 + c*x^2)/2])/8 - (b^2*c^2*Log[1 + c*x^2])/4 -
 (b^2*c*Log[1 + c*x^2])/(4*x^2) - (b^2*c^2*Log[(1 - c*x^2)/2]*Log[1 + c*x^2
])/8 - (b*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2])/(8*x^4) + (b^2*c^2*Log[1
 + c*x^2]^2)/16 - (b^2*Log[1 + c*x^2]^2)/(16*x^4) - (b^2*c^2*PolyLog[2, (1
- c*x^2)/2])/8 - (b^2*c^2*PolyLog[2, (1 + c*x^2)/2])/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^5}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^5} - \frac{b\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{2x^5} + \frac{b^2\log^2 (\ldots}{4\ldots}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^5}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{x^5}\,d\ldots$$

$$= \frac{1}{8}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^3}\,dx, x, x^2\right) - \frac{1}{4}b\,\text{Subst}\left(\int \frac{(-2a+b\log(1-c\ldots}{x^3}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{16x^4} - \frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{8x^4} - \frac{b^2\log^2\left(1+\ldots\right.}{16x^4}$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{16x^4} - \frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{8x^4} - \frac{b^2\log^2\left(1+\ldots\right.}{16x^4}$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{16x^4} - \frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{8x^4} - \frac{b^2\log^2\left(1+\ldots\right.}{16x^4}$$

$$= -\frac{1}{2}abc^2\log(x) - \frac{bc\left(2a-b\log\left(1-cx^2\right)\right)}{8x^2} - \frac{bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)}{8x^2} \ldots$$

$$= \frac{1}{4}b^2c^2\log(x) - \frac{bc\left(2a-b\log\left(1-cx^2\right)\right)}{8x^2} - \frac{bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)}{8x^2} + \ldots$$

$$= \frac{1}{2}b^2c^2\log(x) - \frac{1}{8}b^2c^2\log\left(1-cx^2\right) - \frac{bc\left(2a-b\log\left(1-cx^2\right)\right)}{8x^2} - \frac{bc\left(1-cx^2\right)\left(2\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.10, size = 111, normalized size = 1.26

$$\frac{1}{4}\left(-\frac{a^2}{x^4} - bc^2(a+b)\log\left(1-cx^2\right) + bc^2(a-b)\log\left(cx^2+1\right) - \frac{2abc}{x^2} - \frac{2b\tanh^{-1}\left(cx^2\right)\left(a+bcx^2\right)}{x^4} + \frac{b^2\left(c^2x^4-\ldots\right.}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^5,x]
```

```
[Out] (-(a^2/x^4) - (2*a*b*c)/x^2 - (2*b*(a + b*c*x^2)*ArcTanh[c*x^2])/x^4 + (b^2
*(-1 + c^2*x^4)*ArcTanh[c*x^2]^2)/x^4 + 4*b^2*c^2*Log[x] - b*(a + b)*c^2*Lo
g[1 - c*x^2] + (a - b)*b*c^2*Log[1 + c*x^2])/4
```

fricas [A] time = 0.84, size = 151, normalized size = 1.72

$$\frac{16\, b^2c^2x^4\log(x) + 4\left(ab - b^2\right)c^2x^4\log\left(cx^2+1\right) - 4\left(ab + b^2\right)c^2x^4\log\left(cx^2-1\right) - 8\,abcx^2 + \left(b^2c^2x^4 - b^2\right)\log\left(-\frac{c}{c}\right.}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^2*c^2*x^4*log(x) + 4*(a*b - b^2)*c^2*x^4*log(c*x^2 + 1) - 4*(a*b
 + b^2)*c^2*x^4*log(c*x^2 - 1) - 8*a*b*c*x^2 + (b^2*c^2*x^4 - b^2)*log(-(c*
x^2 + 1)/(c*x^2 - 1))^2 - 4*a^2 - 4*(b^2*c*x^2 + a*b)*log(-(c*x^2 + 1)/(c*x
^2 - 1)))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\operatorname{artanh}\left(cx^2\right) + a\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^5, x)
```

maple [B] time = 0.27, size = 257, normalized size = 2.92

$$\frac{b^2\left(c^2x^4-1\right)\ln\left(c\,x^2+1\right)^2}{16x^4} - \frac{b\left(x^4b\ln\left(-c\,x^2+1\right)c^2 + 2bc\,x^2 - b\ln\left(-c\,x^2+1\right) + 2a\right)\ln\left(c\,x^2+1\right)}{8x^4} + \frac{b^2c^2x^4\ln\left(-c\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^5,x)
```

```
[Out] 1/16*b^2*(c^2*x^4-1)/x^4*ln(c*x^2+1)^2-1/8*b*(x^4*b*ln(-c*x^2+1)*c^2+2*b*c*
x^2-b*ln(-c*x^2+1)+2*a)/x^4*ln(c*x^2+1)+1/16*(b^2*c^2*x^4*ln(-c*x^2+1)^2+4*
b*c^2*ln(c*x^2+1)*x^4*a-4*b^2*c^2*ln(c*x^2+1)*x^4-4*b*c^2*ln(c*x^2-1)*x^4*a
-4*b^2*c^2*ln(c*x^2-1)*x^4+16*b^2*c^2*ln(x)*x^4+4*b^2*c*x^2*ln(-c*x^2+1)-8*
a*b*c*x^2-b^2*ln(-c*x^2+1)^2+4*b*ln(-c*x^2+1)*a-4*a^2)/x^4
```

maxima [B] time = 0.33, size = 175, normalized size = 1.99

$$\frac{1}{4}\left(\left(c\log\left(cx^2+1\right) - c\log\left(cx^2-1\right) - \frac{2}{x^2}\right)c - \frac{2\,\operatorname{artanh}\left(cx^2\right)}{x^4}\right)ab + \frac{1}{16}\left(\left(2\left(\log\left(cx^2-1\right)-2\right)\log\left(cx^2+1\right) - \log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^4
)*a*b + 1/16*((2*(log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 - l
og(c*x^2 - 1)^2 - 4*log(c*x^2 - 1) + 16*log(x))*c^2 + 4*(c*log(c*x^2 + 1) -
 c*log(c*x^2 - 1) - 2/x^2)*c*arctanh(c*x^2))*b^2 - 1/4*b^2*arctanh(c*x^2)^2
/x^4 - 1/4*a^2/x^4
```

mupad [B] time = 1.50, size = 278, normalized size = 3.16

$$\frac{b^2\,c^2\ln\left(c\,x^2+1\right)^2}{16} - \frac{b^2\,c^2\,\ln\left(c\,x^2-1\right)}{4} - \frac{b^2\,c^2\,\ln\left(c\,x^2+1\right)}{4} - \frac{a^2}{4\,x^4} + \frac{b^2\,c^2\ln\left(1-c\,x^2\right)^2}{16} - \frac{b^2\ln\left(c\,x^2+1\right)^2}{16\,x^4} - \frac{b^2\ln\left(1-\right.}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/x^5,x)
```

```
[Out] (b^2*c^2*log(c*x^2 + 1)^2)/16 - (b^2*c^2*log(c*x^2 - 1))/4 - (b^2*c^2*log(c
*x^2 + 1))/4 - a^2/(4*x^4) + (b^2*c^2*log(1 - c*x^2)^2)/16 - (b^2*log(c*x^2
 + 1)^2)/(16*x^4) - (b^2*log(1 - c*x^2)^2)/(16*x^4) + b^2*c^2*log(x) - (a*b
*c^2*log(c*x^2 - 1))/4 + (a*b*c^2*log(c*x^2 + 1))/4 - (a*b*c)/(2*x^2) - (a*
b*log(c*x^2 + 1))/(4*x^4) + (a*b*log(1 - c*x^2))/(4*x^4) - (b^2*c^2*log(c*x
^2 + 1)*log(1 - c*x^2))/8 - (b^2*c*log(c*x^2 + 1))/(4*x^2) + (b^2*c*log(1 -
 c*x^2))/(4*x^2) + (b^2*log(c*x^2 + 1)*log(1 - c*x^2))/(8*x^4)
```

sympy [A] time = 17.02, size = 175, normalized size = 1.99

$$\begin{cases} -\frac{a^2}{4x^4} + \frac{abc^2 \operatorname{atanh}\left(cx^2\right)}{2} - \frac{abc}{2x^2} - \frac{ab \operatorname{atanh}\left(cx^2\right)}{2x^4} + b^2c^2\log\left(x\right) - \frac{b^2c^2\log\left(x - i\sqrt{\frac{1}{c}}\right)}{2} - \frac{b^2c^2\log\left(x + i\sqrt{\frac{1}{c}}\right)}{2} + \frac{b^2c^2\operatorname{atanh}^2\left(cx^2\right)}{4} + \frac{b \cdots}{\cdots} \\ -\frac{a^2}{4x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) + a*b*c**2*atanh(c*x**2)/2 - a*b*c/(2*x**2) - a*b
*atanh(c*x**2)/(2*x**4) + b**2*c**2*log(x) - b**2*c**2*log(x - I*sqrt(1/c))
/2 - b**2*c**2*log(x + I*sqrt(1/c))/2 + b**2*c**2*atanh(c*x**2)**2/4 + b**2
*c**2*atanh(c*x**2)/2 - b**2*c*atanh(c*x**2)/(2*x**2) - b**2*atanh(c*x**2)*
*2/(4*x**4), Ne(c, 0)), (-a**2/(4*x**4), True))
```

3.71 $\int x^4 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=1173

$$\frac{1}{20}\left(2a-b\log\left(1-cx^2\right)\right)^2x^5+\frac{1}{20}b^2\log^2\left(cx^2+1\right)x^5-\frac{2}{25}abx^5+\frac{1}{25}b^2\log\left(1-cx^2\right)x^5+\frac{1}{25}b\left(2a-b\log\left(1-cx^2\right)\right)$$

```
[Out] 1/5*b^2*polylog(2,1-2/(1-x*c^(1/2)))/c^(5/2)+1/5*b^2*polylog(2,1-2/(1+x*c^(
1/2)))/c^(5/2)-1/10*b^2*polylog(2,1+2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-
c^(1/2))/(1+x*c^(1/2)))/c^(5/2)-1/10*b^2*polylog(2,1-2*(1+x*(-c)^(1/2))*c^(
1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))/c^(5/2)-4/15*b^2*arctan(x*c^(1/2))
/c^(5/2)-4/15*b^2*arctanh(x*c^(1/2))/c^(5/2)-1/5*b^2*arctanh(x*c^(1/2))^2/c
^(5/2)+1/25*b^2*x^5*ln(-c*x^2+1)+1/25*b*x^5*(2*a-b*ln(-c*x^2+1))+1/20*b^2*x
^5*ln(c*x^2+1)^2-2/25*a*b*x^5+8/15*b^2*x/c^2-1/15*b^2*x^3*ln(-c*x^2+1)/c-1/
5*b^2*arctan(x*c^(1/2))*ln(-c*x^2+1)/c^(5/2)+1/15*b*x^3*(2*a-b*ln(-c*x^2+1)
)/c-1/5*b*arctanh(x*c^(1/2))*(2*a-b*ln(-c*x^2+1))/c^(5/2)+2/15*b^2*x^3*ln(c
*x^2+1)/c+1/5*a*b*x^5*ln(c*x^2+1)+1/5*b^2*arctan(x*c^(1/2))*ln(c*x^2+1)/c^(
5/2)-1/5*b^2*arctanh(x*c^(1/2))*ln(c*x^2+1)/c^(5/2)-1/10*b^2*x^5*ln(-c*x^2+
1)*ln(c*x^2+1)+1/5*I*b^2*arctan(x*c^(1/2))^2/c^(5/2)+1/5*I*b^2*polylog(2,1-
2/(1-I*x*c^(1/2)))/c^(5/2)+1/5*I*b^2*polylog(2,1-2/(1+I*x*c^(1/2)))/c^(5/2)
+2/5*b^2*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))/c^(5/2)-2/5*b^2*arctan(x*c^
(1/2))*ln(2/(1-I*x*c^(1/2)))/c^(5/2)+1/5*b^2*arctan(x*c^(1/2))*ln((1+I)*(1-
x*c^(1/2))/(1-I*x*c^(1/2)))/c^(5/2)+2/5*b^2*arctan(x*c^(1/2))*ln(2/(1+I*x*c
^(1/2)))/c^(5/2)-2/5*b^2*arctanh(x*c^(1/2))*ln(2/(1+x*c^(1/2)))/c^(5/2)+1/5
*b^2*arctanh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))
/(1+x*c^(1/2)))/c^(5/2)+1/5*b^2*arctanh(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^
(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))/c^(5/2)+1/5*b^2*arctan(x*c^(1/2))
*ln((1-I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))/c^(5/2)-1/10*I*b^2*polylog(2,1-(1+
I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))/c^(5/2)-1/10*I*b^2*polylog(2,1+(-1+I)*(1+
x*c^(1/2))/(1-I*x*c^(1/2)))/c^(5/2)+2/5*a*b*arctan(x*c^(1/2))/c^(5/2)+1/20*
x^5*(2*a-b*ln(-c*x^2+1))^2+2/15*a*b*x^3/c
```

Rubi [A] time = 2.33, antiderivative size = 1173, normalized size of antiderivative = 1.00, number of steps used = 102, number of rules used = 26, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.625, Rules used = {6099, 2457, 2476, 2448, 321, 206, 2455, 302, 2470, 12, 5984, 5918, 2402, 2315, 6742, 203, 30, 2557, 207, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (8*b^2*x)/(15*c^2) + (2*a*b*x^3)/(15*c) - (2*a*b*x^5)/25 + (2*a*b*ArcTan[Sq
rt[c]*x])/(5*c^(5/2)) - (4*b^2*ArcTan[Sqrt[c]*x])/(15*c^(5/2)) + ((I/5)*b^2
*ArcTan[Sqrt[c]*x]^2)/c^(5/2) - (4*b^2*ArcTanh[Sqrt[c]*x])/(15*c^(5/2)) - (
b^2*ArcTanh[Sqrt[c]*x]^2)/(5*c^(5/2)) + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1
 - Sqrt[c]*x)])/(5*c^(5/2)) - (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*
x)])/(5*c^(5/2)) + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1
 - I*Sqrt[c]*x)])/(5*c^(5/2)) + (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c
]*x)])/(5*c^(5/2)) - (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/(5*c
^(5/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[
-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(5*c^(5/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log
[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(5*c
^(5/2)) + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[
c]*x)])/(5*c^(5/2)) - (b^2*x^3*Log[1 - c*x^2])/(15*c) + (b^2*x^5*Log[1 - c*
x^2])/25 - (b^2*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/(5*c^(5/2)) + (b*x^3*(2*a
 - b*Log[1 - c*x^2]))/(15*c) + (b*x^5*(2*a - b*Log[1 - c*x^2]))/25 - (b*Arc
```

```
Tanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/(5*c^(5/2)) + (x^5*(2*a - b*Log[1
 - c*x^2])^2)/20 + (2*b^2*x^3*Log[1 + c*x^2])/(15*c) + (a*b*x^5*Log[1 + c*x
^2])/5 + (b^2*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/(5*c^(5/2)) - (b^2*ArcTanh[
Sqrt[c]*x]*Log[1 + c*x^2])/(5*c^(5/2)) - (b^2*x^5*Log[1 - c*x^2]*Log[1 + c*
x^2])/10 + (b^2*x^5*Log[1 + c*x^2]^2)/20 + (b^2*PolyLog[2, 1 - 2/(1 - Sqrt[
c]*x)])/(5*c^(5/2)) + ((I/5)*b^2*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)])/c^(5/
2) - ((I/10)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)
])/c^(5/2) + ((I/5)*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)])/c^(5/2) + (b^2
*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/(5*c^(5/2)) - (b^2*PolyLog[2, 1 + (2*Sq
rt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(10*c^(5/2
)) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])
*(1 + Sqrt[c]*x))])/(10*c^(5/2)) - ((I/10)*b^2*PolyLog[2, 1 - ((1 - I)*(1 +
 Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx = \int\left(\frac{1}{4}x^4\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx^4\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)\right.$$

$$=\frac{1}{4}\int x^4\left(2a-b\log\left(1-cx^2\right)\right)^2 dx-\frac{1}{2}b\int x^4\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+\right.$$

$$=\frac{1}{20}x^5\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{20}b^2x^5\log^2\left(1+cx^2\right)-\frac{1}{2}b\int\left(-2ax^4\log\left(1-\right.\right.$$

$$=\frac{1}{20}x^5\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{20}b^2x^5\log^2\left(1+cx^2\right)+(ab)\int x^4\log\left(1+cx\right.$$

$$=\frac{1}{20}x^5\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{5}abx^5\log\left(1+cx^2\right)-\frac{1}{10}b^2x^5\log\left(1-cx^2\right)\mathrm{lo}$$

$$=\frac{2abx}{5c^2}+\frac{bx^3\left(2a-b\log\left(1-cx^2\right)\right)}{15c}+\frac{1}{25}bx^5\left(2a-b\log\left(1-cx^2\right)\right)-\frac{b\tanh^{-1}}{}$$

$$=\frac{2b^2x}{5c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5-\frac{b^2x\log\left(1-cx^2\right)}{5c^2}+\frac{bx^3\left(2a-b\log\left(1-cx^2\right)\right)}{15c}+$$

$$=\frac{92b^2x}{75c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{4b^2x^5}{125}+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{2b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}+$$

$$=\frac{92b^2x}{75c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{4b^2x^5}{125}+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{46b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{75c^{5/2}}$$

$$=\frac{32b^2x}{75c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{4b^2x^5}{125}+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{46b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{75c^{5/2}}$$

$$=\frac{8b^2x}{15c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{16b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{75c^{5/2}}+\frac{ib^2\tan^{-1}}{5c}$$

$$=\frac{8b^2x}{15c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{15c^{5/2}}+\frac{ib^2\tan^{-1}}{5c^{5/}}$$

$$=\frac{8b^2x}{15c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{15c^{5/2}}+\frac{ib^2\tan^{-1}}{5c^{5/}}$$

$$=\frac{8b^2x}{15c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{15c^{5/2}}+\frac{ib^2\tan^{-1}}{5c^{5/}}$$

$$=\frac{8b^2x}{15c^2}+\frac{2abx^3}{15c}-\frac{2}{25}abx^5+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{5c^{5/2}}-\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{15c^{5/2}}+\frac{ib^2\tan^{-1}}{5c^{5/}}$$

<u>Mathematica</u> [F] time = 5.62, size = 0, normalized size = 0.00

$$\int x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x^2])^2,x]
[Out] Integrate[x^4*(a + b*ArcTanh[c*x^2])^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2 x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*x^4, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int x^4 \left(a + b \operatorname{arctanh}\left(c\,x^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int(x^4*(a+b*arctanh(c*x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{5}a^2x^5 + \frac{1}{15}\left(6\,x^5 \operatorname{artanh}\left(cx^2\right) + c\left(\frac{4\,x^3}{c^2} + \frac{6\,\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{7}{2}}} + \frac{3\,\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{7}{2}}}\right)\right)ab + \frac{1}{20}\left(x^5 \log\left(-cx^2+1\right)^2 - 5\int -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 + 1/15*(6*x^5*arctanh(c*x^2) + c*(4*x^3/c^2 + 6*arctan(sqrt(c)*
x)/c^(7/2) + 3*log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(7/2)))*a*b + 1/20*(x
^5*log(-c*x^2 + 1)^2 - 5*integrate(-1/5*(5*(c*x^6 - x^4)*log(c*x^2 + 1)^2 -
 2*(2*c*x^6 + 5*(c*x^6 - x^4)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^2 - 1),
 x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(a + b \operatorname{atanh}\left(c\,x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x^2))^2,x)
```

```
[Out] int(x^4*(a + b*atanh(c*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a + b \operatorname{atanh}\left(cx^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c*x**2))**2, x)
```

3.72 $\int x^2 \left(a + b \tanh^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=1129

$$\frac{1}{12}\left(2a - b\log\left(1 - cx^2\right)\right)^2 x^3 + \frac{1}{12}b^2\log^2\left(cx^2 + 1\right)x^3 - \frac{2}{9}abx^3 + \frac{1}{9}b^2\log\left(1 - cx^2\right)x^3 + \frac{1}{9}b\left(2a - b\log\left(1 - cx^2\right)\right)x$$

```
[Out] 1/12*x^3*(2*a-b*ln(-c*x^2+1))^2+4/3*b^2*arctan(x*c^(1/2))/c^(3/2)-4/3*b^2*a
rctanh(x*c^(1/2))/c^(3/2)-1/3*b^2*arctanh(x*c^(1/2))^2/c^(3/2)+1/9*b^2*x^3*
ln(-c*x^2+1)+1/9*b*x^3*(2*a-b*ln(-c*x^2+1))+1/12*b^2*x^3*ln(c*x^2+1)^2+1/3*
b^2*polylog(2,1-2/(1-x*c^(1/2)))/c^(3/2)+1/3*b^2*polylog(2,1-2/(1+x*c^(1/2)
))/c^(3/2)-1/6*b^2*polylog(2,1+2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/
2))/(1+x*c^(1/2)))/c^(3/2)-1/6*b^2*polylog(2,1-2*(1+x*(-c)^(1/2))*c^(1/2)/(
(-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))/c^(3/2)-2/9*a*b*x^3+1/3*b^2*arctanh(x*c^
(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))/c
^(3/2)+1/3*b^2*arctanh(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)
+c^(1/2))/(1+x*c^(1/2)))/c^(3/2)-1/3*b^2*arctan(x*c^(1/2))*ln((1-I)*(1+x*c^
(1/2))/(1-I*x*c^(1/2)))/c^(3/2)-1/3*I*b^2*arctan(x*c^(1/2))^2/c^(3/2)-1/3*I
*b^2*polylog(2,1-2/(1-I*x*c^(1/2)))/c^(3/2)-1/3*I*b^2*polylog(2,1-2/(1+I*x*
c^(1/2)))/c^(3/2)+1/6*I*b^2*polylog(2,1-(1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2))
)/c^(3/2)+1/6*I*b^2*polylog(2,1+(-1+I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))/c^(3/
2)-2/3*a*b*arctan(x*c^(1/2))/c^(3/2)-2/3*b^2*x*ln(-c*x^2+1)/c+1/3*b^2*arcta
n(x*c^(1/2))*ln(-c*x^2+1)/c^(3/2)-1/3*b*arctanh(x*c^(1/2))*(2*a-b*ln(-c*x^2
+1))/c^(3/2)+2/3*b^2*x*ln(c*x^2+1)/c+1/3*a*b*x^3*ln(c*x^2+1)-1/3*b^2*arctan
(x*c^(1/2))*ln(c*x^2+1)/c^(3/2)-1/3*b^2*arctanh(x*c^(1/2))*ln(c*x^2+1)/c^(3
/2)-1/6*b^2*x^3*ln(-c*x^2+1)*ln(c*x^2+1)+2/3*b^2*arctanh(x*c^(1/2))*ln(2/(1
-x*c^(1/2)))/c^(3/2)+2/3*b^2*arctan(x*c^(1/2))*ln(2/(1-I*x*c^(1/2)))/c^(3/2
)-1/3*b^2*arctan(x*c^(1/2))*ln((1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))/c^(3/2)
-2/3*b^2*arctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2)))/c^(3/2)-2/3*b^2*arctanh(x*
c^(1/2))*ln(2/(1+x*c^(1/2)))/c^(3/2)+4/3*a*b*x/c
```

Rubi [A] time = 2.06, antiderivative size = 1129, normalized size of antiderivative = 1.00, number of steps used = 86, number of rules used = 26, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.625, Rules used = {6099, 2457, 2476, 2448, 321, 206, 2455, 302, 2470, 12, 5984, 5918, 2402, 2315, 6742, 203, 30, 2557, 207, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

$$\frac{1}{12}\left(2a - b\log\left(1 - cx^2\right)\right)^2 x^3 + \frac{1}{12}b^2\log^2\left(cx^2 + 1\right)x^3 - \frac{2}{9}abx^3 + \frac{1}{9}b^2\log\left(1 - cx^2\right)x^3 + \frac{1}{9}b\left(2a - b\log\left(1 - cx^2\right)\right)x$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (4*a*b*x)/(3*c) - (2*a*b*x^3)/9 - (2*a*b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) + (
4*b^2*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) - ((I/3)*b^2*ArcTan[Sqrt[c]*x]^2)/c^(3
/2) - (4*b^2*ArcTanh[Sqrt[c]*x])/(3*c^(3/2)) - (b^2*ArcTanh[Sqrt[c]*x]^2)/(
3*c^(3/2)) + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/(3*c^(3/2))
+ (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)) - (b^2*Arc
Tan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)
) - (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/(3*c^(3/2)) - (2*b^2
*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/(3*c^(3/2)) + (b^2*ArcTanh[Sqrt
[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]
*x))])/(3*c^(3/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x
))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(3*c^(3/2)) - (b^2*ArcTan[Sqrt[
c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)) - (2*b^
2*x*Log[1 - c*x^2])/(3*c) + (b^2*x^3*Log[1 - c*x^2])/9 + (b^2*ArcTan[Sqrt[c
]*x]*Log[1 - c*x^2])/(3*c^(3/2)) + (b*x^3*(2*a - b*Log[1 - c*x^2]))/9 - (b*
```

```
ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/(3*c^(3/2)) + (x^3*(2*a - b*Lo
g[1 - c*x^2])^2)/12 + (2*b^2*x*Log[1 + c*x^2])/(3*c) + (a*b*x^3*Log[1 + c*x
^2])/3 - (b^2*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/(3*c^(3/2)) - (b^2*ArcTanh[
Sqrt[c]*x]*Log[1 + c*x^2])/(3*c^(3/2)) - (b^2*x^3*Log[1 - c*x^2]*Log[1 + c*
x^2])/6 + (b^2*x^3*Log[1 + c*x^2]^2)/12 + (b^2*PolyLog[2, 1 - 2/(1 - Sqrt[c
]*x)])/(3*c^(3/2)) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)])/c^(3/2
) + ((I/6)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])
/c^(3/2) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)])/c^(3/2) + (b^2*P
olyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/(3*c^(3/2)) - (b^2*PolyLog[2, 1 + (2*Sqrt
[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(6*c^(3/2))
- (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1
 + Sqrt[c]*x))])/(6*c^(3/2)) + ((I/6)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt
[c]*x))/(1 - I*Sqrt[c]*x)])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x^2\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx^2\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{4}\right.$$

$$=\frac{1}{4}\int x^2\left(2a-b\log\left(1-cx^2\right)\right)^2 dx-\frac{1}{2}b\int x^2\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx\right.$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{12}b^2x^3\log^2\left(1+cx^2\right)-\frac{1}{2}b\int\left(-2ax^2\log\left(1+cx\right.\right.$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{12}b^2x^3\log^2\left(1+cx^2\right)+(ab)\int x^2\log\left(1+cx^2\right)$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{3}abx^3\log\left(1+cx^2\right)-\frac{1}{6}b^2x^3\log\left(1-cx^2\right)\log\left(1\right.$$

$$=\frac{2abx}{3c}+\frac{1}{9}bx^3\left(2a-b\log\left(1-cx^2\right)\right)-\frac{b\tanh^{-1}\left(\sqrt{c}\,x\right)\left(2a-b\log\left(1-cx^2\right)\right)}{3c^{3/2}}+$$

$$=\frac{4abx}{3c}-\frac{2b^2x}{3c}-\frac{2}{9}abx^3-\frac{b^2x\log\left(1-cx^2\right)}{3c}+\frac{1}{9}bx^3\left(2a-b\log\left(1-cx^2\right)\right)-\frac{b\tan}{}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3+\frac{4b^2x^3}{27}-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{2b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\right.}{3c^{3/2}}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3+\frac{4b^2x^3}{27}-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{8b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{9c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\right.}{3c^{3/2}}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3+\frac{4b^2x^3}{27}-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{8b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{9c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\right.}{3c^{3/2}}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{14b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{9c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{3c^{3/2}}-\frac{14}{}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{3c^{3/2}}-\frac{4b^2}{}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{3c^{3/2}}-\frac{4b^2}{}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{3c^{3/2}}-\frac{4b^2}{}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{4b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{3c^{3/2}}-\frac{4b^2}{}$$

Mathematica [F] time = 3.31, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Integrate[x^2*(a + b*ArcTanh[c*x^2])^2, x]
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\operatorname{artanh}\left(cx^2\right)^2 + 2\,abx^2\operatorname{artanh}\left(cx^2\right) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*arctanh(c*x^2)^2 + 2*a*b*x^2*arctanh(c*x^2) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\operatorname{artanh}\left(cx^2\right) + a\right)^2 x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^2*x^2, x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int x^2\left(a + b\operatorname{arctanh}\left(c\,x^2\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^2))^2,x)
[Out] int(x^2*(a+b*arctanh(c*x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^2x^3+\frac{1}{3}\left(2\,x^3\operatorname{artanh}\left(cx^2\right)+c\left(\frac{4\,x}{c^2}-\frac{2\,\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{5}{2}}}+\frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)\right)ab+\frac{1}{12}\left(x^3\log\left(-cx^2+1\right)^2-3\int -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c*x^2) + c*(4*x/c^2 - 2*arctan(sqrt(c)*x)/
c^(5/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(5/2)))*a*b + 1/12*(x^3*lo
g(-c*x^2 + 1)^2 - 3*integrate(-1/3*(3*(c*x^4 - x^2)*log(c*x^2 + 1)^2 - 2*(2
*c*x^4 + 3*(c*x^4 - x^2)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^2 - 1), x))*
b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a + b\operatorname{atanh}\left(c\,x^2\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^2))^2,x)
[Out] int(x^2*(a + b*atanh(c*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a + b\operatorname{atanh}\left(cx^2\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x**2))**2, x)
```

3.73 $\int \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=958

$$xa^2+\frac{2b\tan^{-1}\left(\sqrt{c}\,x\right)a}{\sqrt{c}}-\frac{2b\tanh^{-1}\left(\sqrt{c}\,x\right)a}{\sqrt{c}}-bx\log\left(1-cx^2\right)a+bx\log\left(cx^2+1\right)a+\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}}-\frac{b^2\tanh}{}$$

```
[Out] -1/2*b^2*polylog(2,1-2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c
^(1/2)))/c^(1/2)+1/4*b^2*x*ln(-c*x^2+1)^2+1/4*b^2*x*ln(c*x^2+1)^2-1/2*b^2*p
olylog(2,1+2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))/c
^(1/2)-b^2*arctanh(x*c^(1/2))^2/c^(1/2)+b^2*polylog(2,1-2/(1-x*c^(1/2)))/c^
(1/2)+b^2*polylog(2,1-2/(1+x*c^(1/2)))/c^(1/2)-b^2*arctanh(x*c^(1/2))*ln(c*
x^2+1)/c^(1/2)+b^2*arctan(x*c^(1/2))*ln((1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2))
)/c^(1/2)+b^2*arctanh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)
-c^(1/2))/(1+x*c^(1/2)))/c^(1/2)+b^2*arctanh(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2
))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))/c^(1/2)+b^2*arctan(x*c^(1/2)
)*ln((1-I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))/c^(1/2)+I*b^2*polylog(2,1-2/(1-I*
x*c^(1/2)))/c^(1/2)+I*b^2*polylog(2,1-2/(1+I*x*c^(1/2)))/c^(1/2)-a*b*x*ln(-
c*x^2+1)-1/2*b^2*x*ln(-c*x^2+1)*ln(c*x^2+1)+2*a*b*arctan(x*c^(1/2))/c^(1/2)
-2*a*b*arctanh(x*c^(1/2))/c^(1/2)+2*b^2*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2
)))/c^(1/2)-2*b^2*arctan(x*c^(1/2))*ln(2/(1-I*x*c^(1/2)))/c^(1/2)+2*b^2*arc
tan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2)))/c^(1/2)-2*b^2*arctanh(x*c^(1/2))*ln(2/
(1+x*c^(1/2)))/c^(1/2)-1/2*I*b^2*polylog(2,1-(1+I)*(1-x*c^(1/2))/(1-I*x*c^(
1/2)))/c^(1/2)-1/2*I*b^2*polylog(2,1+(-1+I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))/
c^(1/2)+I*b^2*arctan(x*c^(1/2))^2/c^(1/2)+a*b*x*ln(c*x^2+1)-b^2*arctan(x*c^
(1/2))*ln(-c*x^2+1)/c^(1/2)+b^2*arctanh(x*c^(1/2))*ln(-c*x^2+1)/c^(1/2)+b^2
*arctan(x*c^(1/2))*ln(c*x^2+1)/c^(1/2)+a^2*x
```

Rubi [A] time = 1.47, antiderivative size = 958, normalized size of antiderivative = 1.00, number of steps used = 69, number of rules used = 21, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {6093, 2448, 321, 206, 2450, 2476, 2470, 12, 5984, 5918, 2402, 2315, 203, 2556, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

$$xa^2+\frac{2b\tan^{-1}\left(\sqrt{c}\,x\right)a}{\sqrt{c}}-\frac{2b\tanh^{-1}\left(\sqrt{c}\,x\right)a}{\sqrt{c}}-bx\log\left(1-cx^2\right)a+bx\log\left(cx^2+1\right)a+\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}}-\frac{b^2\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] a^2*x + (2*a*b*ArcTan[Sqrt[c]*x])/Sqrt[c] + (I*b^2*ArcTan[Sqrt[c]*x]^2)/Sqr
t[c] - (2*a*b*ArcTanh[Sqrt[c]*x])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]^2)/Sqrt
[c] + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/Sqrt[c] - (2*b^2*Ar
cTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*
Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (2*b^2*ArcTan[S
qrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/Sqrt[c] - (2*b^2*ArcTanh[Sqrt[c]*x]*Log
[2/(1 + Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 -
 Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/Sqrt[c] + (b^2*ArcTa
nh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + S
qrt[c]*x))])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))
/(1 - I*Sqrt[c]*x)])/Sqrt[c] - a*b*x*Log[1 - c*x^2] - (b^2*ArcTan[Sqrt[c]*x
]*Log[1 - c*x^2])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[1 - c*x^2])/Sqrt[c]
 + (b^2*x*Log[1 - c*x^2]^2)/4 + a*b*x*Log[1 + c*x^2] + (b^2*ArcTan[Sqrt[c]*
x]*Log[1 + c*x^2])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/Sqrt[c
] - (b^2*x*Log[1 - c*x^2]*Log[1 + c*x^2])/2 + (b^2*x*Log[1 + c*x^2]^2)/4 +
(b^2*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 - 2/(
1 - I*Sqrt[c]*x)])/Sqrt[c] - ((I/2)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
```

```
]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*
x)])/Sqrt[c] + (b^2*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/Sqrt[c] - (b^2*PolyL
og[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x
))])/(2*Sqrt[c]) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[
-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(2*Sqrt[c]) - ((I/2)*b^2*PolyLog[2, 1 - (
(1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
```

```
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
```

```
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6093

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Int[Expand
Integrand[(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x] /; Fr
eeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx = \int \left(a^2-ab\log\left(1-cx^2\right)+\frac{1}{4}b^2\log^2\left(1-cx^2\right)+ab\log\left(1+cx^2\right)-\frac{1}{2}b^2\log\left(1-\right.\right.$$

$$= a^2x-(ab)\int\log\left(1-cx^2\right)dx+(ab)\int\log\left(1+cx^2\right)dx+\frac{1}{4}b^2\int\log^2\left(1-cx^2\right.$$

$$= a^2x-abx\log\left(1-cx^2\right)+\frac{1}{4}b^2x\log^2\left(1-cx^2\right)+abx\log\left(1+cx^2\right)-\frac{1}{2}b^2x\log\left(1\right.$$

$$= a^2x-abx\log\left(1-cx^2\right)+\frac{1}{4}b^2x\log^2\left(1-cx^2\right)+abx\log\left(1+cx^2\right)-\frac{1}{2}b^2x\log\left(1\right.$$

$$= a^2x+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-abx\log\left(1-cx^2\right)+\frac{1}{4}b^2x\log^2\left(1\right.$$

$$= a^2x+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-abx\log\left(1-cx^2\right)-b^2x\log\left(1-\right.$$

$$= a^2x+4b^2x+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-abx\log\left(1-cx^2\right)-\frac{b^2\tan}{}$$

$$= a^2x+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{2b^2\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}}-\frac{2ab\tanh^{-1}\left(\right.}{\sqrt{c}}$$

$$= a^2x+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}}-\frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{b^2\tanh^{-1}\left(\right.}{\sqrt{c}}$$

$$= a^2x+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}}-\frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{b^2\tanh^{-1}\left(\right.}{\sqrt{c}}$$

$$= a^2x+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}}-\frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{b^2\tanh^{-1}\left(\right.}{\sqrt{c}}$$

$$= a^2x+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}}-\frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{b^2\tanh^{-1}\left(\right.}{\sqrt{c}}$$

$$= a^2x+\frac{2ab\tan^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2}{\sqrt{c}}-\frac{2ab\tanh^{-1}\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{b^2\tanh^{-1}\left(\right.}{\sqrt{c}}$$

Mathematica [A] time = 2.74, size = 566, normalized size = 0.59

$$\frac{1}{2}x\left(2a^2+4ab\tanh^{-1}\left(cx^2\right)+\frac{4ab\left(\tan^{-1}\left(\sqrt{cx^2}\right)-\tanh^{-1}\left(\sqrt{cx^2}\right)\right)}{\sqrt{cx^2}}+\frac{b^2\left(\mathrm{Li}_2\left(\frac{1}{2}\left(1-\sqrt{cx^2}\right)\right)-\mathrm{Li}_2\left(\left(-\frac{1}{2}-\frac{i}{2}\right.\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (x*(2*a^2 + 4*a*b*ArcTanh[c*x^2] + (4*a*b*(ArcTan[Sqrt[c*x^2]] - ArcTanh[Sq
rt[c*x^2]]))/Sqrt[c*x^2] + (b^2*((-2*I)*ArcTan[Sqrt[c*x^2]]^2 + 4*ArcTan[Sq
rt[c*x^2]]*ArcTanh[c*x^2] + 2*Sqrt[c*x^2]*ArcTanh[c*x^2]^2 + 2*ArcTan[Sqrt[
c*x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c*x^2]])] + 2*ArcTanh[c*x^2]*Log[1 - S
qrt[c*x^2]] - Log[2]*Log[1 - Sqrt[c*x^2]] + Log[1 - Sqrt[c*x^2]]^2/2 - Log[
```

```
1 - Sqrt[c*x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c*x^2])] - 2*ArcTanh[c*x^2]*Log
[1 + Sqrt[c*x^2]] + Log[2]*Log[1 + Sqrt[c*x^2]] + Log[((1 + I) - (1 - I)*Sq
rt[c*x^2])/2]*Log[1 + Sqrt[c*x^2]] + Log[(-1/2 - I/2)*(I + Sqrt[c*x^2])]*Lo
g[1 + Sqrt[c*x^2]] - Log[1 + Sqrt[c*x^2]]^2/2 - Log[1 - Sqrt[c*x^2]]*Log[((
1 + I) + (1 - I)*Sqrt[c*x^2])/2] - (I/2)*PolyLog[2, -E^((4*I)*ArcTan[Sqrt[c
*x^2]])] + PolyLog[2, (1 - Sqrt[c*x^2])/2] - PolyLog[2, (-1/2 - I/2)*(-1 +
Sqrt[c*x^2])] - PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c*x^2])] - PolyLog[2, (1
 + Sqrt[c*x^2])/2] + PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c*x^2])] + PolyLog[2,
 (1/2 + I/2)*(1 + Sqrt[c*x^2])]))/Sqrt[c*x^2]))/2
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2 \operatorname{artanh}\left(cx^2\right)^2 + 2\,ab \operatorname{artanh}\left(cx^2\right) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{arctanh}\left(c\,x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(c\left(\frac{2\arctan\left(\sqrt{c}\,x\right)}{c^{\frac{3}{2}}} + \frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) + 2\,x \operatorname{artanh}\left(cx^2\right)\right)ab + \frac{1}{4}\left(x\log\left(-cx^2+1\right)^2 - \int -\frac{\left(cx^2-1\right)\log\left(cx^2+1\right)^2 -}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] (c*(2*arctan(sqrt(c)*x)/c^(3/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(3
/2)) + 2*x*arctanh(c*x^2))*a*b + 1/4*(x*log(-c*x^2 + 1)^2 - integrate(-((c*
x^2 - 1)*log(c*x^2 + 1)^2 - 2*(2*c*x^2 + (c*x^2 - 1)*log(c*x^2 + 1))*log(-c
*x^2 + 1))/(c*x^2 - 1), x))*b^2 + a^2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b \operatorname{atanh}\left(c\,x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2,x)
```

```
[Out] int((a + b*atanh(c*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atanh}\left(cx^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2, x)
```

3.74 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=942

$$i\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right)^2 b^2+\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)^2 b^2-\frac{\log^2\left(cx^2+1\right)b^2}{4x}-2\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)\log\left(\frac{2}{1-\sqrt{c}\,x}\right)b^2-2\sqrt{c}\tan^{-1}$$

```
[Out] -1/4*b^2*ln(c*x^2+1)^2/x+1/2*b^2*polylog(2,1+2*(1-x*(-c)^(1/2))*c^(1/2)/((-
c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))*c^(1/2)+1/2*b^2*polylog(2,1-2*(1+x*(-c)^(1
/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))*c^(1/2)-b^2*polylog(2,1-2/
(1+x*c^(1/2)))*c^(1/2)+b^2*arctanh(x*c^(1/2))^2*c^(1/2)-b^2*polylog(2,1-2/(
1-x*c^(1/2)))*c^(1/2)-1/4*(2*a-b*ln(-c*x^2+1))^2/x+b^2*arctan(x*c^(1/2))*ln
((1-I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))*c^(1/2)+I*b^2*polylog(2,1-2/(1-I*x*c^
(1/2)))*c^(1/2)+2*b^2*arctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2)))*c^(1/2)+2*b^2
*arctanh(x*c^(1/2))*ln(2/(1+x*c^(1/2)))*c^(1/2)-1/2*I*b^2*polylog(2,1-(1+I)
*(1-x*c^(1/2))/(1-I*x*c^(1/2)))*c^(1/2)-1/2*I*b^2*polylog(2,1+(-1+I)*(1+x*c
^(1/2))/(1-I*x*c^(1/2)))*c^(1/2)+1/2*b^2*ln(-c*x^2+1)*ln(c*x^2+1)/x+2*a*b*a
rctan(x*c^(1/2))*c^(1/2)-2*b^2*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))*c^(1/
2)-2*b^2*arctan(x*c^(1/2))*ln(2/(1-I*x*c^(1/2)))*c^(1/2)+I*b^2*polylog(2,1-
2/(1+I*x*c^(1/2)))*c^(1/2)-a*b*ln(c*x^2+1)/x+I*b^2*arctan(x*c^(1/2))^2*c^(1
/2)-b^2*arctan(x*c^(1/2))*ln(-c*x^2+1)*c^(1/2)+b*arctanh(x*c^(1/2))*(2*a-b*
ln(-c*x^2+1))*c^(1/2)+b^2*arctan(x*c^(1/2))*ln(c*x^2+1)*c^(1/2)+b^2*arctanh
(x*c^(1/2))*ln(c*x^2+1)*c^(1/2)+b^2*arctan(x*c^(1/2))*ln((1+I)*(1-x*c^(1/2)
)/(1-I*x*c^(1/2)))*c^(1/2)-b^2*arctanh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^
(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))*c^(1/2)-b^2*arctanh(x*c^(1/2))*ln
(2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))*c^(1/2)
```

Rubi [A] time = 1.34, antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 21, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.313, Rules used = {6099, 2457, 206, 2470, 12, 5984, 5918, 2402, 2315, 2455, 6742, 203, 30, 2557, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

$$i\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right)^2 b^2+\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)^2 b^2-\frac{\log^2\left(cx^2+1\right)b^2}{4x}-2\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)\log\left(\frac{2}{1-\sqrt{c}\,x}\right)b^2-2\sqrt{c}\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^2,x]
```

```
[Out] 2*a*b*Sqrt[c]*ArcTan[Sqrt[c]*x] + I*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]^2 + b^2*S
qrt[c]*ArcTanh[Sqrt[c]*x]^2 - 2*b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[2/(1 - S
qrt[c]*x)] - 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)] + b^2
*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
 + 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)] + 2*b^2*Sqrt[c]
*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]
*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))]
- b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c
] + Sqrt[c])*(1 + Sqrt[c]*x))] + b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 - I)
*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[1
- c*x^2] + b*Sqrt[c]*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]) - (2*a - b
*Log[1 - c*x^2])^2/(4*x) - (a*b*Log[1 + c*x^2])/x + b^2*Sqrt[c]*ArcTan[Sqrt
[c]*x]*Log[1 + c*x^2] + b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2] + (b^
2*Log[1 - c*x^2]*Log[1 + c*x^2])/(2*x) - (b^2*Log[1 + c*x^2]^2)/(4*x) - b^2
*Sqrt[c]*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2
/(1 - I*Sqrt[c]*x)] - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]*x))/(1 - I*Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)
] - b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)] + (b^2*Sqrt[c]*PolyLog[2,
```

```
 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/
2 + (b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + S
qrt[c])*(1 + Sqrt[c]*x))])/2 - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 - I)*(1
 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^2}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^2} - \frac{b\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{2x^2} + \frac{b^2\log^2\left(1\right.}{4x^2}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^2}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{x^2}\,dx +$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x} - \frac{b^2\log^2\left(1+cx^2\right)}{4x} - \frac{1}{2}b\int\left(-\frac{2a\log\left(1+cx^2\right)}{x^2} + \frac{b\log\left(1\right.}{}\right.$$

$$= b\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)\left(2a-b\log\left(1-cx^2\right)\right) - \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x} + b^2\sqrt{c}\tan^{-1}\left(\right.$$

$$= b\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)\left(2a-b\log\left(1-cx^2\right)\right) - \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x} - \frac{ab\log\left(1+c\right.}{x}$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)^2 + b\sqrt{c}\tanh^{-1}$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b^2\sqrt{c}\tanh$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b^2\sqrt{c}\tanh$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b^2\sqrt{c}\tanh$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b^2\sqrt{c}\tanh$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b^2\sqrt{c}\tanh$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b^2\sqrt{c}\tanh$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}\,x\right)^2 - 2b^2\sqrt{c}\tanh$$

<u>**Mathematica**</u> **[A]** time = 3.72, size = 566, normalized size = 0.60

$$\frac{-2a^2 - 4ab\tanh^{-1}\left(cx^2\right) + 4ab\sqrt{cx^2}\left(\tan^{-1}\left(\sqrt{cx^2}\right) + \tanh^{-1}\left(\sqrt{cx^2}\right)\right) + b^2\sqrt{cx^2}\left(-\text{Li}_2\left(\frac{1}{2}\left(1-\sqrt{cx^2}\right)\right) + \text{Li}_2\left(\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^2,x]
```

```
[Out] (-2*a^2 - 4*a*b*ArcTanh[c*x^2] + 4*a*b*Sqrt[c*x^2]*(ArcTan[Sqrt[c*x^2]] + A
rcTanh[Sqrt[c*x^2]]) + b^2*Sqrt[c*x^2]*((-2*I)*ArcTan[Sqrt[c*x^2]]^2 + 4*Ar
cTan[Sqrt[c*x^2]]*ArcTanh[c*x^2] - (2*ArcTanh[c*x^2]^2)/Sqrt[c*x^2] + 2*Arc
Tan[Sqrt[c*x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c*x^2]])] - 2*ArcTanh[c*x^2]*
Log[1 - Sqrt[c*x^2]] + Log[2]*Log[1 - Sqrt[c*x^2]] - Log[1 - Sqrt[c*x^2]]^2
```

```
/2 + Log[1 - Sqrt[c*x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c*x^2])] + 2*ArcTanh[c
*x^2]*Log[1 + Sqrt[c*x^2]] - Log[2]*Log[1 + Sqrt[c*x^2]] - Log[((1 + I) - (
1 - I)*Sqrt[c*x^2])/2]*Log[1 + Sqrt[c*x^2]] - Log[(-1/2 - I/2)*(I + Sqrt[c*
x^2])]*Log[1 + Sqrt[c*x^2]] + Log[1 + Sqrt[c*x^2]]^2/2 + Log[1 - Sqrt[c*x^2
]]*Log[((1 + I) + (1 - I)*Sqrt[c*x^2])/2] - (I/2)*PolyLog[2, -E^((4*I)*ArcT
an[Sqrt[c*x^2]])] - PolyLog[2, (1 - Sqrt[c*x^2])/2] + PolyLog[2, (-1/2 - I/
2)*(-1 + Sqrt[c*x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c*x^2])] + Poly
Log[2, (1 + Sqrt[c*x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c*x^2])] - P
olyLog[2, (1/2 + I/2)*(1 + Sqrt[c*x^2])]))/(2*x)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(cx^2\right)^2+2\,ab\,\text{artanh}\left(cx^2\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^2, x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan\text{h}\left(c\,x^2\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^2,x)
[Out] int((a+b*arctanh(c*x^2))^2/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(c\left(\frac{2\,\arctan\left(\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{2\,\text{artanh}\left(cx^2\right)}{x}\right)ab-\frac{1}{4}\,b^2\left(\frac{\log\left(-cx^2+1\right)^2}{x}+\int -\frac{\left(cx^2-1\right)\log\left(cx^2+1\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^2,x, algorithm="maxima")
[Out] (c*(2*arctan(sqrt(c)*x)/sqrt(c) - log((c*x - sqrt(c))/(c*x + sqrt(c)))/sqrt
(c)) - 2*arctanh(c*x^2)/x)*a*b - 1/4*b^2*(log(-c*x^2 + 1)^2/x + integrate(-
((c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(2*c*x^2 - (c*x^2 - 1)*log(c*x^2 + 1))*lo
g(-c*x^2 + 1))/(c*x^4 - x^2), x)) - a^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\text{atanh}\left(c\,x^2\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/x^2,x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/x^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{atanh}\left(cx^2\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x**2, x)
```

3.75 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=1102

$$-\frac{1}{3}ic^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)^2b^2+\frac{1}{3}c^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right)^2b^2-\frac{\log^2\left(cx^2+1\right)b^2}{12x^3}+\frac{4}{3}c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)b^2+\frac{4}{3}c^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right.$$

```
[Out] 4/3*b^2*c^(3/2)*arctan(x*c^(1/2))+4/3*b^2*c^(3/2)*arctanh(x*c^(1/2))+1/3*b^
2*c^(3/2)*arctanh(x*c^(1/2))^2-1/12*b^2*ln(c*x^2+1)^2/x^3-1/3*b^2*c^(3/2)*p
olylog(2,1-2/(1-x*c^(1/2)))-1/3*b^2*c^(3/2)*polylog(2,1-2/(1+x*c^(1/2)))+1/
6*b^2*c^(3/2)*polylog(2,1+2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(
1+x*c^(1/2)))+1/6*b^2*c^(3/2)*polylog(2,1-2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^
(1/2)+c^(1/2))/(1+x*c^(1/2)))-1/12*(2*a-b*ln(-c*x^2+1))^2/x^3-1/3*I*b^2*c^(
3/2)*polylog(2,1-2/(1-I*x*c^(1/2)))-1/3*I*b^2*c^(3/2)*polylog(2,1-2/(1+I*x*
c^(1/2)))+1/6*I*b^2*c^(3/2)*polylog(2,1-(1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2))
)+1/6*I*b^2*c^(3/2)*polylog(2,1+(-1+I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))-2/3*a
*b*c^(3/2)*arctan(x*c^(1/2))+1/3*b^2*c*ln(-c*x^2+1)/x+1/3*b^2*c^(3/2)*arcta
n(x*c^(1/2))*ln(-c*x^2+1)-1/3*b*c*(2*a-b*ln(-c*x^2+1))/x+1/3*b*c^(3/2)*arct
anh(x*c^(1/2))*(2*a-b*ln(-c*x^2+1))-1/3*a*b*ln(c*x^2+1)/x^3-2/3*b^2*c*ln(c*
x^2+1)/x-1/3*b^2*c^(3/2)*arctan(x*c^(1/2))*ln(c*x^2+1)+1/3*b^2*c^(3/2)*arct
anh(x*c^(1/2))*ln(c*x^2+1)+1/6*b^2*ln(-c*x^2+1)*ln(c*x^2+1)/x^3-2/3*b^2*c^(
3/2)*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))+2/3*b^2*c^(3/2)*arctan(x*c^(1/2
))*ln(2/(1-I*x*c^(1/2)))-1/3*b^2*c^(3/2)*arctan(x*c^(1/2))*ln((1+I)*(1-x*c^
(1/2))/(1-I*x*c^(1/2)))-2/3*b^2*c^(3/2)*arctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/
2)))+2/3*b^2*c^(3/2)*arctanh(x*c^(1/2))*ln(2/(1+x*c^(1/2)))-1/3*b^2*c^(3/2)
*arctanh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+
x*c^(1/2)))-1/3*b^2*c^(3/2)*arctanh(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2
)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))-1/3*b^2*c^(3/2)*arctan(x*c^(1/2))*ln(
(1-I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))-1/3*I*b^2*c^(3/2)*arctan(x*c^(1/2))^2-
2/3*a*b*c/x
```

Rubi [A] time = 1.84, antiderivative size = 1102, normalized size of antiderivative = 1.00, number of steps used = 64, number of rules used = 24, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {6099, 2457, 2476, 2455, 206, 207, 2470, 12, 5984, 5918, 2402, 2315, 325, 6742, 203, 30, 2557, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

$$-\frac{1}{3}ic^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)^2b^2+\frac{1}{3}c^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right)^2b^2-\frac{\log^2\left(cx^2+1\right)b^2}{12x^3}+\frac{4}{3}c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)b^2+\frac{4}{3}c^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^4,x]
```

```
[Out] (-2*a*b*c)/(3*x) - (2*a*b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 + (4*b^2*c^(3/2)*Arc
Tan[Sqrt[c]*x])/3 - (I/3)*b^2*c^(3/2)*ArcTan[Sqrt[c]*x]^2 + (4*b^2*c^(3/2)*
ArcTanh[Sqrt[c]*x])/3 + (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]^2)/3 - (2*b^2*c^(3/
2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/3 + (2*b^2*c^(3/2)*ArcTan[Sqr
t[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/3 - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[((
1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/3 - (2*b^2*c^(3/2)*ArcTan[Sqrt[
c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/3 + (2*b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]*Log[2
/(1 + Sqrt[c]*x)])/3 - (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 -
 Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/3 - (b^2*c^(3/2)*Arc
Tanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 +
 Sqrt[c]*x))])/3 - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]
*x))/(1 - I*Sqrt[c]*x)])/3 + (b^2*c*Log[1 - c*x^2])/(3*x) + (b^2*c^(3/2)*Ar
cTan[Sqrt[c]*x]*Log[1 - c*x^2])/3 - (b*c*(2*a - b*Log[1 - c*x^2]))/(3*x) +
(b*c^(3/2)*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/3 - (2*a - b*Log[1
```

```
- c*x^2])^2/(12*x^3) - (a*b*Log[1 + c*x^2])/(3*x^3) - (2*b^2*c*Log[1 + c*x^
2])/(3*x) - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/3 + (b^2*c^(3/2)
*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/3 + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])
/(6*x^3) - (b^2*Log[1 + c*x^2]^2)/(12*x^3) - (b^2*c^(3/2)*PolyLog[2, 1 - 2/
(1 - Sqrt[c]*x)])/3 - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)]
 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c
]*x)] - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)] - (b^2*c^(3/2
)*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/3 + (b^2*c^(3/2)*PolyLog[2, 1 + (2*Sqr
t[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/6 + (b^2*c^
(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1
+ Sqrt[c]*x))])/6 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]*
x))/(1 - I*Sqrt[c]*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx = \int\left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^4} - \frac{b\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{2x^4} + \frac{b^2\log^2\left(\right.}{4}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^4}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{x^4}\,dx$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{12x^3} - \frac{b^2\log^2\left(1+cx^2\right)}{12x^3} - \frac{1}{2}b\int\left(-\frac{2a\log\left(1+cx^2\right)}{x^4} + \frac{b\log}{}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{12x^3} - \frac{b^2\log^2\left(1+cx^2\right)}{12x^3} + (ab)\int \frac{\log\left(1+cx^2\right)}{x^4}\,dx - \frac{1}{2}b^2\int$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{12x^3} - \frac{ab\log\left(1+cx^2\right)}{3x^3} + \frac{b^2\log\left(1-cx^2\right)\log\left(1+cx^2\right)}{6x^3} - $$

$$= -\frac{2abc}{3x} - \frac{bc\left(2a-b\log\left(1-cx^2\right)\right)}{3x} + \frac{1}{3}bc^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right)\left(2a-b\log\left(1-cx^2\right)\right)$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{2}{3}b^2c^{3/2}\tanh^{-1}\left(\sqrt{c}\,x\right) - $$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + $$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + $$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + $$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + $$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + $$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + $$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + $$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}\,x\right)^2 + $$

Mathematica [F] time = 2.77, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^4,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/x^4, x]
```

fricas [F] time = 2.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{artanh}\left(cx^2\right)^2+2\,ab\operatorname{artanh}\left(cx^2\right)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^4,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^4,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^4, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\operatorname{arctanh}\left(c\,x^2\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^4,x)
[Out] int((a+b*arctanh(c*x^2))^2/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(\left(2\sqrt{c}\arctan\left(\sqrt{c}\,x\right)+\sqrt{c}\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)+\frac{4}{x}\right)c+\frac{2\operatorname{artanh}\left(cx^2\right)}{x^3}\right)ab-\frac{1}{12}\,b^2\left(\frac{\log\left(-cx^2+1\right)^2}{x^3}+3\int-\frac{3\,(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^4,x, algorithm="maxima")
[Out] -1/3*((2*sqrt(c)*arctan(sqrt(c)*x) + sqrt(c)*log((c*x - sqrt(c))/(c*x + sqr
t(c))) + 4/x)*c + 2*arctanh(c*x^2)/x^3)*a*b - 1/12*b^2*(log(-c*x^2 + 1)^2/x
^3 + 3*integrate(-1/3*(3*(c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(2*c*x^2 - 3*(c*x
^2 - 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^6 - x^4), x)) - 1/3*a^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\operatorname{atanh}\left(c\,x^2\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/x^4,x)
[Out] int((a + b*atanh(c*x^2))^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(cx^2\right)\right)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**4,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x**4, x)
```

3.76 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=1176

$$\frac{1}{5}ib^2\tan^{-1}\left(\sqrt{c}\,x\right)^2c^{5/2}+\frac{1}{5}b^2\tanh^{-1}\left(\sqrt{c}\,x\right)^2c^{5/2}-\frac{4}{15}b^2\tan^{-1}\left(\sqrt{c}\,x\right)c^{5/2}+\frac{2}{5}ab\tan^{-1}\left(\sqrt{c}\,x\right)c^{5/2}+\frac{4}{15}b^2\tanh^{-1}\left(\sqrt{c}\right.$$

```
[Out] -4/15*b^2*c^(5/2)*arctan(x*c^(1/2))+4/15*b^2*c^(5/2)*arctanh(x*c^(1/2))+1/5
*b^2*c^(5/2)*arctanh(x*c^(1/2))^2-1/20*b^2*ln(c*x^2+1)^2/x^5-1/5*b^2*c^(5/2
)*polylog(2,1-2/(1-x*c^(1/2)))-1/5*b^2*c^(5/2)*polylog(2,1-2/(1+x*c^(1/2)))
+1/10*b^2*c^(5/2)*polylog(2,1+2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2
))/(1+x*c^(1/2)))+1/10*b^2*c^(5/2)*polylog(2,1-2*(1+x*(-c)^(1/2))*c^(1/2)/(
(-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))-1/20*(2*a-b*ln(-c*x^2+1))^2/x^5-8/15*b^2
*c^2/x+1/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln(c*x^2+1)+1/5*b^2*c^(5/2)*arctan
h(x*c^(1/2))*ln(c*x^2+1)+1/10*b^2*ln(-c*x^2+1)*ln(c*x^2+1)/x^5-2/5*b^2*c^(5
/2)*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))-2/5*b^2*c^(5/2)*arctan(x*c^(1/2)
)*ln(2/(1-I*x*c^(1/2)))+1/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln((1+I)*(1-x*c^(
1/2))/(1-I*x*c^(1/2)))+2/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2
)))+2/5*b^2*c^(5/2)*arctanh(x*c^(1/2))*ln(2/(1+x*c^(1/2)))-1/5*b^2*c^(5/2)*
arctanh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x
*c^(1/2)))+1/5*I*b^2*c^(5/2)*polylog(2,1-2/(1-I*x*c^(1/2)))+1/5*I*b^2*c^(5/
2)*polylog(2,1-2/(1+I*x*c^(1/2)))+1/5*I*b^2*c^(5/2)*arctan(x*c^(1/2))^2-1/5
*b^2*c^(5/2)*arctanh(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c
^(1/2))/(1+x*c^(1/2)))+1/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln((1-I)*(1+x*c^(1
/2))/(1-I*x*c^(1/2)))-1/10*I*b^2*c^(5/2)*polylog(2,1-(1+I)*(1-x*c^(1/2))/(1
-I*x*c^(1/2)))-1/10*I*b^2*c^(5/2)*polylog(2,1+(-1+I)*(1+x*c^(1/2))/(1-I*x*c
^(1/2)))+2/5*a*b*c^(5/2)*arctan(x*c^(1/2))+1/15*b^2*c*ln(-c*x^2+1)/x^3-1/5*
b^2*c^2*ln(-c*x^2+1)/x-1/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln(-c*x^2+1)-1/15*
b*c*(2*a-b*ln(-c*x^2+1))/x^3-1/5*b*c^2*(2*a-b*ln(-c*x^2+1))/x+1/5*b*c^(5/2)
*arctanh(x*c^(1/2))*(2*a-b*ln(-c*x^2+1))-1/5*a*b*ln(c*x^2+1)/x^5-2/15*b^2*c
*ln(c*x^2+1)/x^3-2/15*a*b*c/x^3+2/5*a*b*c^2/x
```

Rubi [A] time = 1.99, antiderivative size = 1176, normalized size of antiderivative = 1.00, number of steps used = 77, number of rules used = 24, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {6099, 2457, 2476, 2455, 325, 206, 207, 2470, 12, 5984, 5918, 2402, 2315, 6742, 203, 30, 2557, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^6,x]
```

```
[Out] (-2*a*b*c)/(15*x^3) + (2*a*b*c^2)/(5*x) - (8*b^2*c^2)/(15*x) + (2*a*b*c^(5/
2)*ArcTan[Sqrt[c]*x])/5 - (4*b^2*c^(5/2)*ArcTan[Sqrt[c]*x])/15 + (I/5)*b^2*
c^(5/2)*ArcTan[Sqrt[c]*x]^2 + (4*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x])/15 + (b^2*
c^(5/2)*ArcTanh[Sqrt[c]*x]^2)/5 - (2*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[2/(
1 - Sqrt[c]*x)])/5 - (2*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*
x)])/5 + (b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 -
I*Sqrt[c]*x)])/5 + (2*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)
])/5 + (2*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/5 - (b^2*c
^(5/2)*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sq
rt[c])*(1 + Sqrt[c]*x))])/5 - (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c
]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/5 + (b^2*c^(5/
2)*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/5 +
(b^2*c*Log[1 - c*x^2])/(15*x^3) - (b^2*c^2*Log[1 - c*x^2])/(5*x) - (b^2*c^(
5/2)*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/5 - (b*c*(2*a - b*Log[1 - c*x^2]))/(
15*x^3) - (b*c^2*(2*a - b*Log[1 - c*x^2]))/(5*x) + (b*c^(5/2)*ArcTanh[Sqrt[
```

```
c]*x]*(2*a - b*Log[1 - c*x^2]))/5 - (2*a - b*Log[1 - c*x^2])^2/(20*x^5) - (
a*b*Log[1 + c*x^2])/(5*x^5) - (2*b^2*c*Log[1 + c*x^2])/(15*x^3) + (b^2*c^(5
/2)*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/5 + (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*L
og[1 + c*x^2])/5 + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])/(10*x^5) - (b^2*Log[
1 + c*x^2]^2)/(20*x^5) - (b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/5
+ (I/5)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)] - (I/10)*b^2*c^(5/2
)*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] + (I/5)*b^2*c
^(5/2)*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)] - (b^2*c^(5/2)*PolyLog[2, 1 - 2/
(1 + Sqrt[c]*x)])/5 + (b^2*c^(5/2)*PolyLog[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*
x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/10 + (b^2*c^(5/2)*PolyLog[2, 1
 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/10
 - (I/10)*b^2*c^(5/2)*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[
c]*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
```

```
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx=\int\left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^6}-\frac{b\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{2x^6}+\frac{b^2\log^2\left(1\right.}{4x^6}\right.$$

$$=\frac{1}{4}\int\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^6}\,dx-\frac{1}{2}b\int\frac{\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{x^6}\,dx+$$

$$=-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{20x^5}-\frac{b^2\log^2\left(1+cx^2\right)}{20x^5}-\frac{1}{2}b\int\left(-\frac{2a\log\left(1+cx^2\right)}{x^6}+\frac{b\log\left(1\right.}{}\right.$$

$$=-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{20x^5}-\frac{b^2\log^2\left(1+cx^2\right)}{20x^5}+(ab)\int\frac{\log\left(1+cx^2\right)}{x^6}\,dx-\frac{1}{2}b^2\int\frac{\mathrm{lo}}{}$$

$$=-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{20x^5}-\frac{ab\log\left(1+cx^2\right)}{5x^5}+\frac{b^2\log\left(1-cx^2\right)\log\left(1+cx^2\right)}{10x^5}-\frac{b^2\,\mathrm{l}}{}$$

$$=-\frac{2abc}{15x^3}-\frac{bc\left(2a-b\log\left(1-cx^2\right)\right)}{15x^3}-\frac{bc^2\left(2a-b\log\left(1-cx^2\right)\right)}{5x}+\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\sqrt{\ }\right.$$

$$=-\frac{2abc}{15x^3}+\frac{2abc^2}{5x}-\frac{4b^2c^2}{15x}-\frac{2}{5}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{2}{5}b^2c^{5/2}\tanh^{-1}\left(\sqrt{c}\,x\right)-\frac{bc\left(2a-\right.}{}$$

$$=-\frac{2abc}{15x^3}+\frac{2abc^2}{5x}-\frac{4b^2c^2}{15x}+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)-\frac{8}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{5}ib^2c^{5/2}$$

$$=-\frac{2abc}{15x^3}+\frac{2abc^2}{5x}-\frac{4b^2c^2}{15x}+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)-\frac{8}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{5}ib^2c^{5/2}$$

$$=-\frac{2abc}{15x^3}+\frac{2abc^2}{5x}-\frac{8b^2c^2}{15x}+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)-\frac{2}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{5}ib^2c^{5/2}$$

$$=-\frac{2abc}{15x^3}+\frac{2abc^2}{5x}-\frac{8b^2c^2}{15x}+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)-\frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{5}ib^2c^{5/2}$$

$$=-\frac{2abc}{15x^3}+\frac{2abc^2}{5x}-\frac{8b^2c^2}{15x}+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)-\frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{5}ib^2c^{5/2}$$

$$=-\frac{2abc}{15x^3}+\frac{2abc^2}{5x}-\frac{8b^2c^2}{15x}+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)-\frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{5}ib^2c^{5/2}$$

$$=-\frac{2abc}{15x^3}+\frac{2abc^2}{5x}-\frac{8b^2c^2}{15x}+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)-\frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{5}ib^2c^{5/2}$$

$$=-\frac{2abc}{15x^3}+\frac{2abc^2}{5x}-\frac{8b^2c^2}{15x}+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)-\frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}\,x\right)+\frac{1}{5}ib^2c^{5/2}$$

Mathematica [F] time = 3.27, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^6,x]
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/x^6, x]
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(cx^2\right)^2+2\,ab\,\text{artanh}\left(cx^2\right)+a^2}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^6,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^6,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^6, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arctanh}\left(c\,x^2\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^6,x)
[Out] int((a+b*arctanh(c*x^2))^2/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}\left(\left(6\,c^{\frac{3}{2}}\arctan\left(\sqrt{c}\,x\right)-3\,c^{\frac{3}{2}}\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)-\frac{4}{x^3}\right)c-\frac{6\,\text{artanh}\left(cx^2\right)}{x^5}\right)ab-\frac{1}{20}\,b^2\left(\frac{\log\left(-cx^2+1\right)^2}{x^5}+5\int -\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^6,x, algorithm="maxima")
[Out] 1/15*((6*c^(3/2)*arctan(sqrt(c)*x) - 3*c^(3/2)*log((c*x - sqrt(c))/(c*x + s
qrt(c))) - 4/x^3)*c - 6*arctanh(c*x^2)/x^5)*a*b - 1/20*b^2*(log(-c*x^2 + 1)
^2/x^5 + 5*integrate(-1/5*(5*(c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(2*c*x^2 - 5*
(c*x^2 - 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^8 - x^6), x)) - 1/5*a^2/x
^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\text{atanh}\left(c\,x^2\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/x^6,x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(cx^2\right)\right)^2}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**6,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x**6, x)
```

3.77 $\int x^3 \left(a + b\tanh^{-1}\left(cx^2\right)\right)^3 dx$

Optimal. Leaf size=141

$$-\frac{3b^2\log\left(\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{2c^2}-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{4c^2}+\frac{3b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4c^2}+\frac{3bx^2\left(a+b\tanh^{-1}\left(c\right.\right.}{4c}$$

```
[Out] 3/4*b*(a+b*arctanh(c*x^2))^2/c^2+3/4*b*x^2*(a+b*arctanh(c*x^2))^2/c-1/4*(a+
b*arctanh(c*x^2))^3/c^2+1/4*x^4*(a+b*arctanh(c*x^2))^3-3/2*b^2*(a+b*arctanh
(c*x^2))*ln(2/(-c*x^2+1))/c^2-3/4*b^3*polylog(2,1-2/(-c*x^2+1))/c^2
```

Rubi [B] time = 4.22, antiderivative size = 479, normalized size of antiderivative = 3.40, number of steps used = 155, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2439, 2416, 2396, 2433, 2374, 6589, 2411, 43, 2334, 12, 14, 2301, 6742, 2430, 2394, 2393, 2391, 2395, 2375, 2317, 2425}

$$-\frac{3b^3\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)}{8c^2}+\frac{3b^3\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)}{8c^2}-\frac{3b^2\log^2\left(cx^2+1\right)\left(2a-b\log\left(1-cx^2\right)\right)}{32c^2}+\frac{3b^2\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] (-3*b*(1 - c*x^2)*(2*a - b*Log[1 - c*x^2])^2)/(16*c^2) - ((1 - c*x^2)*(2*a
- b*Log[1 - c*x^2])^3)/(16*c^2) + ((1 - c*x^2)^2*(2*a - b*Log[1 - c*x^2])^3
)/(32*c^2) + (3*b^2*(2*a - b*Log[1 - c*x^2])*Log[(1 + c*x^2)/2])/(8*c^2) +
(3*b^3*Log[(1 - c*x^2)/2]*Log[1 + c*x^2])/(8*c^2) + (3*b^2*x^2*(2*a - b*Log
[1 - c*x^2])*Log[1 + c*x^2])/(8*c) - (3*b*(2*a - b*Log[1 - c*x^2])^2*Log[1
+ c*x^2])/(32*c^2) + (3*b*x^4*(2*a - b*Log[1 - c*x^2])^2*Log[1 + c*x^2])/32
 + (3*b^3*(1 + c*x^2)*Log[1 + c*x^2]^2)/(16*c^2) - (3*b^2*(2*a - b*Log[1 -
c*x^2])*Log[1 + c*x^2]^2)/(32*c^2) + (3*b^2*x^4*(2*a - b*Log[1 - c*x^2])*Lo
g[1 + c*x^2]^2)/32 - (b^3*(1 + c*x^2)*Log[1 + c*x^2]^3)/(16*c^2) + (b^3*(1
+ c*x^2)^2*Log[1 + c*x^2]^3)/(32*c^2) - (3*b^3*PolyLog[2, (1 - c*x^2)/2])/(
8*c^2) + (3*b^3*PolyLog[2, (1 + c*x^2)/2])/(8*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 dx = \int \left(\frac{1}{8}x^3\left(2a-b\log\left(1-cx^2\right)\right)^3+\frac{3}{8}bx^3\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)\right.$$

$$=\frac{1}{8}\int x^3\left(2a-b\log\left(1-cx^2\right)\right)^3 dx+\frac{1}{8}(3b)\int x^3\left(-2a+b\log\left(1-cx^2\right)\right)^2\log$$

$$=\frac{1}{16}\text{Subst}\left(\int x(2a-b\log(1-cx))^3 dx,x,x^2\right)+\frac{1}{16}(3b)\text{Subst}\left(\int x(-2a+b\,\mathrm{l}\right.$$

$$=\frac{3}{32}bx^4\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{32}b^2x^4\left(2a-b\log\left(1-cx^2\right)\right)\mathrm{l}$$

$$=\frac{3}{32}bx^4\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{32}b^2x^4\left(2a-b\log\left(1-cx^2\right)\right)\mathrm{l}$$

$$=\frac{3}{32}bx^4\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{32}b^2x^4\left(2a-b\log\left(1-cx^2\right)\right)\mathrm{l}$$

$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c^2}+\frac{\left(1-cx^2\right)^2\left(2a-b\log\left(1-cx^2\right)\right)^3}{32c^2}-\frac{3b\left(\right.}{}$$

$$=-\frac{9b\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32c^2}+\frac{3b\left(1-cx^2\right)^2\left(2a-b\log\left(1-cx^2\right)\right)^2}{64c^2}$$

$$=\frac{9ab^2x^2}{8c}+\frac{9b^3x^2}{16c}+\frac{3b^3\left(1-cx^2\right)^2}{128c^2}-\frac{3b^3\left(1+cx^2\right)^2}{128c^2}+\frac{3b^2\left(1-cx^2\right)^2\left(2a-b\,\mathrm{l}\right.}{64c^2}$$

$$=\frac{9ab^2x^2}{8c}+\frac{9b^3x^2}{8c}+\frac{3b^3\left(1-cx^2\right)^2}{128c^2}-\frac{3b^3\left(1+cx^2\right)^2}{128c^2}+\frac{9b^3\left(1-cx^2\right)\log\left(1-c\right.}{16c^2}$$

$$=\frac{3ab^2x^2}{4c}+\frac{15b^3x^2}{16c}+\frac{9b^3\left(1-cx^2\right)\log\left(1-cx^2\right)}{16c^2}-\frac{3b\left(1-cx^2\right)\left(2a-b\log\left(1\right.\right.}{16c^2}$$

$$=\frac{3ab^2x^2}{4c}+\frac{3b^3x^2}{4c}+\frac{3b^3\left(1-cx^2\right)\log\left(1-cx^2\right)}{8c^2}-\frac{3b\left(1-cx^2\right)\left(2a-b\log\left(1-\right.\right.}{16c^2}$$

Mathematica [A] time = 0.50, size = 185, normalized size = 1.31

$$a\left(2a^2c^2x^4+6abcx^2+3ab\log\left(1-cx^2\right)-3ab\log\left(cx^2+1\right)+6b^2\log\left(1-c^2x^4\right)\right)+6b^2\left(cx^2-1\right)\tanh^{-1}\left(cx^2\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] (6*b^2*(-1 + c*x^2)*(a + b + a*c*x^2)*ArcTanh[c*x^2]^2 + 2*b^3*(-1 + c^2*x^
4)*ArcTanh[c*x^2]^3 + 6*b*ArcTanh[c*x^2]*(a*c*x^2*(2*b + a*c*x^2) - 2*b^2*L
og[1 + E^(-2*ArcTanh[c*x^2])]) + a*(6*a*b*c*x^2 + 2*a^2*c^2*x^4 + 3*a*b*Log
[1 - c*x^2] - 3*a*b*Log[1 + c*x^2] + 6*b^2*Log[1 - c^2*x^4]) + 6*b^3*PolyLo
g[2, -E^(-2*ArcTanh[c*x^2])])/(8*c^2)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^3 \operatorname{artanh}\left(cx^2\right)^3 + 3\,ab^2x^3 \operatorname{artanh}\left(cx^2\right)^2 + 3\,a^2bx^3 \operatorname{artanh}\left(cx^2\right) + a^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctanh(c*x^2)^3 + 3*a*b^2*x^3*arctanh(c*x^2)^2 + 3*a^2*b*
x^3*arctanh(c*x^2) + a^3*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(cx^2\right) + a\right)^3 x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3*x^3, x)
```

maple [C] time = 0.39, size = 748, normalized size = 5.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^2))^3,x)
```

```
[Out] 1/32*b^3*(c^2*x^4-1)/c^2*ln(c*x^2+1)^3+3/32*b^2*(-x^4*b*ln(-c*x^2+1)*c^2+2*
a*c^2*x^4+2*b*c*x^2+b*ln(-c*x^2+1)-2*a+2*b)/c^2*ln(c*x^2+1)^2+(3/32*b^3*(c^
2*x^4-1)/c^2*ln(-c*x^2+1)^2-3/8*b^2*x^2*(a*c*x^2+b)/c*ln(-c*x^2+1)+3/8*b*(a
^2*c^2*x^4+2*a*b*c*x^2+b*ln(-c*x^2+1)*a+b^2*ln(-c*x^2+1))/c^2)*ln(c*x^2+1)+
3/4*b^2/c*Sum(-(ln(x-_alpha)*ln(-c*x^2+1)+2*c*(-1/2*ln(x-_alpha)*(ln((RootO
f(_Z^2*c+2*_Z*_alpha*c-2,index=1)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,i
ndex=1))+ln((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)-x+_alpha)/RootOf(_Z^2*c
+2*_Z*_alpha*c-2,index=2)))/c-1/2*(dilog((RootOf(_Z^2*c+2*_Z*_alpha*c-2,ind
ex=1)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=1))+dilog((RootOf(_Z^2*
c+2*_Z*_alpha*c-2,index=2)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)
))/c))*b/c,_alpha=RootOf(_Z^2*c+1))+3/16*b^2/c^2*a*ln(c*x^2-1)-3/8*a^2*b/c^
2*ln(c*x^2+1)+3/4*a*b^2/c^2*ln(c*x^2+1)+3/8/c^2*b^3*ln(-c*x^2+1)-3/8/c^2*b^
3*ln(c*x^2-1)-1/32*b^3*x^4*ln(-c*x^2+1)^3-3/16*b^3/c^2*ln(-c*x^2+1)^2+1/32*
b^3/c^2*ln(-c*x^2+1)^3-3/16*b^3/c^2+1/4*x^4*a^3+3/4/c*a^2*b*x^2+3/16*b^3/c*
x^2*ln(-c*x^2+1)^2-3/8*a^2*b*x^4*ln(-c*x^2+1)+3/8*a^2*b/c^2*ln(c*x^2-1)+3/1
6*a*b^2*x^4*ln(-c*x^2+1)^2+9/16*a*b^2/c^2*ln(-c*x^2+1)-3/16*a*b^2/c^2*ln(-c
*x^2+1)^2-3/4*a*b^2/c*x^2*ln(-c*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{4}\,ab^2x^4 \operatorname{artanh}\left(cx^2\right)^2+\frac{1}{4}\,a^3x^4+\frac{3}{8}\left(2\,x^4 \operatorname{artanh}\left(cx^2\right) + c\left(\frac{2\,x^2}{c^2} - \frac{\log\left(cx^2+1\right)}{c^3} + \frac{\log\left(cx^2-1\right)}{c^3}\right)\right)a^2b+\frac{3}{16}\left(4\,c\left(\frac{2\,x^2}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctanh(c*x^2)^2 + 1/4*a^3*x^4 + 3/8*(2*x^4*arctanh(c*x^2) +
c*(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3))*a^2*b + 3/16*(4*c*
(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3)*arctanh(c*x^2) - (2*(
log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 - log(c*x^2 - 1)^2 -
4*log(c*x^2 - 1))/c^2)*a*b^2 - 1/128*(4*x^4*log(-c*x^2 + 1)^3 + 3*c^3*(x^4/
c^3 + log(c^2*x^4 - 1)/c^5) - 6*c*((c*x^4 + 2*x^2)/c^2 + 2*log(c*x^2 - 1)/c
^3)*log(-c*x^2 + 1)^2 + 21*c^2*(2*x^2/c^3 - log(c*x^2 + 1)/c^4 + log(c*x^2
- 1)/c^4) + c*(6*(c^2*x^4 + 6*c*x^2 + 2*log(c*x^2 - 1)^2 + 6*log(c*x^2 - 1)
)*log(-c*x^2 + 1)/c^3 - (3*c^2*x^4 + 42*c*x^2 + 4*log(c*x^2 - 1)^3 + 18*log
(c*x^2 - 1)^2 + 42*log(c*x^2 - 1))/c^3) - 1152*c*integrate(1/4*x^3*log(c*x^
2 + 1)/(c^3*x^4 - c), x) - 2*(12*c*x^2*log(c*x^2 + 1)^2 + 2*(c^2*x^4 - 1)*l
og(c*x^2 + 1)^3 - 3*(c^2*x^4 - 2*c*x^2 - 2*(c^2*x^4 - 1)*log(c*x^2 + 1) + 1
)*log(-c*x^2 + 1)^2 + 3*(c^2*x^4 + 6*c*x^2 - 2*(c^2*x^4 - 1)*log(c*x^2 + 1)
^2 - 8*(c*x^2 + 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/c^2 + 18*log(4*c^3*x^4
- 4*c)/c^2 - 384*integrate(1/4*x*log(c*x^2 + 1)/(c^3*x^4 - c), x))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(a + b \operatorname{atanh}\left(c x^2\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^2))^3,x)
```

```
[Out] int(x^3*(a + b*atanh(c*x^2))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b \operatorname{atanh}\left(cx^2\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**2))**3,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*x**2))**3, x)
```

3.78 $\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\,dx$

Optimal. Leaf size=134

$$-\frac{3b^2\text{Li}_2\left(1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3+\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{2c}-\frac{3b\log\left(\frac{2}{1-cx^2}\right)\left(a+b\right.}{2c}$$

```
[Out] 1/2*(a+b*arctanh(c*x^2))^3/c+1/2*x^2*(a+b*arctanh(c*x^2))^3-3/2*b*(a+b*arct
anh(c*x^2))^2*ln(2/(-c*x^2+1))/c-3/2*b^2*(a+b*arctanh(c*x^2))*polylog(2,1-2
/(-c*x^2+1))/c+3/4*b^3*polylog(3,1-2/(-c*x^2+1))/c
```

Rubi [B] time = 2.32, antiderivative size = 390, normalized size of antiderivative = 2.91, number of steps used = 82, number of rules used = 23, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.643, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$-\frac{3b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)\left(2a-b\log\left(1-cx^2\right)\right)}{4c}-\frac{3b^3\text{PolyLog}\left(3,\frac{1}{2}\left(1-cx^2\right)\right)}{4c}-\frac{3b^3\text{PolyLog}\left(3,\frac{1}{2}\left(cx^2+1\right)\right)}{4c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] -((1 - c*x^2)*(2*a - b*Log[1 - c*x^2])^3)/(16*c) + (3*b*(2*a - b*Log[1 - c*
x^2])^2*Log[(1 + c*x^2)/2])/(8*c) - (3*b*(2*a - b*Log[1 - c*x^2])^2*Log[1 +
 c*x^2])/(16*c) + (3*b*x^2*(2*a - b*Log[1 - c*x^2])^2*Log[1 + c*x^2])/16 +
(3*b^3*Log[(1 - c*x^2)/2]*Log[1 + c*x^2]^2)/(8*c) + (3*b^2*(2*a - b*Log[1 -
 c*x^2])*Log[1 + c*x^2]^2)/(16*c) + (3*b^2*x^2*(2*a - b*Log[1 - c*x^2])*Log
[1 + c*x^2]^2)/16 + (b^3*(1 + c*x^2)*Log[1 + c*x^2]^3)/(16*c) - (3*b^2*(2*a
 - b*Log[1 - c*x^2])*PolyLog[2, (1 - c*x^2)/2])/(4*c) + (3*b^3*Log[1 + c*x^
2]*PolyLog[2, (1 + c*x^2)/2])/(4*c) - (3*b^3*PolyLog[3, (1 - c*x^2)/2])/(4*
c) - (3*b^3*PolyLog[3, (1 + c*x^2)/2])/(4*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
```

```
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\,dx=\int\left(\frac{1}{8}x\left(2a-b\log\left(1-cx^2\right)\right)^3+\frac{3}{8}bx\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)-\right.$$

$$=\frac{1}{8}\int x\left(2a-b\log\left(1-cx^2\right)\right)^3\,dx+\frac{1}{8}(3b)\int x\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1\right.$$

$$=\frac{1}{16}\text{Subst}\left(\int(2a-b\log(1-cx))^3\,dx,x,x^2\right)+\frac{1}{16}(3b)\,\text{Subst}\left(\int(-2a+b\log(1\right.$$

$$=\frac{3}{16}bx^2\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{16}b^2x^2\left(2a-b\log\left(1-cx^2\right)\right)\log$$

$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3}{16}bx^2\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx\right.$$

$$=-\frac{3b\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{16c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3}{16}$$

$$=\frac{3}{4}ab^2x^2-\frac{3b^3x^2}{8}-\frac{3b\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{16c}-\frac{\left(1-cx^2\right)\left(2a-b\log\right.}{16c}$$

$$=\frac{3}{4}ab^2x^2+\frac{3b^3\left(1-cx^2\right)\log\left(1-cx^2\right)}{8c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3b}{}$$

$$=\frac{3b^3x^2}{8}+\frac{3b^3\left(1-cx^2\right)\log\left(1-cx^2\right)}{8c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3b\left(\right.}{}$$

$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3b\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(\frac{1}{2}\left(1+cx^2\right.\right.}{8c}$$

$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3b\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(\frac{1}{2}\left(1+cx^2\right.\right.}{8c}$$

Mathematica [A] time = 0.09, size = 213, normalized size = 1.59

$$\frac{a^3x^2}{2}+\frac{3a^2b\log\left(1-c^2x^4\right)}{4c}+\frac{3}{2}a^2bx^2\tanh^{-1}\left(cx^2\right)+\frac{3ab^2\left(\text{Li}_2\left(-e^{-2\tanh^{-1}\left(cx^2\right)}\right)+\tanh^{-1}\left(cx^2\right)\left(cx^2\tanh^{-1}\left(cx^2\right)\right.\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] (a^3*x^2)/2 + (3*a^2*b*x^2*ArcTanh[c*x^2])/2 + (3*a^2*b*Log[1 - c^2*x^4])/(
4*c) + (3*a*b^2*(ArcTanh[c*x^2]*(-ArcTanh[c*x^2] + c*x^2*ArcTanh[c*x^2] - 2
```

```
*Log[1 + E^(-2*ArcTanh[c*x^2])]) + PolyLog[2, -E^(-2*ArcTanh[c*x^2])]))/(2*
c) + (b^3*(ArcTanh[c*x^2]^2*(-ArcTanh[c*x^2] + c*x^2*ArcTanh[c*x^2] - 3*Log
[1 + E^(-2*ArcTanh[c*x^2])]) + 3*ArcTanh[c*x^2]*PolyLog[2, -E^(-2*ArcTanh[c
*x^2])] + (3*PolyLog[3, -E^(-2*ArcTanh[c*x^2])])/2))/(2*c)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^3x\,\mathrm{artanh}\left(cx^2\right)^3+3\,ab^2x\,\mathrm{artanh}\left(cx^2\right)^2+3\,a^2bx\,\mathrm{artanh}\left(cx^2\right)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctanh(c*x^2)^3 + 3*a*b^2*x*arctanh(c*x^2)^2 + 3*a^2*b*x*ar
ctanh(c*x^2) + a^3*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\,\mathrm{artanh}\left(cx^2\right)+a\right)^3x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3*x, x)
```

maple [B] time = 0.20, size = 298, normalized size = 2.22

$$\frac{x^2a^3}{2}+\frac{b^3x^2\,\mathrm{arctanh}\left(c\,x^2\right)^3}{2}+\frac{b^3\,\mathrm{arctanh}\left(c\,x^2\right)^3}{2c}-\frac{3b^3\,\mathrm{arctanh}\left(c\,x^2\right)^2\ln\left(1+\frac{\left(c\,x^2+1\right)^2}{-c^2x^4+1}\right)}{2c}-\frac{3b^3\,\mathrm{arctanh}\left(c\,x^2\right)\mathrm{polylo}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^2))^3,x)
```

```
[Out] 1/2*x^2*a^3+1/2*b^3*x^2*arctanh(c*x^2)^3+1/2/c*b^3*arctanh(c*x^2)^3-3/2/c*b
^3*arctanh(c*x^2)^2*ln(1+(c*x^2+1)^2/(-c^2*x^4+1))-3/2/c*b^3*arctanh(c*x^2)
*polylog(2,-(c*x^2+1)^2/(-c^2*x^4+1))+3/4/c*b^3*polylog(3,-(c*x^2+1)^2/(-c^
2*x^4+1))+3/2*x^2*a*b^2*arctanh(c*x^2)^2+3/2/c*a*b^2*arctanh(c*x^2)^2-3/c*a
rctanh(c*x^2)*ln(1+(c*x^2+1)^2/(-c^2*x^4+1))*a*b^2-3/2/c*polylog(2,-(c*x^2+
1)^2/(-c^2*x^4+1))*a*b^2+3/2*x^2*a^2*b*arctanh(c*x^2)+3/4/c*a^2*b*ln(-c^2*x
^4+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^3x^2+\frac{3\left(2\,cx^2\,\mathrm{artanh}\left(cx^2\right)+\log\left(-c^2x^4+1\right)\right)a^2b}{4\,c}-\frac{\left(b^3cx^2-b^3\right)\log\left(-cx^2+1\right)^3-3\left(2\,ab^2cx^2+\left(b^3cx^2+b^3\right)\mathrm{lo}\right.}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x^2 + 3/4*(2*c*x^2*arctanh(c*x^2) + log(-c^2*x^4 + 1))*a^2*b/c - 1/
16*((b^3*c*x^2 - b^3)*log(-c*x^2 + 1)^3 - 3*(2*a*b^2*c*x^2 + (b^3*c*x^2 + b
^3)*log(c*x^2 + 1))*log(-c*x^2 + 1)^2)/c - integrate(-1/8*((b^3*c*x^3 - b^3
*x)*log(c*x^2 + 1)^3 + 6*(a*b^2*c*x^3 - a*b^2*x)*log(c*x^2 + 1)^2 - 3*(4*a*
b^2*c*x^3 + (b^3*c*x^3 - b^3*x)*log(c*x^2 + 1)^2 + 2*((2*a*b^2*c + b^3*c)*x
^3 - (2*a*b^2 - b^3)*x)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^2))^3,x)
```

```
[Out] int(x*(a + b*atanh(c*x^2))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{atanh}\left(cx^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**2))**3,x)
```

```
[Out] Integral(x*(a + b*atanh(c*x**2))**3, x)
```

3.79 $\int \frac{(a+b\tanh^{-1}(cx^2))^3}{x}\,dx$

Optimal. Leaf size=207

$$\frac{3}{4}b^2\text{Li}_3\left(1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{3}{4}b^2\text{Li}_3\left(\frac{2}{1-cx^2}-1\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{3}{4}b\text{Li}_2\left(1-\frac{2}{1-cx^2}\right)\left(a+b\right.$$

```
[Out] -(a+b*arctanh(c*x^2))^3*arctanh(-1+2/(-c*x^2+1))-3/4*b*(a+b*arctanh(c*x^2))
^2*polylog(2,1-2/(-c*x^2+1))+3/4*b*(a+b*arctanh(c*x^2))^2*polylog(2,-1+2/(-
c*x^2+1))+3/4*b^2*(a+b*arctanh(c*x^2))*polylog(3,1-2/(-c*x^2+1))-3/4*b^2*(a
+b*arctanh(c*x^2))*polylog(3,-1+2/(-c*x^2+1))-3/8*b^3*polylog(4,1-2/(-c*x^2
+1))+3/8*b^3*polylog(4,-1+2/(-c*x^2+1))
```

Rubi [A] time = 0.56, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6095, 5914, 6052, 5948, 6058, 6062, 6610}

$$\frac{3}{4}b^2\text{PolyLog}\left(3,1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{3}{4}b^2\text{PolyLog}\left(3,\frac{2}{1-cx^2}-1\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{3}{4}b\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^3*ArcTanh[1 - 2/(1 - c*x^2)] - (3*b*(a + b*ArcTanh[c
*x^2])^2*PolyLog[2, 1 - 2/(1 - c*x^2)])/4 + (3*b*(a + b*ArcTanh[c*x^2])^2*P
olyLog[2, -1 + 2/(1 - c*x^2)])/4 + (3*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[3,
 1 - 2/(1 - c*x^2)])/4 - (3*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[3, -1 + 2/(1
 - c*x^2)])/4 - (3*b^3*PolyLog[4, 1 - 2/(1 - c*x^2)])/8 + (3*b^3*PolyLog[4,
 -1 + 2/(1 - c*x^2)])/8
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx, x, x^2\right)$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right) - (3bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right.}{1-}\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right) + \frac{1}{2}(3bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(c\right.}{1-c}\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right) - \frac{3}{4}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(1-\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right) - \frac{3}{4}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(1-\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right) - \frac{3}{4}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(1-\right.$$

Mathematica [A] time = 0.21, size = 211, normalized size = 1.02

$$\frac{3}{8}b\left(2\text{Li}_2\left(\frac{cx^2+1}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 - 2\text{Li}_2\left(\frac{cx^2+1}{cx^2-1}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 + b\left(-2\text{Li}_3\left(\frac{cx^2+1}{1-cx^2}\right)\left(a+b\,\text{t}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^3*ArcTanh[1 + 2/(-1 + c*x^2)] + (3*b*(2*(a + b*ArcTa
nh[c*x^2])^2*PolyLog[2, (1 + c*x^2)/(1 - c*x^2)] - 2*(a + b*ArcTanh[c*x^2])
^2*PolyLog[2, (1 + c*x^2)/(-1 + c*x^2)] + b*(-2*(a + b*ArcTanh[c*x^2])*Poly
Log[3, (1 + c*x^2)/(1 - c*x^2)] + 2*(a + b*ArcTanh[c*x^2])*PolyLog[3, (1 +
c*x^2)/(-1 + c*x^2)] + b*(PolyLog[4, (1 + c*x^2)/(1 - c*x^2)] - PolyLog[4,
(1 + c*x^2)/(-1 + c*x^2)]))))/8
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}\left(cx^2\right)^3 + 3\,ab^2 \operatorname{artanh}\left(cx^2\right)^2 + 3\,a^2b \operatorname{artanh}\left(cx^2\right) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh
(c*x^2) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(cx^2\right) + a\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3/x, x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{arctanh}\left(c\,x^2\right)\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^3/x,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \log(x) + \int \frac{b^3\left(\log\left(cx^2+1\right) - \log\left(-cx^2+1\right)\right)^3}{8\,x} + \frac{3\,ab^2\left(\log\left(cx^2+1\right) - \log\left(-cx^2+1\right)\right)^2}{4\,x} + \frac{3\,a^2b\left(\log\left(cx^2+1\right) - \right.}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c*x^2 + 1) - log(-c*x^2 + 1))^3/x + 3/4
*a*b^2*(log(c*x^2 + 1) - log(-c*x^2 + 1))^2/x + 3/2*a^2*b*(log(c*x^2 + 1) -
 log(-c*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(c\,x^2\right)\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^3/x,x)
```

```
[Out] int((a + b*atanh(c*x^2))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(cx^2\right)\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**3/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**3/x, x)
```

3.80 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=125

$$-\frac{3}{2}b^2c\text{Li}_2\left(\frac{2}{cx^2+1}-1\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{1}{2}c\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{2x^2}+\frac{3}{2}bc\log\left(2-\frac{2}{cx^2}\right.$$

```
[Out] 1/2*c*(a+b*arctanh(c*x^2))^3-1/2*(a+b*arctanh(c*x^2))^3/x^2+3/2*b*c*(a+b*ar
ctanh(c*x^2))^2*ln(2-2/(c*x^2+1))-3/2*b^2*c*(a+b*arctanh(c*x^2))*polylog(2,
-1+2/(c*x^2+1))-3/4*b^3*c*polylog(3,-1+2/(c*x^2+1))
```

Rubi [F] time = 0.78, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^2])^3/x^3,x]
```

```
[Out] (3*b*c*Log[c*x^2]*(2*a - b*Log[1 - c*x^2])^2)/16 - ((1 - c*x^2)*(2*a - b*Lo
g[1 - c*x^2])^3)/(16*x^2) + (3*b^3*c*Log[-(c*x^2)]*Log[1 + c*x^2]^2)/16 - (
b^3*(1 + c*x^2)*Log[1 + c*x^2]^3)/(16*x^2) - (3*b^2*c*(2*a - b*Log[1 - c*x^
2])*PolyLog[2, 1 - c*x^2])/8 + (3*b^3*c*Log[1 + c*x^2]*PolyLog[2, 1 + c*x^2
])/8 - (3*b^3*c*PolyLog[3, 1 - c*x^2])/8 - (3*b^3*c*PolyLog[3, 1 + c*x^2])/
8 + (3*b*Defer[Subst][Defer[Int][((-2*a + b*Log[1 - c*x])^2*Log[1 + c*x])/x
^2, x], x, x^2])/16 - (3*b^2*Defer[Subst][Defer[Int][((-2*a + b*Log[1 - c*x
])*Log[1 + c*x]^2)/x^2, x], x, x^2])/16
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^3}\,dx=\int\left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{8x^3}+\frac{3b\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)}{8x^3}-\frac{3b^2\left(-2a\right.}{}\right.$$

$$=\frac{1}{8}\int\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{x^3}\,dx+\frac{1}{8}(3b)\int\frac{\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)}{x^3}\,d$$

$$=\frac{1}{16}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^3}{x^2}\,dx,x,x^2\right)+\frac{1}{16}(3b)\,\text{Subst}\left(\int\frac{(-2a+b\log(1-}{x}\right.$$

$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2}-\frac{b^3\left(1+cx^2\right)\log^3\left(1+cx^2\right)}{16x^2}+\frac{1}{16}(3b)\,\text{Subst}\left(\int\right.$$

$$=\frac{3}{16}bc\log\left(cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2}+\frac{3}{16}b^3c$$

$$=\frac{3}{16}bc\log\left(cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2}+\frac{3}{16}b^3c$$

$$=\frac{3}{16}bc\log\left(cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2}+\frac{3}{16}b^3c$$

$$=\frac{3}{16}bc\log\left(cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2}+\frac{3}{16}b^3c$$

Mathematica [C] time = 0.43, size = 222, normalized size = 1.78

$$\frac{1}{4}\left(-\frac{2a^3}{x^2}-3a^2bc\log\left(1-c^2x^4\right)-\frac{6a^2b\tanh^{-1}\left(cx^2\right)}{x^2}+12a^2bc\log(x)+6ab^2c\left(\tanh^{-1}\left(cx^2\right)\left(\left(1-\frac{1}{cx^2}\right)\tanh^{-1}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^3/x^3,x]
```

```
[Out] ((-2*a^3)/x^2 - (6*a^2*b*ArcTanh[c*x^2])/x^2 + 12*a^2*b*c*Log[x] - 3*a^2*b*
c*Log[1 - c^2*x^4] + 6*a*b^2*c*(ArcTanh[c*x^2]*((1 - 1/(c*x^2))*ArcTanh[c*x
^2] + 2*Log[1 - E^(-2*ArcTanh[c*x^2])]) - PolyLog[2, E^(-2*ArcTanh[c*x^2])]
) + 2*b^3*c*((I/8)*Pi^3 - ArcTanh[c*x^2]^3 - ArcTanh[c*x^2]^3/(c*x^2) + 3*A
rcTanh[c*x^2]^2*Log[1 - E^(2*ArcTanh[c*x^2])] + 3*ArcTanh[c*x^2]*PolyLog[2,
 E^(2*ArcTanh[c*x^2])] - (3*PolyLog[3, E^(2*ArcTanh[c*x^2])])/2))/4
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\operatorname{artanh}\left(cx^2\right)^3+3\,ab^2\operatorname{artanh}\left(cx^2\right)^2+3\,a^2b\operatorname{artanh}\left(cx^2\right)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh
(c*x^2) + a^3)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3/x^3, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\operatorname{arctanh}\left(c\,x^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^3/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^3/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{4}\left(c\left(\log\left(c^2x^4-1\right)-\log\left(x^4\right)\right)+\frac{2\operatorname{artanh}\left(cx^2\right)}{x^2}\right)a^2b-\frac{a^3}{2\,x^2}-\frac{\left(b^3cx^2-b^3\right)\log\left(-cx^2+1\right)^3+3\left(2\,ab^2+\left(b^3cx^2\right.\right.}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/4*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*a^2*b - 1/2*a
^3/x^2 - 1/16*((b^3*c*x^2 - b^3)*log(-c*x^2 + 1)^3 + 3*(2*a*b^2 + (b^3*c*x^
2 + b^3)*log(c*x^2 + 1))*log(-c*x^2 + 1)^2)/x^2 - integrate(-1/8*((b^3*c*x^
2 - b^3)*log(c*x^2 + 1)^3 + 6*(a*b^2*c*x^2 - a*b^2)*log(c*x^2 + 1)^2 + 3*(4
*a*b^2*c*x^2 - (b^3*c*x^2 - b^3)*log(c*x^2 + 1)^2 + 2*(b^3*c^2*x^4 + 2*a*b^
2 - (2*a*b^2*c - b^3*c)*x^2)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^5 - x^3)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^3/x^3,x)
```

```
[Out] int((a + b*atanh(c*x^2))^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(cx^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**3/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**3/x**3, x)
```

3.81 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx$

Optimal. Leaf size=139

$$\frac{3}{2}b^2c^2\log\left(2-\frac{2}{cx^2+1}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{3}{4}bc^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{1}{4}c^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3-\frac{3bc\left(a+\right.}{}$$

```
[Out] 3/4*b*c^2*(a+b*arctanh(c*x^2))^2-3/4*b*c*(a+b*arctanh(c*x^2))^2/x^2+1/4*c^2
*(a+b*arctanh(c*x^2))^3-1/4*(a+b*arctanh(c*x^2))^3/x^4+3/2*b^2*c^2*(a+b*arc
tanh(c*x^2))*ln(2-2/(c*x^2+1))-3/4*b^3*c^2*polylog(2,-1+2/(c*x^2+1))
```

Rubi [F] time = 1.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^2])^3/x^5,x]
```

```
[Out] (3*a*b^2*c^2*Log[x])/4 - (3*b*c*(1 - c*x^2)*(2*a - b*Log[1 - c*x^2])^2)/(32
*x^2) + (3*b*c^2*Log[c*x^2]*(2*a - b*Log[1 - c*x^2])^2)/32 + (c^2*(2*a - b*
Log[1 - c*x^2])^3)/32 - (2*a - b*Log[1 - c*x^2])^3/(32*x^4) - (3*b^3*c*(1 +
 c*x^2)*Log[1 + c*x^2]^2)/(32*x^2) - (3*b^3*c^2*Log[-(c*x^2)]*Log[1 + c*x^2
]^2)/32 + (b^3*c^2*Log[1 + c*x^2]^3)/32 - (b^3*Log[1 + c*x^2]^3)/(32*x^4) -
 (3*b^3*c^2*PolyLog[2, -(c*x^2)])/16 + (3*b^3*c^2*PolyLog[2, c*x^2])/16 - (
3*b^2*c^2*(2*a - b*Log[1 - c*x^2])*PolyLog[2, 1 - c*x^2])/16 - (3*b^3*c^2*L
og[1 + c*x^2]*PolyLog[2, 1 + c*x^2])/16 - (3*b^3*c^2*PolyLog[3, 1 - c*x^2])
/16 + (3*b^3*c^2*PolyLog[3, 1 + c*x^2])/16 + (3*b*Defer[Subst][Defer[Int][(
(-2*a + b*Log[1 - c*x])^2*Log[1 + c*x])/x^3, x], x, x^2])/16 - (3*b^2*Defer
[Subst][Defer[Int][((-2*a + b*Log[1 - c*x])*Log[1 + c*x]^2)/x^3, x], x, x^2
])/16
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{8x^5}+\frac{3b\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)}{8x^5}-\frac{3b^2\left(-2a\right.}{}\right.$$

$$=\frac{1}{8}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{x^5}\,dx+\frac{1}{8}(3b)\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)}{x^5}\,d$$

$$=\frac{1}{16}\,\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^3}{x^3}\,dx,x,x^2\right)+\frac{1}{16}(3b)\,\text{Subst}\left(\int \frac{(-2a+b\log(1-}{x}\right.$$

$$=-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{32x^4}-\frac{b^3\log^3\left(1+cx^2\right)}{32x^4}+\frac{1}{16}(3b)\,\text{Subst}\left(\int \frac{(-2a+b\log(1-}{x}\right.$$

$$=-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{32x^4}-\frac{b^3\log^3\left(1+cx^2\right)}{32x^4}-\frac{1}{32}(3b)\,\text{Subst}\left(\int \frac{(2a-b\log(x))^2}{x\left(\frac{1}{c}-\frac{x}{c}\right)^2}\,d\right.$$

$$=-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{32x^4}-\frac{b^3\log^3\left(1+cx^2\right)}{32x^4}-\frac{1}{32}(3b)\,\text{Subst}\left(\int \frac{(2a-b\log(x))^2}{\left(\frac{1}{c}-\frac{x}{c}\right)^2}\,d\right.$$

$$=-\frac{3bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32x^2}-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{32x^4}-\frac{3b^3c\left(1+cx^2\right)\mathrm{lo}}{32x^2}$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{3bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32x^2}+\frac{3}{32}bc^2\log\left(cx^2\right)\left(2a-b\log\right.$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{3bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32x^2}+\frac{3}{32}bc^2\log\left(cx^2\right)\left(2a-b\log\right.$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{3bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32x^2}+\frac{3}{32}bc^2\log\left(cx^2\right)\left(2a-b\log\right.$$

Mathematica [A] time = 0.29, size = 218, normalized size = 1.57

$$\frac{a\left(-2a^2-3abc^2x^4\log\left(1-cx^2\right)+3abc^2x^4\log\left(cx^2+1\right)-6abcx^2+12b^2c^2x^4\log\left(\frac{cx^2}{\sqrt{1-c^2x^4}}\right)\right)-6b\tanh^{-1}\left(cx^2\right)\left(a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^3/x^5,x]
```

```
[Out] (6*b^2*(-1 + c*x^2)*(a + a*c*x^2 + b*c*x^2)*ArcTanh[c*x^2]^2 + 2*b^3*(-1 +
c^2*x^4)*ArcTanh[c*x^2]^3 - 6*b*ArcTanh[c*x^2]*(a^2 + 2*a*b*c*x^2 - 2*b^2*c
^2*x^4*Log[1 - E^(-2*ArcTanh[c*x^2])]) + a*(-2*a^2 - 6*a*b*c*x^2 - 3*a*b*c^
2*x^4*Log[1 - c*x^2] + 3*a*b*c^2*x^4*Log[1 + c*x^2] + 12*b^2*c^2*x^4*Log[(c
*x^2)/Sqrt[1 - c^2*x^4]]) - 6*b^3*c^2*x^4*PolyLog[2, E^(-2*ArcTanh[c*x^2])]
)/(8*x^4)
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{artanh}\left(cx^2\right)^3+3\,ab^2\,\text{artanh}\left(cx^2\right)^2+3\,a^2b\,\text{artanh}\left(cx^2\right)+a^3}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh
(c*x^2) + a^3)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(cx^2\right) + a\right)^3}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3/x^5, x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{arctanh}\left(c\,x^2\right)\right)^3}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^3/x^5,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^3/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{8}\left(\left(c\log\left(cx^2+1\right)-c\log\left(cx^2-1\right)-\frac{2}{x^2}\right)c-\frac{2\ \operatorname{artanh}\left(cx^2\right)}{x^4}\right)a^2b+\frac{3}{16}\left(\left(2\left(\log\left(cx^2-1\right)-2\right)\log\left(cx^2+1\right)-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="maxima")
```

```
[Out] 3/8*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^4
)*a^2*b + 3/16*((2*(log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 -
 log(c*x^2 - 1)^2 - 4*log(c*x^2 - 1) + 16*log(x))*c^2 + 4*(c*log(c*x^2 + 1)
 - c*log(c*x^2 - 1) - 2/x^2)*c*arctanh(c*x^2))*a*b^2 - 1/32*b^3*(((c^2*x^4
- 1)*log(-c*x^2 + 1)^3 + 3*(2*c*x^2 - (c^2*x^4 - 1)*log(c*x^2 + 1))*log(-c*
x^2 + 1)^2)/x^4 + 4*integrate(-((c*x^2 - 1)*log(c*x^2 + 1)^3 + 3*(2*c^2*x^4
 - (c*x^2 - 1)*log(c*x^2 + 1)^2 - (c^3*x^6 - c*x^2)*log(c*x^2 + 1))*log(-c*
x^2 + 1))/(c*x^7 - x^5), x)) - 3/4*a*b^2*arctanh(c*x^2)^2/x^4 - 1/4*a^3/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\operatorname{atanh}\left(c\,x^2\right)\right)^3}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^3/x^5,x)
```

```
[Out] int((a + b*atanh(c*x^2))^3/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\operatorname{atanh}\left(cx^2\right)\right)^3}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**3/x**5,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**3/x**5, x)
```

3.82 $\int (dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=317

$$\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{bd^{5/2}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}c^{7/4}}+\frac{bd^{5/2}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{}\right.}{7\sqrt{2}c^{7/4}}$$

```
[Out] 8/21*b*d*(d*x)^(3/2)/c+2/7*b*d^(5/2)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/c^
(7/4)+2/7*(d*x)^(7/2)*(a+b*arctanh(c*x^2))/d-2/7*b*d^(5/2)*arctanh(c^(1/4)*
(d*x)^(1/2)/d^(1/2))/c^(7/4)-1/14*b*d^(5/2)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^
(1/4)*2^(1/2)*(d*x)^(1/2))/c^(7/4)*2^(1/2)+1/14*b*d^(5/2)*ln(d^(1/2)+x*c^(1
/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)^(1/2))/c^(7/4)*2^(1/2)-1/7*b*d^(5/2)*arct
an(-1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(7/4)-1/7*b*d^(5/2)*ar
ctan(1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(7/4)
```

Rubi [A] time = 0.33, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6097, 16, 321, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{bd^{5/2}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}c^{7/4}}+\frac{bd^{5/2}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{}\right.}{7\sqrt{2}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (8*b*d*(d*x)^(3/2))/(21*c) + (2*b*d^(5/2)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d
]])/(7*c^(7/4)) + (Sqrt[2]*b*d^(5/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])
/Sqrt[d]])/(7*c^(7/4)) - (Sqrt[2]*b*d^(5/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[d*x])/Sqrt[d]])/(7*c^(7/4)) + (2*(d*x)^(7/2)*(a + b*ArcTanh[c*x^2]))/(7*d
) - (2*b*d^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/4)) - (b*d^(
5/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sqrt[
2]*c^(7/4)) + (b*d^(5/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*
Sqrt[d*x]])/(7*Sqrt[2]*c^(7/4))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx = \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{(4bc)\int\frac{x(dx)^{7/2}}{1-c^2x^4}dx}{7d}$$

$$= \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{(4bc)\int\frac{(dx)^{9/2}}{1-c^2x^4}dx}{7d^2}$$

$$= \frac{8bd(dx)^{3/2}}{21c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{\left(4bd^2\right)\int\frac{\sqrt{dx}}{1-c^2x^4}dx}{7c}$$

$$= \frac{8bd(dx)^{3/2}}{21c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{(8bd)\,\mathrm{Subst}\left(\int\frac{x^2}{1-\frac{c^2x^8}{d^4}}dx,x,\right.}{7c}$$

$$= \frac{8bd(dx)^{3/2}}{21c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{\left(4bd^3\right)\mathrm{Subst}\left(\int\frac{x^2}{d^2-cx^4}dx,x,\right.}{7c}$$

$$= \frac{8bd(dx)^{3/2}}{21c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{\left(2bd^3\right)\mathrm{Subst}\left(\int\frac{1}{d-\sqrt{c}x^2}dx,x\right.}{7c^{3/2}}$$

$$= \frac{8bd(dx)^{3/2}}{21c}+\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{2bd}{}$$

$$= \frac{8bd(dx)^{3/2}}{21c}+\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{2bd}{}$$

$$= \frac{8bd(dx)^{3/2}}{21c}+\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}+\frac{\sqrt{2}\,bd^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}-\frac{\sqrt{}}{}$$

Mathematica [A] time = 0.13, size = 241, normalized size = 0.76

$$\frac{(dx)^{5/2}\left(12ac^{7/4}x^{7/2}+16bc^{3/4}x^{3/2}+12bc^{7/4}x^{7/2}\tanh^{-1}\left(cx^2\right)+6b\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-6b\log\left(\sqrt[4]{c}\sqrt{x}+1\right)-3\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] ((d*x)^(5/2)*(16*b*c^(3/4)*x^(3/2) + 12*a*c^(7/4)*x^(7/2) + 6*Sqrt[2]*b*Arc
Tan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]] - 6*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*S
qrt[x]] + 12*b*ArcTan[c^(1/4)*Sqrt[x]] + 12*b*c^(7/4)*x^(7/2)*ArcTanh[c*x^2
] + 6*b*Log[1 - c^(1/4)*Sqrt[x]] - 6*b*Log[1 + c^(1/4)*Sqrt[x]] - 3*Sqrt[2]
*b*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 3*Sqrt[2]*b*Log[1 + Sqrt[
2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/(42*c^(7/4)*x^(5/2))
```

fricas [A] time = 1.31, size = 57, normalized size = 0.18

$$\frac{\left(3\,bcd^2x^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)+6\,acd^2x^3+8\,bd^2x\right)\sqrt{dx}}{21\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/21*(3*b*c*d^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a*c*d^2*x^3 + 8*b*d^2
*x)*sqrt(d*x)/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{5}{2}}\left(b\operatorname{artanh}\left(cx^2\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*(b*arctanh(c*x^2) + a), x)
```

maple [A] time = 0.06, size = 302, normalized size = 0.95

$$\frac{2\,(dx)^{\frac{7}{2}}\,a}{7d}+\frac{2b\,(dx)^{\frac{7}{2}}\operatorname{arctanh}\left(c\,x^2\right)}{7d}+\frac{8bd\,(dx)^{\frac{3}{2}}}{21c}+\frac{2d^3b\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{7c^2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}-\frac{d^3b\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{7c^2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}-\frac{d^3b\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{d}}{dx+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{d}}\right.}{14c^2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 2/7/d*(d*x)^(7/2)*a+2/7/d*b*(d*x)^(7/2)*arctanh(c*x^2)+8/21*b*d*(d*x)^(3/2)
/c+2/7*d^3*b/c^2/(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))-1/7*d^3*b/
c^2/(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)
))-1/14*d^3*b/c^2/(d^2/c)^(1/4)*2^(1/2)*ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(d^2/c)^(1/2))/(d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))
)-1/7*d^3*b/c^2/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1
/2)+1)-1/7*d^3*b/c^2/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*
x)^(1/2)-1)
```

maxima [A] time = 0.42, size = 316, normalized size = 1.00

$$\frac{12\,(dx)^{\frac{7}{2}}a + \left(12\,(dx)^{\frac{7}{2}}\operatorname{artanh}\left(cx^2\right) - \frac{3\,d^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}\,\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}\,d\cdots\right.}{\cdots}\cdots\right)}{c^2}\cdots\right.}{42\,d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/42*(12*(d*x)^(7/2)*a + (12*(d*x)^(7/2)*arctanh(c*x^2) - (3*d^6*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*x)*sqrt(c))/sqrt(sq
rt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2
)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*
sqrt(c)) - sqrt(2)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)
/(c^(3/4)*sqrt(d)) + sqrt(2)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^(1/4)*sq
rt(d) + d)/(c^(3/4)*sqrt(d)))/c^2 - 6*d^6*(2*arctan(sqrt(d*x)*sqrt(c)/sqrt(
sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + log((sqrt(d*x)*sqrt(c) - sqrt(sqrt(
c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/(sqrt(sqrt(c)*d)*sqrt(c)))/c^
2 - 16*(d*x)^(3/2)*d^4/c^2)*c/d^2)*b)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^{5/2}\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a + b*atanh(c*x^2)),x)
```

```
[Out] int((d*x)^(5/2)*(a + b*atanh(c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(a+b*atanh(c*x**2)),x)
```

```
[Out] Timed out
```

3.83 $\int (dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=317

$$\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}+\frac{bd^{3/2}\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}\,c^{5/4}}-\frac{bd^{3/2}\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}\,c^{5/4}}$$

```
[Out] -2/5*b*d^(3/2)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/c^(5/4)+2/5*(d*x)^(5/2)*
(a+b*arctanh(c*x^2))/d-2/5*b*d^(3/2)*arctanh(c^(1/4)*(d*x)^(1/2)/d^(1/2))/c
^(5/4)+1/10*b*d^(3/2)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(1
/2))/c^(5/4)*2^(1/2)-1/10*b*d^(3/2)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)+c^(1/4)*2^
(1/2)*(d*x)^(1/2))/c^(5/4)*2^(1/2)-1/5*b*d^(3/2)*arctan(-1+c^(1/4)*2^(1/2)*
(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(5/4)-1/5*b*d^(3/2)*arctan(1+c^(1/4)*2^(1/2)
*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(5/4)+8/5*b*d*(d*x)^(1/2)/c
```

Rubi [A] time = 0.30, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6097, 16, 321, 329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}+\frac{bd^{3/2}\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}\,c^{5/4}}-\frac{bd^{3/2}\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}\,c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (8*b*d*Sqrt[d*x])/(5*c) - (2*b*d^(3/2)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])
/(5*c^(5/4)) + (Sqrt[2]*b*d^(3/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sq
rt[d]])/(5*c^(5/4)) - (Sqrt[2]*b*d^(3/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d
*x])/Sqrt[d]])/(5*c^(5/4)) + (2*(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]))/(5*d) -
 (2*b*d^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/4)) + (b*d^(3/2
)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(5*Sqrt[2]*
c^(5/4)) - (b*d^(3/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqr
t[d*x]])/(5*Sqrt[2]*c^(5/4))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx = \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}-\frac{(4bc)\int\frac{x(dx)^{5/2}}{1-c^2x^4}\,dx}{5d}$$

$$=\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}-\frac{(4bc)\int\frac{(dx)^{7/2}}{1-c^2x^4}\,dx}{5d^2}$$

$$=\frac{8bd\sqrt{dx}}{5c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}-\frac{\left(4bd^2\right)\int\frac{1}{\sqrt{dx}\left(1-c^2x^4\right)}\,dx}{5c}$$

$$=\frac{8bd\sqrt{dx}}{5c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}-\frac{(8bd)\,\mathrm{Subst}\left(\int\frac{1}{1-\frac{c^2x^8}{d^4}}\,dx,x,\sqrt{d\right.}}{5c}$$

$$=\frac{8bd\sqrt{dx}}{5c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}-\frac{\left(4bd^3\right)\mathrm{Subst}\left(\int\frac{1}{d^2-cx^4}\,dx,x,\right.}{5c}$$

$$=\frac{8bd\sqrt{dx}}{5c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}-\frac{\left(2bd^2\right)\mathrm{Subst}\left(\int\frac{1}{d-\sqrt{c}\,x^2}\,dx,x,\right.}{5c}$$

$$=\frac{8bd\sqrt{dx}}{5c}-\frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}-\frac{2bd^{3/2}}{}$$

$$=\frac{8bd\sqrt{dx}}{5c}-\frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}-\frac{2bd^{3/2}}{}$$

$$=\frac{8bd\sqrt{dx}}{5c}-\frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}}+\frac{\sqrt{2}\,bd^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}}-\frac{\sqrt{2}}{}$$

Mathematica [A] time = 0.10, size = 240, normalized size = 0.76

$$\frac{(dx)^{3/2}\left(4ac^{5/4}x^{5/2}+4bc^{5/4}x^{5/2}\tanh^{-1}\left(cx^2\right)+16b\sqrt[4]{c}\sqrt{x}+2b\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2b\log\left(\sqrt[4]{c}\sqrt{x}+1\right)+\sqrt{2}\,b\,\mathrm{l}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] ((d*x)^(3/2)*(16*b*c^(1/4)*Sqrt[x] + 4*a*c^(5/4)*x^(5/2) + 2*Sqrt[2]*b*ArcT
an[1 - Sqrt[2]*c^(1/4)*Sqrt[x]] - 2*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sq
rt[x]] - 4*b*ArcTan[c^(1/4)*Sqrt[x]] + 4*b*c^(5/4)*x^(5/2)*ArcTanh[c*x^2] +
 2*b*Log[1 - c^(1/4)*Sqrt[x]] - 2*b*Log[1 + c^(1/4)*Sqrt[x]] + Sqrt[2]*b*Lo
g[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Sqrt[2]*b*Log[1 + Sqrt[2]*c^(1
/4)*Sqrt[x] + Sqrt[c]*x]))/(10*c^(5/4)*x^(3/2))
```

fricas [A] time = 2.03, size = 49, normalized size = 0.15

$$\frac{\left(bcdx^{2}\log\left(-\frac{cx^{2}+1}{cx^{2}-1}\right)+2\,acdx^{2}+8\,bd\right)\sqrt{dx}}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/5*(b*c*d*x^2*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*d*x^2 + 8*b*d)*sqrt(d*
x)/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(dx)^{\frac{3}{2}}\left(b\operatorname{artanh}\left(cx^{2}\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b*arctanh(c*x^2) + a), x)
```

maple [A] time = 0.04, size = 292, normalized size = 0.92

$$\frac{2\,(dx)^{\frac{5}{2}}\,a}{5d}+\frac{2b\,(dx)^{\frac{5}{2}}\operatorname{arctanh}\left(c\,x^{2}\right)}{5d}+\frac{8bd\sqrt{dx}}{5c}-\frac{db\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{5c}-\frac{2db\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{5c}-\frac{db\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 2/5/d*(d*x)^(5/2)*a+2/5/d*b*(d*x)^(5/2)*arctanh(c*x^2)+8/5*b*d*(d*x)^(1/2)/
c-1/5*d*b/c*(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/
c)^(1/4)))-2/5*d*b/c*(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))-1/10*d
*b/c*(d^2/c)^(1/4)*2^(1/2)*ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c
)^(1/2))/(d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))-1/5*d*b/c*(
d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)-1/5*d*b/c*
(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 0.43, size = 310, normalized size = 0.98

$$\frac{4\,(dx)^{\frac{5}{2}}\,a+\left(4\,(dx)^{\frac{5}{2}}\operatorname{artanh}\left(cx^{2}\right)+\frac{\left(\frac{16\,\sqrt{dx}\,d^{4}}{c^{2}}-\frac{\frac{2\,\sqrt{2}\,d^{5}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\,\sqrt{d}+2\,\sqrt{dx}\,\sqrt{c}\right)}{2\,\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}+\frac{2\,\sqrt{2}\,d^{5}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\,\sqrt{d}-2\,\sqrt{dx}\,\sqrt{c}\right)}{2\,\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}+\sqrt{2}\,d^{\frac{9}{2}}\cdots}{c^{2}}\right)}{\cdots}\right)\cdots}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/10*(4*(d*x)^(5/2)*a + (4*(d*x)^(5/2)*arctanh(c*x^2) + (16*sqrt(d*x)*d^4/c
^2 - (2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*
x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) + 2*sqrt(2)*d^5*arctan(-1/2*sq
rt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt
(sqrt(c)*d) + sqrt(2)*d^(9/2)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*s
qrt(d) + d)/c^(1/4) - sqrt(2)*d^(9/2)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c
^(1/4)*sqrt(d) + d)/c^(1/4))/c^2 - 2*(2*d^5*arctan(sqrt(d*x)*sqrt(c)/sqrt(s
qrt(c)*d))/sqrt(sqrt(c)*d) - d^5*log((sqrt(d*x)*sqrt(c) - sqrt(sqrt(c)*d))/
(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/sqrt(sqrt(c)*d))/c^2)*c/d^2)*b)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^{3/2}\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a + b*atanh(c*x^2)),x)
```

```
[Out] int((d*x)^(3/2)*(a + b*atanh(c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \left(a + b \operatorname{atanh}\left(cx^{2}\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(a+b*atanh(c*x**2)),x)
```

```
[Out] Integral((d*x)**(3/2)*(a + b*atanh(c*x**2)), x)
```

3.84 $\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx$

Optimal. Leaf size=301

$$\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d}+\frac{b\sqrt{d}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}c^{3/4}}-\frac{b\sqrt{d}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}c^{3/4}}$$

```
[Out] 2/3*(d*x)^(3/2)*(a+b*arctanh(c*x^2))/d+2/3*b*arctan(c^(1/4)*(d*x)^(1/2)/d^(
1/2))*d^(1/2)/c^(3/4)-2/3*b*arctanh(c^(1/4)*(d*x)^(1/2)/d^(1/2))*d^(1/2)/c^
(3/4)+1/6*b*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/
2)/c^(3/4)*2^(1/2)-1/6*b*ln(d^(1/2)+x*c^(1/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)
^(1/2))*d^(1/2)/c^(3/4)*2^(1/2)+1/3*b*arctan(-1+c^(1/4)*2^(1/2)*(d*x)^(1/2)
/d^(1/2))*2^(1/2)*d^(1/2)/c^(3/4)+1/3*b*arctan(1+c^(1/4)*2^(1/2)*(d*x)^(1/2
)/d^(1/2))*2^(1/2)*d^(1/2)/c^(3/4)
```

Rubi [A] time = 0.25, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6097, 16, 329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d}+\frac{b\sqrt{d}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}c^{3/4}}-\frac{b\sqrt{d}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*Sqrt[d]*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4)) - (Sqrt[2]*b*
Sqrt[d]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4)) + (Sqr
t[2]*b*Sqrt[d]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4))
 + (2*(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]))/(3*d) - (2*b*Sqrt[d]*ArcTanh[(c^(
1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4)) + (b*Sqrt[d]*Log[Sqrt[d] + Sqrt[c]*Sq
rt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*c^(3/4)) - (b*Sqrt[d]*Log[
Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*c^(3/4
))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
```

```
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx = \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{(4bc)\int \frac{x(dx)^{3/2}}{1-c^2x^4}\,dx}{3d}$$
$$= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{(4bc)\int \frac{(dx)^{5/2}}{1-c^2x^4}\,dx}{3d^2}$$
$$= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{(8bc)\,\text{Subst}\left(\int \frac{x^6}{1-\frac{c^2x^8}{d^4}}\,dx, x, \sqrt{dx}\right)}{3d^3}$$
$$= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{1}{3}(4bd)\,\text{Subst}\left(\int \frac{x^2}{d^2-cx^4}\,dx, x, \sqrt{dx}\right) + \frac{1}{3}(4b$$
$$= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{(2bd)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{c}x^2}\,dx, x, \sqrt{dx}\right)}{3\sqrt{c}} + \frac{(2bd)\,\text{Su}}{}$$
$$= \frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{2b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right.}{3c^{3/4}}$$
$$= \frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{2b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right.}{3c^{3/4}}$$
$$= \frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} - \frac{\sqrt{2}b\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{\sqrt{2}b\sqrt{d}\tan^{-1}\left(1+\right.}{3c^{3/4}}$$

Mathematica [A] time = 0.08, size = 227, normalized size = 0.75

$$\frac{\sqrt{dx}\left(4ac^{3/4}x^{3/2}+4bc^{3/4}x^{3/2}\tanh^{-1}\left(cx^2\right)+2b\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2b\log\left(\sqrt[4]{c}\sqrt{x}+1\right)+\sqrt{2}b\log\left(\sqrt{c}x-\sqrt{2}\sqrt[4]{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (Sqrt[d*x]*(4*a*c^(3/4)*x^(3/2) - 2*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*c^(1/4)*Sq
rt[x]] + 2*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] + 4*b*ArcTan[c^(1/
4)*Sqrt[x]] + 4*b*c^(3/4)*x^(3/2)*ArcTanh[c*x^2] + 2*b*Log[1 - c^(1/4)*Sqrt
[x]] - 2*b*Log[1 + c^(1/4)*Sqrt[x]] + Sqrt[2]*b*Log[1 - Sqrt[2]*c^(1/4)*Sqr
t[x] + Sqrt[c]*x] - Sqrt[2]*b*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
)/(6*c^(3/4)*Sqrt[x])
```

fricas [A] time = 1.29, size = 34, normalized size = 0.11

$$\frac{1}{3}\left(bx\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,ax\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*x)*sqrt(d*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\left(b\operatorname{artanh}\left(cx^2\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b*arctanh(c*x^2) + a), x)
```

maple [A] time = 0.04, size = 280, normalized size = 0.93

$$\frac{2\,(dx)^{\frac{3}{2}}\,a}{3d}+\frac{2b\,(dx)^{\frac{3}{2}}\operatorname{arctanh}\left(c\,x^2\right)}{3d}+\frac{2db\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{3c\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}-\frac{db\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{3c\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}+\frac{db\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}{dx+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}\right)}{6c\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 2/3/d*(d*x)^(3/2)*a+2/3/d*b*(d*x)^(3/2)*arctanh(c*x^2)+2/3*d*b/c/(d^2/c)^(1
/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))-1/3*d*b/c/(d^2/c)^(1/4)*ln(((d*x)^(1/
2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)))+1/6*d*b/c/(d^2/c)^(1/4)*2^(1
/2)*ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x+(d^2/c)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+1/3*d*b/c/(d^2/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+1/3*d*b/c/(d^2/c)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 0.42, size = 301, normalized size = 1.00

$$\frac{4\,(dx)^{\frac{3}{2}}a + \left(4\,(dx)^{\frac{3}{2}}\operatorname{artanh}\left(cx^2\right) + \frac{\frac{d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}dx+\sqrt{2}\sqrt{d}\cdots\right)}{c^{\frac{3}{4}}\sqrt{d}}\cdots\right)}{c}\cdots}{d^2}\cdots\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/6*(4*(d*x)^(3/2)*a + (4*(d*x)^(3/2)*arctanh(c*x^2) + (d^4*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)
*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(
1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(
c)) - sqrt(2)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/(c^(
3/4)*sqrt(d)) + sqrt(2)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d)
 + d)/(c^(3/4)*sqrt(d)))/c + 2*d^4*(2*arctan(sqrt(d*x)*sqrt(c)/sqrt(sqrt(c)
*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + log((sqrt(d*x)*sqrt(c) - sqrt(sqrt(c)*d))/
(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/(sqrt(sqrt(c)*d)*sqrt(c)))/c)*c/d^2)
*b)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d\,x}\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a + b*atanh(c*x^2)),x)
```

```
[Out] int((d*x)^(1/2)*(a + b*atanh(c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(a+b*atanh(c*x**2)),x)
```

```
[Out] Timed out
```

3.85 $\int \frac{a+b\tanh^{-1}(cx^2)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=285

$$\frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d}-\frac{b\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{b\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{2b\tan^{-1}}{\sqrt[4]{}}$$

```
[Out] -2*b*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/c^(1/4)/d^(1/2)-2*b*arctanh(c^(1/4
)*(d*x)^(1/2)/d^(1/2))/c^(1/4)/d^(1/2)-1/2*b*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c
^(1/4)*2^(1/2)*(d*x)^(1/2))/c^(1/4)*2^(1/2)/d^(1/2)+1/2*b*ln(d^(1/2)+x*c^(1
/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)^(1/2))/c^(1/4)*2^(1/2)/d^(1/2)+b*arctan(-
1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(1/4)/d^(1/2)+b*arctan(1+c
^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(1/4)/d^(1/2)+2*(a+b*arctanh(
c*x^2))*(d*x)^(1/2)/d
```

Rubi [A] time = 0.24, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6097, 16, 329, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$\frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d}-\frac{b\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{b\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{2b\tan^{-1}}{\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/Sqrt[d*x],x]
```

```
[Out] (-2*b*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(c^(1/4)*Sqrt[d]) - (Sqrt[2]*b*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(c^(1/4)*Sqrt[d]) + (Sqrt[2
]*b*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(c^(1/4)*Sqrt[d]) + (2
*Sqrt[d*x]*(a + b*ArcTanh[c*x^2]))/d - (2*b*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqr
t[d]])/(c^(1/4)*Sqrt[d]) - (b*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(
1/4)*Sqrt[d*x]])/(Sqrt[2]*c^(1/4)*Sqrt[d]) + (b*Log[Sqrt[d] + Sqrt[c]*Sqrt[
d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(Sqrt[2]*c^(1/4)*Sqrt[d])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
```

```
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - \frac{(4bc)\int \frac{x\sqrt{dx}}{1-c^2x^4}\,dx}{d}$$

$$= \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - \frac{(4bc)\int \frac{(dx)^{3/2}}{1-c^2x^4}\,dx}{d^2}$$

$$= \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - \frac{(8bc)\,\text{Subst}\left(\int \frac{x^4}{1-\frac{c^2x^8}{d^4}}\,dx, x, \sqrt{dx}\right)}{d^3}$$

$$= \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - (4bd)\,\text{Subst}\left(\int \frac{1}{d^2-cx^4}\,dx, x, \sqrt{dx}\right) + (4bd)\,\text{Subst}\left(\int$$

$$= \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - (2b)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{c}\,x^2}\,dx, x, \sqrt{dx}\right) - (2b)\,\text{Subst}\left(\int$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - \frac{2b\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{b\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - \frac{2b\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} - \frac{b\log\left(\sqrt{d}\right.}{}$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} - \frac{\sqrt{2}\,b\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{\sqrt{2}\,b\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{2\sqrt{}}{}$$

Mathematica [A] time = 0.06, size = 227, normalized size = 0.80

$$\frac{\sqrt{x}\left(4a\sqrt[4]{c}\sqrt{x} + 4b\sqrt[4]{c}\sqrt{x}\tanh^{-1}\left(cx^2\right) + 2b\log\left(1-\sqrt[4]{c}\sqrt{x}\right) - 2b\log\left(\sqrt[4]{c}\sqrt{x}+1\right) - \sqrt{2}\,b\log\left(\sqrt{c}\,x - \sqrt{2}\sqrt[4]{c}\sqrt{\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/Sqrt[d*x],x]
```

```
[Out] (Sqrt[x]*(4*a*c^(1/4)*Sqrt[x] - 2*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt
[x]] + 2*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] - 4*b*ArcTan[c^(1/4)
*Sqrt[x]] + 4*b*c^(1/4)*Sqrt[x]*ArcTanh[c*x^2] + 2*b*Log[1 - c^(1/4)*Sqrt[x
]] - 2*b*Log[1 + c^(1/4)*Sqrt[x]] - Sqrt[2]*b*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[
x] + Sqrt[c]*x] + Sqrt[2]*b*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/
(2*c^(1/4)*Sqrt[d*x])
```

fricas [A] time = 1.96, size = 34, normalized size = 0.12

$$\frac{\sqrt{dx}\left(b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(d*x)*(b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/d
```

giac [B] time = 0.16, size = 493, normalized size = 1.73

$$\frac{cd^2\left(\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}+\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}-\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+2\right.}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right.}{c^2d^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((c*d^2*(2*sqrt(2)*(c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^
(1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^2*d^2) + 2*sqrt(2)*(c^3*d^2)^(1/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^
2*d^2) - 2*sqrt(2)*(-c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1
/4) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c^2*d^2) - 2*sqrt(2)*(-c^3*d^2)^(1/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(
c^2*d^2) + sqrt(2)*(c^3*d^2)^(1/4)*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4
) + sqrt(d^2/c))/(c^2*d^2) - sqrt(2)*(c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt
(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^2*d^2) - sqrt(2)*(-c^3*d^2)^(1/4)*log
(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c^2*d^2) + sqrt(2)
*(-c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c)
)/(c^2*d^2)) + 2*sqrt(d*x)*log(-(c*x^2 + 1)/(c*x^2 - 1)))*b + 4*sqrt(d*x)*a
)/d
```

maple [A] time = 0.04, size = 273, normalized size = 0.96

$$\frac{2a\sqrt{dx}}{d}+\frac{2b\sqrt{dx}\,\operatorname{arctanh}\left(c\,x^2\right)}{d}-\frac{b\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d}-\frac{2b\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d}+\frac{b\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{}}{dx-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{}}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(1/2),x)
```

```
[Out] 2/d*a*(d*x)^(1/2)+2/d*b*(d*x)^(1/2)*arctanh(c*x^2)-1/d*b*(d^2/c)^(1/4)*ln((
(d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)))-2/d*b*(d^2/c)^(1/4)
*arctan((d*x)^(1/2)/(d^2/c)^(1/4))+1/2/d*b*(d^2/c)^(1/4)*2^(1/2)*ln((d*x+(d
^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x-(d^2/c)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(d^2/c)^(1/2)))+1/d*b*(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/
c)^(1/4)*(d*x)^(1/2)+1)+1/d*b*(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^
(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 0.42, size = 296, normalized size = 1.04

$$\frac{4\sqrt{dx}\,\operatorname{artanh}\left(cx^2\right) + \frac{c\left(\frac{\frac{2\sqrt{2}d^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}} + \frac{2\sqrt{2}d^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}} + \frac{\sqrt{2}d^{\frac{5}{2}}\log\left(\sqrt{c}dx+\sqrt{2}\sqrt{dx}c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{1}{4}}} - \cdots}{c}\right)}{d^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*((4*sqrt(d*x)*arctanh(c*x^2) + c*((2*sqrt(2)*d^3*arctan(1/2*sqrt(2)*(sq
rt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*
d) + 2*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*
x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) + sqrt(2)*d^(5/2)*log(sqrt(c)*
d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/c^(1/4) - sqrt(2)*d^(5/2)*log(
sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/c^(1/4))/c - 2*(2*d^3*
arctan(sqrt(d*x)*sqrt(c)/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) - d^3*log((sqrt(d
*x)*sqrt(c) - sqrt(sqrt(c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/sqrt(
sqrt(c)*d))/c)/d^2)*b + 4*sqrt(d*x)*a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\operatorname{atanh}\left(c\,x^2\right)}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/(d*x)^(1/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))/(d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\operatorname{atanh}\left(cx^2\right)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(1/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))/sqrt(d*x), x)
```

3.86 $\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=285

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{b\sqrt[4]{c}\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\,d^{3/2}}-\frac{b\sqrt[4]{c}\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\,d^{3/2}}-$$

```
[Out] -2*b*c^(1/4)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+2*b*c^(1/4)*arctan
h(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+1/2*b*c^(1/4)*ln(d^(1/2)+x*c^(1/2)*d
^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(1/2))/d^(3/2)*2^(1/2)-1/2*b*c^(1/4)*ln(d^(1/2
)+x*c^(1/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)^(1/2))/d^(3/2)*2^(1/2)+b*c^(1/4)*
arctan(-1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)+b*c^(1/4)*ar
ctan(1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)-2*(a+b*arctanh(
c*x^2))/d/(d*x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6097, 16, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{b\sqrt[4]{c}\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\,d^{3/2}}-\frac{b\sqrt[4]{c}\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\,d^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(3/2),x]
```

```
[Out] (-2*b*c^(1/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[2]*b*c^(
1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[2]*b*
c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*(a +
b*ArcTanh[c*x^2]))/(d*Sqrt[d*x]) + (2*b*c^(1/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])
/Sqrt[d]])/d^(3/2) + (b*c^(1/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c
^(1/4)*Sqrt[d*x]])/(Sqrt[2]*d^(3/2)) - (b*c^(1/4)*Log[Sqrt[d] + Sqrt[c]*Sqr
t[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(Sqrt[2]*d^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
```

```
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{(dx)^{3/2}}\,dx &= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{(4bc)\int \frac{x}{\sqrt{dx}\left(1-c^2x^4\right)}\,dx}{d}\\
&= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{(4bc)\int \frac{\sqrt{dx}}{1-c^2x^4}\,dx}{d^2}\\
&= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{(8bc)\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^8}{d^4}}\,dx,x,\sqrt{dx}\right)}{d^3}\\
&= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{(4bc)\,\mathrm{Subst}\left(\int \frac{x^2}{d^2-cx^4}\,dx,x,\sqrt{dx}\right)}{d}+\frac{(4bc)\,\mathrm{Subst}\left(\int \frac{x^2}{d^2+c\cdots}\right.}{d}\\
&= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{\left(2b\sqrt{c}\right)\mathrm{Subst}\left(\int \frac{1}{d-\sqrt{c}x^2}\,dx,x,\sqrt{dx}\right)}{d}-\frac{\left(2b\sqrt{c}\right)\mathrm{Subst}\left(\cdots\right.}{\cdots}\\
&= -\frac{2b\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{2b\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\left(b\sqrt[4]{c}\cdots\right.}{\cdots}\\
&= -\frac{2b\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{2b\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{b\sqrt[4]{c}\cdots}{\cdots}\\
&= -\frac{2b\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{2}\,b\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{2}\,b\sqrt[4]{c}\tan^{-1}\left(1+\frac{\sqrt{2}\cdots}{\cdots}\right)}{d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 268, normalized size = 0.94

$$-\frac{x\left(4a+4b\tanh^{-1}\left(cx^2\right)+2b\sqrt[4]{c}\sqrt{x}\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2b\sqrt[4]{c}\sqrt{x}\log\left(\sqrt[4]{c}\sqrt{x}+1\right)-\sqrt{2}\,b\sqrt[4]{c}\sqrt{x}\log\left(\sqrt{c}\,x-\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(3/2),x]
```

```
[Out] -1/2*(x*(4*a + 2*Sqrt[2]*b*c^(1/4)*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[
x]] - 2*Sqrt[2]*b*c^(1/4)*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] + 4*b
*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]] + 4*b*ArcTanh[c*x^2] + 2*b*c^(1/4)
*Sqrt[x]*Log[1 - c^(1/4)*Sqrt[x]] - 2*b*c^(1/4)*Sqrt[x]*Log[1 + c^(1/4)*Sqr
t[x]] - Sqrt[2]*b*c^(1/4)*Sqrt[x]*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]
*x] + Sqrt[2]*b*c^(1/4)*Sqrt[x]*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x
]))/(d*x)^(3/2)
```

fricas [A] time = 1.83, size = 38, normalized size = 0.13

$$-\frac{\sqrt{dx}\left(b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a\right)}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(d*x)*(b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/(d^2*x)
```

giac [B] time = 0.85, size = 505, normalized size = 1.77

$$-\frac{\frac{2\,b\log\left(-\frac{cd^2x^2+d^2}{cd^2x^2-d^2}\right)}{\sqrt{dx}}+\frac{4\,a}{\sqrt{dx}}-\frac{2\,\sqrt{2}\left(c^3d^2\right)^{\frac{3}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}-\frac{2\,\sqrt{2}\left(c^3d^2\right)^{\frac{3}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}-\frac{2\,\sqrt{2}\left(-c^3d^2\right)^{\frac{3}{4}}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/sqrt(d*x) + 4*a/sqrt(d*
x) - 2*sqrt(2)*(c^3*d^2)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4)
+ 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^2*d^2) - 2*sqrt(2)*(c^3*d^2)^(3/4)*b*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^2*d^
2) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4
) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c^2*d^2) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*b*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(
c^2*d^2) + sqrt(2)*(c^3*d^2)^(3/4)*b*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1
/4) + sqrt(d^2/c))/(c^2*d^2) - sqrt(2)*(c^3*d^2)^(3/4)*b*log(d*x - sqrt(2)*
sqrt(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^2*d^2) + sqrt(2)*(-c^3*d^2)^(3/4)
*b*log(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c^2*d^2) - s
qrt(2)*(-c^3*d^2)^(3/4)*b*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt
(-d^2/c))/(c^2*d^2))/d
```

maple [A] time = 0.03, size = 272, normalized size = 0.95

$$-\frac{2a}{d\sqrt{dx}}-\frac{2b\,\mathrm{arctanh}\left(c\,x^2\right)}{d\sqrt{dx}}-\frac{2b\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}+\frac{b\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}{dx+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}\right)}{2d\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\,\arctan}{d\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(3/2),x)
```

```
[Out] -2/d*a/(d*x)^(1/2)-2/d*b/(d*x)^(1/2)*arctanh(c*x^2)-2/d*b/(d^2/c)^(1/4)*arc
tan((d*x)^(1/2)/(d^2/c)^(1/4))+1/d*b/(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^
(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)))+1/2/d*b/(d^2/c)^(1/4)*2^(1/2)*ln((d*x-(
d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x+(d^2/c)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(d^2/c)^(1/2)))+1/d*b/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2
/c)^(1/4)*(d*x)^(1/2)+1)+1/d*b/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)
^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 0.41, size = 296, normalized size = 1.04

$$-\frac{b\left(\frac{4\,\operatorname{artanh}\left(cx^2\right)}{\sqrt{dx}}-\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}dx+\sqrt{2}\sqrt{dx}c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}}+\frac{\sqrt{2}\log\left(\sqrt{\right.}}{}\right.}{d^2}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(4*arctanh(c*x^2)/sqrt(d*x) - (d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(s
qrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c
)*d)*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*
sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) - sqrt(2)*log
(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/(c^(3/4)*sqrt(d)) + s
qrt(2)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/(c^(3/4)*sq
rt(d))) - 2*d^2*(2*arctan(sqrt(d*x)*sqrt(c)/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*
d)*sqrt(c)) + log((sqrt(d*x)*sqrt(c) - sqrt(sqrt(c)*d))/(sqrt(d*x)*sqrt(c)
+ sqrt(sqrt(c)*d)))/(sqrt(sqrt(c)*d)*sqrt(c))))*c/d^2) + 4*a/sqrt(d*x))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\operatorname{atanh}\left(c\,x^2\right)}{(d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/(d*x)^(3/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))/(d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\operatorname{atanh}\left(cx^2\right)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(3/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))/(d*x)**(3/2), x)
```

3.87 $\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{5/2}} \, dx$

Optimal. Leaf size=301

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}-\frac{bc^{3/4}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}d^{5/2}}+\frac{bc^{3/4}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}d^{5/2}}+\frac{2bc}{}$$

```
[Out] 2/3*b*c^(3/4)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(5/2)-2/3*(a+b*arctanh(
c*x^2))/d/(d*x)^(3/2)+2/3*b*c^(3/4)*arctanh(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^
(5/2)-1/6*b*c^(3/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(1/2
))/d^(5/2)*2^(1/2)+1/6*b*c^(3/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)+c^(1/4)*2^(1/
2)*(d*x)^(1/2))/d^(5/2)*2^(1/2)+1/3*b*c^(3/4)*arctan(-1+c^(1/4)*2^(1/2)*(d*
x)^(1/2)/d^(1/2))*2^(1/2)/d^(5/2)+1/3*b*c^(3/4)*arctan(1+c^(1/4)*2^(1/2)*(d
*x)^(1/2)/d^(1/2))*2^(1/2)/d^(5/2)
```

Rubi [A] time = 0.25, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6097, 16, 329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}-\frac{bc^{3/4}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}d^{5/2}}+\frac{bc^{3/4}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}d^{5/2}}+\frac{2bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(5/2),x]
```

```
[Out] (2*b*c^(3/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) - (Sqrt[2]*b*
c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) + (Sqr
t[2]*b*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2))
 - (2*(a + b*ArcTanh[c*x^2]))/(3*d*(d*x)^(3/2)) + (2*b*c^(3/4)*ArcTanh[(c^(
1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) - (b*c^(3/4)*Log[Sqrt[d] + Sqrt[c]*Sq
rt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*d^(5/2)) + (b*c^(3/4)*Log[
Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*d^(5/2
))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
```

```
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{(dx)^{5/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{(4bc)\int \frac{x}{(dx)^{3/2}\left(1-c^2x^4\right)}\,dx}{3d}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{(4bc)\int \frac{1}{\sqrt{dx}\left(1-c^2x^4\right)}\,dx}{3d^2}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{(8bc)\,\text{Subst}\left(\int \frac{1}{1-\frac{c^2x^8}{d^4}}\,dx,x,\sqrt{dx}\right)}{3d^3}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{(4bc)\,\text{Subst}\left(\int \frac{1}{d^2-cx^4}\,dx,x,\sqrt{dx}\right)}{3d}+\frac{(4bc)\,\text{Subst}\left(\int \frac{1}{d^2+cx^4}\right.}{3d}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{(2bc)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{c}x^2}\,dx,x,\sqrt{dx}\right)}{3d^2}+\frac{(2bc)\,\text{Subst}\left(\int \frac{1}{d+\sqrt{c}\,x}\right.}{3d^2}$$

$$= \frac{2bc^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{2bc^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{\left(bc^{3/4}\right)\text{Su}}{}$$

$$= \frac{2bc^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{2bc^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{bc^{3/4}\log\left(\right.}{}$$

$$= \frac{2bc^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{\sqrt{2}\,bc^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{\sqrt{2}\,bc^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{d}}{\sqrt{d}}\right.}{3d^{5/2}}$$

Mathematica [A] time = 0.11, size = 268, normalized size = 0.89

$$-\frac{x\left(4a+2bc^{3/4}x^{3/2}\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2bc^{3/4}x^{3/2}\log\left(\sqrt[4]{c}\sqrt{x}+1\right)+\sqrt{2}\,bc^{3/4}x^{3/2}\log\left(\sqrt{c}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{x}+1\right)-\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(5/2),x]
```

```
[Out] -1/6*(x*(4*a + 2*Sqrt[2]*b*c^(3/4)*x^(3/2)*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[
x]] - 2*Sqrt[2]*b*c^(3/4)*x^(3/2)*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] - 4*b
*c^(3/4)*x^(3/2)*ArcTan[c^(1/4)*Sqrt[x]] + 4*b*ArcTanh[c*x^2] + 2*b*c^(3/4)
*x^(3/2)*Log[1 - c^(1/4)*Sqrt[x]] - 2*b*c^(3/4)*x^(3/2)*Log[1 + c^(1/4)*Sqr
t[x]] + Sqrt[2]*b*c^(3/4)*x^(3/2)*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]
*x] - Sqrt[2]*b*c^(3/4)*x^(3/2)*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x
]))/(d*x)^(5/2)
```

fricas [A] time = 0.89, size = 38, normalized size = 0.13

$$-\frac{\sqrt{dx}\left(b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a\right)}{3\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(d*x)*(b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/(d^3*x^2)
```

giac [B] time = 0.73, size = 516, normalized size = 1.71

$$bcd^2\left(\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{cd^4}+\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{cd^4}+\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+2\right.}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right.}{cd^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(b*c*d^2*(2*sqrt(2)*(c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)
^(1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d^4) + 2*sqrt(2)*(c^3*d^2)^(1/4)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d
^4) + 2*sqrt(2)*(-c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4)
 + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c*d^4) + 2*sqrt(2)*(-c^3*d^2)^(1/4)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c*d^4
) + sqrt(2)*(c^3*d^2)^(1/4)*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4) + sqr
t(d^2/c))/(c*d^4) - sqrt(2)*(c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt(d*x)*(d^
2/c)^(1/4) + sqrt(d^2/c))/(c*d^4) + sqrt(2)*(-c^3*d^2)^(1/4)*log(d*x + sqrt
(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c*d^4) - sqrt(2)*(-c^3*d^2)^(
1/4)*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c*d^4)) -
2*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*x)*d*x) - 4*a/(sqrt(d
*x)*d*x))/d
```

maple [A] time = 0.04, size = 280, normalized size = 0.93

$$-\frac{2a}{3d\,(dx)^{\frac{3}{2}}}-\frac{2b\,\mathrm{arctanh}\left(c\,x^2\right)}{3d\,(dx)^{\frac{3}{2}}}+\frac{bc\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{3d^3}+\frac{2bc\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{3d^3}+\frac{bc\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\cdots}{dx-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\cdots}\right.}{6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(5/2),x)
```

```
[Out] -2/3/d*a/(d*x)^(3/2)-2/3/d*b/(d*x)^(3/2)*arctanh(c*x^2)+1/3/d^3*b*c*(d^2/c)
^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)))+2/3/d^3*
b*c*(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))+1/6/d^3*b*c*(d^2/c)^(1/
4)*2^(1/2)*ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x-(d
^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+1/3/d^3*b*c*(d^2/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+1/3/d^3*b*c*(d^2/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)
```

maxima [A] time = 0.44, size = 277, normalized size = 0.92

$$\frac{b\left(\frac{\left(\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}+\frac{2\sqrt{2}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}\,d}}\right)}{\sqrt{\sqrt{c}\,d}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{c}\,dx+\sqrt{2}\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{c}\,dx-\sqrt{2}\sqrt{dx}\,c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{1}{4}}}\cdots}{d^2}\cdots\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*((2*sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d
*x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) + 2*sqrt(2)*d*arctan(-1/2*sqr
t(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(
sqrt(c)*d) + sqrt(2)*sqrt(d)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sq
rt(d) + d)/c^(1/4) - sqrt(2)*sqrt(d)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^
(1/4)*sqrt(d) + d)/c^(1/4) + 4*d*arctan(sqrt(d*x)*sqrt(c)/sqrt(sqrt(c)*d))/
sqrt(sqrt(c)*d) - 2*d*log((sqrt(d*x)*sqrt(c) - sqrt(sqrt(c)*d))/(sqrt(d*x)*
sqrt(c) + sqrt(sqrt(c)*d)))/sqrt(sqrt(c)*d))*c/d^2 - 4*arctanh(c*x^2)/(d*x)
^(3/2)) - 4*a/(d*x)^(3/2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x^2\right)}{(d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/(d*x)^(5/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))/(d*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{atanh}\left(cx^2\right)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(5/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))/(d*x)**(5/2), x)
```

3.88 $\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}}-\frac{bc^{5/4}\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}\,d^{7/2}}+\frac{bc^{5/4}\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}\,d^{7/2}}-$$

```
[Out] -2/5*b*c^(5/4)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(7/2)-2/5*(a+b*arctanh
(c*x^2))/d/(d*x)^(5/2)+2/5*b*c^(5/4)*arctanh(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d
^(7/2)-1/10*b*c^(5/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(1
/2))/d^(7/2)*2^(1/2)+1/10*b*c^(5/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)+c^(1/4)*2^
(1/2)*(d*x)^(1/2))/d^(7/2)*2^(1/2)-1/5*b*c^(5/4)*arctan(-1+c^(1/4)*2^(1/2)*
(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(7/2)-1/5*b*c^(5/4)*arctan(1+c^(1/4)*2^(1/2)
*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(7/2)-8/5*b*c/d^3/(d*x)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6097, 16, 325, 329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}}-\frac{bc^{5/4}\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}\,d^{7/2}}+\frac{bc^{5/4}\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}\,d^{7/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(7/2),x]
```

```
[Out] (-8*b*c)/(5*d^3*Sqrt[d*x]) - (2*b*c^(5/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d
]])/(5*d^(7/2)) + (Sqrt[2]*b*c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])
/Sqrt[d]])/(5*d^(7/2)) - (Sqrt[2]*b*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[d*x])/Sqrt[d]])/(5*d^(7/2)) - (2*(a + b*ArcTanh[c*x^2]))/(5*d*(d*x)^(5/2)
) + (2*b*c^(5/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*d^(7/2)) - (b*c^(
5/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(5*Sqrt[
2]*d^(7/2)) + (b*c^(5/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*
Sqrt[d*x]])/(5*Sqrt[2]*d^(7/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{(dx)^{7/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}} + \frac{(4bc)\int \frac{x}{(dx)^{5/2}\left(1-c^2x^4\right)}\,dx}{5d}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}} + \frac{(4bc)\int \frac{1}{(dx)^{3/2}\left(1-c^2x^4\right)}\,dx}{5d^2}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}} + \frac{\left(4bc^3\right)\int \frac{(dx)^{5/2}}{1-c^2x^4}\,dx}{5d^6}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}} + \frac{\left(8bc^3\right)\text{Subst}\left(\int \frac{x^6}{1-\frac{c^2x^8}{d^4}}\,dx, x, \sqrt{dx}\right)}{5d^7}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}} + \frac{\left(4bc^2\right)\text{Subst}\left(\int \frac{x^2}{d^2-cx^4}\,dx, x, \sqrt{dx}\right)}{5d^3} - \frac{\left(4bc^2\right)\ldots}{}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}} + \frac{\left(2bc^{3/2}\right)\text{Subst}\left(\int \frac{1}{d-\sqrt{c}x^2}\,dx, x, \sqrt{dx}\right)}{5d^3} - \frac{(2b\ldots}{}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2bc^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}} + \frac{2bc^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{\ldots}}{\sqrt{d}}\right.}{5d^{7/2}}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2bc^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}} + \frac{2bc^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{\ldots}}{\sqrt{d}}\right.}{5d^{7/2}}$$

$$= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2bc^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{\sqrt{2}\,bc^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{\sqrt{2}\,bc^{5/4}\tan\ldots}{}$$

Mathematica [A] time = 0.09, size = 275, normalized size = 0.87

$$\frac{x\left(-4a-2bc^{5/4}x^{5/2}\log\left(1-\sqrt[4]{c}\sqrt{x}\right)+2bc^{5/4}x^{5/2}\log\left(\sqrt[4]{c}\sqrt{x}+1\right)-\sqrt{2}\,bc^{5/4}x^{5/2}\log\left(\sqrt{c}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{x}+1\right)+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(7/2),x]

[Out] (x*(-4*a - 16*b*c*x^2 + 2*Sqrt[2]*b*c^(5/4)*x^(5/2)*ArcTan[1 - Sqrt[2]*c^(1
/4)*Sqrt[x]] - 2*Sqrt[2]*b*c^(5/4)*x^(5/2)*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[
```

```
x]] - 4*b*c^(5/4)*x^(5/2)*ArcTan[c^(1/4)*Sqrt[x]] - 4*b*ArcTanh[c*x^2] - 2*
b*c^(5/4)*x^(5/2)*Log[1 - c^(1/4)*Sqrt[x]] + 2*b*c^(5/4)*x^(5/2)*Log[1 + c^
(1/4)*Sqrt[x]] - Sqrt[2]*b*c^(5/4)*x^(5/2)*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x] + Sqrt[2]*b*c^(5/4)*x^(5/2)*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] +
Sqrt[c]*x]))/(10*(d*x)^(7/2))
```

fricas [A] time = 1.12, size = 45, normalized size = 0.14

$$-\frac{\left(8\,bcx^2+b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a\right)\sqrt{dx}}{5\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(8*b*c*x^2 + b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)*sqrt(d*x)/(d^4*x^3
)
```

giac [B] time = 2.61, size = 532, normalized size = 1.68

$$-\frac{\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{3}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{cd^4}+\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{3}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{cd^4}-\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac{3}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{cd^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/10*(2*sqrt(2)*(c^3*d^2)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4
) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d^4) + 2*sqrt(2)*(c^3*d^2)^(3/4)*b*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d^4)
 - 2*sqrt(2)*(-c^3*d^2)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4)
+ 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c*d^4) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*b*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c*d^
4) - sqrt(2)*(c^3*d^2)^(3/4)*b*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4) +
sqrt(d^2/c))/(c*d^4) + sqrt(2)*(c^3*d^2)^(3/4)*b*log(d*x - sqrt(2)*sqrt(d*x
)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c*d^4) + sqrt(2)*(-c^3*d^2)^(3/4)*b*log(d*x
 + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c*d^4) - sqrt(2)*(-c^3
*d^2)^(3/4)*b*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c
*d^4) + 2*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*x)*d^2*x^2) +
 4*(4*b*c*d^2*x^2 + a*d^2)/(sqrt(d*x)*d^4*x^2))/d
```

maple [A] time = 0.04, size = 292, normalized size = 0.92

$$-\frac{2a}{5d\left(dx\right)^{\frac{5}{2}}}-\frac{2b\operatorname{arctanh}\left(c\,x^2\right)}{5d\left(dx\right)^{\frac{5}{2}}}-\frac{2bc\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{5d^3\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}+\frac{bc\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{5d^3\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}-\frac{8bc}{5d^3\sqrt{dx}}-\frac{bc\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}{dx+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^2}{c}}}\right)}{10d^3\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(7/2),x)
```

```
[Out] -2/5/d*a/(d*x)^(5/2)-2/5/d*b/(d*x)^(5/2)*arctanh(c*x^2)-2/5/d^3*b*c/(d^2/c)
^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))+1/5/d^3*b*c/(d^2/c)^(1/4)*ln(((d*x
```

```
)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)))-8/5*b*c/d^3/(d*x)^(1/2)
-1/10/d^3*b*c/(d^2/c)^(1/4)*2^(1/2)*ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(d^2/c)^(1/2))/(d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))-1/
5/d^3*b*c/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)
-1/5/d^3*b*c/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)
-1)
```

maxima [A] time = 0.42, size = 298, normalized size = 0.94

$$-\frac{b\left(\frac{\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}dx+\sqrt{2}\sqrt{dx}c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}}+\frac{\sqrt{2}\log\left(\sqrt{c}dx-\sqrt{2}\sqrt{dx}c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}}\cdots\right)\right)}{d^2}\cdots\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/10*(b*((c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqr
t(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c
)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) - sqrt(2)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*
x)*c^(1/4)*sqrt(d) + d)/(c^(3/4)*sqrt(d)) + sqrt(2)*log(sqrt(c)*d*x - sqrt(
2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/(c^(3/4)*sqrt(d))) + 2*c*(2*arctan(sqrt(d
*x)*sqrt(c)/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + log((sqrt(d*x)*sqr
t(c) - sqrt(sqrt(c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/(sqrt(sqrt(c
)*d)*sqrt(c))) + 16/sqrt(d*x))*c/d^2 + 4*arctanh(c*x^2)/(d*x)^(5/2)) + 4*a/
(d*x)^(5/2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x^2\right)}{{\left(d\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/(d*x)^(7/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))/(d*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

3.89 $\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{9/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{bc^{7/4}\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}\,d^{9/2}}-\frac{bc^{7/4}\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}\,d^{9/2}}+\frac{2bc}{}$$

```
[Out] -8/21*b*c/d^3/(d*x)^(3/2)+2/7*b*c^(7/4)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))
/d^(9/2)-2/7*(a+b*arctanh(c*x^2))/d/(d*x)^(7/2)+2/7*b*c^(7/4)*arctanh(c^(1/
4)*(d*x)^(1/2)/d^(1/2))/d^(9/2)+1/14*b*c^(7/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)
-c^(1/4)*2^(1/2)*(d*x)^(1/2))/d^(9/2)*2^(1/2)-1/14*b*c^(7/4)*ln(d^(1/2)+x*c
^(1/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)^(1/2))/d^(9/2)*2^(1/2)-1/7*b*c^(7/4)*a
rctan(-1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(9/2)-1/7*b*c^(7/4)
*arctan(1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(9/2)
```

Rubi [A] time = 0.29, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6097, 16, 325, 329, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{bc^{7/4}\log\left(\sqrt{c}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}\,d^{9/2}}-\frac{bc^{7/4}\log\left(\sqrt{c}\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}\,d^{9/2}}+\frac{2bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(9/2),x]
```

```
[Out] (-8*b*c)/(21*d^3*(d*x)^(3/2)) + (2*b*c^(7/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqr
t[d]])/(7*d^(9/2)) + (Sqrt[2]*b*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*
x])/Sqrt[d]])/(7*d^(9/2)) - (Sqrt[2]*b*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) - (2*(a + b*ArcTanh[c*x^2]))/(7*d*(d*x)^(7
/2)) + (2*b*c^(7/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) + (b*
c^(7/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sq
rt[2]*d^(9/2)) - (b*c^(7/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/
4)*Sqrt[d*x]])/(7*Sqrt[2]*d^(9/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{(dx)^{9/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{(4bc)\int \frac{x}{(dx)^{7/2}\left(1-c^2x^4\right)}\,dx}{7d}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{(4bc)\int \frac{1}{(dx)^{5/2}\left(1-c^2x^4\right)}\,dx}{7d^2}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{\left(4bc^3\right)\int \frac{(dx)^{3/2}}{1-c^2x^4}\,dx}{7d^6}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{\left(8bc^3\right)\text{Subst}\left(\int \frac{x^4}{1-\frac{c^2x^8}{d^4}}\,dx,x,\sqrt{dx}\right)}{7d^7}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{\left(4bc^2\right)\text{Subst}\left(\int \frac{1}{d^2-cx^4}\,dx,x,\sqrt{dx}\right)}{7d^3}-\frac{\left(4bc^2\right)}{}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{\left(2bc^2\right)\text{Subst}\left(\int \frac{1}{d-\sqrt{c}\,x^2}\,dx,x,\sqrt{dx}\right)}{7d^4}+\frac{\left(2bc^2\right.}{}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}+\frac{2bc^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{2bc^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{}}{\sqrt{d}}\right.}{7d^{9/2}}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}+\frac{2bc^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{2bc^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{}}{\sqrt{d}}\right.}{7d^{9/2}}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}+\frac{2bc^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}+\frac{\sqrt{2}\,bc^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{\sqrt{2}\,bc^{7/4}\tan}{}$$

Mathematica [A] time = 0.11, size = 281, normalized size = 0.89

$$\sqrt{dx}\,\left(-12a-6bc^{7/4}x^{7/2}\log\left(1-\sqrt[4]{c}\,\sqrt{x}\right)+6bc^{7/4}x^{7/2}\log\left(\sqrt[4]{c}\,\sqrt{x}\,+1\right)+3\sqrt{2}\,bc^{7/4}x^{7/2}\log\left(\sqrt{c}\,x-\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}\,+1\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(9/2),x]
```

```
[Out] (Sqrt[d*x]*(-12*a - 16*b*c*x^2 + 6*Sqrt[2]*b*c^(7/4)*x^(7/2)*ArcTan[1 - Sqr
t[2]*c^(1/4)*Sqrt[x]] - 6*Sqrt[2]*b*c^(7/4)*x^(7/2)*ArcTan[1 + Sqrt[2]*c^(1
/4)*Sqrt[x]] + 12*b*c^(7/4)*x^(7/2)*ArcTan[c^(1/4)*Sqrt[x]] - 12*b*ArcTanh[
```

```
c*x^2] - 6*b*c^(7/4)*x^(7/2)*Log[1 - c^(1/4)*Sqrt[x]] + 6*b*c^(7/4)*x^(7/2)
*Log[1 + c^(1/4)*Sqrt[x]] + 3*Sqrt[2]*b*c^(7/4)*x^(7/2)*Log[1 - Sqrt[2]*c^(
1/4)*Sqrt[x] + Sqrt[c]*x] - 3*Sqrt[2]*b*c^(7/4)*x^(7/2)*Log[1 + Sqrt[2]*c^(
1/4)*Sqrt[x] + Sqrt[c]*x]))/(42*d^5*x^4)
```

fricas [A] time = 0.70, size = 46, normalized size = 0.15

$$-\frac{\left(8\,bcx^2+3\,b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+6\,a\right)\sqrt{dx}}{21\,d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/21*(8*b*c*x^2 + 3*b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a)*sqrt(d*x)/(d^5*
x^4)
```

giac [B] time = 7.78, size = 519, normalized size = 1.64

$$-\frac{\frac{6\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}bc\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d^4}+\frac{6\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}bc\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d^4}-\frac{6\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}bc\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+\cdots\right)}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d^4}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/42*(6*sqrt(2)*(c^3*d^2)^(1/4)*b*c*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1
/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/d^4 + 6*sqrt(2)*(c^3*d^2)^(1/4)*b*c*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/d^4 - 6
*sqrt(2)*(-c^3*d^2)^(1/4)*b*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) +
2*sqrt(d*x))/(-d^2/c)^(1/4))/d^4 - 6*sqrt(2)*(-c^3*d^2)^(1/4)*b*c*arctan(-1
/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/d^4 + 3*s
qrt(2)*(c^3*d^2)^(1/4)*b*c*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4) + sqrt
(d^2/c))/d^4 - 3*sqrt(2)*(c^3*d^2)^(1/4)*b*c*log(d*x - sqrt(2)*sqrt(d*x)*(d
^2/c)^(1/4) + sqrt(d^2/c))/d^4 - 3*sqrt(2)*(-c^3*d^2)^(1/4)*b*c*log(d*x + s
qrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/d^4 + 3*sqrt(2)*(-c^3*d^2)^
(1/4)*b*c*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/d^4 +
6*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*x)*d^3*x^3) + 4*(4*b*
c*d^2*x^2 + 3*a*d^2)/(sqrt(d*x)*d^5*x^3))/d
```

maple [A] time = 0.04, size = 302, normalized size = 0.95

$$-\frac{2a}{7d\,(dx)^{\frac{7}{2}}}-\frac{2b\,\mathrm{arctanh}\left(c\,x^2\right)}{7d\,(dx)^{\frac{7}{2}}}+\frac{b\,c^2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{7d^5}+\frac{2b\,c^2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{7d^5}-\frac{8bc}{21d^3\,(dx)^{\frac{3}{2}}}-\frac{b\,c^2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(9/2),x)
```

```
[Out] -2/7/d*a/(d*x)^(7/2)-2/7/d*b/(d*x)^(7/2)*arctanh(c*x^2)+1/7/d^5*b*c^2*(d^2/
c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)))+2/7/d^
5*b*c^2*(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))-8/21*b*c/d^3/(d*x)^
(3/2)-1/14/d^5*b*c^2*(d^2/c)^(1/4)*2^(1/2)*ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2
```

```
)*2^(1/2)+(d^2/c)^(1/2))/(d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/
2)))-1/7/d^5*b*c^2*(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)
^(1/2)+1)-1/7/d^5*b*c^2*(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*
(d*x)^(1/2)-1)
```

maxima [A] time = 0.41, size = 297, normalized size = 0.94

$$-\frac{b\left(\frac{c\left(\frac{6\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\,d}+\frac{6\sqrt{2}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\,d}+\frac{3\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{c}dx+\sqrt{2}\sqrt{dx}c^{\frac{1}{4}}\sqrt{d}+d\right)}{d^{\frac{3}{2}}}-\frac{3\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{c}dx-\sqrt{2}\sqrt{dx}c^{\frac{1}{4}}\cdots\right.}{d^{\frac{3}{2}}}\cdots\right)}{d^{2}}\cdots\right)}{42\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/42*(b*(c*(6*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sq
rt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*d) + 6*sqrt(2)*c*arctan(
-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d
))/(sqrt(sqrt(c)*d)*d) + 3*sqrt(2)*c^(3/4)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d
*x)*c^(1/4)*sqrt(d) + d)/d^(3/2) - 3*sqrt(2)*c^(3/4)*log(sqrt(c)*d*x - sqrt
(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/d^(3/2) - 12*c*arctan(sqrt(d*x)*sqrt(c)/
sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*d) + 6*c*log((sqrt(d*x)*sqrt(c) - sqrt(sq
rt(c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/(sqrt(sqrt(c)*d)*d) + 16/(
d*x)^(3/2))/d^2 + 12*arctanh(c*x^2)/(d*x)^(7/2)) + 12*a/(d*x)^(7/2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{atanh}\left(c\,x^2\right)}{{\left(d\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))/(d*x)^(9/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))/(d*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(9/2),x)
```

```
[Out] Timed out
```

3.90 $\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=6327

result too large to display

```
[Out] 1/6*x*(2*a-b*ln(-c*x^2+1))^2*(d*x)^(1/2)+2/3*b^2*arctanh((-c)^(1/4)*x^(1/2)
)*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)
^(1/4)+(-c^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-2/3*b^2*arctanh(c^
(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2)
)/(c^(1/4)+(-c^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*I*b^2*arctan(
(-c)^(1/4)*x^(1/2))^2*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-2/3*I*b^2*arctan(c^(1/
4)*x^(1/2))^2*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*I*b^2*polylog(2,1-2/(1-I*(-c)
^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-1/3*I*b^2*polylog(2,1-(1+I)
*(1-(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^
(1/2)-2/3*I*b^2*polylog(2,1-2/(1+I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3
/4)/x^(1/2)+2/3*b^2*arctan((-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)*(d*x)^(1/2)/(-c
)^(3/4)/x^(1/2)-2/3*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)*(d*x)^(1/2
)/(-c)^(3/4)/x^(1/2)+2/3*b*arctan(c^(1/4)*x^(1/2))*(2*a-b*ln(-c*x^2+1))*(d*
x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b*arctanh(c^(1/4)*x^(1/2))*(2*a-b*ln(-c*x^2+1)
)*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b^2*arctan((-c)^(1/4)*x^(1/2))*ln(c*x^2+1
)*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*arctan(c^(1/4)*x^(1/2))*ln(c*x^2+1
)*(d*x)^(1/2)/c^(3/4)/x^(1/2)+2/3*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(c*x^2+
1)*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-2/3*b^2*arctanh(c^(1/4)*x^(1/2))*ln(c*x^2
+1)*(d*x)^(1/2)/c^(3/4)/x^(1/2)+4/3*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(2/(1
-(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+4/3*b^2*arctan((-c)^(1
/4)*x^(1/2))*ln(2/(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+
2/3*b^2*arctan((-c)^(1/4)*x^(1/2))*ln((1+I)*(1-(-c)^(1/4)*x^(1/2))/(1-I*(-c
)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-4/3*b^2*arctan((-c)^(1/4)*
x^(1/2))*ln(2/(1+I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-4/3*
b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(2/(1+(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-
c)^(3/4)/x^(1/2)+2/3*b^2*arctan((-c)^(1/4)*x^(1/2))*ln((1-I)*(1+(-c)^(1/4)*
x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+4/3*b^2*a
rctanh(c^(1/4)*x^(1/2))*ln(2/(1-c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/
2)-2/3*b^2*arctan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/(
(-c)^(1/4)-I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1
/2)+2/3*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))
/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2
)+4/3*b^2*arctan(c^(1/4)*x^(1/2))*ln(2/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c
^(3/4)/x^(1/2)-2/3*b^2*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*
x^(1/2))/(I*(-c)^(1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/
x^(1/2)-2/3*b^2*arctan(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))
/(I*(-c)^(1/4)+c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+
2/3*b^2*arctan(c^(1/4)*x^(1/2))*ln((1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x
^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-4/3*b^2*arctan(c^(1/4)*x^(1/2))*ln(2/(
1+I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+1/3*I*b^2*polylog(2,1-2*(
-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2
)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+1/3*I*b^2*polylog(2,1+2*c^(1/4)*(1-(-c)^
(1/4)*x^(1/2))/(I*(-c)^(1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^
(3/4)/x^(1/2)+1/3*I*b^2*polylog(2,1-2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/(I*(-c
)^(1/4)+c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+1/3*I*b
^2*polylog(2,1-2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*c^(1/4))/(1-I
*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+1/3*I*b^2*polylog(2,1+
2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I
*(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+1/3*I*b^2*polylog(2,1-2*
c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(c^(1/4)+I*(-
(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+1/3*I*b^2*polylog(2,1+2*(-c
)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+
I*(-c^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+1/3*I*b^2*polylog(2,1-2
```

```
*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/((-c)^(1/
4)+I*(-c^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-1/3*I*b^2*polylog(2,
1+(-1+I)*(1+(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^
(3/4)/x^(1/2)-2/3*I*b^2*polylog(2,1-2/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^
(3/4)/x^(1/2)-1/3*I*b^2*polylog(2,1-(1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^(1/4)*
x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*I*b^2*polylog(2,1-2/(1+I*c^(1/4)*
x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/3*I*b^2*polylog(2,1+(-1+I)*(1+c^(1/
4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-4/3*b^2*arct
anh(c^(1/4)*x^(1/2))*ln(2/(1+c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+
2/3*b^2*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/((-c)
^(1/4)-c^(1/4))/(1+c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+2/3*b^2*ar
ctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1
/4))/(1+c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b^2*arctan((-c)^(
1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*c^(1/4))/(1
-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*arctanh((-c)
^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1
+(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*arctan(c^(1/4)
*x^(1/2))*ln((1-I)*(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c
^(3/4)/x^(1/2)-2/3*b^2*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-(
-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*(
d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(
1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-
(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*arctanh(c^(1/4)*
x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/
(-c^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b^2*arctan(
c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*
x^(1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b
^2*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/
2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/(-
c)^(3/4)/x^(1/2)+2/3*b^2*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(
-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x
)^(1/2)/c^(3/4)/x^(1/2)-2/3*b^2*arctan((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)
*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+I*(-c^(
1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*arctanh((-c)^(1/4)*x^(
1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/
(-(-c)^(1/4)+(-c^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-2/3*b^2*arct
anh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*
x^(1/2))/(-c^(1/4)+(-c^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*b^2*a
rctan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1-I
*(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+I*(-c^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4
)/x^(1/2)-2/3*b^2*arctanh(c^(1/4)*x^(1/2))^2*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/
3*b^2*arctanh((-c)^(1/4)*x^(1/2))^2*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)-1/3*b^2*
x*ln(-c*x^2+1)*ln(c*x^2+1)*(d*x)^(1/2)+2/3*a*b*x*ln(c*x^2+1)*(d*x)^(1/2)-8/
9*a*b*x*(d*x)^(1/2)+4/9*b^2*x*ln(-c*x^2+1)*(d*x)^(1/2)+4/9*b*x*(2*a-b*ln(-c
*x^2+1))*(d*x)^(1/2)+1/6*b^2*x*ln(c*x^2+1)^2*(d*x)^(1/2)+2/3*a*b*arctan(-1+
c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*(d*x)^(1/2)/c^(3/4)/x^(1/2)+2/3*a*b*arctan
(1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*(d*x)^(1/2)/c^(3/4)/x^(1/2)+1/3*a*b*ln(
1+x*c^(1/2)-c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/
3*a*b*ln(1+x*c^(1/2)+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*(d*x)^(1/2)/c^(3/4)/x
^(1/2)-1/3*b^2*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(
1/4)*x^(1/2))/(c^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/
3*b^2*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x
^(1/2))/(-(-c)^(1/4)+(-c^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+1/3*
b^2*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/
(-c^(1/4)+(-c^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/3*b^2*polylog(2,
1-2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1
/4)+(-c^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+1/3*b^2*polylog(2,1-2
*c^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2
))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+2/3*b^2*polylog(2,1-2/(1-(-c)^(1/4)*
```

```
x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*polylog(2,1-2/(1+(-c)^(1/4
)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+2/3*b^2*polylog(2,1-2/(1-c^(1/4)
*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/3*b^2*polylog(2,1-2*(-c)^(1/4)*(1-
c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(
-c)^(3/4)/x^(1/2)+2/3*b^2*polylog(2,1-2/(1+c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^
(3/4)/x^(1/2)-1/3*b^2*polylog(2,1+2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/((-c)^(1
/4)-c^(1/4))/(1+c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/3*b^2*polyl
og(2,1-2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+c^(1/4)*x^(
1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/3*b^2*polylog(2,1-2*(-c)^(1/4)*(1+c^(1
/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^
(3/4)/x^(1/2)+1/3*b^2*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/
2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/(
-c)^(3/4)/x^(1/2)-1/3*b^2*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1
/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4
)/x^(1/2)+1/3*b^2*polylog(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/
(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(
3/4)/x^(1/2)
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{dx} \left(a + b \tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2,x]
[Out] Defer[Int][Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2, x]
```

Rubi steps

$$\int \sqrt{dx} \left(a + b \tanh^{-1}\left(cx^2\right)\right)^2 dx = \int \sqrt{dx} \left(a + b \tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Mathematica [F] time = 63.72, size = 0, normalized size = 0.00

$$\int \sqrt{dx} \left(a + b \tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2,x]
[Out] Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2, x]
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \operatorname{artanh}\left(cx^2\right)^2 + 2\,ab \operatorname{artanh}\left(cx^2\right) + a^2\right)\sqrt{dx}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx} \left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b*arctanh(c*x^2) + a)^2, x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,\left(a + b\operatorname{arctanh}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\,b^2\sqrt{d}\,x^{\frac{3}{2}}\log\left(-cx^2+1\right)^2+\frac{1}{6}\,a^2c\sqrt{d}\left(\frac{4\,x^{\frac{3}{2}}}{c}-\frac{3\left(\frac{i\left(\log\left(i\,c^{\frac{1}{4}}\,\sqrt{x}+1\right)-\log\left(-i\,c^{\frac{1}{4}}\,\sqrt{x}+1\right)\right)}{c^{\frac{3}{4}}}-\frac{\log\left(\frac{\sqrt{c}\,\sqrt{x}-c^{\frac{1}{4}}}{\sqrt{c}\,\sqrt{x}+c^{\frac{1}{4}}}\right)}{c^{\frac{3}{4}}}\right)}{c}\right)+3\,b^2c\sqrt{d}\int\frac{x^{\frac{5}{2}}\,\mathrm{lo}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*sqrt(d)*x^(3/2)*log(-c*x^2 + 1)^2 + 1/6*a^2*c*sqrt(d)*(4*x^(3/2)/c
- 3*(I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))/c^(3/4) -
 log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(3/4))/c) +
 3*b^2*c*sqrt(d)*integrate(1/12*x^(5/2)*log(c*x^2 + 1)^2/(c*x^2 - 1), x) -
6*b^2*c*sqrt(d)*integrate(1/12*x^(5/2)*log(c*x^2 + 1)*log(-c*x^2 + 1)/(c*x^
2 - 1), x) + 12*a*b*c*sqrt(d)*integrate(1/12*x^(5/2)*log(c*x^2 + 1)/(c*x^2
- 1), x) - 12*a*b*c*sqrt(d)*integrate(1/12*x^(5/2)*log(-c*x^2 + 1)/(c*x^2 -
 1), x) - 8*b^2*c*sqrt(d)*integrate(1/12*x^(5/2)*log(-c*x^2 + 1)/(c*x^2 - 1
), x) + 1/2*a^2*sqrt(d)*(I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqr
t(x) + 1))/c^(3/4) - log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(
1/4)))/c^(3/4)) - 3*b^2*sqrt(d)*integrate(1/12*sqrt(x)*log(c*x^2 + 1)^2/(c*
x^2 - 1), x) + 6*b^2*sqrt(d)*integrate(1/12*sqrt(x)*log(c*x^2 + 1)*log(-c*x
^2 + 1)/(c*x^2 - 1), x) - 12*a*b*sqrt(d)*integrate(1/12*sqrt(x)*log(c*x^2 +
 1)/(c*x^2 - 1), x) + 12*a*b*sqrt(d)*integrate(1/12*sqrt(x)*log(-c*x^2 + 1)
/(c*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d\,x}\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a + b*atanh(c*x^2))^2,x)
```

```
[Out] int((d*x)^(1/2)*(a + b*atanh(c*x^2))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(a+b*atanh(c*x**2))**2,x)

[Out] Timed out
```

3.91 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$

Optimal. Leaf size=6177

result too large to display

```
[Out] 2*a^2*x/(d*x)^(1/2)+2*b^2*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)
*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-(-c)^(1/2))^(1/2)))*x
^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctan(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^
(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(
1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-2*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(
-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4
)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*arctanh(c^(1/4
)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))
/(c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctan((-
c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(
1/4)*x^(1/2))/(-(-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1
/2)+2*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2)
)^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/(-c
)^(1/4)/(d*x)^(1/2)-2*b^2*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)
*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)
/c^(1/4)/(d*x)^(1/2)+2*b^2*arctan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^
(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+I*(-c^(1/2))^(
1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(
2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4
)+(-c^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-b^2*x*ln(-c*x^2+1)*ln(c
*x^2+1)/(d*x)^(1/2)-2*b^2*arctanh(c^(1/4)*x^(1/2))^2*x^(1/2)/c^(1/4)/(d*x)^
(1/2)-2*b^2*arctanh((-c)^(1/4)*x^(1/2))^2*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-b^
2*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/
2))/(-(-c)^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+b^2*poly
log(2,1+2*c^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4
)+(-c^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-b^2*polylog(2,1-2*(-c)^(1/
4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-c^(1/2
))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+b^2*polylog(2,1-2*c^(1/4)*(1+x^(1
/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1/2)))*x^(1/
2)/c^(1/4)/(d*x)^(1/2)-2*a*b*x*ln(-c*x^2+1)/(d*x)^(1/2)+2*a*b*x*ln(c*x^2+1)
/(d*x)^(1/2)-b^2*polylog(2,1-2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+c
^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+b^2*polylog(
2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(
-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-b^2*polylog(
2,1+2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)
+(-(-c)^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+b^2*polylog(2,1-2*(-c)^(
1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-(
-c)^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-b^2*polylog(2,1-2*c^(1/4)
*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-(-c)^(1/2))
^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*polylog(2,1-2/(1-(-c)^(1/4)*x^(1
/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*polylog(2,1-2/(1+(-c)^(1/4)*x^(1
/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*polylog(2,1-2/(1-c^(1/4)*x^(1/2)
))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-b^2*polylog(2,1-2*(-c)^(1/4)*(1-c^(1/4)*x^(1
/2))/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^
(1/2)+2*b^2*polylog(2,1-2/(1+c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-
b^2*polylog(2,1+2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+c^
(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-b^2*polylog(2,1-2*c^(1/4)*(1+(-
c)^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)
/(d*x)^(1/2)+1/2*b^2*x*ln(-c*x^2+1)^2/(d*x)^(1/2)+1/2*b^2*x*ln(c*x^2+1)^2/(
d*x)^(1/2)+2*a*b*arctan(-1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/c^(1/4)
/(d*x)^(1/2)+2*a*b*arctan(1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/c^(1/4
)/(d*x)^(1/2)-a*b*ln(1+x*c^(1/2)-c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/c
```

```
^(1/4)/(d*x)^(1/2)+a*b*ln(1+x*c^(1/2)+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1
/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(
1-c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c
)^(1/4)/(d*x)^(1/2)-4*b^2*arctan(c^(1/4)*x^(1/2))*ln(2/(1-I*c^(1/4)*x^(1/2)
))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*
(1-(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2
)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctan(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1
/4)*x^(1/2))/(I*(-c)^(1/4)+c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/
(d*x)^(1/2)-2*b^2*arctan(c^(1/4)*x^(1/2))*ln((1+I)*(1-c^(1/4)*x^(1/2))/(1-I
*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+4*b^2*arctan(c^(1/4)*x^(1/2)
)*ln(2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-4*b^2*arctanh(c^(
1/4)*x^(1/2))*ln(2/(1+c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*a
rctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/((-c)^(1/4)-c^
(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctanh(c^(1/
4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+c^(
1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctan((-c)^(1/4)*x^(1/2))
*ln(2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*c^(1/4))/(1-I*(-c)^(1/4)
*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*arctanh((-c)^(1/4)*x^(1/2))
*ln(2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+(-c)^(1/4)*x^(
1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-2*b^2*arctan(c^(1/4)*x^(1/2))*ln((1-I
)*(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*
b^2*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(
1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x
)^(1/2)-2*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-(-c
)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x
^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-2*b^2*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(
1+x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1/2)))
*x^(1/2)/c^(1/4)/(d*x)^(1/2)+I*b^2*polylog(2,1-(1+I)*(1-(-c)^(1/4)*x^(1/2))
/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+I*b^2*polylog(2,1
+(-1+I)*(1+(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)
/(d*x)^(1/2)+I*b^2*polylog(2,1-(1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/
2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+I*b^2*polylog(2,1+(-1+I)*(1+c^(1/4)*x^(1/2
))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-I*b^2*polylog(2,1-2*(
-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2
)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-I*b^2*polylog(2,1+2*c^(1/4)*(1-(-c)^(1/4
)*x^(1/2))/(I*(-c)^(1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d
*x)^(1/2)-I*b^2*polylog(2,1-2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)+
c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-I*b^2*polylog(2
,1-2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*c^(1/4))/(1-I*(-c)^(1/4)*
x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-I*b^2*polylog(2,1+2*c^(1/4)*(1-x^(
1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(
1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-I*b^2*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(
-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*
x^(1/2)/c^(1/4)/(d*x)^(1/2)-I*b^2*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-c^(
1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(
1/2)/(-c)^(1/4)/(d*x)^(1/2)-I*b^2*polylog(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-c^(
1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(1
/2)/(-c)^(1/4)/(d*x)^(1/2)+2*I*b^2*arctan((-c)^(1/4)*x^(1/2))^2*x^(1/2)/(-c
)^(1/4)/(d*x)^(1/2)+2*I*b^2*arctan(c^(1/4)*x^(1/2))^2*x^(1/2)/c^(1/4)/(d*x)
^(1/2)+2*I*b^2*polylog(2,1-2/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(
d*x)^(1/2)+2*I*b^2*polylog(2,1-2/(1+I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/
4)/(d*x)^(1/2)+2*I*b^2*polylog(2,1-2/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)
/(d*x)^(1/2)+2*I*b^2*polylog(2,1-2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(
d*x)^(1/2)-4*a*b*arctan(c^(1/4)*x^(1/2))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-4*a*b*
arctanh(c^(1/4)*x^(1/2))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-2*b^2*arctan((-c)^(1/4
)*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*arctan(c^(1/4)
*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/c^(1/4)/(d*x)^(1/2)-2*b^2*arctanh((-c)^(1/4)
*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*b^2*arctanh(c^(1/4)
*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctan((-c)^(1/4)*
```

```
x^(1/2))*ln(c*x^2+1)*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-2*b^2*arctan(c^(1/4)*x^
(1/2))*ln(c*x^2+1)*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*b^2*arctanh((-c)^(1/4)*x^(
1/2))*ln(c*x^2+1)*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-2*b^2*arctanh(c^(1/4)*x^(1
/2))*ln(c*x^2+1)*x^(1/2)/c^(1/4)/(d*x)^(1/2)+4*b^2*arctanh((-c)^(1/4)*x^(1/
2))*ln(2/(1-(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-4*b^2*arcta
n((-c)^(1/4)*x^(1/2))*ln(2/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*
x)^(1/2)-2*b^2*arctan((-c)^(1/4)*x^(1/2))*ln((1+I)*(1-(-c)^(1/4)*x^(1/2))/(
1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+4*b^2*arctan((-c)^(
1/4)*x^(1/2))*ln(2/(1+I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)
-4*b^2*arctanh((-c)^(1/4)*x^(1/2))*ln(2/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c
)^(1/4)/(d*x)^(1/2)-2*b^2*arctan((-c)^(1/4)*x^(1/2))*ln((1-I)*(1+(-c)^(1/4)
*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+4*b^2*ar
ctanh(c^(1/4)*x^(1/2))*ln(2/(1-c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2
)+2*b^2*arctan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c
)^(1/4)-I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x],x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x], x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$$

Mathematica [F] time = 62.67, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x],x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x], x]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^2\,\mathrm{artanh}\left(cx^2\right)^2+2\,ab\,\mathrm{artanh}\left(cx^2\right)+a^2\right)\sqrt{dx}}{dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d*x
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^2/sqrt(d*x), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{arctanh}\left(c\, x^2\right)\right)^2}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x)
[Out] int((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} a^2 c \left(-\frac{\frac{i\left(\log\left(i c^{\frac{1}{4}} \sqrt{x}+1\right)-\log\left(-i c^{\frac{1}{4}} \sqrt{x}+1\right)\right)}{c^{\frac{1}{4}}} - \frac{\log\left(\frac{\sqrt{c}\sqrt{x}-c^{\frac{1}{4}}}{\sqrt{c}\sqrt{x}+c^{\frac{1}{4}}}\right)}{c^{\frac{1}{4}}}}{c\sqrt{d}} - \frac{4\sqrt{x}}{c\sqrt{d}} \right) + b^2 c \int \frac{x^{\frac{3}{2}} \log\left(cx^2+1\right)^2}{4\left(c\sqrt{d}\, x^2 - \sqrt{d}\right)}\, dx - 2\, b^2 c \int \frac{x^{\frac{3}{2}} \log\left(\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x, algorithm="maxima")
[Out] -1/2*a^2*c*((-I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))/
c^(1/4) - log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(1
/4))/(c*sqrt(d)) - 4*sqrt(x)/(c*sqrt(d))) + b^2*c*integrate(1/4*x^(3/2)*log
(c*x^2 + 1)^2/(c*sqrt(d)*x^2 - sqrt(d)), x) - 2*b^2*c*integrate(1/4*x^(3/2)
*log(c*x^2 + 1)*log(-c*x^2 + 1)/(c*sqrt(d)*x^2 - sqrt(d)), x) + 4*a*b*c*int
egrate(1/4*x^(3/2)*log(c*x^2 + 1)/(c*sqrt(d)*x^2 - sqrt(d)), x) - 4*a*b*c*i
ntegrate(1/4*x^(3/2)*log(-c*x^2 + 1)/(c*sqrt(d)*x^2 - sqrt(d)), x) - 8*b^2*
c*integrate(1/4*x^(3/2)*log(-c*x^2 + 1)/(c*sqrt(d)*x^2 - sqrt(d)), x) + 1/2
*b^2*sqrt(x)*log(-c*x^2 + 1)^2/sqrt(d) - b^2*integrate(1/4*log(c*x^2 + 1)^2
/((c*sqrt(d)*x^2 - sqrt(d))*sqrt(x)), x) + 2*b^2*integrate(1/4*log(c*x^2 +
1)*log(-c*x^2 + 1)/((c*sqrt(d)*x^2 - sqrt(d))*sqrt(x)), x) - 4*a*b*integrat
e(1/4*log(c*x^2 + 1)/((c*sqrt(d)*x^2 - sqrt(d))*sqrt(x)), x) + 4*a*b*integr
ate(1/4*log(-c*x^2 + 1)/((c*sqrt(d)*x^2 - sqrt(d))*sqrt(x)), x) + 1/2*a^2*(
-I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))/c^(1/4) - log
((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(1/4))/sqrt(d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(c\, x^2\right)\right)^2}{\sqrt{d\, x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/(d*x)^(1/2),x)
[Out] int((a + b*atanh(c*x^2))^2/(d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{atanh}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/(d*x)**(1/2),x)
[Out] Integral((a + b*atanh(c*x**2))**2/sqrt(d*x), x)
```

3.92 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=6334

result too large to display

```
[Out] -1/2*(2*a-b*ln(-c*x^2+1))^2/d/(d*x)^(1/2)+2*a*b*c^(1/4)*arctan(-1+c^(1/4)*2
^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d/(d*x)^(1/2)+2*a*b*c^(1/4)*arctan(1+c^(1/4
)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d/(d*x)^(1/2)+a*b*c^(1/4)*ln(1+x*c^(1/2)
-c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d/(d*x)^(1/2)-a*b*c^(1/4)*ln(1+x*
c^(1/2)+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d/(d*x)^(1/2)-1/2*b^2*ln(c
*x^2+1)^2/d/(d*x)^(1/2)-I*b^2*c^(1/4)*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-(-
c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^
(1/2)/d/(d*x)^(1/2)-I*b^2*c^(1/4)*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-(-c)^(
1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)
/d/(d*x)^(1/2)-I*b^2*(-c)^(1/4)*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/
2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(1/
2)/d/(d*x)^(1/2)-I*b^2*(-c)^(1/4)*polylog(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-c^(
1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(1
/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1
/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2
)/d/(d*x)^(1/2)-4*b^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln(2/(1-I*c^(1/4)*x^(
1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln(-2*c^
(1/4)*(1-(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*
x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1
+(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)+c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/
d/(d*x)^(1/2)-2*b^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln((1+I)*(1-c^(1/4)*x^(
1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+4*b^2*c^(1/4)*arctan(c^(
1/4)*x^(1/2))*ln(2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+4*b^2*c^(1/
4)*arctanh(c^(1/4)*x^(1/2))*ln(2/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)
-2*b^2*c^(1/4)*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2)
)/((-c)^(1/4)-c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*c^(
1/4)*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/
4)+c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*arc
tan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*c
^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*ar
ctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+c
^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*c^(1/4)*arctan(
c^(1/4)*x^(1/2))*ln((1-I)*(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2
)/d/(d*x)^(1/2)+2*b^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1
/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1
/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*ln
(-2*(-c)^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c
)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*c^(1/4)*arctanh(c
^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^
(1/2))/(-c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1/4)*
arctan(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c
^(1/4)*x^(1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^
2*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^
(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1
/2)/d/(d*x)^(1/2)-2*b^2*c^(1/4)*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^
(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-(-c)^(1/2))^(1/2)
))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))*ln(-2*
(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^(1/
4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*arctanh((-c)
^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)
*x^(1/2))/(-(-c)^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1/
4)*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c
^(1/4)*x^(1/2))/(-c^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-
```

```
c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(
1/2))/(1-I*(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/d/(
d*x)^(1/2)-2*b^2*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+
x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-c^(1/2))^(1/
2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1/4)*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1
/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1
/2)))*x^(1/2)/d/(d*x)^(1/2)+I*b^2*(-c)^(1/4)*polylog(2,1-(1+I)*(1-(-c)^(1/4
)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+I*b^2*(-c)^(1/4)
*polylog(2,1+(-1+I)*(1+(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2
)/d/(d*x)^(1/2)+I*b^2*c^(1/4)*polylog(2,1-(1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^
(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+I*b^2*c^(1/4)*polylog(2,1+(-1+I)*(1+c
^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-I*b^2*(-c)^(1/
4)*polylog(2,1-2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-I*c^(1/4))/(1-I
*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-I*b^2*c^(1/4)*polylog(2,1+2*c^(
1/4)*(1-(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x
^(1/2)/d/(d*x)^(1/2)-I*b^2*c^(1/4)*polylog(2,1-2*c^(1/4)*(1+(-c)^(1/4)*x^(1
/2))/(I*(-c)^(1/4)+c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-I*
b^2*(-c)^(1/4)*polylog(2,1-2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*c
^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*I*b^2*(-c)^(1/4)*
polylog(2,1-2/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*I*b^2*(-c)^
(1/4)*polylog(2,1-2/(1+I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*I*b^2
*c^(1/4)*polylog(2,1-2/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*I*b^2
*c^(1/4)*polylog(2,1-2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*I*b^2
*(-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))^2*x^(1/2)/d/(d*x)^(1/2)+2*I*b^2*c^(1
/4)*arctan(c^(1/4)*x^(1/2))^2*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*arctan
((-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*ar
ctanh((-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/d/(d*x)^(1/2)-2*b*c^(1/4)*ar
ctan(c^(1/4)*x^(1/2))*(2*a-b*ln(-c*x^2+1))*x^(1/2)/d/(d*x)^(1/2)+2*b*c^(1/4
)*arctanh(c^(1/4)*x^(1/2))*(2*a-b*ln(-c*x^2+1))*x^(1/2)/d/(d*x)^(1/2)+2*b^2
*(-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d/(d*x)^(1/2)-2*
b^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d/(d*x)^(1/2)-2*b^2
*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d/(d*x)^(1/2)+2
*b^2*c^(1/4)*arctanh(c^(1/4)*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d/(d*x)^(1/2)-4*b
^2*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(2/(1-(-c)^(1/4)*x^(1/2)))*x^(1
/2)/d/(d*x)^(1/2)-4*b^2*(-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2/(1-I*(-c
)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*arctan((-c)^(1/4)*
x^(1/2))*ln((1+I)*(1-(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/
d/(d*x)^(1/2)+4*b^2*(-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2/(1+I*(-c)^(1
/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+4*b^2*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(
1/2))*ln(2/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*a
rctan((-c)^(1/4)*x^(1/2))*ln((1-I)*(1+(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x
^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-4*b^2*c^(1/4)*arctanh(c^(1/4)*x^(1/2))*ln(2/
(1-c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*arctan((-c)^(1/
4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-I*c^(1/4))/(1-I
*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*arctanh((-c)^(
1/4)*x^(1/2))^2*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1/4)*arctanh(c^(1/4)*x^(1/2)
)^2*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*polylog(2,1-2/(1-(-c)^(1/4)*x^(1
/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*polylog(2,1-2/(1+(-c)^(1/4)*x^
(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*c^(1/4)*polylog(2,1-2/(1-c^(1/4)*x^(1/2
)))*x^(1/2)/d/(d*x)^(1/2)+b^2*(-c)^(1/4)*polylog(2,1-2*(-c)^(1/4)*(1-c^(1/4
)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/
2)-2*b^2*c^(1/4)*polylog(2,1-2/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+b
^2*c^(1/4)*polylog(2,1+2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4)
)/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+b^2*c^(1/4)*polylog(2,1-2*c^(1
/4)*(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2
)/d/(d*x)^(1/2)+b^2*(-c)^(1/4)*polylog(2,1-2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))
/((-c)^(1/4)+c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-b^2*(-c
)^(1/4)*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1
/4)*x^(1/2))/(-(-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+b^2*c
```

```
^(1/4)*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(
1/2))/(-c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-b^2*(-c)^(1/4)*
polylog(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1
/2))/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+b^2*c^(1/4)*po
lylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^
(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+b^2*(-c)^(1/4)*polylog(2,
1+2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(
1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-b^2*c^(1/4)*polylog(2,1+2*c^(
1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-c^(1/2))^
(1/2)))*x^(1/2)/d/(d*x)^(1/2)+b^2*(-c)^(1/4)*polylog(2,1-2*(-c)^(1/4)*(1+x^
(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-c^(1/2))^(1/2)
))*x^(1/2)/d/(d*x)^(1/2)-b^2*c^(1/4)*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-c^(
1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x
)^(1/2)-2*a*b*ln(c*x^2+1)/d/(d*x)^(1/2)+b^2*ln(-c*x^2+1)*ln(c*x^2+1)/d/(d*x
)^(1/2)
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2),x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2), x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx$$

Mathematica [F] time = 99.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2), x]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\,\text{artanh}\left(cx^2\right)^2+2\,ab\,\text{artanh}\left(cx^2\right)+a^2\right)\sqrt{dx}}{d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d^2
*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/(d*x)^(3/2), x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{arctanh}\left(c\,x^2\right)\right)^2}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2c \int \frac{x^{\frac{3}{2}} \log\left(cx^2+1\right)^2}{4\left(cd^{\frac{3}{2}}x^3 - d^{\frac{3}{2}}x\right)}\,dx - 2\,b^2c \int \frac{x^{\frac{3}{2}} \log\left(cx^2+1\right)\log\left(-cx^2+1\right)}{4\left(cd^{\frac{3}{2}}x^3 - d^{\frac{3}{2}}x\right)}\,dx + 4\,abc \int \frac{x^{\frac{3}{2}} \log\left(cx^2+1\right)}{4\left(cd^{\frac{3}{2}}x^3 - d^{\frac{3}{2}}x\right)}\,dx - 4\,abc \int \frac{x^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*c*integrate(1/4*x^(3/2)*log(c*x^2 + 1)^2/(c*d^(3/2)*x^3 - d^(3/2)*x), x
) - 2*b^2*c*integrate(1/4*x^(3/2)*log(c*x^2 + 1)*log(-c*x^2 + 1)/(c*d^(3/2)
*x^3 - d^(3/2)*x), x) + 4*a*b*c*integrate(1/4*x^(3/2)*log(c*x^2 + 1)/(c*d^(
3/2)*x^3 - d^(3/2)*x), x) - 4*a*b*c*integrate(1/4*x^(3/2)*log(-c*x^2 + 1)/(
c*d^(3/2)*x^3 - d^(3/2)*x), x) + 8*b^2*c*integrate(1/4*x^(3/2)*log(-c*x^2 +
 1)/(c*d^(3/2)*x^3 - d^(3/2)*x), x) + 1/2*a^2*(c*(I*(log(I*c^(1/4)*sqrt(x)
+ 1) - log(-I*c^(1/4)*sqrt(x) + 1))/c^(3/4) - log((sqrt(c)*sqrt(x) - c^(1/4
))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(3/4))/d^(3/2) - 4/(d^(3/2)*sqrt(x))) - b
^2*integrate(1/4*log(c*x^2 + 1)^2/((c*d^(3/2)*x^3 - d^(3/2)*x)*sqrt(x)), x)
 + 2*b^2*integrate(1/4*log(c*x^2 + 1)*log(-c*x^2 + 1)/((c*d^(3/2)*x^3 - d^(
3/2)*x)*sqrt(x)), x) - 4*a*b*integrate(1/4*log(c*x^2 + 1)/((c*d^(3/2)*x^3 -
 d^(3/2)*x)*sqrt(x)), x) + 4*a*b*integrate(1/4*log(-c*x^2 + 1)/((c*d^(3/2)*
x^3 - d^(3/2)*x)*sqrt(x)), x) - 1/2*a^2*c*(I*(log(I*c^(1/4)*sqrt(x) + 1) -
log(-I*c^(1/4)*sqrt(x) + 1))/c^(3/4) - log((sqrt(c)*sqrt(x) - c^(1/4))/(sqr
t(c)*sqrt(x) + c^(1/4)))/c^(3/4))/d^(3/2) - 1/2*b^2*log(-c*x^2 + 1)^2/(d^(3
/2)*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(c\,x^2\right)\right)^2}{(d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/(d*x)^(3/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/(d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(cx^2\right)\right)^2}{(dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/(d*x)**(3/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/(d*x)**(3/2), x)
```

3.93 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=6520

result too large to display

```
[Out] 2/3*a*b*c^(3/4)*arctan(-1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d^2/(d*x
)^(1/2)+2/3*a*b*c^(3/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/d
^2/(d*x)^(1/2)-1/3*a*b*c^(3/4)*ln(1+x*c^(1/2)-c^(1/4)*2^(1/2)*x^(1/2))*2^(1
/2)*x^(1/2)/d^2/(d*x)^(1/2)+1/3*a*b*c^(3/4)*ln(1+x*c^(1/2)+c^(1/4)*2^(1/2)*
x^(1/2))*2^(1/2)*x^(1/2)/d^2/(d*x)^(1/2)-1/6*(2*a-b*ln(-c*x^2+1))^2/d^2/x/(
d*x)^(1/2)+2/3*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2))*ln((1+I)*(1-c^(1/4)*x^(1
/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-4/3*b^2*c^(3/4)*arctan(
c^(1/4)*x^(1/2))*ln(2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+4/3*b^
2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(2/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d
*x)^(1/2)-2/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1
/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1
/2)-2/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(
1/2))/((-c)^(1/4)+c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3
*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+c^(1/4)*x^(1/
2))/((-c)^(1/4)+I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2
)-2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+c^(1/4)
*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1
/2)+2/3*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2))*ln((1-I)*(1+c^(1/4)*x^(1/2))/(1
-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*arctan(c^(1/4)
*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2
))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*(-c)^(
3/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^
(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d^
2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(
1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-(-c)^(1/2))^(1/2)
))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2))*ln(2*c^(
1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(c^(1/4)+I*(-(-c
)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1
/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*
x^(1/2))/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*
c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2
))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/
2)-2/3*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2
)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+I*(-c^(1/2))^(1/2
)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*x^(1/2))*
ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)
^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*c^(3/4)*arctanh(c
^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/
2))/(-c^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)
*arctan((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1
-I*(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+I*(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(
1/2)-2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1
/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-c^(1/2))^(1/2)))
*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1
/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1
/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*I*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2
))^2*x^(1/2)/d^2/(d*x)^(1/2)-2/3*I*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2))^2*x^
(1/2)/d^2/(d*x)^(1/2)-2/3*I*b^2*(-c)^(3/4)*polylog(2,1-2/(1-I*(-c)^(1/4)*x^
(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*I*b^2*(-c)^(3/4)*polylog(2,1-(1+I)*(1-(
-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*I*
b^2*(-c)^(3/4)*polylog(2,1-2/(1+I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1
/2)-1/3*I*b^2*(-c)^(3/4)*polylog(2,1+(-1+I)*(1+(-c)^(1/4)*x^(1/2))/(1-I*(-c
)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*I*b^2*c^(3/4)*polylog(2,1-2/(
```

```
1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*I*b^2*c^(3/4)*polylog(2,1
-2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*I*b^2*c^(3/4)*polylog
(2,1+(-1+I)*(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1
/2)-1/3*I*b^2*c^(3/4)*polylog(2,1-(1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^
(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*(-c)^(3/4)*polylog(2,1-2*(-c)^(1/
4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(
1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*c^(3/4)*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-(
-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x
^(1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*c^(3/4)*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(
-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*
x^(1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*(-c)^(3/4)*polylog(2,1+2*(-c)^(1/4)*(1-x^
(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+I*(-c^(1/2))^
(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*(-c)^(3/4)*polylog(2,1-2*(-c)^(1/
4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+I*(-c^
(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*(-c)^(3/4)*polylog(2,1-2*(
-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2
)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*I*b^2*c^(3/4)*polylog(2,1+2*c^(1/4)*(1-(-c)
^(1/4)*x^(1/2))/(I*(-c)^(1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(
d*x)^(1/2)+1/3*I*b^2*c^(3/4)*polylog(2,1-2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/(
I*(-c)^(1/4)+c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^
2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/d^2/(d*x)^(1/2
)+2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(-c*x^2+1)*x^(1/2)/d^2/(
d*x)^(1/2)+2/3*b*c^(3/4)*arctan(c^(1/4)*x^(1/2))*(2*a-b*ln(-c*x^2+1))*x^(1/
2)/d^2/(d*x)^(1/2)+2/3*b*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*(2*a-b*ln(-c*x^2+
1))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*l
n(c*x^2+1)*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2))*
ln(c*x^2+1)*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*x
^(1/2))*ln(c*x^2+1)*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*c^(3/4)*arctanh(c^(1/4)
*x^(1/2))*ln(c*x^2+1)*x^(1/2)/d^2/(d*x)^(1/2)-4/3*b^2*(-c)^(3/4)*arctanh((-
c)^(1/4)*x^(1/2))*ln(2/(1-(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+4/3*
b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2/(1-I*(-c)^(1/4)*x^(1/2)))*x^
(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln((1+I
)*(1-(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-
4/3*b^2*(-c)^(3/4)*arctan((-c)^(1/4)*x^(1/2))*ln(2/(1+I*(-c)^(1/4)*x^(1/2))
)*x^(1/2)/d^2/(d*x)^(1/2)+4/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)*x^(1/2))*ln
(2/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*(-c)^(3/4)*arcta
n((-c)^(1/4)*x^(1/2))*ln((1-I)*(1+(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/
2)))*x^(1/2)/d^2/(d*x)^(1/2)-4/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(2/
(1-c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctan((-c)
^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-I*c^(1/4))/
(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctan
h((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/
4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+4/3*b^2*c^(3/4)*arctan(
c^(1/4)*x^(1/2))*ln(2/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^
2*c^(3/4)*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))/(I*(
-c)^(1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c
^(3/4)*arctan(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/(I*(-c)^
(1/4)+c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*ln(-c
*x^2+1)*ln(c*x^2+1)/d^2/x/(d*x)^(1/2)+2/3*b^2*(-c)^(3/4)*arctanh((-c)^(1/4)
*x^(1/2))^2*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2)
)^2*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*polylog(2,1-2/(1-(-c)^(1/4)*
x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*polylog(2,1-2/(1+(-c)^
(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*polylog(2,1-2/(1-c^
(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*(-c)^(3/4)*polylog(2,1-2*(-
c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^(1/2)))*x
^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*c^(3/4)*polylog(2,1-2/(1+c^(1/4)*x^(1/2)))*x
^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*c^(3/4)*polylog(2,1+2*c^(1/4)*(1-(-c)^(1/4)*
x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+
1/3*b^2*c^(3/4)*polylog(2,1-2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+c^
```

```
(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*(-c)^(3/4)*poly
log(2,1-2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/4)+c^(1/4))/(1+(-c)^(1/4)
*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*b^2*(-c)^(3/4)*polylog(2,1+2*(-c)^(1
/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-(
-c)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*c^(3/4)*polylog(2,1+2*c^
(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-(-c)^
(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*b^2*(-c)^(3/4)*polylog(2,1-2*(-c
)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+
(-(-c)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*c^(3/4)*polylog(2,1-2
*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-(-c
)^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*(-c)^(3/4)*polylog(2,1+2*(
-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+
(-c^(1/2))^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*b^2*c^(3/4)*polylog(2,1+2*c^
(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1/4)+(-c^(1/2))
^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+1/3*b^2*(-c)^(3/4)*polylog(2,1-2*(-c)^(1/4
)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-c^(1/2)
)^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-1/3*b^2*c^(3/4)*polylog(2,1-2*c^(1/4)*(1+
x^(1/2)*(-c^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1/2)))*x
^(1/2)/d^2/(d*x)^(1/2)-2/3*a*b*ln(c*x^2+1)/d^2/x/(d*x)^(1/2)-1/6*b^2*ln(c*x
^2+1)^2/d^2/x/(d*x)^(1/2)
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2),x]
[Out] Defer[Int][(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2), x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx$$

Mathematica [F] time = 52.22, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2),x]
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2), x]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\operatorname{artanh}\left(cx^2\right)^2+2\,ab\operatorname{artanh}\left(cx^2\right)+a^2\right)\sqrt{dx}}{d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d^3
*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2}{(dx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/(d*x)^(5/2), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{arctanh}\left(c\, x^2\right)\right)^2}{(dx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$3\, b^2 c \int \frac{x^{\frac{3}{2}} \log\left(cx^2+1\right)^2}{12\left(cd^{\frac{5}{2}}x^4 - d^{\frac{5}{2}}x^2\right)}\, dx - 6\, b^2 c \int \frac{x^{\frac{3}{2}} \log\left(cx^2+1\right) \log\left(-cx^2+1\right)}{12\left(cd^{\frac{5}{2}}x^4 - d^{\frac{5}{2}}x^2\right)}\, dx + 12\, abc \int \frac{x^{\frac{3}{2}} \log\left(cx^2+1\right)}{12\left(cd^{\frac{5}{2}}x^4 - d^{\frac{5}{2}}x^2\right)}\, dx - 12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 3*b^2*c*integrate(1/12*x^(3/2)*log(c*x^2 + 1)^2/(c*d^(5/2)*x^4 - d^(5/2)*x^
2), x) - 6*b^2*c*integrate(1/12*x^(3/2)*log(c*x^2 + 1)*log(-c*x^2 + 1)/(c*d
^(5/2)*x^4 - d^(5/2)*x^2), x) + 12*a*b*c*integrate(1/12*x^(3/2)*log(c*x^2 +
 1)/(c*d^(5/2)*x^4 - d^(5/2)*x^2), x) - 12*a*b*c*integrate(1/12*x^(3/2)*log
(-c*x^2 + 1)/(c*d^(5/2)*x^4 - d^(5/2)*x^2), x) + 8*b^2*c*integrate(1/12*x^(
3/2)*log(-c*x^2 + 1)/(c*d^(5/2)*x^4 - d^(5/2)*x^2), x) + 1/6*a^2*(3*(-I*c^(
3/4)*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1)) - c^(3/4)*l
og((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4))))/d^(5/2) - 4/(d
^(5/2)*x^(3/2))) - 3*b^2*integrate(1/12*log(c*x^2 + 1)^2/((c*d^(5/2)*x^4 -
d^(5/2)*x^2)*sqrt(x)), x) + 6*b^2*integrate(1/12*log(c*x^2 + 1)*log(-c*x^2
+ 1)/((c*d^(5/2)*x^4 - d^(5/2)*x^2)*sqrt(x)), x) - 12*a*b*integrate(1/12*lo
g(c*x^2 + 1)/((c*d^(5/2)*x^4 - d^(5/2)*x^2)*sqrt(x)), x) + 12*a*b*integrate
(1/12*log(-c*x^2 + 1)/((c*d^(5/2)*x^4 - d^(5/2)*x^2)*sqrt(x)), x) - 1/2*a^2
*c*(-I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))/c^(1/4) -
 log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(1/4))/d^(5
/2) - 1/6*b^2*log(-c*x^2 + 1)^2/(d^(5/2)*x^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(c\, x^2\right)\right)^2}{(d\, x)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^2))^2/(d*x)^(5/2),x)
```

```
[Out] int((a + b*atanh(c*x^2))^2/(d*x)^(5/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{atanh}\left(cx^2\right)\right)^2}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/(d*x)**(5/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/(d*x)**(5/2), x)
```

3.94 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \, dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x^2))^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^2])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^2])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \, dx = \int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \, dx$$

Mathematica [A] time = 1.85, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^3, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3 \operatorname{artanh}\left(cx^2\right)^3 + 3\,ab^2 \operatorname{artanh}\left(cx^2\right)^2 + 3\,a^2b \operatorname{artanh}\left(cx^2\right) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")
[Out] integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh
(c*x^2) + a^3)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(cx^2\right) + a\right)^3 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^3*(d*x)^m, x)
```

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b \operatorname{arctanh}\left(c\,x^2\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^2))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^2))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3d^mxx^m\log\left(-cx^2+1\right)^3}{8\,(m+1)}+\frac{(dx)^{m+1}\,a^3}{d(m+1)}+\int\frac{\left(b^3cd^m(m+1)x^2-b^3d^m(m+1)\right)x^m\log\left(cx^2+1\right)^3+6\left(ab^2cd^m(m+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*d^m*x*x^m*log(-c*x^2 + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1)
) + integrate(1/8*((b^3*c*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(c*x^2
+ 1)^3 + 6*(a*b^2*c*d^m*(m + 1)*x^2 - a*b^2*d^m*(m + 1))*x^m*log(c*x^2 + 1)
^2 + 12*(a^2*b*c*d^m*(m + 1)*x^2 - a^2*b*d^m*(m + 1))*x^m*log(c*x^2 + 1) +
3*((b^3*c*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(c*x^2 + 1) - 2*(a*b^2*
d^m*(m + 1) - (a*b^2*c*d^m*(m + 1) + b^3*c*d^m)*x^2)*x^m)*log(-c*x^2 + 1)^2
 - 3*((b^3*c*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(c*x^2 + 1)^2 + 4*(a
*b^2*c*d^m*(m + 1)*x^2 - a*b^2*d^m*(m + 1))*x^m*log(c*x^2 + 1) + 4*(a^2*b*c
*d^m*(m + 1)*x^2 - a^2*b*d^m*(m + 1))*x^m)*log(-c*x^2 + 1))/(c*(m + 1)*x^2
- m - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^2))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^2))^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a+b\,\mathrm{atanh}\left(cx^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**2))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x**2))**3, x)
```

3.95 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 \, dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x^2))^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^2])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^2])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 \, dx = \int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 \, dx$$

Mathematica [A] time = 1.25, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^2, x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \operatorname{artanh}\left(cx^2\right)^2 + 2\,ab \operatorname{artanh}\left(cx^2\right) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^2) + a)^2*(d*x)^m, x)
```

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\operatorname{arctanh}\left(c\,x^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^2))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2 d^m x x^m \log\left(-cx^2+1\right)^2}{4\,(m+1)} + \frac{(dx)^{m+1}\,a^2}{d(m+1)} - \int -\frac{\left(b^2 c d^m (m+1)x^2 - b^2 d^m (m+1)\right)x^m \log\left(cx^2+1\right)^2 + 4\left(abcd^m(m+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d^m*x*x^m*log(-c*x^2 + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
 - integrate(-1/4*((b^2*c*d^m*(m + 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(c*x^2
+ 1)^2 + 4*(a*b*c*d^m*(m + 1)*x^2 - a*b*d^m*(m + 1))*x^m*log(c*x^2 + 1) - 2
*((b^2*c*d^m*(m + 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(c*x^2 + 1) - 2*(a*b*d^m
*(m + 1) - (a*b*c*d^m*(m + 1) + b^2*c*d^m)*x^2)*x^m)*log(-c*x^2 + 1))/(c*(m
 + 1)*x^2 - m - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^2))^2,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^2))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b\,\mathrm{atanh}\left(cx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x**2))**2, x)
```

3.96 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=74

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{4};\frac{m+7}{4};c^2x^4\right)}{d^3(m+1)(m+3)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctanh(c*x^2))/d/(1+m)-2*b*c*(d*x)^(3+m)*hypergeom([1, 3/
4+1/4*m],[7/4+1/4*m],c^2*x^4)/d^3/(1+m)/(3+m)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6097, 16, 364}

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{4};\frac{m+7}{4};c^2x^4\right)}{d^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x^2]))/(d*(1 + m)) - (2*b*c*(d*x)^(3 + m)*H
ypergeometric2F1[1, (3 + m)/4, (7 + m)/4, c^2*x^4])/(d^3*(1 + m)*(3 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{(2bc)\int \frac{x(dx)^{1+m}}{1-c^2x^4}\,dx}{d(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{(2bc)\int \frac{(dx)^{2+m}}{1-c^2x^4}\,dx}{d^2(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{2bc(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{4};\frac{7+m}{4};c^2x^4\right)}{d^3(1+m)(3+m)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.86

$$-\frac{x(dx)^m\left(2bcx^2\,{}_2F_1\left(1,\frac{m+3}{4};\frac{m+7}{4};c^2x^4\right)-(m+3)\left(a+b\tanh^{-1}\left(cx^2\right)\right)\right)}{(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] -((x*(d*x)^m*(-((3 + m)*(a + b*ArcTanh[c*x^2])) + 2*b*c*x^2*Hypergeometric2
F1[1, (3 + m)/4, (7 + m)/4, c^2*x^4]))/((1 + m)*(3 + m)))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\,\text{artanh}\left(cx^2\right)+a\right)(dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^2) + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\,\text{artanh}\left(cx^2\right)+a\right)(dx)^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)*(d*x)^m, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int(dx)^m\left(a+b\,\text{arctanh}\left(c\,x^2\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^2)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(4\,cd^m\int\frac{x^2x^m}{c^2(m+1)x^4-m-1}\,dx+\frac{d^mxx^m\log\left(cx^2+1\right)-d^mxx^m\log\left(-cx^2+1\right)}{m+1}\right)b+\frac{(dx)^{m+1}\,a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*c*d^m*integrate(x^2*x^m/(c^2*(m + 1)*x^4 - m - 1), x) + (d^m*x*x^m*l
og(c*x^2 + 1) - d^m*x*x^m*log(-c*x^2 + 1))/(m + 1))*b + (d*x)^(m + 1)*a/(d*
(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int(d\,x)^m\,\left(a+b\,\text{atanh}\left(c\,x^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^2)),x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b \operatorname{atanh}\left(cx^2\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**2)),x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x**2)), x)
```

3.97 $\int \frac{(dx)^m}{a+b \tanh^{-1}(cx^2)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x^2)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^2]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^2]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)}\,dx$$

Mathematica [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2]), x]
```

fricas [A] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b\,\text{artanh}\left(cx^2\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctanh(c*x^2) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\,\text{artanh}\left(cx^2\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctanh(c*x^2) + a), x)
```

maple [A] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\,\mathrm{arctanh}\left(c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^2)),x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x^2)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\,\mathrm{artanh}\left(cx^2\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^2) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\,\mathrm{atanh}\left(c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x^2)),x)
```

```
[Out] int((d*x)^m/(a + b*atanh(c*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\,\mathrm{atanh}\left(cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**2)),x)
```

```
[Out] Integral((d*x)**m/(a + b*atanh(c*x**2)), x)
```

3.98 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x^2))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^2])^2,x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^2])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}\,dx$$

Mathematica [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2])^2, x]
```

fricas [A] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b^2\,\text{artanh}\left(cx^2\right)^2+2\,ab\,\text{artanh}\left(cx^2\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctanh(c*x^2) + a)^2, x)
```

maple [A] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{m}}{\left(a+b\,\mathrm{arctanh}\left(c\,x^{2}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^2))^2,x)
[Out] int((d*x)^m/(a+b*arctanh(c*x^2))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(c^{2}d^{m}x^{4}-d^{m}\right)x^{m}}{b^{2}cx\log\left(cx^{2}+1\right)-b^{2}cx\log\left(-cx^{2}+1\right)+2\,abcx}+\int -\frac{\left(c^{2}d^{m}(m+3)x^{4}-d^{m}(m-1)\right)x^{m}}{b^{2}cx^{2}\log\left(cx^{2}+1\right)-b^{2}cx^{2}\log\left(-cx^{2}+1\right)+2\,abcx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
[Out] (c^2*d^m*x^4 - d^m)*x^m/(b^2*c*x*log(c*x^2 + 1) - b^2*c*x*log(-c*x^2 + 1) +
 2*a*b*c*x) + integrate(-(c^2*d^m*(m + 3)*x^4 - d^m*(m - 1))*x^m/(b^2*c*x^2
*log(c*x^2 + 1) - b^2*c*x^2*log(-c*x^2 + 1) + 2*a*b*c*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^{m}}{\left(a+b\,\mathrm{atanh}\left(c\,x^{2}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x^2))^2,x)
[Out] int((d*x)^m/(a + b*atanh(c*x^2))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**2))**2,x)
[Out] Timed out
```

3.99 $\int x^{11} \left(a + b \tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=54

$$\frac{1}{12}x^{12}\left(a + b\tanh^{-1}\left(cx^3\right)\right) - \frac{b\tanh^{-1}\left(cx^3\right)}{12c^4} + \frac{bx^3}{12c^3} + \frac{bx^9}{36c}$$

```
[Out] 1/12*b*x^3/c^3+1/36*b*x^9/c-1/12*b*arctanh(c*x^3)/c^4+1/12*x^12*(a+b*arctan
h(c*x^3))
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = $\{6097, 275, 302, 206\}$

$$\frac{1}{12}x^{12}\left(a + b\tanh^{-1}\left(cx^3\right)\right) + \frac{bx^3}{12c^3} - \frac{b\tanh^{-1}\left(cx^3\right)}{12c^4} + \frac{bx^9}{36c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^3)/(12*c^3) + (b*x^9)/(36*c) - (b*ArcTanh[c*x^3])/(12*c^4) + (x^12*(a
+ b*ArcTanh[c*x^3]))/12
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^{11}\left(a+b\tanh^{-1}\left(cx^3\right)\right)dx=\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{4}(bc)\int\frac{x^{14}}{1-c^2x^6}dx$$
$$=\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{12}(bc)\,\text{Subst}\left(\int\frac{x^4}{1-c^2x^2}dx,x,x^3\right)$$
$$=\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{12}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^4}-\frac{x^2}{c^2}+\frac{1}{c^4\left(1-c^2x^2\right)}\right)dx\right.$$
$$=\frac{bx^3}{12c^3}+\frac{bx^9}{36c}+\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1-c^2x^2}dx,x,x^3\right)}{12c^3}$$
$$=\frac{bx^3}{12c^3}+\frac{bx^9}{36c}-\frac{b\tanh^{-1}\left(cx^3\right)}{12c^4}+\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)$$

Mathematica [A] time = 0.02, size = 78, normalized size = 1.44

$$\frac{ax^{12}}{12}+\frac{b\log\left(1-cx^3\right)}{24c^4}-\frac{b\log\left(cx^3+1\right)}{24c^4}+\frac{bx^3}{12c^3}+\frac{bx^9}{36c}+\frac{1}{12}bx^{12}\tanh^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^3)/(12*c^3) + (b*x^9)/(36*c) + (a*x^12)/12 + (b*x^12*ArcTanh[c*x^3])/1
2 + (b*Log[1 - c*x^3])/(24*c^4) - (b*Log[1 + c*x^3])/(24*c^4)
```

fricas [A] time = 0.60, size = 64, normalized size = 1.19

$$\frac{6\,ac^4x^{12}+2\,bc^3x^9+6\,bcx^3+3\left(bc^4x^{12}-b\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{72\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/72*(6*a*c^4*x^12 + 2*b*c^3*x^9 + 6*b*c*x^3 + 3*(b*c^4*x^12 - b)*log(-(c*x
^3 + 1)/(c*x^3 - 1)))/c^4
```

giac [A] time = 0.14, size = 78, normalized size = 1.44

$$\frac{1}{24}\,bx^{12}\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{12}\,ax^{12}+\frac{bx^9}{36\,c}+\frac{bx^3}{12\,c^3}-\frac{b\log\left(cx^3+1\right)}{24\,c^4}+\frac{b\log\left(cx^3-1\right)}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/24*b*x^12*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/12*a*x^12 + 1/36*b*x^9/c + 1/
12*b*x^3/c^3 - 1/24*b*log(c*x^3 + 1)/c^4 + 1/24*b*log(c*x^3 - 1)/c^4
```

maple [A] time = 0.03, size = 66, normalized size = 1.22

$$\frac{x^{12}a}{12}+\frac{b\,x^{12}\operatorname{arctanh}\left(c\,x^3\right)}{12}+\frac{b\,x^9}{36c}+\frac{b\,x^3}{12c^3}-\frac{b\ln\left(c\,x^3+1\right)}{24c^4}+\frac{b\ln\left(c\,x^3-1\right)}{24c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/12*x^12*a+1/12*b*x^12*arctanh(c*x^3)+1/36*b*x^9/c+1/12*b*x^3/c^3-1/24*b/c
^4*ln(c*x^3+1)+1/24*b/c^4*ln(c*x^3-1)
```

maxima [A] time = 0.30, size = 69, normalized size = 1.28

$$\frac{1}{12}\,ax^{12}+\frac{1}{72}\left(6\,x^{12}\,\mathrm{artanh}\left(cx^3\right)+c\left(\frac{2\left(c^2x^9+3\,x^3\right)}{c^4}-\frac{3\,\log\left(cx^3+1\right)}{c^5}+\frac{3\,\log\left(cx^3-1\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/12*a*x^12 + 1/72*(6*x^12*arctanh(c*x^3) + c*(2*(c^2*x^9 + 3*x^3)/c^4 - 3*
log(c*x^3 + 1)/c^5 + 3*log(c*x^3 - 1)/c^5))*b
```

mupad [B] time = 1.11, size = 69, normalized size = 1.28

$$\frac{a\,x^{12}}{12}+\frac{b\,x^3}{12\,c^3}+\frac{b\,x^9}{36\,c}+\frac{b\,x^{12}\,\ln\left(c\,x^3+1\right)}{24}-\frac{b\,x^{12}\,\ln\left(1-c\,x^3\right)}{24}+\frac{b\,\mathrm{atan}\left(c\,x^3\,1\mathrm{i}\right)\,1\mathrm{i}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^12)/12 + (b*x^3)/(12*c^3) + (b*x^9)/(36*c) + (b*atan(c*x^3*1i)*1i)/(12
*c^4) + (b*x^12*log(c*x^3 + 1))/24 - (b*x^12*log(1 - c*x^3))/24
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

3.100 $\int x^8 \left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=48

$$\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{18c^3}+\frac{bx^6}{18c}$$

```
[Out] 1/18*b*x^6/c+1/9*x^9*(a+b*arctanh(c*x^3))+1/18*b*ln(-c^2*x^6+1)/c^3
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6097, 266, 43}

$$\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{18c^3}+\frac{bx^6}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^6)/(18*c) + (x^9*(a + b*ArcTanh[c*x^3]))/9 + (b*Log[1 - c^2*x^6])/(18*
c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx &= \frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{3}(bc)\int\frac{x^{11}}{1-c^2x^6}\,dx\\
&= \frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{18}(bc)\,\text{Subst}\left(\int\frac{x}{1-c^2x}\,dx,x,x^6\right)\\
&= \frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{18}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^2}-\frac{1}{c^2\left(-1+c^2x\right)}\right)dx,x,x^6\right)\\
&= \frac{bx^6}{18c}+\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{18c^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.10

$$\frac{ax^9}{9}+\frac{b\log\left(1-c^2x^6\right)}{18c^3}+\frac{bx^6}{18c}+\frac{1}{9}bx^9\tanh^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^6)/(18*c) + (a*x^9)/9 + (b*x^9*ArcTanh[c*x^3])/9 + (b*Log[1 - c^2*x^6]
)/(18*c^3)
```

fricas [A] time = 0.57, size = 62, normalized size = 1.29

$$\frac{bc^3x^9\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2\,ac^3x^9+bc^2x^6+b\log\left(c^2x^6-1\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/18*(b*c^3*x^9*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a*c^3*x^9 + b*c^2*x^6 + b
*log(c^2*x^6 - 1))/c^3
```

giac [A] time = 0.16, size = 57, normalized size = 1.19

$$\frac{1}{18}\,bx^9\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{9}\,ax^9+\frac{bx^6}{18\,c}+\frac{b\log\left(c^2x^6-1\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/18*b*x^9*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/9*a*x^9 + 1/18*b*x^6/c + 1/18*
b*log(c^2*x^6 - 1)/c^3
```

maple [A] time = 0.02, size = 45, normalized size = 0.94

$$\frac{x^9a}{9}+\frac{b\,x^9\operatorname{arctanh}\left(c\,x^3\right)}{9}+\frac{b\,x^6}{18c}+\frac{b\ln\left(c^2x^6-1\right)}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/9*x^9*a+1/9*b*x^9*arctanh(c*x^3)+1/18*b*x^6/c+1/18*b/c^3*ln(c^2*x^6-1)
```

maxima [A] time = 0.31, size = 46, normalized size = 0.96

$$\frac{1}{9}\,ax^9+\frac{1}{18}\left(2\,x^9\operatorname{artanh}\left(cx^3\right)+\left(\frac{x^6}{c^2}+\frac{\log\left(c^2x^6-1\right)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/9*a*x^9 + 1/18*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)*c
)*b
```

mupad [B] time = 0.82, size = 61, normalized size = 1.27

$$\frac{a\,x^9}{9}+\frac{b\,\ln\left(c^2\,x^6-1\right)}{18\,c^3}+\frac{b\,x^6}{18\,c}+\frac{b\,x^9\,\ln\left(c\,x^3+1\right)}{18}-\frac{b\,x^9\,\ln\left(1-c\,x^3\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^9)/9 + (b*log(c^2*x^6 - 1))/(18*c^3) + (b*x^6)/(18*c) + (b*x^9*log(c*x
^3 + 1))/18 - (b*x^9*log(1 - c*x^3))/18
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

3.101 $\int x^5 \left(a + b \tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=43

$$\frac{1}{6}x^6 \left(a + b \tanh^{-1}\left(cx^3\right)\right) - \frac{b \tanh^{-1}\left(cx^3\right)}{6c^2} + \frac{bx^3}{6c}$$

```
[Out] 1/6*b*x^3/c-1/6*b*arctanh(c*x^3)/c^2+1/6*x^6*(a+b*arctanh(c*x^3))
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6097, 275, 321, 206}

$$\frac{1}{6}x^6 \left(a + b \tanh^{-1}\left(cx^3\right)\right) - \frac{b \tanh^{-1}\left(cx^3\right)}{6c^2} + \frac{bx^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^3)/(6*c) - (b*ArcTanh[c*x^3])/(6*c^2) + (x^6*(a + b*ArcTanh[c*x^3]))/6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^5 \left(a+b\tanh^{-1}\left(cx^3\right)\right) dx = \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right) - \frac{1}{2}(bc)\int \frac{x^8}{1-c^2x^6}\,dx$$
$$= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right) - \frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{x^2}{1-c^2x^2}\,dx, x, x^3\right)$$
$$= \frac{bx^3}{6c} + \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right) - \frac{b\,\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, x^3\right)}{6c}$$
$$= \frac{bx^3}{6c} - \frac{b\tanh^{-1}\left(cx^3\right)}{6c^2} + \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)$$

Mathematica [A] time = 0.01, size = 67, normalized size = 1.56

$$\frac{ax^6}{6} + \frac{b\log\left(1-cx^3\right)}{12c^2} - \frac{b\log\left(cx^3+1\right)}{12c^2} + \frac{bx^3}{6c} + \frac{1}{6}bx^6\tanh^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^3)/(6*c) + (a*x^6)/6 + (b*x^6*ArcTanh[c*x^3])/6 + (b*Log[1 - c*x^3])/(
12*c^2) - (b*Log[1 + c*x^3])/(12*c^2)
```

fricas [A] time = 0.71, size = 54, normalized size = 1.26

$$\frac{2\,ac^2x^6 + 2\,bcx^3 + \left(bc^2x^6 - b\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{12\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*a*c^2*x^6 + 2*b*c*x^3 + (b*c^2*x^6 - b)*log(-(c*x^3 + 1)/(c*x^3 - 1
)))/c^2
```

giac [B] time = 0.14, size = 181, normalized size = 4.21

$$\frac{1}{3}c\left(\frac{\left(cx^3+1\right)b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{\left(cx^3-1\right)\left(\frac{\left(cx^3+1\right)^2c^3}{\left(cx^3-1\right)^2} - \frac{2\left(cx^3+1\right)c^3}{cx^3-1} + c^3\right)} + \frac{\frac{2\left(cx^3+1\right)a}{cx^3-1} + \frac{\left(cx^3+1\right)b}{cx^3-1} - b}{\frac{\left(cx^3+1\right)^2c^3}{\left(cx^3-1\right)^2} - \frac{2\left(cx^3+1\right)c^3}{cx^3-1} + c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/3*c*((c*x^3 + 1)*b*log(-(c*x^3 + 1)/(c*x^3 - 1))/((c*x^3 - 1)*((c*x^3 + 1
)^2*c^3/(c*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3 - 1) + c^3)) + (2*(c*x^3 +
 1)*a/(c*x^3 - 1) + (c*x^3 + 1)*b/(c*x^3 - 1) - b)/((c*x^3 + 1)^2*c^3/(c*x^
3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3 - 1) + c^3))
```

maple [A] time = 0.03, size = 57, normalized size = 1.33

$$\frac{x^6a}{6} + \frac{b\,x^6\,\text{arctanh}\left(c\,x^3\right)}{6} + \frac{b\,x^3}{6c} - \frac{b\ln\left(c\,x^3+1\right)}{12c^2} + \frac{b\ln\left(c\,x^3-1\right)}{12c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctanh(c*x^3)+1/6*b*x^3/c-1/12*b/c^2*ln(c*x^3+1)+1/12*
b/c^2*ln(c*x^3-1)
```

maxima [A] time = 0.30, size = 58, normalized size = 1.35

$$\frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\,\mathrm{artanh}\left(cx^3\right)+c\left(\frac{2\,x^3}{c^2}-\frac{\log\left(cx^3+1\right)}{c^3}+\frac{\log\left(cx^3-1\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctanh(c*x^3) + c*(2*x^3/c^2 - log(c*x^3 + 1)/c^3
+ log(c*x^3 - 1)/c^3))*b
```

mupad [B] time = 0.96, size = 60, normalized size = 1.40

$$\frac{a\,x^6}{6}+\frac{b\,x^3}{6\,c}+\frac{b\,x^6\,\ln\left(c\,x^3+1\right)}{12}-\frac{b\,x^6\,\ln\left(1-c\,x^3\right)}{12}+\frac{b\,\mathrm{atan}\left(c\,x^3\,1\mathrm{i}\right)\,1\mathrm{i}}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^6)/6 + (b*x^3)/(6*c) + (b*atan(c*x^3*1i)*1i)/(6*c^2) + (b*x^6*log(c*x^
3 + 1))/12 - (b*x^6*log(1 - c*x^3))/12
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

3.102 $\int x^2 \left(a+b\tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=37

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{6c}$$

```
[Out] 1/3*x^3*(a+b*arctanh(c*x^3))+1/6*b*ln(-c^2*x^6+1)/c
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 260}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (x^3*(a + b*ArcTanh[c*x^3]))/3 + (b*Log[1 - c^2*x^6])/(6*c)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right) dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)-(bc)\int \frac{x^5}{1-c^2x^6} dx\\ &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{6c}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.14

$$\frac{ax^3}{3}+\frac{b\log\left(1-c^2x^6\right)}{6c}+\frac{1}{3}bx^3\tanh^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*ArcTanh[c*x^3])/3 + (b*Log[1 - c^2*x^6])/(6*c)
```

fricas [A] time = 0.66, size = 50, normalized size = 1.35

$$\frac{bcx^3\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2\,acx^3+b\log\left(c^2x^6-1\right)}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/6*(b*c*x^3*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a*c*x^3 + b*log(c^2*x^6 - 1)
)/c
```

giac [B] time = 0.16, size = 188, normalized size = 5.08

$$\frac{1}{3}ax^3 + \frac{1}{3}bc\left(\frac{\log\left(\frac{|-cx^3-1|}{|cx^3-1|}\right)}{c^2} - \frac{\log\left(\left|-\frac{cx^3+1}{cx^3-1}+1\right|\right)}{c^2} + \frac{\log\left(-\frac{\frac{c\left(\frac{cx^3+1}{cx^3-1}+1\right)}{\frac{(cx^3+1)c}{cx^3-1}-c}+1}{\frac{c\left(\frac{cx^3+1}{cx^3-1}+1\right)}{\frac{(cx^3+1)c}{cx^3-1}-c}-1}\right)}{c^2\left(\frac{cx^3+1}{cx^3-1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 + 1/3*b*c*(log(abs(-c*x^3 - 1)/abs(c*x^3 - 1))/c^2 - log(abs(-(c*
x^3 + 1)/(c*x^3 - 1) + 1))/c^2 + log(-(c*((c*x^3 + 1)/(c*x^3 - 1) + 1)/((c*
x^3 + 1)*c/(c*x^3 - 1) - c) + 1)/(c*((c*x^3 + 1)/(c*x^3 - 1) + 1)/((c*x^3 +
 1)*c/(c*x^3 - 1) - c) - 1))/(c^2*((c*x^3 + 1)/(c*x^3 - 1) - 1)))
```

maple [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{x^3 a}{3} + \frac{b\,x^3 \operatorname{arctanh}\left(c\,x^3\right)}{3} + \frac{b \ln\left(-c^2 x^6 + 1\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c*x^3)+1/6*b*ln(-c^2*x^6+1)/c
```

maxima [A] time = 0.30, size = 37, normalized size = 1.00

$$\frac{1}{3}\,ax^3 + \frac{\left(2\,cx^3 \operatorname{artanh}\left(cx^3\right) + \log\left(-c^2x^6 + 1\right)\right)b}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*c*x^3*arctanh(c*x^3) + log(-c^2*x^6 + 1))*b/c
```

mupad [B] time = 0.78, size = 52, normalized size = 1.41

$$\frac{a\,x^3}{3} + \frac{b\,\ln\left(c^2\,x^6 - 1\right)}{6\,c} + \frac{b\,x^3\,\ln\left(c\,x^3 + 1\right)}{6} - \frac{b\,x^3\,\ln\left(1 - c\,x^3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^3)/3 + (b*log(c^2*x^6 - 1))/(6*c) + (b*x^3*log(c*x^3 + 1))/6 - (b*x^3*
log(1 - c*x^3))/6
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

3.103 $\int \frac{a+b\tanh^{-1}(cx^3)}{x}\,dx$

Optimal. Leaf size=30

$$a\log(x)-\frac{1}{6}b\text{Li}_2\left(-cx^3\right)+\frac{1}{6}b\text{Li}_2\left(cx^3\right)$$

```
[Out] a*ln(x)-1/6*b*polylog(2,-c*x^3)+1/6*b*polylog(2,c*x^3)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6095, 5912}

$$-\frac{1}{6}b\text{PolyLog}\left(2,-cx^3\right)+\frac{1}{6}b\text{PolyLog}\left(2,cx^3\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x,x]
```

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^3)])/6 + (b*PolyLog[2, c*x^3])/6
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,x^3\right)\\ &= a\log(x)-\frac{1}{6}b\text{Li}_2\left(-cx^3\right)+\frac{1}{6}b\text{Li}_2\left(cx^3\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.93

$$a\log(x)+\frac{1}{6}b\left(\text{Li}_2\left(cx^3\right)-\text{Li}_2\left(-cx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x,x]
```

```
[Out] a*Log[x] + (b*(-PolyLog[2, -(c*x^3)] + PolyLog[2, c*x^3]))/6
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx^3\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x,x, algorithm="fricas")
[Out] integral((b*arctanh(c*x^3) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{artanh}\left(cx^3\right) + a}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^3) + a)/x, x)
```

maple [C] time = 0.10, size = 92, normalized size = 3.07

$$a \ln(x)+b \ln(x) \operatorname{arctanh}\left(c\, x^3\right)+\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(c\_Z^3-1\right)}\left(\ln(x) \ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}-\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(c\_Z^3+1\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x,x)
[Out] a*ln(x)+b*ln(x)*arctanh(c*x^3)+1/2*b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x
)/_R1),_R1=RootOf(_Z^3*c-1))-1/2*b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1),_R1=RootOf(_Z^3*c+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\, b \int \frac{\log\left(cx^3+1\right) - \log\left(-cx^3+1\right)}{x}\, dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x,x, algorithm="maxima")
[Out] 1/2*b*integrate((log(c*x^3 + 1) - log(-c*x^3 + 1))/x, x) + a*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{atanh}\left(c\, x^3\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x,x)
[Out] int((a + b*atanh(c*x^3))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x,x)
[Out] Timed out
```

3.104 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^4}\,dx$

Optimal. Leaf size=40

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{3x^3}-\frac{1}{6}bc\log\left(1-c^2x^6\right)+bc\log(x)$$

[Out] 1/3*(-a-b*arctanh(c*x^3))/x^3+b*c*ln(x)-1/6*b*c*ln(-c^2*x^6+1)

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 266, 36, 29, 31}

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{3x^3}-\frac{1}{6}bc\log\left(1-c^2x^6\right)+bc\log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTanh[c*x^3])/x^4,x]

[Out] -(a + b*ArcTanh[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 - c^2*x^6])/6

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^4}\,dx = -\frac{a+b\tanh^{-1}\left(cx^3\right)}{3x^3} + (bc)\int \frac{1}{x\left(1-c^2x^6\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{3x^3} + \frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1-c^2x\right)}\,dx, x, x^6\right)$$
$$= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{3x^3} + \frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^6\right) + \frac{1}{6}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x}\,dx\right.$$
$$= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{3x^3} + bc\log(x) - \frac{1}{6}bc\log\left(1-c^2x^6\right)$$

Mathematica [A] time = 0.01, size = 45, normalized size = 1.12

$$-\frac{a}{3x^3} - \frac{1}{6}bc\log\left(1-c^2x^6\right) - \frac{b\tanh^{-1}\left(cx^3\right)}{3x^3} + bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*ArcTanh[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 - c^2*x^6
])/6
```

fricas [A] time = 0.64, size = 55, normalized size = 1.38

$$-\frac{bcx^3\log\left(c^2x^6-1\right) - 6\,bcx^3\log(x) + b\log\left(-\frac{cx^3+1}{cx^3-1}\right) + 2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c*x^3*log(c^2*x^6 - 1) - 6*b*c*x^3*log(x) + b*log(-(c*x^3 + 1)/(c*x
^3 - 1)) + 2*a)/x^3
```

giac [A] time = 0.15, size = 51, normalized size = 1.28

$$-\frac{1}{6}\,bc\log\left(c^2x^6-1\right) + bc\log(x) - \frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{6\,x^3} - \frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*b*c*log(c^2*x^6 - 1) + b*c*log(x) - 1/6*b*log(-(c*x^3 + 1)/(c*x^3 - 1)
)/x^3 - 1/3*a/x^3
```

maple [A] time = 0.03, size = 49, normalized size = 1.22

$$-\frac{a}{3x^3} - \frac{b\,\mathrm{arctanh}\left(c\,x^3\right)}{3x^3} + bc\ln(x) - \frac{bc\ln\left(c\,x^3-1\right)}{6} - \frac{bc\ln\left(c\,x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x^3)+b*c*ln(x)-1/6*b*c*ln(c*x^3-1)-1/6*b*c*l
n(c*x^3+1)
```

maxima [A] time = 0.31, size = 41, normalized size = 1.02

$$-\frac{1}{6}\left(c\left(\log\left(c^2x^6-1\right)-\log\left(x^6\right)\right)+\frac{2\ \mathrm{artanh}\left(cx^3\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*(log(c^2*x^6 - 1) - log(x^6)) + 2*arctanh(c*x^3)/x^3)*b - 1/3*a/x^3
```

mupad [B] time = 0.85, size = 55, normalized size = 1.38

$$b\,c\,\ln(x)-\frac{a}{3\,x^3}-\frac{b\,c\,\ln\left(c^2\,x^6-1\right)}{6}-\frac{b\,\ln\left(c\,x^3+1\right)}{6\,x^3}+\frac{b\,\ln\left(1-c\,x^3\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^4,x)
```

```
[Out] b*c*log(x) - a/(3*x^3) - (b*c*log(c^2*x^6 - 1))/6 - (b*log(c*x^3 + 1))/(6*x
^3) + (b*log(1 - c*x^3))/(6*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**4,x)
```

```
[Out] Timed out
```

3.105 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^7} \, dx$

Optimal. Leaf size=41

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6}+\frac{1}{6}bc^2\tanh^{-1}\left(cx^3\right)-\frac{bc}{6x^3}$$

```
[Out] -1/6*b*c/x^3+1/6*b*c^2*arctanh(c*x^3)+1/6*(-a-b*arctanh(c*x^3))/x^6
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 275, 325, 206}

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6}+\frac{1}{6}bc^2\tanh^{-1}\left(cx^3\right)-\frac{bc}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^7,x]
```

```
[Out] -(b*c)/(6*x^3) + (b*c^2*ArcTanh[c*x^3])/6 - (a + b*ArcTanh[c*x^3])/(6*x^6)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^7}\,dx = -\frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6} + \frac{1}{2}(bc)\int \frac{1}{x^4\left(1-c^2x^6\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6} + \frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1-c^2x^2\right)}\,dx, x, x^3\right)$$
$$= -\frac{bc}{6x^3} - \frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6} + \frac{1}{6}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, x^3\right)$$
$$= -\frac{bc}{6x^3} + \frac{1}{6}bc^2\tanh^{-1}\left(cx^3\right) - \frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 1.59

$$-\frac{a}{6x^6} - \frac{1}{12}bc^2\log\left(1-cx^3\right) + \frac{1}{12}bc^2\log\left(cx^3+1\right) - \frac{bc}{6x^3} - \frac{b\tanh^{-1}\left(cx^3\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^7,x]
```

```
[Out] -1/6*a/x^6 - (b*c)/(6*x^3) - (b*ArcTanh[c*x^3])/(6*x^6) - (b*c^2*Log[1 - c*
x^3])/12 + (b*c^2*Log[1 + c*x^3])/12
```

fricas [A] time = 0.56, size = 49, normalized size = 1.20

$$-\frac{2\,bcx^3 - \left(bc^2x^6 - b\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right) + 2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(2*b*c*x^3 - (b*c^2*x^6 - b)*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a)/x^6
```

giac [A] time = 0.15, size = 67, normalized size = 1.63

$$\frac{1}{12}\,bc^2\log\left(cx^3+1\right) - \frac{1}{12}\,bc^2\log\left(cx^3-1\right) - \frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{12\,x^6} - \frac{bcx^3+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="giac")
```

```
[Out] 1/12*b*c^2*log(c*x^3 + 1) - 1/12*b*c^2*log(c*x^3 - 1) - 1/12*b*log(-(c*x^3
+ 1)/(c*x^3 - 1))/x^6 - 1/6*(b*c*x^3 + a)/x^6
```

maple [A] time = 0.03, size = 55, normalized size = 1.34

$$-\frac{a}{6x^6} - \frac{b\,\text{arctanh}\left(c\,x^3\right)}{6x^6} - \frac{bc}{6x^3} - \frac{b\,c^2\ln\left(c\,x^3-1\right)}{12} + \frac{b\,c^2\ln\left(c\,x^3+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^7,x)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctanh(c*x^3)-1/6*b*c/x^3-1/12*b*c^2*ln(c*x^3-1)+1/12
*b*c^2*ln(c*x^3+1)
```

maxima [A] time = 0.31, size = 51, normalized size = 1.24

$$\frac{1}{12}\left(\left(c\log\left(cx^3+1\right)-c\log\left(cx^3-1\right)-\frac{2}{x^3}\right)c-\frac{2\,\operatorname{artanh}\left(cx^3\right)}{x^6}\right)b-\frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^
6)*b - 1/6*a/x^6
```

mupad [B] time = 1.02, size = 52, normalized size = 1.27

$$\frac{b\,c^2\,\operatorname{atanh}\left(c\,x^3\right)}{6}-\frac{\frac{a}{6}+\frac{b\,\ln\left(c\,x^3+1\right)}{12}-\frac{b\,\ln\left(1-c\,x^3\right)}{12}+\frac{b\,c\,x^3}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^7,x)
```

```
[Out] (b*c^2*atanh(c*x^3))/6 - (a/6 + (b*log(c*x^3 + 1))/12 - (b*log(1 - c*x^3))/
12 + (b*c*x^3)/6)/x^6
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**7,x)
```

```
[Out] Timed out
```

3.106 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^{10}}\,dx$

Optimal. Leaf size=56

$$-\frac{a+b\tanh^{-1}(cx^3)}{9x^9}+\frac{1}{3}bc^3\log(x)-\frac{1}{18}bc^3\log\left(1-c^2x^6\right)-\frac{bc}{18x^6}$$

```
[Out] -1/18*b*c/x^6+1/9*(-a-b*arctanh(c*x^3))/x^9+1/3*b*c^3*ln(x)-1/18*b*c^3*ln(-c^2*x^6+1)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = $\{6097, 266, 44\}$

$$-\frac{a+b\tanh^{-1}(cx^3)}{9x^9}-\frac{1}{18}bc^3\log\left(1-c^2x^6\right)+\frac{1}{3}bc^3\log(x)-\frac{bc}{18x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^10,x]
```

```
[Out] -(b*c)/(18*x^6) - (a + b*ArcTanh[c*x^3])/(9*x^9) + (b*c^3*Log[x])/3 - (b*c^3*Log[1 - c^2*x^6])/18
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx^3)}{x^{10}}\,dx &= -\frac{a+b\tanh^{-1}(cx^3)}{9x^9}+\frac{1}{3}(bc)\int \frac{1}{x^7\left(1-c^2x^6\right)}\,dx\\
&= -\frac{a+b\tanh^{-1}(cx^3)}{9x^9}+\frac{1}{18}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1-c^2x\right)}\,dx,x,x^6\right)\\
&= -\frac{a+b\tanh^{-1}(cx^3)}{9x^9}+\frac{1}{18}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{c^2}{x}-\frac{c^4}{-1+c^2x}\right)dx,x,x^6\right)\\
&= -\frac{bc}{18x^6}-\frac{a+b\tanh^{-1}(cx^3)}{9x^9}+\frac{1}{3}bc^3\log(x)-\frac{1}{18}bc^3\log\left(1-c^2x^6\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 1.09

$$-\frac{a}{9x^9}+\frac{1}{3}bc^3\log(x)-\frac{1}{18}bc^3\log\left(1-c^2x^6\right)-\frac{bc}{18x^6}-\frac{b\tanh^{-1}\left(cx^3\right)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^10,x]
```

```
[Out] -1/9*a/x^9 - (b*c)/(18*x^6) - (b*ArcTanh[c*x^3])/(9*x^9) + (b*c^3*Log[x])/3
 - (b*c^3*Log[1 - c^2*x^6])/18
```

fricas [A] time = 0.68, size = 65, normalized size = 1.16

$$-\frac{bc^3x^9\log\left(c^2x^6-1\right)-6\,bc^3x^9\log(x)+bcx^3+b\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2\,a}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="fricas")
```

```
[Out] -1/18*(b*c^3*x^9*log(c^2*x^6 - 1) - 6*b*c^3*x^9*log(x) + b*c*x^3 + b*log(-(
c*x^3 + 1)/(c*x^3 - 1)) + 2*a)/x^9
```

giac [A] time = 0.13, size = 65, normalized size = 1.16

$$-\frac{1}{18}\,bc^3\log\left(c^2x^6-1\right)+\frac{1}{3}\,bc^3\log(x)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{18\,x^9}-\frac{bcx^3+2\,a}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="giac")
```

```
[Out] -1/18*b*c^3*log(c^2*x^6 - 1) + 1/3*b*c^3*log(x) - 1/18*b*log(-(c*x^3 + 1)/(
c*x^3 - 1))/x^9 - 1/18*(b*c*x^3 + 2*a)/x^9
```

maple [A] time = 0.04, size = 63, normalized size = 1.12

$$-\frac{a}{9x^9}-\frac{b\,\mathrm{arctanh}\left(c\,x^3\right)}{9x^9}-\frac{bc}{18x^6}+\frac{b\,c^3\ln(x)}{3}-\frac{b\,c^3\ln\left(c\,x^3-1\right)}{18}-\frac{b\,c^3\ln\left(c\,x^3+1\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^10,x)
```

```
[Out] -1/9*a/x^9-1/9*b/x^9*arctanh(c*x^3)-1/18*b*c/x^6+1/3*b*c^3*ln(x)-1/18*b*c^3
*ln(c*x^3-1)-1/18*b*c^3*ln(c*x^3+1)
```

maxima [A] time = 0.30, size = 51, normalized size = 0.91

$$-\frac{1}{18}\left(\left(c^2\log\left(c^2x^6-1\right)-c^2\log\left(x^6\right)+\frac{1}{x^6}\right)c+\frac{2\,\mathrm{artanh}\left(cx^3\right)}{x^9}\right)b-\frac{a}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="maxima")
```

```
[Out] -1/18*((c^2*log(c^2*x^6 - 1) - c^2*log(x^6) + 1/x^6)*c + 2*arctanh(c*x^3)/x
^9)*b - 1/9*a/x^9
```

mupad [**B**] time = 0.90, size = 67, normalized size = 1.20

$$\frac{b\,c^3\,\ln(x)}{3}-\frac{b\,c^3\,\ln\left(c^2\,x^6-1\right)}{18}-\frac{a}{9\,x^9}-\frac{b\,c}{18\,x^6}-\frac{b\,\ln\left(c\,x^3+1\right)}{18\,x^9}+\frac{b\,\ln\left(1-c\,x^3\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^10,x)
```

```
[Out] (b*c^3*log(x))/3 - (b*c^3*log(c^2*x^6 - 1))/18 - a/(9*x^9) - (b*c)/(18*x^6)
 - (b*log(c*x^3 + 1))/(18*x^9) + (b*log(1 - c*x^3))/(18*x^9)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**10,x)
```

```
[Out] Timed out
```

3.107 $\int x^3 \left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=174

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)}{16c^{4/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)}{16c^{4/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{8c^{4/3}}-\frac{\sqrt{3}\,b}{}$$

```
[Out] 3/4*b*x/c-1/4*b*arctanh(c^(1/3)*x)/c^(4/3)+1/4*x^4*(a+b*arctanh(c*x^3))+1/1
6*b*ln(1-c^(1/3)*x+c^(2/3)*x^2)/c^(4/3)-1/16*b*ln(1+c^(1/3)*x+c^(2/3)*x^2)/
c^(4/3)-1/8*b*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)-1/
8*b*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)
```

Rubi [A] time = 0.22, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6097, 321, 210, 634, 618, 204, 628, 206}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)}{16c^{4/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)}{16c^{4/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{8c^{4/3}}-\frac{\sqrt{3}\,b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x)/(4*c) + (Sqrt[3]*b*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/(8*c^
(4/3)) - (Sqrt[3]*b*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(8*c^(4/3))
- (b*ArcTanh[c^(1/3)*x])/(4*c^(4/3)) + (x^4*(a + b*ArcTanh[c*x^3]))/4 + (b*
Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(16*c^(4/3)) - (b*Log[1 + c^(1/3)*x + c^(
2/3)*x^2])/(16*c^(4/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)dx=\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{4}(3bc)\int\frac{x^6}{1-c^2x^6}dx$$
$$=\frac{3bx}{4c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{(3b)\int\frac{1}{1-c^2x^6}dx}{4c}$$
$$=\frac{3bx}{4c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\int\frac{1}{1-c^{2/3}x^2}dx}{4c}-\frac{b\int\frac{1-\frac{\sqrt[3]{c}x}{2}}{1-\sqrt[3]{c}x+c^{2/3}x^2}dx}{4c}-\frac{b\int\frac{1}{1+\sqrt[3]{}}$$
$$=\frac{3bx}{4c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}x+c^{2/3}x^2}dx}{16c^{4/3}}-\frac{b\int\frac{}{1+}}{}$$
$$=\frac{3bx}{4c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}-$$
$$=\frac{3bx}{4c}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{8c^{4/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{8c^{4/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a\right.$$

Mathematica [A] time = 0.04, size = 196, normalized size = 1.13

$$\frac{ax^4}{4}+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{16c^{4/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{16c^{4/3}}+\frac{b\log\left(1-\sqrt[3]{c}x\right)}{8c^{4/3}}-\frac{b\log\left(\sqrt[3]{c}x+1\right)}{8c^{4/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{2\sqrt[3]{c}x-}{\sqrt{3}}\right.}{8c^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x)/(4*c) + (a*x^4)/4 - (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/
(8*c^(4/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(8*c^(4/3)) + (
b*x^4*ArcTanh[c*x^3])/4 + (b*Log[1 - c^(1/3)*x])/(8*c^(4/3)) - (b*Log[1 + c
```

```
^(1/3)*x])/(8*c^(4/3)) + (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(16*c^(4/3))
- (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(16*c^(4/3))
```

fricas [A] time = 0.75, size = 981, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 + sqrt(3)*b*
c*sqrt((-c)^(1/3)/c)*log((2*c*x^3 - sqrt(3)*(2*c*x^2 + (-c)^(2/3)*x + (-c)^
(1/3))*sqrt((-c)^(1/3)/c) + 3*(-c)^(1/3)*x - 1)/(c*x^3 + 1)) + sqrt(3)*b*c*
sqrt(-1/c^(2/3))*log((2*c*x^3 - sqrt(3)*(2*c*x^2 - c^(2/3)*x - c^(1/3))*sqr
t(-1/c^(2/3)) - 3*c^(1/3)*x + 1)/(c*x^3 - 1)) + 12*b*c*x + b*(-c)^(2/3)*log
(c*x^2 - (-c)^(2/3)*x - (-c)^(1/3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(
1/3)) - 2*b*(-c)^(2/3)*log(c*x + (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3
)))/c^2, 1/16*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 - 2*
sqrt(3)*b*c*sqrt(-(-c)^(1/3)/c)*arctan(1/3*sqrt(3)*(2*(-c)^(2/3)*x + (-c)^(
1/3))*sqrt(-(-c)^(1/3)/c)) + sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c*x^3 - sq
rt(3)*(2*c*x^2 - c^(2/3)*x - c^(1/3))*sqrt(-1/c^(2/3)) - 3*c^(1/3)*x + 1)/(
c*x^3 - 1)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c)^(2/3)*x - (-c)^(1/3)
) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(2/3)*log(c*x + (
-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2, 1/16*(2*b*c^2*x^4*log(-(c
*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 + sqrt(3)*b*c*sqrt((-c)^(1/3)/c)*log((
2*c*x^3 - sqrt(3)*(2*c*x^2 + (-c)^(2/3)*x + (-c)^(1/3))*sqrt((-c)^(1/3)/c)
+ 3*(-c)^(1/3)*x - 1)/(c*x^3 + 1)) - 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)
*(2*c^(2/3)*x + c^(1/3))/c^(1/3)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c
)^(2/3)*x - (-c)^(1/3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*
(-c)^(2/3)*log(c*x + (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2, 1/1
6*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 - 2*sqrt(3)*b*c*
sqrt(-(-c)^(1/3)/c)*arctan(1/3*sqrt(3)*(2*(-c)^(2/3)*x + (-c)^(1/3))*sqrt(-
(-c)^(1/3)/c)) - 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1
/3))/c^(1/3)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c)^(2/3)*x - (-c)^(1/
3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(2/3)*log(c*x +
 (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2]
```

giac [A] time = 0.20, size = 207, normalized size = 1.19

$$\frac{1}{16}\,bc^7\left(\frac{2\left(-\frac{1}{c}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right|\right)}{c^8}-\frac{2\sqrt{3}\,|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,c^{\frac{1}{3}}\left(2x+\frac{1}{c^{\frac{1}{3}}}\right)\right)}{c^9}-\frac{2\sqrt{3}\left(-c^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{1}{c}\right)\right.}{3\left(-\frac{1}{c}\right)^{\frac{1}{3}}}\right.}{c^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/16*b*c^7*(2*(-1/c)^(1/3)*log(abs(x - (-1/c)^(1/3)))/c^8 - 2*sqrt(3)*abs(c
)^(2/3)*arctan(1/3*sqrt(3)*c^(1/3)*(2*x + 1/c^(1/3)))/c^9 - 2*sqrt(3)*(-c^2
)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-1/c)^(1/3))/(-1/c)^(1/3))/c^9 - abs(c)^
(2/3)*log(x^2 + x/c^(1/3) + 1/c^(2/3))/c^9 + 2*log(abs(x - 1/c^(1/3)))/c^(2
5/3) - (-c^2)^(1/3)*log(x^2 + x*(-1/c)^(1/3) + (-1/c)^(2/3))/c^9) + 1/8*b*x
^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/4*a*x^4 + 3/4*b*x/c
```

maple [A] time = 0.04, size = 184, normalized size = 1.06

$$\frac{x^4 a}{4}+\frac{b\,x^4 \operatorname{arctanh}\left(c\,x^3\right)}{4}+\frac{3bx}{4c}+\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x\right.}{8c^2\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x^3)+3/4*b*x/c+1/8*b/c^2/(1/c)^(2/3)*ln(x-(1/
c)^(1/3))-1/16*b/c^2/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))-1/8*b/c^
2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/8*b/c^2/(1/
c)^(2/3)*ln(x+(1/c)^(1/3))+1/16*b/c^2/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c
)^(2/3))-1/8*b/c^2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-
1))
```

maxima [A] time = 0.40, size = 162, normalized size = 0.93

$$\frac{1}{4}\,ax^4+\frac{1}{16}\left(4\,x^4\operatorname{artanh}\left(cx^3\right)-c\left(\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}-\frac{12\,x}{c^2}+\frac{\log\left(c^{\frac{2}{3}}x^2+\right.}{c^{\frac{7}{3}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/16*(4*x^4*arctanh(c*x^3) - c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2
*c^(2/3)*x + c^(1/3))/c^(1/3))/c^(7/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^
(2/3)*x - c^(1/3))/c^(1/3))/c^(7/3) - 12*x/c^2 + log(c^(2/3)*x^2 + c^(1/3)*
x + 1)/c^(7/3) - log(c^(2/3)*x^2 - c^(1/3)*x + 1)/c^(7/3) + 2*log((c^(1/3)*
x + 1)/c^(1/3))/c^(7/3) - 2*log((c^(1/3)*x - 1)/c^(1/3))/c^(7/3)))*b
```

mupad [B] time = 1.57, size = 125, normalized size = 0.72

$$\frac{a\,x^4}{4}+\frac{b\left(-\frac{\operatorname{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\operatorname{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\operatorname{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)1\mathrm{i}}{4\,c^{4/3}}+\frac{3\,b\,x}{4\,c}+\frac{b\,x^4\,\ln\left(c\,x^3+1\right)}{8}-\frac{b\,x^4\,\ln\left(1-c\,x^3\right)}{8}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^4)/4 + (b*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)/2 - atan((c^(1/3)*x*(3^(
1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/(4*c^(4/3)) + (3*b*x)/(4*c) + (b
*x^4*log(c*x^3 + 1))/8 - (b*x^4*log(1 - c*x^3))/8 - (3^(1/2)*b*(atan((c^(1/
3)*x*(3^(1/2) - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/(8*c^(4/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

3.108 $\int \left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=101

$$ax+\frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{b\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}+bx\tanh^{-1}\left(cx^3\right)$$

```
[Out] a*x+b*x*arctanh(c*x^3)+1/2*b*ln(1-c^(2/3)*x^2)/c^(1/3)-1/4*b*ln(1+c^(2/3)*x
^2+c^(4/3)*x^4)/c^(1/3)+1/2*b*arctan(1/3*(1+2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)
/c^(1/3)
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6091, 275, 292, 31, 634, 617, 204, 628}

$$ax+\frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+bx\tanh^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x^3],x]
```

```
[Out] a*x + (Sqrt[3]*b*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c^(1/3)) + b*x*Arc
Tanh[c*x^3] + (b*Log[1 - c^(2/3)*x^2])/(2*c^(1/3)) - (b*Log[1 + c^(2/3)*x^2
 + c^(4/3)*x^4])/(4*c^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(cx^3\right)\right)dx = ax+b\int \tanh^{-1}\left(cx^3\right)dx$$
$$= ax+bx\tanh^{-1}\left(cx^3\right)-(3bc)\int \frac{x^3}{1-c^2x^6}\,dx$$
$$= ax+bx\tanh^{-1}\left(cx^3\right)-\frac{1}{2}(3bc)\,\text{Subst}\left(\int \frac{x}{1-c^2x^3}\,dx,x,x^2\right)$$
$$= ax+bx\tanh^{-1}\left(cx^3\right)-\frac{1}{2}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x}\,dx,x,x^2\right)+\frac{1}{2}\left(b\sqrt[3]{c}\right)\text{Subst}\,($$
$$= ax+bx\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\,\text{Subst}\left(\int \frac{c^{2/3}+2c^{4/3}x}{1+c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right)}{4\sqrt[3]{c}}+$$
$$= ax+bx\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}}-\frac{(3b)\,\text{Su}}{}$$
$$= ax+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+bx\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1+c^{}\right.}{4}$$

Mathematica [A] time = 0.04, size = 136, normalized size = 1.35

$$ax-\frac{b\left(\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)+\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)-2\log\left(1-\sqrt[3]{c}\,x\right)-2\log\left(\sqrt[3]{c}\,x+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{}}{}\right.\right.}{4\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x^3],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^3] - (b*(-2*Sqrt[3]*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]
] + 2*Sqrt[3]*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]] - 2*Log[1 - c^(1/3)*x] - 2*
Log[1 + c^(1/3)*x] + Log[1 - c^(1/3)*x + c^(2/3)*x^2] + Log[1 + c^(1/3)*x +
 c^(2/3)*x^2]))/(4*c^(1/3))
```

fricas [A] time = 1.00, size = 260, normalized size = 2.57

$$\left[\frac{\sqrt{3}\,bc\sqrt{-\frac{1}{c^{\frac{2}{3}}}}\log\left(\frac{2\,c^{2}x^{6}-3\,c^{\frac{2}{3}}x^{2}+\sqrt{3}\left(2\,c^{\frac{5}{3}}x^{4}-cx^{2}-c^{\frac{1}{3}}\right)\sqrt{-\frac{1}{c^{\frac{2}{3}}}}+1}{c^{2}x^{6}-1}\right)+2\,bcx\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)+4\,acx-bc^{\frac{2}{3}}\log\left(c^{2}x^{4}+c^{\frac{4}{3}}x^{2}+c^{\frac{2}{3}}\right.}{4\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c^2*x^6 - 3*c^(2/3)*x^2 + sqrt(3)
*(2*c^(5/3)*x^4 - c*x^2 - c^(1/3))*sqrt(-1/c^(2/3)) + 1)/(c^2*x^6 - 1)) + 2
*b*c*x*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 + c^
(4/3)*x^2 + c^(2/3)) + 2*b*c^(2/3)*log(c*x^2 - c^(1/3)))/c, 1/4*(2*b*c*x*lo
g(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)*(2*c*x
^2 + c^(1/3))/c^(1/3)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 + c^(4/3)*x^2 + c^
(2/3)) + 2*b*c^(2/3)*log(c*x^2 - c^(1/3)))/c]
```

giac [A] time = 0.13, size = 109, normalized size = 1.08

$$\frac{1}{4}\left(\left(c\left(\frac{2\sqrt{3}\,|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{2}+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{2}}-\frac{|c|^{\frac{2}{3}}\log\left(x^{4}+\frac{x^{2}}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{2}}+\frac{2\log\left(\left|x^{2}-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{4}{3}}}\right)+2\,x\log\left(-\frac{cx^{3}+}{cx^{3}-}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^3),x, algorithm="giac")
```

```
[Out] 1/4*(c*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2/3))*
abs(c)^(2/3))/c^2 - abs(c)^(2/3)*log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3
))/c^2 + 2*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(4/3)) + 2*x*log(-(c*x^3 +
 1)/(c*x^3 - 1)))*b + a*x
```

maple [A] time = 0.03, size = 99, normalized size = 0.98

$$ax+bx\,\mathrm{arctanh}\left(c\,x^{3}\right)+\frac{b\ln\left(x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{4}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^3),x)
```

```
[Out] a*x+b*x*arctanh(c*x^3)+1/2*b/c/(1/c^2)^(1/3)*ln(x^2-(1/c^2)^(1/3))-1/4*b/c/
(1/c^2)^(1/3)*ln(x^4+(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b*3^(1/2)/c/(1/c^
2)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2+1))
```

maxima [A] time = 0.40, size = 90, normalized size = 0.89

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}-\frac{\log\left(c^{\frac{4}{3}}x^4+c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{4}{3}}}+\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2-1}{c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}\right)+4x\operatorname{artanh}\left(cx^3\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^3),x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3))/c^(2/3))/c^(
4/3) - log(c^(4/3)*x^4 + c^(2/3)*x^2 + 1)/c^(4/3) + 2*log((c^(2/3)*x^2 - 1)
/c^(2/3))/c^(4/3)) + 4*x*arctanh(c*x^3))*b + a*x
```

mupad [B] time = 2.76, size = 107, normalized size = 1.06

$$ax+\frac{b\ln\left(c^{2/3}x^2-1\right)}{2c^{1/3}}-\frac{\ln\left(4c^{2/3}x^2+2-\sqrt{3}\,2\mathrm{i}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{4c^{1/3}}-\frac{\ln\left(4c^{2/3}x^2+2+\sqrt{3}\,2\mathrm{i}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{4c^{1/3}}+\frac{b\,x\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c*x^3),x)
```

```
[Out] a*x + (b*log(c^(2/3)*x^2 - 1))/(2*c^(1/3)) - (log(4*c^(2/3)*x^2 - 3^(1/2)*2
i + 2)*(b + 3^(1/2)*b*1i))/(4*c^(1/3)) - (log(3^(1/2)*2i + 4*c^(2/3)*x^2 +
2)*(b - 3^(1/2)*b*1i))/(4*c^(1/3)) + (b*x*log(c*x^3 + 1))/2 - (b*x*log(1 -
c*x^3))/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**3),x)
```

```
[Out] Timed out
```

3.109 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^3}\,dx$

Optimal. Leaf size=165

$$-\frac{a+b\tanh^{-1}(cx^3)}{2x^2}-\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)+\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)-\frac{1}{4}\sqrt{3}\,bc^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}}{\sqrt{3}}\right.$$

```
[Out] 1/2*b*c^(2/3)*arctanh(c^(1/3)*x)+1/2*(-a-b*arctanh(c*x^3))/x^2-1/8*b*c^(2/3
)*ln(1-c^(1/3)*x+c^(2/3)*x^2)+1/8*b*c^(2/3)*ln(1+c^(1/3)*x+c^(2/3)*x^2)+1/4
*b*c^(2/3)*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)+1/4*b*c^(2/3)
*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6097, 210, 634, 618, 204, 628, 206}

$$-\frac{a+b\tanh^{-1}(cx^3)}{2x^2}-\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)+\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)-\frac{1}{4}\sqrt{3}\,bc^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}}{\sqrt{3}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^3,x]
```

```
[Out] -(Sqrt[3]*b*c^(2/3)*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/4 + (Sqrt[3]
*b*c^(2/3)*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/4 + (b*c^(2/3)*ArcTan
h[c^(1/3)*x])/2 - (a + b*ArcTanh[c*x^3])/(2*x^2) - (b*c^(2/3)*Log[1 - c^(1/
3)*x + c^(2/3)*x^2])/8 + (b*c^(2/3)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^3}\,dx = -\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}+\frac{1}{2}(3bc)\int \frac{1}{1-c^2x^6}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}+\frac{1}{2}(bc)\int \frac{1}{1-c^{2/3}x^2}\,dx+\frac{1}{2}(bc)\int \frac{1-\frac{\sqrt[3]{c}\,x}{2}}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx+\frac{1}{2}($$
$$= \frac{1}{2}bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}\left(bc^{2/3}\right)\int \frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx+\frac{1}{8}$$
$$= \frac{1}{2}bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}bc^{2/3}\log\left(1-\sqrt[3]{c}\,x+c^{2/3}x^2\right)+\frac{1}{8}bc^{2}$$
$$= -\frac{1}{4}\sqrt{3}\,bc^{2/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\,x}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{3}\,bc^{2/3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)+\frac{1}{2}bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\right.$$

Mathematica [A] time = 0.06, size = 187, normalized size = 1.13

$$-\frac{a}{2x^2}-\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)+\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)-\frac{1}{4}bc^{2/3}\log\left(1-\sqrt[3]{c}\,x\right)+\frac{1}{4}bc^{2/3}\log\left(\sqrt[3]{c}\,x+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (Sqrt[3]*b*c^(2/3)*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/4 + (Sq
rt[3]*b*c^(2/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/4 - (b*ArcTanh[c*x^3])/(
2*x^2) - (b*c^(2/3)*Log[1 - c^(1/3)*x])/4 + (b*c^(2/3)*Log[1 + c^(1/3)*x])/
4 - (b*c^(2/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/8 + (b*c^(2/3)*Log[1 + c^(
1/3)*x + c^(2/3)*x^2])/8
```

fricas [A] time = 0.65, size = 228, normalized size = 1.38

$$-\frac{2\sqrt{3}\left(-c^2\right)^{\frac{1}{3}}bx^2\arctan\left(\frac{2\sqrt{3}\left(-c^2\right)^{\frac{2}{3}}x+\sqrt{3}c}{3c}\right)-2\sqrt{3}b\left(c^2\right)^{\frac{1}{3}}x^2\arctan\left(\frac{2\sqrt{3}\left(c^2\right)^{\frac{2}{3}}x-\sqrt{3}c}{3c}\right)+\left(-c^2\right)^{\frac{1}{3}}bx^2\log\left(c^2x^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(3)*(-c^2)^(1/3)*b*x^2*arctan(1/3*(2*sqrt(3)*(-c^2)^(2/3)*x + s
qrt(3)*c)/c) - 2*sqrt(3)*b*(c^2)^(1/3)*x^2*arctan(1/3*(2*sqrt(3)*(c^2)^(2/3
)*x - sqrt(3)*c)/c) + (-c^2)^(1/3)*b*x^2*log(c^2*x^2 - (-c^2)^(1/3)*c*x + (
-c^2)^(2/3)) + b*(c^2)^(1/3)*x^2*log(c^2*x^2 - (c^2)^(1/3)*c*x + (c^2)^(2/3
)) - 2*(-c^2)^(1/3)*b*x^2*log(c*x + (-c^2)^(1/3)) - 2*b*(c^2)^(1/3)*x^2*log
(c*x + (c^2)^(1/3)) + 2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a)/x^2
```

giac [A] time = 0.21, size = 165, normalized size = 1.00

$$\frac{1}{8}\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{\log\left(x^2+\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}-\frac{\log\left(x^2-\right.}{|c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1/abs(c)^(1/3))*abs(c)^(1/3))/abs(
c)^(1/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1/abs(c)^(1/3))*abs(c)^(1/3)
)/abs(c)^(1/3) + log(x^2 + x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) -
log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) + 2*log(abs(x + 1/a
bs(c)^(1/3)))/abs(c)^(1/3) - 2*log(abs(x - 1/abs(c)^(1/3)))/abs(c)^(1/3))*b
*c - 1/4*b*log(-(c*x^3 + 1)/(c*x^3 - 1))/x^2 - 1/2*a/x^2
```

maple [A] time = 0.03, size = 159, normalized size = 0.96

$$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}\left(c\,x^3\right)}{2x^2}-\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right.}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x^3)-1/4*b/(1/c)^(2/3)*ln(x-(1/c)^(1/3))+1/8
*b/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b/(1/c)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))+1/4*b/(1/c)^(2/3)*ln(x+(1/c)^(1/3))
-1/8*b/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b/(1/c)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

maxima [A] time = 0.40, size = 155, normalized size = 0.94

$$\frac{1}{8}\left(\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\frac{\log\left(c^{\frac{2}{3}}x^2+c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}-\frac{\log\left(c^{\frac{2}{3}}x^2-c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*((2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/3))/c^(1/3)
 + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x - c^(1/3))/c^(1/3))/c^(1/3) +
log(c^(2/3)*x^2 + c^(1/3)*x + 1)/c^(1/3) - log(c^(2/3)*x^2 - c^(1/3)*x + 1)
/c^(1/3) + 2*log((c^(1/3)*x + 1)/c^(1/3))/c^(1/3) - 2*log((c^(1/3)*x - 1)/c
^(1/3))/c^(1/3))*c - 4*arctanh(c*x^3)/x^2)*b - 1/2*a/x^2
```

mupad [B] time = 1.47, size = 118, normalized size = 0.72

$$\frac{b\,\ln\left(1-c\,x^3\right)}{4\,x^2}-\frac{b\,c^{2/3}\,\left(-\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\mathrm{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)\,1\mathrm{i}}{2}-\frac{b\,\ln\left(c\,x^3+1\right)}{4\,x^2}-\frac{a}{2\,x^2}+\frac{\sqrt{3}\,b\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^3,x)
```

```
[Out] (b*log(1 - c*x^3))/(4*x^2) - (b*c^(2/3)*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)
/2 - atan((c^(1/3)*x*(3^(1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/2 - (b*
log(c*x^3 + 1))/(4*x^2) - a/(2*x^2) + (3^(1/2)*b*c^(2/3)*(atan((c^(1/3)*x*(
3^(1/2) - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**3,x)
```

```
[Out] Timed out
```

3.110 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^6}\,dx$

Optimal. Leaf size=115

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)-\frac{1}{10}\sqrt{3}\,bc^{5/3}\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)+\frac{1}{20}bc^{5/3}\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)-$$

```
[Out] -3/10*b*c/x^2+1/5*(-a-b*arctanh(c*x^3))/x^5-1/10*b*c^(5/3)*ln(1-c^(2/3)*x^2
)+1/20*b*c^(5/3)*ln(1+c^(2/3)*x^2+c^(4/3)*x^4)-1/10*b*c^(5/3)*arctan(1/3*(1
+2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6097, 275, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}bc^{5/3}\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)-\frac{1}{10}\sqrt{3}\,bc^{5/3}\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^6,x]
```

```
[Out] (-3*b*c)/(10*x^2) - (Sqrt[3]*b*c^(5/3)*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])
/10 - (a + b*ArcTanh[c*x^3])/(5*x^5) - (b*c^(5/3)*Log[1 - c^(2/3)*x^2])/10
+ (b*c^(5/3)*Log[1 + c^(2/3)*x^2 + c^(4/3)*x^4])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^6}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{5}(3bc)\int \frac{1}{x^3\left(1-c^2x^6\right)}\,dx \\
&= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}(3bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1-c^2x^3\right)}\,dx,x,x^2\right) \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}\left(3bc^3\right)\text{Subst}\left(\int \frac{x}{1-c^2x^3}\,dx,x,x^2\right) \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}\left(bc^{7/3}\right)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x}\,dx,x,x^2\right)-\frac{1}{10}\left(bc^{7/3}\right. \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}\left(bc^{5/3}\right)\text{Subst}\left(\int \frac{c^2}{1+c}\right. \\
&= -\frac{3bc}{10x^2}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}bc^{5/3}\log\left(1+c^{2/3}x^2+\right. \\
&= -\frac{3bc}{10x^2}-\frac{1}{10}\sqrt{3}\,bc^{5/3}\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c\right.
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 196, normalized size = 1.70

$$
-\frac{a}{5x^5}+\frac{1}{20}bc^{5/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)+\frac{1}{20}bc^{5/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)-\frac{1}{10}bc^{5/3}\log\left(1-\sqrt[3]{c}\,x\right)-\frac{1}{10}bc^{5/3}\log\left(\sqrt[3]{\right.}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (3*b*c)/(10*x^2) - (Sqrt[3]*b*c^(5/3)*ArcTan[(-1 + 2*c^(1/3)*x
)/Sqrt[3]])/10 + (Sqrt[3]*b*c^(5/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/10 -
 (b*ArcTanh[c*x^3])/(5*x^5) - (b*c^(5/3)*Log[1 - c^(1/3)*x])/10 - (b*c^(5/3
)*Log[1 + c^(1/3)*x])/10 + (b*c^(5/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/20
+ (b*c^(5/3)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/20
```

fricas [A] time = 0.97, size = 151, normalized size = 1.31

$$-\frac{2\sqrt{3}\left(-c^2\right)^{\frac{1}{3}}bcx^5\arctan\left(\frac{2}{3}\sqrt{3}\left(-c^2\right)^{\frac{1}{3}}x^2-\frac{1}{3}\sqrt{3}\right)+\left(-c^2\right)^{\frac{1}{3}}bcx^5\log\left(c^2x^4+\left(-c^2\right)^{\frac{2}{3}}x^2-\left(-c^2\right)^{\frac{1}{3}}\right)-2\left(-c^2\right)^{\frac{1}{3}}b\cdots}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(3)*(-c^2)^(1/3)*b*c*x^5*arctan(2/3*sqrt(3)*(-c^2)^(1/3)*x^2 -
 1/3*sqrt(3)) + (-c^2)^(1/3)*b*c*x^5*log(c^2*x^4 + (-c^2)^(2/3)*x^2 - (-c^2
)^(1/3)) - 2*(-c^2)^(1/3)*b*c*x^5*log(c^2*x^2 - (-c^2)^(2/3)) + 6*b*c*x^3 +
 2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a)/x^5
```

giac [A] time = 0.17, size = 125, normalized size = 1.09

$$-\frac{1}{20}bc^3\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}-\frac{|c|^{\frac{2}{3}}\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}+\frac{2\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{4}{3}}}\right)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{10\,x^5}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*b*c^3*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2
/3))*abs(c)^(2/3))/c^2 - abs(c)^(2/3)*log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)
^(4/3))/c^2 + 2*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(4/3)) - 1/10*b*log(-
(c*x^3 + 1)/(c*x^3 - 1))/x^5 - 1/10*(3*b*c*x^3 + 2*a)/x^5
```

maple [A] time = 0.04, size = 172, normalized size = 1.50

$$-\frac{a}{5x^5}-\frac{b\,\mathrm{arctanh}\left(c\,x^3\right)}{5x^5}-\frac{3bc}{10x^2}-\frac{bc\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{bc\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{10\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{bc\ln\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctanh(c*x^3)-3/10*b*c/x^2-1/10*b*c/(1/c)^(2/3)*ln(x-
(1/c)^(1/3))+1/20*b*c/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/10*b*
c/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/10*b*c/(1/c
)^(2/3)*ln(x+(1/c)^(1/3))+1/20*b*c/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(
2/3))-1/10*b*c/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

maxima [A] time = 0.42, size = 100, normalized size = 0.87

$$-\frac{1}{20}\left(\left(\left(2\sqrt{3}c^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)-c^{\frac{2}{3}}\log\left(c^{\frac{4}{3}}x^4+c^{\frac{2}{3}}x^2+1\right)+2c^{\frac{2}{3}}\log\left(\frac{c^{\frac{2}{3}}x^2-1}{c^{\frac{2}{3}}}\right)+\frac{6}{x^2}\right)c+\frac{4\ \mathrm{artan}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*sqrt(3)*c^(2/3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3))/c^(2
/3)) - c^(2/3)*log(c^(4/3)*x^4 + c^(2/3)*x^2 + 1) + 2*c^(2/3)*log((c^(2/3)*
x^2 - 1)/c^(2/3)) + 6/x^2)*c + 4*arctanh(c*x^3)/x^5)*b - 1/5*a/x^5
```

mupad [B] time = 3.12, size = 135, normalized size = 1.17

$$\frac{b\ \ln\left(1-c\,x^3\right)}{10\,x^5}-\frac{b\,c^{5/3}\ \ln\left(c^{2/3}\,x^2-1\right)}{10}-\frac{b\ \ln\left(c\,x^3+1\right)}{10\,x^5}-\frac{\frac{3\,b\,c\,x^3}{2}+a}{5\,x^5}+\frac{b\,c^{5/3}\ \ln\left(\sqrt{3}\ c^{2/3}\,x^2-c^{2/3}\,x^2\,1\mathrm{i}-2\mathrm{i}\right)\ \left(1+\sqrt{\right.}\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^6,x)
```

```
[Out] (b*log(1 - c*x^3))/(10*x^5) - (b*c^(5/3)*log(c^(2/3)*x^2 - 1))/10 - (b*log(
c*x^3 + 1))/(10*x^5) - (a + (3*b*c*x^3)/2)/(5*x^5) + (b*c^(5/3)*log(3^(1/2)
*c^(2/3)*x^2 - c^(2/3)*x^2*1i - 2i)*(3^(1/2)*1i + 1))/20 - (b*c^(5/3)*log(-
 c^(2/3)*x^2*1i - 3^(1/2)*c^(2/3)*x^2 - 2i)*(3^(1/2)*1i - 1))/20
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**6,x)
```

```
[Out] Timed out
```

3.111 $\int x^7 \left(a + b \tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=176

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{\sqrt{3}b\tan\cdots}{}$$

```
[Out] 3/40*b*x^5/c-1/8*b*arctanh(c^(1/3)*x)/c^(8/3)+1/8*x^8*(a+b*arctanh(c*x^3))+
1/32*b*ln(1-c^(1/3)*x+c^(2/3)*x^2)/c^(8/3)-1/32*b*ln(1+c^(1/3)*x+c^(2/3)*x^
2)/c^(8/3)+1/16*b*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3
)+1/16*b*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3)
```

Rubi [A] time = 0.28, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6097, 321, 296, 634, 618, 204, 628, 206}

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{\sqrt{3}b\tan\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x^5)/(40*c) - (Sqrt[3]*b*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/(1
6*c^(8/3)) + (Sqrt[3]*b*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(16*c^(8
/3)) - (b*ArcTanh[c^(1/3)*x])/(8*c^(8/3)) + (x^8*(a + b*ArcTanh[c*x^3]))/8
+ (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(32*c^(8/3)) - (b*Log[1 + c^(1/3)*x
+ c^(2/3)*x^2])/(32*c^(8/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^7\left(a+b\tanh^{-1}\left(cx^3\right)\right)dx=\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{8}(3bc)\int\frac{x^{10}}{1-c^2x^6}\,dx$$

$$=\frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{(3b)\int\frac{x^4}{1-c^2x^6}\,dx}{8c}$$

$$=\frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\int\frac{1}{1-c^{2/3}x^2}\,dx}{8c^{7/3}}-\frac{b\int\frac{-\frac{1}{2}-\frac{\sqrt[3]{c}\,x}{2}}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx}{8c^{7/3}}-\frac{b\int$$

$$=\frac{3bx^5}{40c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx}{32c^{8/3}}-\frac{b}{}$$

$$=\frac{3bx^5}{40c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{32c^{8/3}}$$

$$=\frac{3bx^5}{40c}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{16c^{8/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}+\frac{1}{8}x^8$$

Mathematica [A] time = 0.03, size = 198, normalized size = 1.12

$$\frac{ax^8}{8}+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)}{32c^{8/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)}{32c^{8/3}}+\frac{b\log\left(1-\sqrt[3]{c}\,x\right)}{16c^{8/3}}-\frac{b\log\left(\sqrt[3]{c}\,x+1\right)}{16c^{8/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{2\sqrt[3]{}}{}\right.}{16c^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x^5)/(40*c) + (a*x^8)/8 + (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]
])/(16*c^(8/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(16*c^(8/3)
```

```
) + (b*x^8*ArcTanh[c*x^3])/8 + (b*Log[1 - c^(1/3)*x])/(16*c^(8/3)) - (b*Log
[1 + c^(1/3)*x])/(16*c^(8/3)) + (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(32*c^
(8/3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(32*c^(8/3))
```

fricas [A] time = 0.64, size = 248, normalized size = 1.41

$$10\,bc^4x^8\log\left(-\frac{cx^3+1}{cx^3-1}\right)+20\,ac^4x^8+12\,bc^3x^5+10\,\sqrt{3}\,bc\sqrt{-\left(-c^2\right)^{\frac{1}{3}}}\,\arctan\left(\frac{\sqrt{3}\left(2\,cx+\left(-c^2\right)^{\frac{1}{3}}\right)\sqrt{-\left(-c^2\right)^{\frac{1}{3}}}}{3\,c}\right)+10\,\sqrt{3}\,b\left(c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/160*(10*b*c^4*x^8*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 20*a*c^4*x^8 + 12*b*c^3
*x^5 + 10*sqrt(3)*b*c*sqrt(-(-c^2)^(1/3))*arctan(1/3*sqrt(3)*(2*c*x + (-c^2
)^(1/3))*sqrt(-(-c^2)^(1/3))/c) + 10*sqrt(3)*b*(c^2)^(1/6)*c*arctan(1/3*sqr
t(3)*(c^2)^(1/6)*(2*c*x + (c^2)^(1/3))/c) + 5*(-c^2)^(2/3)*b*log(c^2*x^2 +
(-c^2)^(1/3)*c*x + (-c^2)^(2/3)) - 5*b*(c^2)^(2/3)*log(c^2*x^2 + (c^2)^(1/3
)*c*x + (c^2)^(2/3)) - 10*(-c^2)^(2/3)*b*log(c*x - (-c^2)^(1/3)) + 10*b*(c^
2)^(2/3)*log(c*x - (c^2)^(1/3)))/c^4
```

giac [A] time = 0.29, size = 208, normalized size = 1.18

$$\frac{1}{16}\,bx^8\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{8}\,ax^8+\frac{3\,bx^5}{40\,c}-\frac{b\left(-\frac{1}{c}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right|\right)}{16\,c^2}+\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{16\,\left(-c^2\right)^{\frac{1}{3}}\,c^2}+\frac{\sqrt{3}\,b|c|^{\frac{4}{3}}\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/16*b*x^8*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/8*a*x^8 + 3/40*b*x^5/c - 1/16*
b*(-1/c)^(2/3)*log(abs(x - (-1/c)^(1/3)))/c^2 + 1/16*sqrt(3)*b*arctan(1/3*s
qrt(3)*(2*x + (-1/c)^(1/3))/(-1/c)^(1/3))/((-c^2)^(1/3)*c^2) + 1/16*sqrt(3)
*b*abs(c)^(4/3)*arctan(1/3*sqrt(3)*c^(1/3)*(2*x + 1/c^(1/3)))/c^4 - 1/32*b*
log(x^2 + x*(-1/c)^(1/3) + (-1/c)^(2/3))/((-c^2)^(1/3)*c^2) - 1/32*b*log(x^
2 + x/c^(1/3) + 1/c^(2/3))/(c^2*abs(c)^(2/3)) + 1/16*b*log(abs(x - 1/c^(1/3
)))/c^(8/3)
```

maple [A] time = 0.03, size = 186, normalized size = 1.06

$$\frac{x^8a}{8}+\frac{b\,x^8\,\mathrm{arctanh}\left(c\,x^3\right)}{8}+\frac{3b\,x^5}{40c}+\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{16c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{32c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{16c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(\right.}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/8*x^8*a+1/8*b*x^8*arctanh(c*x^3)+3/40*b*x^5/c+1/16*b/c^3/(1/c)^(1/3)*ln(x
-(1/c)^(1/3))-1/32*b/c^3/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/16
*b/c^3*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/16*b/c
^3/(1/c)^(1/3)*ln(x+(1/c)^(1/3))+1/32*b/c^3/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*
x+(1/c)^(2/3))+1/16*b/c^3*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(
1/3)*x-1))
```

maxima [A] time = 0.41, size = 164, normalized size = 0.93

$$\frac{1}{8}ax^8+\frac{1}{160}\left(20x^8\operatorname{artanh}\left(cx^3\right)+\left(\frac{12x^5}{c^2}+\frac{10\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}+\frac{10\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}-\frac{5\,\mathrm{l}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/160*(20*x^8*arctanh(c*x^3) + (12*x^5/c^2 + 10*sqrt(3)*arctan(
1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/3))/c^(11/3) + 10*sqrt(3)*arctan(1
/3*sqrt(3)*(2*c^(2/3)*x - c^(1/3))/c^(1/3))/c^(11/3) - 5*log(c^(2/3)*x^2 +
c^(1/3)*x + 1)/c^(11/3) + 5*log(c^(2/3)*x^2 - c^(1/3)*x + 1)/c^(11/3) - 10*
log((c^(1/3)*x + 1)/c^(1/3))/c^(11/3) + 10*log((c^(1/3)*x - 1)/c^(1/3))/c^(
11/3))*c)*b
```

mupad [B] time = 1.26, size = 127, normalized size = 0.72

$$\frac{a\,x^8}{8}+\frac{b\left(-\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\mathrm{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)\,1\mathrm{i}}{8\,c^{8/3}}+\frac{3\,b\,x^5}{40\,c}+\frac{b\,x^8\,\ln\left(c\,x^3+1\right)}{16}-\frac{b\,x^8\,\ln\left(1-c\,x^3\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^8)/8 + (b*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)/2 - atan((c^(1/3)*x*(3^(
1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/(8*c^(8/3)) + (3*b*x^5)/(40*c) +
 (b*x^8*log(c*x^3 + 1))/16 - (b*x^8*log(1 - c*x^3))/16 + (3^(1/2)*b*(atan((
c^(1/3)*x*(3^(1/2) - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/(16*c^(
8/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

3.112 $\int x^4 \left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=117

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{10c^{5/3}}-\frac{b\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)}{20c^{5/3}}+\frac{3bx^2}{10c}$$

```
[Out] 3/10*b*x^2/c+1/5*x^5*(a+b*arctanh(c*x^3))+1/10*b*ln(1-c^(2/3)*x^2)/c^(5/3)-
1/20*b*ln(1+c^(2/3)*x^2+c^(4/3)*x^4)/c^(5/3)-1/10*b*arctan(1/3*(1+2*c^(2/3)
*x^2)*3^(1/2))*3^(1/2)/c^(5/3)
```

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6097, 275, 321, 200, 31, 634, 617, 204, 628}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)}{20c^{5/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{3bx^2}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x^2)/(10*c) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/(10*c^(5
/3)) + (x^5*(a + b*ArcTanh[c*x^3]))/5 + (b*Log[1 - c^(2/3)*x^2])/(10*c^(5/3
)) - (b*Log[1 + c^(2/3)*x^2 + c^(4/3)*x^4])/(20*c^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)dx=\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{5}(3bc)\int\frac{x^7}{1-c^2x^6}\,dx$$

$$=\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{10}(3bc)\,\text{Subst}\left(\int\frac{x^3}{1-c^2x^3}\,dx,x,x^2\right)$$

$$=\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{(3b)\,\text{Subst}\left(\int\frac{1}{1-c^2x^3}\,dx,x,x^2\right)}{10c}$$

$$=\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1-c^{2/3}x}\,dx,x,x^2\right)}{10c}-\frac{b\,\text{Subst}\left(\int\right.}{}$$

$$=\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\,\text{Subst}\left(\int\frac{c^{2/3}+2c^{4/3}x}{1+c^{2/3}x+c^{4/3}x}\right.}{20c^{5/3}}$$

$$=\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1+c^{2/3}x^2+c^{4/3}\right.}{20c^{5/3}}$$

$$=\frac{3bx^2}{10c}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}$$

Mathematica [A] time = 0.03, size = 198, normalized size = 1.69

$$\frac{ax^5}{5}-\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)}{20c^{5/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)}{20c^{5/3}}+\frac{b\log\left(1-\sqrt[3]{c}\,x\right)}{10c^{5/3}}+\frac{b\log\left(\sqrt[3]{c}\,x+1\right)}{10c^{5/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{2\sqrt[3]{c}}{\sqrt{}}\right.}{10c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x^2)/(10*c) + (a*x^5)/5 - (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]
])/(10*c^(5/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(10*c^(5/3)
) + (b*x^5*ArcTanh[c*x^3])/5 + (b*Log[1 - c^(1/3)*x])/(10*c^(5/3)) + (b*Log
[1 + c^(1/3)*x])/(10*c^(5/3)) - (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(20*c^
(5/3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(20*c^(5/3))
```

fricas [A] time = 0.77, size = 166, normalized size = 1.42

$$\frac{2\,bc^3x^5\log\left(-\frac{cx^3+1}{cx^3-1}\right)+4\,ac^3x^5+6\,bc^2x^2-2\,\sqrt{3}\,b\left(c^2\right)^{\frac{1}{6}}c\arctan\left(-\frac{\sqrt{3}\left(4\,c^2x^4-2\left(c^2\right)^{\frac{2}{3}}x^2+\left(c^2\right)^{\frac{1}{3}}\right)\left(c^2\right)^{\frac{1}{6}}}{8\,c^3x^6+c}\right)-b\left(c^2\right)^{\frac{2}{3}}\log\left(c^2x^4\right.}{20\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/20*(2*b*c^3*x^5*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^3*x^5 + 6*b*c^2*x^2
 - 2*sqrt(3)*b*(c^2)^(1/6)*c*arctan(-sqrt(3)*(4*c^2*x^4 - 2*(c^2)^(2/3)*x^2
 + (c^2)^(1/3))*(c^2)^(1/6)/(8*c^3*x^6 + c)) - b*(c^2)^(2/3)*log(c^2*x^4 +
(c^2)^(2/3)*x^2 + (c^2)^(1/3)) + 2*b*(c^2)^(2/3)*log(c^2*x^2 - (c^2)^(2/3))
)/c^3
```

giac [A] time = 0.15, size = 126, normalized size = 1.08

$$-\frac{1}{20}\,bc^9\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{10}|c|^{\frac{2}{3}}}+\frac{\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}-\frac{2\,\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{c^{10}|c|^{\frac{2}{3}}}\right)+\frac{1}{10}\,bx^5\log\left(-\frac{cx^3+}{cx^3-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] -1/20*b*c^9*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2/3))*abs(c)^(
2/3))/(c^10*abs(c)^(2/3)) + log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/(c
^10*abs(c)^(2/3)) - 2*log(abs(x^2 - 1/abs(c)^(2/3)))/(c^10*abs(c)^(2/3))) +
 1/10*b*x^5*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/5*a*x^5 + 3/10*b*x^2/c
```

maple [A] time = 0.03, size = 114, normalized size = 0.97

$$\frac{a\,x^5}{5}+\frac{b\,x^5\,\mathrm{arctanh}\left(c\,x^3\right)}{5}+\frac{3b\,x^2}{10c}+\frac{b\ln\left(x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^4+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/5*a*x^5+1/5*b*x^5*arctanh(c*x^3)+3/10*b*x^2/c+1/10*b/c^3/(1/c^2)^(2/3)*ln
(x^2-(1/c^2)^(1/3))-1/20*b/c^3/(1/c^2)^(2/3)*ln(x^4+(1/c^2)^(1/3)*x^2+(1/c^
2)^(2/3))-1/10*b/c^3/(1/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1
/3)*x^2+1))
```

maxima [A] time = 0.41, size = 103, normalized size = 0.88

$$\frac{1}{5}ax^5+\frac{1}{20}\left(4x^5\operatorname{artanh}\left(cx^3\right)+c\left(\frac{6x^2}{c^2}-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{8}{3}}}-\frac{\log\left(c^{\frac{4}{3}}x^4+c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{8}{3}}}+\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2-}{c^{\frac{2}{3}}}\right.}{c^{\frac{8}{3}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctanh(c*x^3) + c*(6*x^2/c^2 - 2*sqrt(3)*arctan(1/
3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3))/c^(2/3))/c^(8/3) - log(c^(4/3)*x^4 + c^
(2/3)*x^2 + 1)/c^(8/3) + 2*log((c^(2/3)*x^2 - 1)/c^(2/3))/c^(8/3)))*b
```

mupad [B] time = 2.44, size = 124, normalized size = 1.06

$$\frac{a\,x^5}{5}+\frac{b\,\ln\left(1-c^{2/3}\,x^2\right)}{10\,c^{5/3}}+\frac{3\,b\,x^2}{10\,c}-\frac{\ln\left(2\,c^{2/3}\,x^2+1-\sqrt{3}\,1\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}}-\frac{\ln\left(2\,c^{2/3}\,x^2+1+\sqrt{3}\,1\mathrm{i}\right)\,\left(b+\sqrt{3}\,b\,1\right.}{20\,c^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^5)/5 + (b*log(1 - c^(2/3)*x^2))/(10*c^(5/3)) + (3*b*x^2)/(10*c) - (log
(2*c^(2/3)*x^2 - 3^(1/2)*1i + 1)*(b - 3^(1/2)*b*1i))/(20*c^(5/3)) - (log(3^
(1/2)*1i + 2*c^(2/3)*x^2 + 1)*(b + 3^(1/2)*b*1i))/(20*c^(5/3)) + (b*x^5*log
(c*x^3 + 1))/10 - (b*x^5*log(1 - c*x^3))/10
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

3.113 $\int x\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=165

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)}{8c^{2/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)}{8c^{2/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{\sqrt{3}\,b\,\mathrm{ta}}{}$$

```
[Out] -1/2*b*arctanh(c^(1/3)*x)/c^(2/3)+1/2*x^2*(a+b*arctanh(c*x^3))+1/8*b*ln(1-c
^(1/3)*x+c^(2/3)*x^2)/c^(2/3)-1/8*b*ln(1+c^(1/3)*x+c^(2/3)*x^2)/c^(2/3)+1/4
*b*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)+1/4*b*arctan(
1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)
```

Rubi [A] time = 0.25, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6097, 296, 634, 618, 204, 628, 206}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)}{8c^{2/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)}{8c^{2/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}\,x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{\sqrt{3}\,b\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] -(Sqrt[3]*b*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (Sqrt[
3]*b*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) - (b*ArcTanh[c^
(1/3)*x])/(2*c^(2/3)) + (x^2*(a + b*ArcTanh[c*x^3]))/2 + (b*Log[1 - c^(1/3)
*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(8*c^
(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}(3bc)\int\frac{x^4}{1-c^2x^6}\,dx$$

$$=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\int\frac{1}{1-c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}-\frac{\sqrt[3]{c}x}{2}}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}+\frac{\sqrt[3]{c}x}{2}}{1+\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}$$

$$=-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8c^{2/3}}-\frac{b\int\frac{\sqrt[3]{c}+\ldots}{1+\sqrt[3]{c}\ldots}}{8c\ldots}$$

$$=-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}-\frac{b\,\mathrm{l}\ldots}{\ldots}$$

$$=-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\ldots\right.$$

Mathematica [A] time = 0.03, size = 187, normalized size = 1.13

$$\frac{ax^2}{2}+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{8c^{2/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{8c^{2/3}}+\frac{b\log\left(1-\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{b\log\left(\sqrt[3]{c}x+1\right)}{4c^{2/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{2\sqrt[3]{\ldots}}{\ldots}\right.}{4c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (a*x^2)/2 + (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (S
qrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (b*x^2*ArcTanh[c*
x^3])/2 + (b*Log[1 - c^(1/3)*x])/(4*c^(2/3)) - (b*Log[1 + c^(1/3)*x])/(4*c^
(2/3)) + (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*Log[1 + c^(1
/3)*x + c^(2/3)*x^2])/(8*c^(2/3))
```

fricas [A] time = 0.61, size = 238, normalized size = 1.44

$$2\,bc^2x^2\log\left(-\frac{cx^3+1}{cx^3-1}\right)+4\,ac^2x^2+2\,\sqrt{3}\,bc\sqrt{-\left(-c^2\right)^{\frac{1}{3}}}\,\arctan\left(\frac{\sqrt{3}\left(2\,cx+\left(-c^2\right)^{\frac{1}{3}}\right)\sqrt{-\left(-c^2\right)^{\frac{1}{3}}}}{3\,c}\right)+2\,\sqrt{3}\,b\left(c^2\right)^{\frac{1}{6}}c\arctan\left(\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*c^2*x^2*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^2 + 2*sqrt(3)*b*
c*sqrt(-(-c^2)^(1/3))*arctan(1/3*sqrt(3)*(2*c*x + (-c^2)^(1/3))*sqrt(-(-c^2
)^(1/3))/c) + 2*sqrt(3)*b*(c^2)^(1/6)*c*arctan(1/3*sqrt(3)*(c^2)^(1/6)*(2*c
*x + (c^2)^(1/3))/c) + (-c^2)^(2/3)*b*log(c^2*x^2 + (-c^2)^(1/3)*c*x + (-c^
2)^(2/3)) - b*(c^2)^(2/3)*log(c^2*x^2 + (c^2)^(1/3)*c*x + (c^2)^(2/3)) - 2*
(-c^2)^(2/3)*b*log(c*x - (-c^2)^(1/3)) + 2*b*(c^2)^(2/3)*log(c*x - (c^2)^(1
/3)))/c^2
```

giac [A] time = 0.44, size = 179, normalized size = 1.08

$$\frac{1}{4}\,bx^2\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{2}\,ax^2+\frac{\sqrt{3}\,b|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{4\,c}+\frac{\sqrt{3}\,b|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{4\,c}-\frac{bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^2*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/2*a*x^2 + 1/4*sqrt(3)*b*abs(c)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + 1/abs(c)^(1/3))*abs(c)^(1/3))/c + 1/4*sqrt(
3)*b*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*x - 1/abs(c)^(1/3))*abs(c)^(1/3))/c
 - 1/8*b*c*log(x^2 + x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(5/3) + 1/8*b*
c*log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(5/3) - 1/4*b*c*log(abs
(x + 1/abs(c)^(1/3)))/abs(c)^(5/3) + 1/4*b*abs(c)^(1/3)*log(abs(x - 1/abs(c
)^(1/3)))/c
```

maple [A] time = 0.03, size = 177, normalized size = 1.07

$$\frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{arctanh}\left(c\,x^3\right)}{2}+\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8c\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right.\right.}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c*x^3)+1/4*b/c/(1/c)^(1/3)*ln(x-(1/c)^(1/3))-1/
8*b/c/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b*3^(1/2)/c/(1/c)^(
1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/4*b/c/(1/c)^(1/3)*ln(x+(1/c)
^(1/3))+1/8*b/c/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b*3^(1/2)
/c/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

maxima [A] time = 0.41, size = 155, normalized size = 0.94

$$\frac{1}{2}\,ax^2+\frac{1}{8}\left(4\,x^2\,\mathrm{artanh}\left(cx^3\right)+c\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}+\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}-\frac{\log\left(c^{\frac{2}{3}}x^2+c^{\frac{1}{3}}x+1\right.}{c^{\frac{5}{3}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/8*(4*x^2*arctanh(c*x^3) + c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*
c^(2/3)*x + c^(1/3))/c^(1/3))/c^(5/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(
2/3)*x - c^(1/3))/c^(1/3))/c^(5/3) - log(c^(2/3)*x^2 + c^(1/3)*x + 1)/c^(5/
3) + log(c^(2/3)*x^2 - c^(1/3)*x + 1)/c^(5/3) - 2*log((c^(1/3)*x + 1)/c^(1/
3))/c^(5/3) + 2*log((c^(1/3)*x - 1)/c^(1/3))/c^(5/3)))*b
```

mupad [B] time = 1.25, size = 118, normalized size = 0.72

$$\frac{a\,x^2}{2}+\frac{b\,\left(-\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\mathrm{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)\,1\mathrm{i}}{2\,c^{2/3}}+\frac{b\,x^2\,\ln\left(c\,x^3+1\right)}{4}-\frac{b\,x^2\,\ln\left(1-c\,x^3\right)}{4}+\frac{\sqrt{3}\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^3)),x)
```

```
[Out] (a*x^2)/2 + (b*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)/2 - atan((c^(1/3)*x*(3^(
1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/(2*c^(2/3)) + (b*x^2*log(c*x^3 +
 1))/4 - (b*x^2*log(1 - c*x^3))/4 + (3^(1/2)*b*(atan((c^(1/3)*x*(3^(1/2) -
1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/(4*c^(2/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

3.114 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^2}\,dx$

Optimal. Leaf size=104

$$-\frac{a+b\tanh^{-1}(cx^3)}{x}-\frac{1}{2}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2\right)+\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)+\frac{1}{4}b\sqrt[3]{c}\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)$$

```
[Out] (-a-b*arctanh(c*x^3))/x-1/2*b*c^(1/3)*ln(1-c^(2/3)*x^2)+1/4*b*c^(1/3)*ln(1+
c^(2/3)*x^2+c^(4/3)*x^4)+1/2*b*c^(1/3)*arctan(1/3*(1+2*c^(2/3)*x^2)*3^(1/2)
)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6097, 275, 200, 31, 634, 617, 204, 628}

$$-\frac{a+b\tanh^{-1}(cx^3)}{x}-\frac{1}{2}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2\right)+\frac{1}{4}b\sqrt[3]{c}\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)+\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^2,x]
```

```
[Out] (Sqrt[3]*b*c^(1/3)*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/2 - (a + b*ArcTanh[
c*x^3])/x - (b*c^(1/3)*Log[1 - c^(2/3)*x^2])/2 + (b*c^(1/3)*Log[1 + c^(2/3)
*x^2 + c^(4/3)*x^4])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}+(3bc)\int \frac{x}{1-c^2x^6}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}+\frac{1}{2}(3bc)\operatorname{Subst}\left(\int \frac{1}{1-c^2x^3}\,dx,x,x^2\right)\\
&= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}+\frac{1}{2}(bc)\operatorname{Subst}\left(\int \frac{1}{1-c^{2/3}x}\,dx,x,x^2\right)+\frac{1}{2}(bc)\operatorname{Subst}\left(\int \frac{2}{1+c\right.\\
&= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}-\frac{1}{2}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2\right)+\frac{1}{4}\left(b\sqrt[3]{c}\right)\operatorname{Subst}\left(\int \frac{c^{2/3}+2c^{4/3}x}{1+c^{2/3}x+c^{4/3}\right.\\
&= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}-\frac{1}{2}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2\right)+\frac{1}{4}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)-\frac{1}{2}\\
&= \frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}-\frac{1}{2}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2\right)+\frac{1}{4}b\end{aligned}$$

Mathematica [A] time = 0.03, size = 183, normalized size = 1.76

$$-\frac{a}{x}+\frac{1}{4}b\sqrt[3]{c}\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)+\frac{1}{4}b\sqrt[3]{c}\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)-\frac{b\tanh^{-1}\left(cx^3\right)}{x}-\frac{1}{2}b\sqrt[3]{c}\log\left(1-\sqrt[3]{c}x\right)-\frac{1}{2}b$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^2,x]
```

```
[Out] -(a/x) + (Sqrt[3]*b*c^(1/3)*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/2 - (Sqrt[3
]*b*c^(1/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/2 - (b*ArcTanh[c*x^3])/x - (
b*c^(1/3)*Log[1 - c^(1/3)*x])/2 - (b*c^(1/3)*Log[1 + c^(1/3)*x])/2 + (b*c^(
1/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/4 + (b*c^(1/3)*Log[1 + c^(1/3)*x + c
^(2/3)*x^2])/4
```

fricas [A] time = 0.73, size = 117, normalized size = 1.12

$$-\frac{2\sqrt{3}\,b(-c)^{\frac{1}{3}}x\arctan\left(\frac{2}{3}\sqrt{3}(-c)^{\frac{2}{3}}x^2+\frac{1}{3}\sqrt{3}\right)+b(-c)^{\frac{1}{3}}x\log\left(c^2x^4-(-c)^{\frac{1}{3}}cx^2+(-c)^{\frac{2}{3}}\right)-2b(-c)^{\frac{1}{3}}x\log\left(c\right.}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*b*(-c)^(1/3)*x*arctan(2/3*sqrt(3)*(-c)^(2/3)*x^2 + 1/3*sqrt
(3)) + b*(-c)^(1/3)*x*log(c^2*x^4 - (-c)^(1/3)*c*x^2 + (-c)^(2/3)) - 2*b*(-
c)^(1/3)*x*log(c*x^2 + (-c)^(1/3)) + 2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*
a)/x
```

giac [A] time = 0.15, size = 106, normalized size = 1.02

$$\frac{1}{4}\,bc\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{|c|^{\frac{2}{3}}}+\frac{\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{|c|^{\frac{2}{3}}}-\frac{2\,\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{2}{3}}}\right)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{2\,x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2/3))*abs(c)^(2/3)
)/abs(c)^(2/3) + log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/abs(c)^(2/3)
- 2*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(2/3)) - 1/2*b*log(-(c*x^3 + 1)/(
c*x^3 - 1))/x - a/x
```

maple [A] time = 0.03, size = 105, normalized size = 1.01

$$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(c\,x^3\right)}{x}-\frac{b\ln\left(x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x^4+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c*x^3)-1/2*b/c/(1/c^2)^(2/3)*ln(x^2-(1/c^2)^(1/3))+1/4*b/c
/(1/c^2)^(2/3)*ln(x^4+(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b/c/(1/c^2)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2+1))
```

maxima [A] time = 0.41, size = 94, normalized size = 0.90

$$\frac{1}{4}\left(c\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^4+c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{2}{3}}}-\frac{2\,\log\left(\frac{c^{\frac{2}{3}}x^2-1}{c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}\right)-\frac{4\,\mathrm{artanh}\left(cx^3\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3))/c^(2/3))/c^(
2/3) + log(c^(4/3)*x^4 + c^(2/3)*x^2 + 1)/c^(2/3) - 2*log((c^(2/3)*x^2 - 1)
/c^(2/3))/c^(2/3)) - 4*arctanh(c*x^3)/x)*b - a/x
```

mupad [B] time = 2.39, size = 117, normalized size = 1.12

$$\frac{b\ln\left(1-c\,x^3\right)}{2\,x}-\frac{b\,c^{1/3}\ln\left(1-c^{2/3}\,x^2\right)}{2}-\frac{b\ln\left(c\,x^3+1\right)}{2\,x}-\frac{a}{x}-\frac{b\,c^{1/3}\ln\left(-\sqrt{3}-c^{2/3}\,x^2\,2\mathrm{i}-\mathrm{i}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}+\frac{b\,c^{1/3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^2,x)
```

```
[Out] (b*log(1 - c*x^3))/(2*x) - (b*c^(1/3)*log(1 - c^(2/3)*x^2))/2 - (b*log(c*x^
3 + 1))/(2*x) - a/x - (b*c^(1/3)*log(- 3^(1/2) - c^(2/3)*x^2*2i - 1i)*(3^(1
/2)*1i - 1))/4 + (b*c^(1/3)*log(c^(2/3)*x^2*2i - 3^(1/2) + 1i)*(3^(1/2)*1i
+ 1))/4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**2,x)
```

```
[Out] Timed out
```

3.115 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^5}\,dx$

Optimal. Leaf size=174

$$-\frac{a+b\tanh^{-1}(cx^3)}{4x^4}-\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)+\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)+\frac{1}{8}\sqrt{3}\,bc^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\cdot}{\cdot}\right.$$

```
[Out] -3/4*b*c/x+1/4*b*c^(4/3)*arctanh(c^(1/3)*x)+1/4*(-a-b*arctanh(c*x^3))/x^4-1
/16*b*c^(4/3)*ln(1-c^(1/3)*x+c^(2/3)*x^2)+1/16*b*c^(4/3)*ln(1+c^(1/3)*x+c^(
2/3)*x^2)-1/8*b*c^(4/3)*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)-
1/8*b*c^(4/3)*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6097, 325, 296, 634, 618, 204, 628, 206}

$$-\frac{a+b\tanh^{-1}(cx^3)}{4x^4}-\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)+\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)+\frac{1}{8}\sqrt{3}\,bc^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\cdot}{\cdot}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^5,x]
```

```
[Out] (-3*b*c)/(4*x) + (Sqrt[3]*b*c^(4/3)*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3
]])/8 - (Sqrt[3]*b*c^(4/3)*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/8 + (
b*c^(4/3)*ArcTanh[c^(1/3)*x])/4 - (a + b*ArcTanh[c*x^3])/(4*x^4) - (b*c^(4/
3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/16 + (b*c^(4/3)*Log[1 + c^(1/3)*x + c^
(2/3)*x^2])/16
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^5}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{4x^4}+\frac{1}{4}(3bc)\int \frac{1}{x^2\left(1-c^2x^6\right)}\,dx\\ &= -\frac{3bc}{4x}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{4x^4}+\frac{1}{4}\left(3bc^3\right)\int \frac{x^4}{1-c^2x^6}\,dx\\ &= -\frac{3bc}{4x}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{4x^4}+\frac{1}{4}\left(bc^{5/3}\right)\int \frac{1}{1-c^{2/3}x^2}\,dx+\frac{1}{4}\left(bc^{5/3}\right)\int \frac{-\frac{1}{2}-\frac{\sqrt[3]{c}\,x}{2}}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\\ &= -\frac{3bc}{4x}+\frac{1}{4}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{16}\left(bc^{4/3}\right)\int \frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}\,x+c^{2/3}x}\\ &= -\frac{3bc}{4x}+\frac{1}{4}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{16}bc^{4/3}\log\left(1-\sqrt[3]{c}\,x+c^{2/3}x^2\right.\\ &= -\frac{3bc}{4x}+\frac{1}{8}\sqrt{3}\,bc^{4/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\,x}{\sqrt{3}}\right)-\frac{1}{8}\sqrt{3}\,bc^{4/3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\,x}{\sqrt{3}}\right)+\frac{1}{4}bc^{4/3}\tanh\end{aligned}$$

Mathematica [A] time = 0.05, size = 196, normalized size = 1.13

$$-\frac{a}{4x^4}-\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}\,x+1\right)+\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}\,x+1\right)-\frac{1}{8}bc^{4/3}\log\left(1-\sqrt[3]{c}\,x\right)+\frac{1}{8}bc^{4/3}\log\left(\sqrt[3]{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (3*b*c)/(4*x) - (Sqrt[3]*b*c^(4/3)*ArcTan[(-1 + 2*c^(1/3)*x)/S
qrt[3]])/8 - (Sqrt[3]*b*c^(4/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/8 - (b*A
rcTanh[c*x^3])/(4*x^4) - (b*c^(4/3)*Log[1 - c^(1/3)*x])/8 + (b*c^(4/3)*Log[
1 + c^(1/3)*x])/8 - (b*c^(4/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/16 + (b*c^
(4/3)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/16
```

fricas [A] time = 0.58, size = 196, normalized size = 1.13

$$-\frac{2\sqrt{3}\,b\,(-c)^{\frac{1}{3}}\,cx^4\arctan\left(\frac{2}{3}\sqrt{3}\,(-c)^{\frac{1}{3}}\,x-\frac{1}{3}\sqrt{3}\right)+2\sqrt{3}\,bc^{\frac{4}{3}}x^4\arctan\left(\frac{2}{3}\sqrt{3}\,c^{\frac{1}{3}}x-\frac{1}{3}\sqrt{3}\right)+b\,(-c)^{\frac{1}{3}}\,cx^4\log\left(cx^2+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(2*sqrt(3)*b*(-c)^(1/3)*c*x^4*arctan(2/3*sqrt(3)*(-c)^(1/3)*x - 1/3*s
qrt(3)) + 2*sqrt(3)*b*c^(4/3)*x^4*arctan(2/3*sqrt(3)*c^(1/3)*x - 1/3*sqrt(3
)) + b*(-c)^(1/3)*c*x^4*log(c*x^2 + (-c)^(2/3)*x - (-c)^(1/3)) + b*c^(4/3)*
x^4*log(c*x^2 - c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(1/3)*c*x^4*log(c*x - (-c)^
(2/3)) - 2*b*c^(4/3)*x^4*log(c*x + c^(2/3)) + 12*b*c*x^3 + 2*b*log(-(c*x^3
+ 1)/(c*x^3 - 1)) + 4*a)/x^4
```

giac [A] time = 0.44, size = 187, normalized size = 1.07

$$-\frac{1}{8}\sqrt{3}\,bc|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)-\frac{1}{8}\sqrt{3}\,bc|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)+\frac{bc^3\log\left(x^2+\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{16\,|c|^{\frac{5}{3}}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(3)*b*c*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*x + 1/abs(c)^(1/3))*abs
(c)^(1/3)) - 1/8*sqrt(3)*b*c*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*x - 1/abs(c
)^(1/3))*abs(c)^(1/3)) + 1/16*b*c^3*log(x^2 + x/abs(c)^(1/3) + 1/abs(c)^(2/
3))/abs(c)^(5/3) - 1/16*b*c^3*log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(2/3))/ab
s(c)^(5/3) + 1/8*b*c*abs(c)^(1/3)*log(abs(x + 1/abs(c)^(1/3))) - 1/8*b*c^3*
log(abs(x - 1/abs(c)^(1/3)))/abs(c)^(5/3) - 1/8*b*log(-(c*x^3 + 1)/(c*x^3 -
 1))/x^4 - 1/4*(3*b*c*x^3 + a)/x^4
```

maple [A] time = 0.04, size = 172, normalized size = 0.99

$$-\frac{a}{4x^4}-\frac{b\,\mathrm{arctanh}\left(c\,x^3\right)}{4x^4}-\frac{3bc}{4x}-\frac{bc\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{bc\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{bc\ln\left(\cdots\right.}{8\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctanh(c*x^3)-3/4*b*c/x-1/8*b*c/(1/c)^(1/3)*ln(x-(1/c
)^(1/3))+1/16*b*c/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))-1/8*b*c*3^(
1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))+1/8*b*c/(1/c)^(1/3
)*ln(x+(1/c)^(1/3))-1/16*b*c/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))-
1/8*b*c*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

maxima [A] time = 0.42, size = 160, normalized size = 0.92

$$-\frac{1}{16}\left(\left(2\sqrt{3}c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)+2\sqrt{3}c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)-c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}x^2+c^{\frac{1}{3}}x+1\right)+c^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*((2*sqrt(3)*c^(1/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/3
)) + 2*sqrt(3)*c^(1/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x - c^(1/3))/c^(1/3))
- c^(1/3)*log(c^(2/3)*x^2 + c^(1/3)*x + 1) + c^(1/3)*log(c^(2/3)*x^2 - c^(1
/3)*x + 1) - 2*c^(1/3)*log((c^(1/3)*x + 1)/c^(1/3)) + 2*c^(1/3)*log((c^(1/3
)*x - 1)/c^(1/3)) + 12/x)*c + 4*arctanh(c*x^3)/x^4)*b - 1/4*a/x^4
```

mupad [B] time = 1.28, size = 125, normalized size = 0.72

$$\frac{b\ln\left(1-cx^3\right)}{8x^4}-\frac{bc^{4/3}\left(-\frac{\operatorname{atan}\left(\frac{c^{1/3}x\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\operatorname{atan}\left(\frac{c^{1/3}x\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\operatorname{atan}\left(c^{1/3}x\,1\mathrm{i}\right)\right)1\mathrm{i}}{4}-\frac{3bc}{4x}-\frac{b\ln\left(cx^3+1\right)}{8x^4}-\frac{a}{4x^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))/x^5,x)
```

```
[Out] (b*log(1 - c*x^3))/(8*x^4) - (b*c^(4/3)*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)
/2 - atan((c^(1/3)*x*(3^(1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/4 - (3*
b*c)/(4*x) - (b*log(c*x^3 + 1))/(8*x^4) - a/(4*x^4) - (3^(1/2)*b*c^(4/3)*(a
tan((c^(1/3)*x*(3^(1/2) - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/8
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**5,x)
```

```
[Out] Timed out
```

3.116 $\int x^{11} \left(a + b \tanh^{-1}\left(cx^3\right)\right)^2 \, dx$

Optimal. Leaf size=125

$$-\frac{\left(a + b \tanh^{-1}\left(cx^3\right)\right)^2}{12c^4} + \frac{abx^3}{6c^3} + \frac{1}{12}x^{12}\left(a + b \tanh^{-1}\left(cx^3\right)\right)^2 + \frac{bx^9\left(a + b \tanh^{-1}\left(cx^3\right)\right)}{18c} + \frac{b^2x^3 \tanh^{-1}\left(cx^3\right)}{6c^3} + \frac{b^2x^6}{36c^2}$$

```
[Out] 1/6*a*b*x^3/c^3+1/36*b^2*x^6/c^2+1/6*b^2*x^3*arctanh(c*x^3)/c^3+1/18*b*x^9*
(a+b*arctanh(c*x^3))/c-1/12*(a+b*arctanh(c*x^3))^2/c^4+1/12*x^12*(a+b*arcta
nh(c*x^3))^2+1/9*b^2*ln(-c^2*x^6+1)/c^4
```

Rubi [C] time = 1.55, antiderivative size = 636, normalized size of antiderivative = 5.09, number of steps used = 62, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {6099, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$\frac{b^2\text{PolyLog}\left(2, \frac{1}{2}\left(1 - cx^3\right)\right)}{24c^4} + \frac{b^2\text{PolyLog}\left(2, \frac{1}{2}\left(cx^3 + 1\right)\right)}{24c^4} + \frac{abx^3}{12c^3} - \frac{bx^6\left(2a - b\log\left(1 - cx^3\right)\right)}{48c^2} - \frac{1}{288}b\left(-\frac{3\left(1 - cx^3\right)^4}{c^4}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^11*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (a*b*x^3)/(12*c^3) + (23*b^2*x^3)/(288*c^3) + (b^2*x^6)/(192*c^2) - (7*b^2*
x^9)/(864*c) - (b^2*x^12)/384 + (b^2*(1 - c*x^3)^2)/(16*c^4) - (b^2*(1 - c*
x^3)^3)/(54*c^4) + (b^2*(1 - c*x^3)^4)/(384*c^4) - (5*b^2*Log[1 - c*x^3])/(
288*c^4) + (b^2*(1 - c*x^3)*Log[1 - c*x^3])/(24*c^4) + (b^2*Log[1 - c*x^3]^
2)/(48*c^4) - (b*x^6*(2*a - b*Log[1 - c*x^3]))/(48*c^2) + (b*x^9*(2*a - b*L
og[1 - c*x^3]))/(72*c) - (b*x^12*(2*a - b*Log[1 - c*x^3]))/96 + (x^12*(2*a
- b*Log[1 - c*x^3])^2)/48 - (b*(2*a - b*Log[1 - c*x^3])*((48*(1 - c*x^3))/c
^4 - (36*(1 - c*x^3)^2)/c^4 + (16*(1 - c*x^3)^3)/c^4 - (3*(1 - c*x^3)^4)/c^
4 - (12*Log[1 - c*x^3])/c^4))/288 - (b*(2*a - b*Log[1 - c*x^3])*Log[(1 + c*
x^3)/2])/(24*c^4) + (b^2*Log[1 + c*x^3])/(36*c^4) + (b^2*x^9*Log[1 + c*x^3]
)/(36*c) + (b^2*(1 + c*x^3)*Log[1 + c*x^3])/(12*c^4) + (b^2*Log[(1 - c*x^3)
/2]*Log[1 + c*x^3])/(24*c^4) + (b*x^12*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x
^3])/24 - (b^2*Log[1 + c*x^3]^2)/(48*c^4) + (b^2*x^12*Log[1 + c*x^3]^2)/48
+ (b^2*PolyLog[2, (1 - c*x^3)/2])/(24*c^4) + (b^2*PolyLog[2, (1 + c*x^3)/2]
)/(24*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x^{11}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx = \int \left(\frac{1}{4}x^{11}\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{2}bx^{11}\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx\right.\right.$$
$$=\frac{1}{4}\int x^{11}\left(2a-b\log\left(1-cx^3\right)\right)^2 dx-\frac{1}{2}b\int x^{11}\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(\right.$$
$$=\frac{1}{12}\text{Subst}\left(\int x^3(2a-b\log(1-cx))^2 dx, x, x^3\right)-\frac{1}{6}b\,\text{Subst}\left(\int x^3(-2a+b\log\right.$$
$$=\frac{1}{48}x^{12}\left(2a-b\log\left(1-cx^3\right)\right)^2+\frac{1}{24}bx^{12}\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)$$
$$=\frac{1}{48}x^{12}\left(2a-b\log\left(1-cx^3\right)\right)^2+\frac{1}{24}bx^{12}\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)$$
$$=\frac{1}{48}x^{12}\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{288}b\left(2a-b\log\left(1-cx^3\right)\right)\left(\frac{48\left(1-cx^3\right)}{c^4}-\right.$$
$$=\frac{abx^3}{12c^3}-\frac{bx^6\left(2a-b\log\left(1-cx^3\right)\right)}{48c^2}+\frac{bx^9\left(2a-b\log\left(1-cx^3\right)\right)}{72c}-\frac{1}{96}bx^{12}\left(2a\right.$$
$$=\frac{abx^3}{12c^3}-\frac{bx^6\left(2a-b\log\left(1-cx^3\right)\right)}{48c^2}+\frac{bx^9\left(2a-b\log\left(1-cx^3\right)\right)}{72c}-\frac{1}{96}bx^{12}\left(2a\right.$$
$$=\frac{abx^3}{12c^3}+\frac{55b^2x^3}{288c^3}-\frac{5b^2x^6}{576c^2}+\frac{b^2x^9}{864c}-\frac{b^2x^{12}}{384}+\frac{b^2\left(1-cx^3\right)^2}{16c^4}-\frac{b^2\left(1-cx^3\right)^3}{54c^4}+$$
$$=\frac{abx^3}{12c^3}+\frac{55b^2x^3}{288c^3}-\frac{5b^2x^6}{576c^2}+\frac{b^2x^9}{864c}-\frac{b^2x^{12}}{384}+\frac{b^2\left(1-cx^3\right)^2}{16c^4}-\frac{b^2\left(1-cx^3\right)^3}{54c^4}+$$

Mathematica [A] time = 0.08, size = 146, normalized size = 1.17

$$\frac{3a^2c^4x^{12}+2abc^3x^9+2bcx^3\tanh^{-1}\left(cx^3\right)\left(3ac^3x^9+b\left(c^2x^6+3\right)\right)+6abcx^3+b(3a+4b)\log\left(1-cx^3\right)-3ab\log}{36c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (6*a*b*c*x^3 + b^2*c^2*x^6 + 2*a*b*c^3*x^9 + 3*a^2*c^4*x^12 + 2*b*c*x^3*(3*
a*c^3*x^9 + b*(3 + c^2*x^6))*ArcTanh[c*x^3] + 3*b^2*(-1 + c^4*x^12)*ArcTanh
[c*x^3]^2 + b*(3*a + 4*b)*Log[1 - c*x^3] - 3*a*b*Log[1 + c*x^3] + 4*b^2*Log
[1 + c*x^3])/(36*c^4)
```

fricas [A] time = 0.57, size = 176, normalized size = 1.41

$$\frac{12\,a^2c^4x^{12}+8\,abc^3x^9+4\,b^2c^2x^6+24\,abcx^3+3\left(b^2c^4x^{12}-b^2\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2-4\left(3\,ab-4\,b^2\right)\log\left(cx^3+1\right)+}{144\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/144*(12*a^2*c^4*x^12 + 8*a*b*c^3*x^9 + 4*b^2*c^2*x^6 + 24*a*b*c*x^3 + 3*(
b^2*c^4*x^12 - b^2)*log(-(c*x^3 + 1)/(c*x^3 - 1))^2 - 4*(3*a*b - 4*b^2)*log
(c*x^3 + 1) + 4*(3*a*b + 4*b^2)*log(c*x^3 - 1) + 4*(3*a*b*c^4*x^12 + b^2*c^
3*x^9 + 3*b^2*c*x^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)))/c^4
```

giac [A] time = 0.30, size = 175, normalized size = 1.40

$$\frac{1}{12}a^2x^{12}+\frac{abx^9}{18c}+\frac{b^2x^6}{36c^2}+\frac{1}{48}\left(b^2x^{12}-\frac{b^2}{c^4}\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2+\frac{abx^3}{6c^3}+\frac{1}{36}\left(3abx^{12}+\frac{b^2x^9}{c}+\frac{3b^2x^3}{c^3}\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/12*a^2*x^12 + 1/18*a*b*x^9/c + 1/36*b^2*x^6/c^2 + 1/48*(b^2*x^12 - b^2/c^
4)*log(-(c*x^3 + 1)/(c*x^3 - 1))^2 + 1/6*a*b*x^3/c^3 + 1/36*(3*a*b*x^12 + b
^2*x^9/c + 3*b^2*x^3/c^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)) - 1/36*(3*a*b - 4*b
^2)*log(c*x^3 + 1)/c^4 + 1/36*(3*a*b + 4*b^2)*log(c*x^3 - 1)/c^4
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^{11}\left(a+b\,\mathrm{arctanh}\left(c\,x^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] int(x^11*(a+b*arctanh(c*x^3))^2,x)
```

maxima [A] time = 0.32, size = 217, normalized size = 1.74

$$\frac{1}{12}b^2x^{12}\,\mathrm{artanh}\left(cx^3\right)^2+\frac{1}{12}a^2x^{12}+\frac{1}{36}\left(6x^{12}\,\mathrm{artanh}\left(cx^3\right)+c\left(\frac{2\left(c^2x^9+3x^3\right)}{c^4}-\frac{3\log\left(cx^3+1\right)}{c^5}+\frac{3\log\left(cx^3-1\right.}{c^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*x^12*arctanh(c*x^3)^2 + 1/12*a^2*x^12 + 1/36*(6*x^12*arctanh(c*x^3
) + c*(2*(c^2*x^9 + 3*x^3)/c^4 - 3*log(c*x^3 + 1)/c^5 + 3*log(c*x^3 - 1)/c^
5))*a*b + 1/144*(4*c*(2*(c^2*x^9 + 3*x^3)/c^4 - 3*log(c*x^3 + 1)/c^5 + 3*lo
g(c*x^3 - 1)/c^5)*arctanh(c*x^3) + (4*c^2*x^6 - 2*(3*log(c*x^3 - 1) - 8)*lo
g(c*x^3 + 1) + 3*log(c*x^3 + 1)^2 + 3*log(c*x^3 - 1)^2 + 16*log(c*x^3 - 1))
/c^4)*b^2
```

mupad [B] time = 1.60, size = 335, normalized size = 2.68

$$\frac{a^2\,x^{12}}{12}+\frac{b^2\,\ln\left(c\,x^3-1\right)}{9\,c^4}+\frac{b^2\,\ln\left(c\,x^3+1\right)}{9\,c^4}-\frac{b^2\ln\left(c\,x^3+1\right)^2}{48\,c^4}-\frac{b^2\ln\left(1-c\,x^3\right)^2}{48\,c^4}+\frac{b^2\,x^6}{36\,c^2}+\frac{b^2\,x^{12}\ln\left(c\,x^3+1\right)^2}{48}+\frac{b^2\,x^{12}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*atanh(c*x^3))^2,x)
```

```
[Out] (a^2*x^12)/12 + (b^2*log(c*x^3 - 1))/(9*c^4) + (b^2*log(c*x^3 + 1))/(9*c^4)
 - (b^2*log(c*x^3 + 1)^2)/(48*c^4) - (b^2*log(1 - c*x^3)^2)/(48*c^4) + (b^2
*x^6)/(36*c^2) + (b^2*x^12*log(c*x^3 + 1)^2)/48 + (b^2*x^12*log(1 - c*x^3)^
2)/48 + (b^2*x^3*log(c*x^3 + 1))/(12*c^3) - (b^2*x^3*log(1 - c*x^3))/(12*c^
3) + (b^2*x^9*log(c*x^3 + 1))/(36*c) - (b^2*x^9*log(1 - c*x^3))/(36*c) + (a
*b*log(c*x^3 - 1))/(12*c^4) - (a*b*log(c*x^3 + 1))/(12*c^4) + (a*b*x^12*log
```

```
(c*x^3 + 1))/12 - (a*b*x^12*log(1 - c*x^3))/12 + (b^2*log(c*x^3 + 1)*log(1
- c*x^3))/(24*c^4) + (a*b*x^3)/(6*c^3) + (a*b*x^9)/(18*c) - (b^2*x^12*log(c
*x^3 + 1)*log(1 - c*x^3))/24
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

3.117 $\int x^8 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=146

$$\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{9c^3}-\frac{2b\log\left(\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{9c^3}+\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{bx^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{9c}-$$

```
[Out] 1/9*b^2*x^3/c^2-1/9*b^2*arctanh(c*x^3)/c^3+1/9*b*x^6*(a+b*arctanh(c*x^3))/c
+1/9*(a+b*arctanh(c*x^3))^2/c^3+1/9*x^9*(a+b*arctanh(c*x^3))^2-2/9*b*(a+b*a
rctanh(c*x^3))*ln(2/(-c*x^3+1))/c^3-1/9*b^2*polylog(2,1-2/(-c*x^3+1))/c^3
```

Rubi [B] time = 1.31, antiderivative size = 536, normalized size of antiderivative = 3.67, number of steps used = 53, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {6099, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)}{18c^3}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^3+1\right)\right)}{18c^3}-\frac{abx^3}{9c^2}-\frac{1}{108}b\left(\frac{2\left(1-cx^3\right)^3}{c^3}-\frac{9\left(1-cx^3\right)^2}{c^3}+\frac{18\left(1-cx\right.}{c^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^8*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] -(a*b*x^3)/(9*c^2) + (19*b^2*x^3)/(108*c^2) - (5*b^2*x^6)/(216*c) - (b^2*x^
9)/162 + (b^2*(1 - c*x^3)^2)/(24*c^3) - (b^2*(1 - c*x^3)^3)/(162*c^3) + (b^
2*Log[1 - c*x^3])/(108*c^3) - (b^2*(1 - c*x^3)*Log[1 - c*x^3])/(18*c^3) + (
b^2*Log[1 - c*x^3]^2)/(36*c^3) + (b*x^6*(2*a - b*Log[1 - c*x^3]))/(36*c) -
(b*x^9*(2*a - b*Log[1 - c*x^3]))/54 + (x^9*(2*a - b*Log[1 - c*x^3])^2)/36 -
 (b*(2*a - b*Log[1 - c*x^3])*((18*(1 - c*x^3))/c^3 - (9*(1 - c*x^3)^2)/c^3
+ (2*(1 - c*x^3)^3)/c^3 - (6*Log[1 - c*x^3])/c^3))/108 + (b*(2*a - b*Log[1
- c*x^3])*Log[(1 + c*x^3)/2])/(18*c^3) - (b^2*Log[1 + c*x^3])/(18*c^3) + (b
^2*x^6*Log[1 + c*x^3])/(18*c) + (b^2*Log[(1 - c*x^3)/2]*Log[1 + c*x^3])/(18
*c^3) + (b*x^9*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3])/18 + (b^2*Log[1 + c
*x^3]^2)/(36*c^3) + (b^2*x^9*Log[1 + c*x^3]^2)/36 - (b^2*PolyLog[2, (1 - c*
x^3)/2])/(18*c^3) + (b^2*PolyLog[2, (1 + c*x^3)/2])/(18*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
```

```
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x^8 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx = \int \left(\frac{1}{4}x^8\left(2a-b\log\left(1-cx^3\right)\right)^2 - \frac{1}{2}bx^8\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)\right.$$
$$= \frac{1}{4}\int x^8\left(2a-b\log\left(1-cx^3\right)\right)^2 dx - \frac{1}{2}b\int x^8\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+\right.$$
$$= \frac{1}{12}\operatorname{Subst}\left(\int x^2(2a-b\log(1-cx))^2\,dx, x, x^3\right) - \frac{1}{6}b\operatorname{Subst}\left(\int x^2(-2a+b\log\right.$$
$$= \frac{1}{36}x^9\left(2a-b\log\left(1-cx^3\right)\right)^2 + \frac{1}{18}bx^9\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right) + \frac{1}{3}$$
$$= \frac{1}{36}x^9\left(2a-b\log\left(1-cx^3\right)\right)^2 + \frac{1}{18}bx^9\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right) + \frac{1}{3}$$
$$= \frac{1}{36}x^9\left(2a-b\log\left(1-cx^3\right)\right)^2 - \frac{1}{108}b\left(2a-b\log\left(1-cx^3\right)\right)\left(\frac{18\left(1-cx^3\right)}{c^3} - \frac{9}{}\right.$$
$$= -\frac{abx^3}{9c^2} + \frac{bx^6\left(2a-b\log\left(1-cx^3\right)\right)}{36c} - \frac{1}{54}bx^9\left(2a-b\log\left(1-cx^3\right)\right) + \frac{1}{36}x^9\left(2\right.$$
$$= -\frac{abx^3}{9c^2} + \frac{bx^6\left(2a-b\log\left(1-cx^3\right)\right)}{36c} - \frac{1}{54}bx^9\left(2a-b\log\left(1-cx^3\right)\right) + \frac{1}{36}x^9\left(2\right.$$
$$= -\frac{abx^3}{9c^2} + \frac{13b^2x^3}{108c^2} + \frac{b^2x^6}{216c} - \frac{b^2x^9}{162} + \frac{b^2\left(1-cx^3\right)^2}{24c^3} - \frac{b^2\left(1-cx^3\right)^3}{162c^3} + \frac{b^2\log\left(1-\right.}{108c^3}$$
$$= -\frac{abx^3}{9c^2} + \frac{13b^2x^3}{108c^2} + \frac{b^2x^6}{216c} - \frac{b^2x^9}{162} + \frac{b^2\left(1-cx^3\right)^2}{24c^3} - \frac{b^2\left(1-cx^3\right)^3}{162c^3} + \frac{b^2\log\left(1-\right.}{108c^3}$$

Mathematica [A] time = 0.29, size = 132, normalized size = 0.90

$$\frac{a^2c^3x^9 + abc^2x^6 + ab\log\left(c^2x^6-1\right) + b\tanh^{-1}\left(cx^3\right)\left(2ac^3x^9 + bc^2x^6 - 2b\log\left(e^{-2\tanh^{-1}\left(cx^3\right)}+1\right) - b\right) + b^2\left(c^3\right.}{9c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (b^2*c*x^3 + a*b*c^2*x^6 + a^2*c^3*x^9 + b^2*(-1 + c^3*x^9)*ArcTanh[c*x^3]^
2 + b*ArcTanh[c*x^3]*(-b + b*c^2*x^6 + 2*a*c^3*x^9 - 2*b*Log[1 + E^(-2*ArcT
anh[c*x^3])]) + a*b*Log[-1 + c^2*x^6] + b^2*PolyLog[2, -E^(-2*ArcTanh[c*x^3
])])/(9*c^3)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^2x^8\operatorname{artanh}\left(cx^3\right)^2 + 2\,abx^8\operatorname{artanh}\left(cx^3\right) + a^2x^8, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^8*arctanh(c*x^3)^2 + 2*a*b*x^8*arctanh(c*x^3) + a^2*x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(cx^3\right) + a\right)^2 x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2*x^8, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^8 \left(a + b \operatorname{arctanh}\left(c\,x^3\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] int(x^8*(a+b*arctanh(c*x^3))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{9}a^2x^9 + \frac{1}{9}\left(2\,x^9 \operatorname{artanh}\left(cx^3\right) + \left(\frac{x^6}{c^2} + \frac{\log\left(c^2x^6 - 1\right)}{c^4}\right)c\right)ab + \frac{1}{648}\left(18\,x^9 \log\left(-cx^3 + 1\right)^2 - 2\,c^4\left(\frac{2\left(c^2x^9 + 3\,x^3\right)}{c^6} - \frac{3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/9*a^2*x^9 + 1/9*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)*
c)*a*b + 1/648*(18*x^9*log(-c*x^3 + 1)^2 - 2*c^4*(2*(c^2*x^9 + 3*x^3)/c^6 -
 3*log(c*x^3 + 1)/c^7 + 3*log(c*x^3 - 1)/c^7) + 3*(x^6/c^4 + log(c^2*x^6 -
1)/c^6)*c^3 + 1944*c^3*integrate(1/9*x^11*log(c*x^3 + 1)/(c^4*x^6 - c^2), x
) - 9*c^2*(2*x^3/c^4 - log(c*x^3 + 1)/c^5 + log(c*x^3 - 1)/c^5) - 6*c*((2*c
^2*x^9 + 3*c*x^6 + 6*x^3)/c^3 + 6*log(c*x^3 - 1)/c^4)*log(-c*x^3 + 1) + 972
*c*integrate(1/9*x^5*log(c*x^3 + 1)/(c^4*x^6 - c^2), x) + 6*(3*c^3*x^9*log(
c*x^3 + 1)^2 + (2*c^3*x^9 - 3*c^2*x^6 + 6*c*x^3 - 6*(c^3*x^9 + 1)*log(c*x^3
 + 1))*log(-c*x^3 + 1))/c^3 + (4*c^3*x^9 + 15*c^2*x^6 + 66*c*x^3 + 18*log(c
*x^3 - 1)^2 + 66*log(c*x^3 - 1))/c^3 - 18*log(9*c^4*x^6 - 9*c^2)/c^3 + 972*
integrate(1/9*x^2*log(c*x^3 + 1)/(c^4*x^6 - c^2), x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8 \left(a + b \operatorname{atanh}\left(c\,x^3\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*atanh(c*x^3))^2,x)
```

```
[Out] int(x^8*(a + b*atanh(c*x^3))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

3.118 $\int x^5 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=91

$$-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6c^2}+\frac{abx^3}{3c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{b^2\log\left(1-c^2x^6\right)}{6c^2}+\frac{b^2x^3\tanh^{-1}\left(cx^3\right)}{3c}$$

```
[Out] 1/3*a*b*x^3/c+1/3*b^2*x^3*arctanh(c*x^3)/c-1/6*(a+b*arctanh(c*x^3))^2/c^2+1
/6*x^6*(a+b*arctanh(c*x^3))^2+1/6*b^2*ln(-c^2*x^6+1)/c^2
```

Rubi [C] time = 0.99, antiderivative size = 524, normalized size of antiderivative = 5.76, number of steps used = 44, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 2439, 2416, 2394, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)}{12c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^3+1\right)\right)}{12c^2}+\frac{\left(1-cx^3\right)^2\left(2a-b\log\left(1-cx^3\right)\right)^2}{24c^2}-\frac{\left(1-cx^3\right)\left(2a-b\right.}{12\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (a*b*x^3)/(2*c) - (b^2*x^6)/24 + (b^2*(1 - c*x^3)^2)/(48*c^2) + (b^2*(1 + c
*x^3)^2)/(48*c^2) - (b^2*Log[1 - c*x^3])/(24*c^2) + (b^2*(1 - c*x^3)*Log[1
- c*x^3])/(4*c^2) - (b*x^6*(2*a - b*Log[1 - c*x^3]))/24 + (b*(1 - c*x^3)^2*
(2*a - b*Log[1 - c*x^3]))/(24*c^2) - ((1 - c*x^3)*(2*a - b*Log[1 - c*x^3])^
2)/(12*c^2) + ((1 - c*x^3)^2*(2*a - b*Log[1 - c*x^3])^2)/(24*c^2) - (b*(2*a
 - b*Log[1 - c*x^3])*Log[(1 + c*x^3)/2])/(12*c^2) - (b^2*Log[1 + c*x^3])/(2
4*c^2) + (b^2*x^6*Log[1 + c*x^3])/24 + (b^2*(1 + c*x^3)*Log[1 + c*x^3])/(4*
c^2) - (b^2*(1 + c*x^3)^2*Log[1 + c*x^3])/(24*c^2) + (b^2*Log[(1 - c*x^3)/2
]*Log[1 + c*x^3])/(12*c^2) + (b*x^6*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3]
)/12 - (b^2*(1 + c*x^3)*Log[1 + c*x^3]^2)/(12*c^2) + (b^2*(1 + c*x^3)^2*Log
[1 + c*x^3]^2)/(24*c^2) + (b^2*PolyLog[2, (1 - c*x^3)/2])/(12*c^2) + (b^2*P
olyLog[2, (1 + c*x^3)/2])/(12*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\,dx=\int\left(\frac{1}{4}x^5\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{2}bx^5\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)$$

$$=\frac{1}{4}\int x^5\left(2a-b\log\left(1-cx^3\right)\right)^2\,dx-\frac{1}{2}b\int x^5\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+\right.$$

$$=\frac{1}{12}\,\mathrm{Subst}\left(\int x(2a-b\log(1-cx))^2\,dx,x,x^3\right)-\frac{1}{6}b\,\mathrm{Subst}\left(\int x(-2a+b\log(1\right.$$

$$=\frac{1}{12}bx^6\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{12}\,\mathrm{Subst}\left(\int\left(\frac{(2a-b\log(1-c}{c}\right.\right.$$

$$=\frac{1}{12}bx^6\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{\mathrm{Subst}\left(\int(2a-b\log(1-cx))^2\,dx\right.}{12c}$$

$$=\frac{1}{12}bx^6\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{12}b\,\mathrm{Subst}\left(\int x(-2a+b\log(1-\right.$$

$$=\frac{abx^3}{6c}-\frac{1}{24}bx^6\left(2a-b\log\left(1-cx^3\right)\right)-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c^2}+\frac{(1}{}$$

$$=\frac{abx^3}{2c}-\frac{b^2x^3}{4c}+\frac{b^2\left(1-cx^3\right)^2}{48c^2}+\frac{b^2\left(1+cx^3\right)^2}{48c^2}-\frac{1}{24}bx^6\left(2a-b\log\left(1-cx^3\right)\right)$$

$$=\frac{abx^3}{2c}-\frac{b^2x^6}{24}+\frac{b^2\left(1-cx^3\right)^2}{48c^2}+\frac{b^2\left(1+cx^3\right)^2}{48c^2}-\frac{b^2\log\left(1-cx^3\right)}{24c^2}+\frac{b^2\left(1-cx^3\right.}{}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 1.16

$$\frac{a^2c^2x^6+2abcx^3+b(a+b)\log\left(1-cx^3\right)-ab\log\left(cx^3+1\right)+2bcx^3\tanh^{-1}\left(cx^3\right)\left(acx^3+b\right)+b^2\left(c^2x^6-1\right)\tanh^{-1}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (2*a*b*c*x^3 + a^2*c^2*x^6 + 2*b*c*x^3*(b + a*c*x^3)*ArcTanh[c*x^3] + b^2*(
-1 + c^2*x^6)*ArcTanh[c*x^3]^2 + b*(a + b)*Log[1 - c*x^3] - a*b*Log[1 + c*x
^3] + b^2*Log[1 + c*x^3])/(6*c^2)
```

fricas [A] time = 0.55, size = 138, normalized size = 1.52

$$\frac{4\,a^2c^2x^6+8\,abcx^3+\left(b^2c^2x^6-b^2\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2-4\left(ab-b^2\right)\log\left(cx^3+1\right)+4\left(ab+b^2\right)\log\left(cx^3-1\right)+4\left(abc^2x\right.}{24\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(4*a^2*c^2*x^6 + 8*a*b*c*x^3 + (b^2*c^2*x^6 - b^2)*log(-(c*x^3 + 1)/(c
*x^3 - 1))^2 - 4*(a*b - b^2)*log(c*x^3 + 1) + 4*(a*b + b^2)*log(c*x^3 - 1)
+ 4*(a*b*c^2*x^6 + b^2*c*x^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)))/c^2
```

giac [B] time = 0.25, size = 361, normalized size = 3.97

$$\frac{1}{6}\left(\frac{\left(cx^3+1\right)b^2\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2}{\left(cx^3-1\right)\left(\frac{\left(cx^3+1\right)^2c^3}{\left(cx^3-1\right)^2}-\frac{2\left(cx^3+1\right)c^3}{cx^3-1}+c^3\right)}+\frac{2\left(\frac{2\left(cx^3+1\right)ab}{cx^3-1}+\frac{\left(cx^3+1\right)b^2}{cx^3-1}-b^2\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{\frac{\left(cx^3+1\right)^2c^3}{\left(cx^3-1\right)^2}-\frac{2\left(cx^3+1\right)c^3}{cx^3-1}+c^3}+\frac{4\left(\frac{\left(cx^3+1\right)a^2}{cx^3-1}+\frac{\left(cx^3+1\right)ab}{cx^3-1}-\right.}{\frac{\left(cx^3+1\right)^2c^3}{\left(cx^3-1\right)^2}-\frac{2\left(cx^3+1\right)c^3}{cx^3-1}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/6*((c*x^3 + 1)*b^2*log(-(c*x^3 + 1)/(c*x^3 - 1))^2/((c*x^3 - 1)*((c*x^3 +
 1)^2*c^3/(c*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3 - 1) + c^3)) + 2*(2*(c*x
^3 + 1)*a*b/(c*x^3 - 1) + (c*x^3 + 1)*b^2/(c*x^3 - 1) - b^2)*log(-(c*x^3 +
1)/(c*x^3 - 1))/((c*x^3 + 1)^2*c^3/(c*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3
 - 1) + c^3) + 4*((c*x^3 + 1)*a^2/(c*x^3 - 1) + (c*x^3 + 1)*a*b/(c*x^3 - 1)
 - a*b)/((c*x^3 + 1)^2*c^3/(c*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3 - 1) +
c^3) - 2*b^2*log(-(c*x^3 + 1)/(c*x^3 - 1) + 1)/c^3 + 2*b^2*log(-(c*x^3 + 1)
/(c*x^3 - 1))/c^3)*c
```

maple [B] time = 0.28, size = 247, normalized size = 2.71

$$\frac{b^2\left(c^2x^6-1\right)\ln\left(c\,x^3+1\right)^2}{24c^2}+\frac{b\left(-x^6b\ln\left(-c\,x^3+1\right)c^2+2a\,c^2x^6+2bc\,x^3+b\ln\left(-c\,x^3+1\right)\right)\ln\left(c\,x^3+1\right)}{12c^2}+\frac{b^2x^6\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] 1/24*b^2*(c^2*x^6-1)/c^2*ln(c*x^3+1)^2+1/12*b*(-x^6*b*ln(-c*x^3+1)*c^2+2*a*
c^2*x^6+2*b*c*x^3+b*ln(-c*x^3+1))/c^2*ln(c*x^3+1)+1/24*b^2*x^6*ln(-c*x^3+1)
^2-1/6*a*b*x^6*ln(-c*x^3+1)+1/6*x^6*a^2-1/6/c*b^2*x^3*ln(-c*x^3+1)+1/3*a*b*
x^3/c-1/24/c^2*b^2*ln(-c*x^3+1)^2+1/6/c^2*b*ln(-c*x^3+1)*a+1/6/c^2*b^2*ln(-
c*x^3+1)-1/6/c^2*b*ln(-c*x^3-1)*a+1/6/c^2*b^2*ln(-c*x^3-1)
```

maxima [B] time = 0.32, size = 186, normalized size = 2.04

$$\frac{1}{6}\,b^2x^6\,\operatorname{artanh}\left(cx^3\right)^2+\frac{1}{6}\,a^2x^6+\frac{1}{6}\left(2\,x^6\,\operatorname{artanh}\left(cx^3\right)+c\left(\frac{2\,x^3}{c^2}-\frac{\log\left(cx^3+1\right)}{c^3}+\frac{\log\left(cx^3-1\right)}{c^3}\right)\right)ab+\frac{1}{24}\left(4\,c\left(\frac{2\,x^3}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctanh(c*x^3)^2 + 1/6*a^2*x^6 + 1/6*(2*x^6*arctanh(c*x^3) + c*
(2*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3))*a*b + 1/24*(4*c*(2*x
^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3)*arctanh(c*x^3) - (2*(log(
c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 - log(c*x^3 - 1)^2 - 4*lo
g(c*x^3 - 1))/c^2)*b^2
```

mupad [B] time = 1.25, size = 275, normalized size = 3.02

$$\frac{a^2\,x^6}{6}+\frac{b^2\,\ln\left(c\,x^3-1\right)}{6\,c^2}+\frac{b^2\,\ln\left(c\,x^3+1\right)}{6\,c^2}-\frac{b^2\ln\left(c\,x^3+1\right)^2}{24\,c^2}-\frac{b^2\ln\left(1-c\,x^3\right)^2}{24\,c^2}+\frac{b^2\,x^6\ln\left(c\,x^3+1\right)^2}{24}+\frac{b^2\,x^6\ln\left(1-\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x^3))^2,x)
```

```
[Out] (a^2*x^6)/6 + (b^2*log(c*x^3 - 1))/(6*c^2) + (b^2*log(c*x^3 + 1))/(6*c^2) -
 (b^2*log(c*x^3 + 1)^2)/(24*c^2) - (b^2*log(1 - c*x^3)^2)/(24*c^2) + (b^2*x
^6*log(c*x^3 + 1)^2)/24 + (b^2*x^6*log(1 - c*x^3)^2)/24 + (b^2*x^3*log(c*x^
3 + 1))/(6*c) - (b^2*x^3*log(1 - c*x^3))/(6*c) + (a*b*log(c*x^3 - 1))/(6*c^
2) - (a*b*log(c*x^3 + 1))/(6*c^2) + (a*b*x^6*log(c*x^3 + 1))/6 - (a*b*x^6*l
og(1 - c*x^3))/6 + (b^2*log(c*x^3 + 1)*log(1 - c*x^3))/(12*c^2) + (a*b*x^3)
/(3*c) - (b^2*x^6*log(c*x^3 + 1)*log(1 - c*x^3))/12
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

3.119 $\int x^2 \left(a+b \tanh^{-1}\left(c x^3\right)\right)^2 dx$

Optimal. Leaf size=96

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{3c}-\frac{2b\log\left(\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{3c}-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx^3}\right)}{3c}$$

```
[Out] 1/3*(a+b*arctanh(c*x^3))^2/c+1/3*x^3*(a+b*arctanh(c*x^3))^2-2/3*b*(a+b*arct
anh(c*x^3))*ln(2/(-c*x^3+1))/c-1/3*b^2*polylog(2,1-2/(-c*x^3+1))/c
```

Rubi [B] time = 0.59, antiderivative size = 207, normalized size of antiderivative = 2.16, number of steps used = 28, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)}{6c}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^3+1\right)\right)}{6c}+\frac{1}{6}bx^3\log\left(cx^3+1\right)\left(2a-b\log\left(1-cx^3\right)\right)-\frac{\left(1-cx^3\right)\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] -((1 - c*x^3)*(2*a - b*Log[1 - c*x^3])^2)/(12*c) + (b*(2*a - b*Log[1 - c*x^
3])*Log[(1 + c*x^3)/2])/(6*c) + (b^2*Log[(1 - c*x^3)/2]*Log[1 + c*x^3])/(6*
c) + (b*x^3*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3])/6 + (b^2*(1 + c*x^3)*L
og[1 + c*x^3]^2)/(12*c) - (b^2*PolyLog[2, (1 - c*x^3)/2])/(6*c) + (b^2*Poly
Log[2, (1 + c*x^3)/2])/(6*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2dx=\int\left(\frac{1}{4}x^2\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{2}bx^2\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{4}\right.$$
$$=\frac{1}{4}\int x^2\left(2a-b\log\left(1-cx^3\right)\right)^2dx-\frac{1}{2}b\int x^2\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx\right.$$
$$=\frac{1}{12}\text{Subst}\left(\int(2a-b\log(1-cx))^2\,dx,x,x^3\right)-\frac{1}{6}b\,\text{Subst}\left(\int(-2a+b\log(1-cx)\right.$$
$$=\frac{1}{6}bx^3\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)-\frac{\text{Subst}\left(\int(2a-b\log(x))^2\,dx,x,1-c\right.}{12c}$$
$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}+\frac{1}{6}bx^3\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+$$
$$=\frac{1}{3}abx^3+\frac{b^2x^3}{6}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}-\frac{b^2\left(1+cx^3\right)\log\left(1+cx^3\right)}{6c}+$$
$$=\frac{b^2x^3}{3}+\frac{b^2\left(1-cx^3\right)\log\left(1-cx^3\right)}{6c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}+\frac{b\left(2a-b\right.}{}$$
$$=\frac{b^2x^3}{6}+\frac{b^2\left(1-cx^3\right)\log\left(1-cx^3\right)}{6c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}+\frac{b\left(2a-b\right.}{}$$
$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}+\frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(\frac{1}{2}\left(1+cx^3\right)\right)}{6c}+$$

Mathematica [A] time = 0.16, size = 99, normalized size = 1.03

$$\frac{a\left(acx^3+b\log\left(1-c^2x^6\right)\right)+2b\tanh^{-1}\left(cx^3\right)\left(acx^3-b\log\left(e^{-2\tanh^{-1}\left(cx^3\right)}+1\right)\right)+b^2\text{Li}_2\left(-e^{-2\tanh^{-1}\left(cx^3\right)}\right)+b^2\left(cx^3\right.}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (b^2*(-1 + c*x^3)*ArcTanh[c*x^3]^2 + 2*b*ArcTanh[c*x^3]*(a*c*x^3 - b*Log[1
+ E^(-2*ArcTanh[c*x^3])]) + a*(a*c*x^3 + b*Log[1 - c^2*x^6]) + b^2*PolyLog[
2, -E^(-2*ArcTanh[c*x^3])])/(3*c)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\,\text{artanh}\left(cx^3\right)^2+2\,abx^2\,\text{artanh}\left(cx^3\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c*x^3)^2 + 2*a*b*x^2*arctanh(c*x^3) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\,\text{artanh}\left(cx^3\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2*x^2, x)
```

maple [A] time = 0.18, size = 145, normalized size = 1.51

$$\frac{x^3b^2\operatorname{arctanh}\left(cx^3\right)^2}{3}+\frac{2x^3ab\operatorname{arctanh}\left(cx^3\right)}{3}+\frac{x^3a^2}{3}+\frac{b^2\operatorname{arctanh}\left(cx^3\right)^2}{3c}-\frac{2\operatorname{arctanh}\left(cx^3\right)\ln\left(1+\frac{\left(cx^3+1\right)^2}{-c^2x^6+1}\right)b^2}{3c}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] 1/3*x^3*b^2*arctanh(c*x^3)^2+2/3*x^3*a*b*arctanh(c*x^3)+1/3*x^3*a^2+1/3/c*b
^2*arctanh(c*x^3)^2-2/3/c*arctanh(c*x^3)*ln(1+(c*x^3+1)^2/(-c^2*x^6+1))*b^2
+1/3/c*a*b*ln(-c^2*x^6+1)-1/3/c*polylog(2,-(c*x^3+1)^2/(-c^2*x^6+1))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2x^3+\frac{1}{12}\left(x^3\log\left(-cx^3+1\right)^2-c^2\left(\frac{2x^3}{c^2}-\frac{\log\left(cx^3+1\right)}{c^3}+\frac{\log\left(cx^3-1\right)}{c^3}\right)-2\left(\frac{x^3}{c}+\frac{\log\left(cx^3-1\right)}{c^2}\right)c\log\left(-cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/12*(x^3*log(-c*x^3 + 1)^2 - c^2*(2*x^3/c^2 - log(c*x^3 + 1)
/c^3 + log(c*x^3 - 1)/c^3) - 2*(x^3/c + log(c*x^3 - 1)/c^2)*c*log(-c*x^3 +
1) + 18*c*integrate(x^5*log(c*x^3 + 1)/(c^2*x^6 - 1), x) + (c*x^3*log(c*x^3
 + 1)^2 + 2*(c*x^3 - (c*x^3 + 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/c + (2*c*
x^3 + log(c*x^3 - 1)^2 + 2*log(c*x^3 - 1))/c - log(c^2*x^6 - 1)/c + 6*integ
rate(x^2*log(c*x^3 + 1)/(c^2*x^6 - 1), x))*b^2 + 1/3*(2*c*x^3*arctanh(c*x^3
) + log(-c^2*x^6 + 1))*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+b\operatorname{atanh}\left(cx^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^3))^2,x)
```

```
[Out] int(x^2*(a + b*atanh(c*x^3))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

3.120 $\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x}\,dx$

Optimal. Leaf size=140

$$-\frac{1}{3}b\text{Li}_2\left(1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{3}b\text{Li}_2\left(\frac{2}{1-cx^3}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{2}{3}\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)\left(a\right.$$

```
[Out] -2/3*(a+b*arctanh(c*x^3))^2*arctanh(-1+2/(-c*x^3+1))-1/3*b*(a+b*arctanh(c*x
^3))*polylog(2,1-2/(-c*x^3+1))+1/3*b*(a+b*arctanh(c*x^3))*polylog(2,-1+2/(-
c*x^3+1))+1/6*b^2*polylog(3,1-2/(-c*x^3+1))-1/6*b^2*polylog(3,-1+2/(-c*x^3+
1))
```

Rubi [A] time = 0.33, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-\frac{1}{3}b\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{3}b\text{PolyLog}\left(2,\frac{2}{1-cx^3}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{6}b^2\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])^2/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^3])^2*ArcTanh[1 - 2/(1 - c*x^3)])/3 - (b*(a + b*ArcTa
nh[c*x^3])*PolyLog[2, 1 - 2/(1 - c*x^3)])/3 + (b*(a + b*ArcTanh[c*x^3])*Pol
yLog[2, -1 + 2/(1 - c*x^3)])/3 + (b^2*PolyLog[3, 1 - 2/(1 - c*x^3)])/6 - (b
^2*PolyLog[3, -1 + 2/(1 - c*x^3)])/6
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, x^3\right)$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) - \frac{1}{3}(4bc)\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}\right.}{}\right.$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) + \frac{1}{3}(2bc)\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}\right.}{1-}\right.$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) - \frac{1}{3}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)\operatorname{Li}_2\left(1-\right.$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) - \frac{1}{3}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)\operatorname{Li}_2\left(1-\right.$$

Mathematica [A] time = 0.07, size = 183, normalized size = 1.31

$$\frac{1}{3}\left(2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 - 4bc\left(\frac{1}{2}\left(\frac{\operatorname{Li}_2\left(\frac{-cx^3-1}{cx^3-1}\right)\left(-a-b\tanh^{-1}\left(cx^3\right)\right)}{2c} + \frac{b\operatorname{Li}_3\left(\frac{-cx^3-1}{cx^3-1}\right)}{4c}\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^2/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^3])^2*ArcTanh[1 - 2/(1 - c*x^3)] - 4*b*c*((((-a - b*A
rcTanh[c*x^3])*PolyLog[2, (-1 - c*x^3)/(-1 + c*x^3)])/(2*c) + (b*PolyLog[3,
 (-1 - c*x^3)/(-1 + c*x^3)])/(4*c))/2 + (-1/2*((-a - b*ArcTanh[c*x^3])*Poly
Log[2, (1 + c*x^3)/(-1 + c*x^3)])/c - (b*PolyLog[3, (1 + c*x^3)/(-1 + c*x^3
)])/(4*c))/2))/3
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2\operatorname{artanh}\left(cx^3\right)^2 + 2\,ab\operatorname{artanh}\left(cx^3\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x, x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(cx^3\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^3) + a)^2/x, x)
```

maple **[F]** time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arctanh}\left(c\,x^3\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^2/x,x)
[Out] int((a+b*arctanh(c*x^3))^2/x,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$a^2\log(x)+\int \frac{b^2\left(\log\left(cx^3+1\right)-\log\left(-cx^3+1\right)\right)^2}{4\,x}+\frac{ab\left(\log\left(cx^3+1\right)-\log\left(-cx^3+1\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="maxima")
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c*x^3 + 1) - log(-c*x^3 + 1))^2/x + a*b
*(log(c*x^3 + 1) - log(-c*x^3 + 1))/x, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\text{atanh}\left(c\,x^3\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^2/x,x)
[Out] int((a + b*atanh(c*x^3))^2/x, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x,x)
[Out] Timed out
```

3.121 $\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=90

$$\frac{1}{3}c\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{3x^3}+\frac{2}{3}bc\log\left(2-\frac{2}{cx^3+1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{3}b^2c\text{Li}_2\left(\frac{2}{cx^3+\right.$$

```
[Out] 1/3*c*(a+b*arctanh(c*x^3))^2-1/3*(a+b*arctanh(c*x^3))^2/x^3+2/3*b*c*(a+b*ar
ctanh(c*x^3))*ln(2-2/(c*x^3+1))-1/3*b^2*c*polylog(2,-1+2/(c*x^3+1))
```

Rubi [B] time = 0.62, antiderivative size = 237, normalized size of antiderivative = 2.63, number of steps used = 24, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {6099, 2454, 2397, 2392, 2391, 2395, 36, 29, 31, 2439, 2416, 2394, 2393}

$$-\frac{1}{3}b^2c\text{PolyLog}\left(2,-cx^3\right)+\frac{1}{3}b^2c\text{PolyLog}\left(2,cx^3\right)+\frac{1}{6}b^2c\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)-\frac{1}{6}b^2c\text{PolyLog}\left(2,\frac{1}{2}\left(cx^3+1\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c*x^3])^2/x^4,x]
```

```
[Out] 2*a*b*c*Log[x] - ((1 - c*x^3)*(2*a - b*Log[1 - c*x^3])^2)/(12*x^3) - (b*c*(
2*a - b*Log[1 - c*x^3])*Log[(1 + c*x^3)/2])/6 - (b^2*c*Log[(1 - c*x^3)/2]*L
og[1 + c*x^3])/6 - (b*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3])/(6*x^3) - (b
^2*(1 + c*x^3)*Log[1 + c*x^3]^2)/(12*x^3) - (b^2*c*PolyLog[2, -(c*x^3)])/3
+ (b^2*c*PolyLog[2, c*x^3])/3 + (b^2*c*PolyLog[2, (1 - c*x^3)/2])/6 - (b^2*
c*PolyLog[2, (1 + c*x^3)/2])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^4}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{4x^4} - \frac{b\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{2x^4} + \frac{b^2\log^2}{4}\right.$$
$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{x^4}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{x^4}\,d$$
$$= \frac{1}{12}\operatorname{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^2}\,dx, x, x^3\right) - \frac{1}{6}b\operatorname{Subst}\left(\int \frac{(-2a+b\log(1-c}{x^2}\right.$$
$$= -\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{6x^3} - \frac{b^2}{}$$
$$= abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+\right.}{6x^3}$$
$$= abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+\right.}{6x^3}$$
$$= 2abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{1}{6}bc\left(2a-b\log\left(1-cx^3\right)\right)\log$$
$$= 2abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{1}{6}bc\left(2a-b\log\left(1-cx^3\right)\right)\log$$
$$= 2abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{1}{6}bc\left(2a-b\log\left(1-cx^3\right)\right)\log$$

Mathematica [A] time = 0.16, size = 117, normalized size = 1.30

$$\frac{-a\left(a+bcx^3\log\left(1-c^2x^6\right)-2bcx^3\log\left(cx^3\right)\right)+2b\tanh^{-1}\left(cx^3\right)\left(bcx^3\log\left(1-e^{-2\tanh^{-1}\left(cx^3\right)}\right)-a\right)-b^2cx^3\mathrm{Li}_2}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^2/x^4,x]
```

```
[Out] (b^2*(-1 + c*x^3)*ArcTanh[c*x^3]^2 + 2*b*ArcTanh[c*x^3]*(-a + b*c*x^3*Log[1
 - E^(-2*ArcTanh[c*x^3])]) - a*(a - 2*b*c*x^3*Log[c*x^3] + b*c*x^3*Log[1 -
c^2*x^6]) - b^2*c*x^3*PolyLog[2, E^(-2*ArcTanh[c*x^3])])/(3*x^3)
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2\operatorname{artanh}\left(cx^3\right)^2+2\,ab\operatorname{artanh}\left(cx^3\right)+a^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\operatorname{artanh}\left(cx^3\right)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x^4, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{arctanh}\left(c\,x^{3}\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^2/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^2/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(c\left(\log\left(c^{2}x^{6}-1\right)-\log\left(x^{6}\right)\right)+\frac{2\,\operatorname{artanh}\left(cx^{3}\right)}{x^{3}}\right)ab-\frac{1}{12}\,b^{2}\left(\frac{\log\left(-cx^{3}+1\right)^{2}}{x^{3}}+3\int -\frac{\left(cx^{3}-1\right)\log\left(cx^{3}+1\right)^{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(c*(log(c^2*x^6 - 1) - log(x^6)) + 2*arctanh(c*x^3)/x^3)*a*b - 1/12*b^
2*(log(-c*x^3 + 1)^2/x^3 + 3*integrate(-((c*x^3 - 1)*log(c*x^3 + 1)^2 + 2*(
c*x^3 - (c*x^3 - 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/(c*x^7 - x^4), x)) - 1
/3*a^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\operatorname{atanh}\left(c\,x^{3}\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^2/x^4,x)
```

```
[Out] int((a + b*atanh(c*x^3))^2/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x**4,x)
```

```
[Out] Timed out
```

3.122 $\int \frac{\left(a+b\tanh^{-1}(cx^3)\right)^2}{x^7}\,dx$

Optimal. Leaf size=88

$$\frac{1}{6}c^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{bc\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{3x^3}-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6x^6}-\frac{1}{6}b^2c^2\log\left(1-c^2x^6\right)+b^2c^2\log(x)$$

```
[Out] -1/3*b*c*(a+b*arctanh(c*x^3))/x^3+1/6*c^2*(a+b*arctanh(c*x^3))^2-1/6*(a+b*a
rctanh(c*x^3))^2/x^6+b^2*c^2*ln(x)-1/6*b^2*c^2*ln(-c^2*x^6+1)
```

Rubi [C] time = 1.06, antiderivative size = 360, normalized size of antiderivative = 4.09, number of steps used = 46, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2395, 44, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{12}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)-\frac{1}{12}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^3+1\right)\right)+\frac{1}{12}bc^2\log\left(\frac{1}{2}\left(cx^3+1\right)\right)\left(2a-b\log\left(1-\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c*x^3])^2/x^7,x]
```

```
[Out] b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c*x^3])/12 - (b*c*(2*a - b*Log[1 - c*x^3]
))/(12*x^3) - (b*c*(1 - c*x^3)*(2*a - b*Log[1 - c*x^3]))/(12*x^3) + (c^2*(2
*a - b*Log[1 - c*x^3])^2)/24 - (2*a - b*Log[1 - c*x^3])^2/(24*x^6) + (b*c^2
*(2*a - b*Log[1 - c*x^3])*Log[(1 + c*x^3)/2])/12 - (b^2*c^2*Log[1 + c*x^3])
/6 - (b^2*c*Log[1 + c*x^3])/(6*x^3) - (b^2*c^2*Log[(1 - c*x^3)/2]*Log[1 + c
*x^3])/12 - (b*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3])/(12*x^6) + (b^2*c^2
*Log[1 + c*x^3]^2)/24 - (b^2*Log[1 + c*x^3]^2)/(24*x^6) - (b^2*c^2*PolyLog[
2, (1 - c*x^3)/2])/12 - (b^2*c^2*PolyLog[2, (1 + c*x^3)/2])/12
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^7}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{4x^7} - \frac{b\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{2x^7} + \frac{b^2\log^2\left(1\right.}{4x^7}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{x^7}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{x^7}\,dx +$$

$$= \frac{1}{12}\,\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^3}\,dx, x, x^3\right) - \frac{1}{6}b\,\text{Subst}\left(\int \frac{(-2a+b\log(1-cx))}{x^3}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{24x^6} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{12x^6} - \frac{b^2\log^2\left(1+cx^3\right.}{24x^6}$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{24x^6} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{12x^6} - \frac{b^2\log^2\left(1+cx^3\right.}{24x^6}$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{24x^6} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{12x^6} - \frac{b^2\log^2\left(1+cx^3\right.}{24x^6}$$

$$= -\frac{1}{2}abc^2\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{12x^3} - \frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)}{12x^3} - \left(\right.$$

$$= \frac{1}{4}b^2c^2\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{12x^3} - \frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)}{12x^3} + \frac{1}{24}$$

$$= \frac{1}{2}b^2c^2\log(x) - \frac{1}{12}b^2c^2\log\left(1-cx^3\right) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{12x^3} - \frac{bc\left(1-cx^3\right)\left(2a\right.}{12}$$

Mathematica [A] time = 0.09, size = 111, normalized size = 1.26

$$\frac{1}{6}\left(-\frac{a^2}{x^6} - bc^2(a+b)\log\left(1-cx^3\right) + bc^2(a-b)\log\left(cx^3+1\right) - \frac{2abc}{x^3} - \frac{2b\tanh^{-1}\left(cx^3\right)\left(a+bcx^3\right)}{x^6} + \frac{b^2\left(c^2x^6-1\right)}{x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^2/x^7,x]
```

```
[Out] (-(a^2/x^6) - (2*a*b*c)/x^3 - (2*b*(a + b*c*x^3)*ArcTanh[c*x^3])/x^6 + (b^2
*(-1 + c^2*x^6)*ArcTanh[c*x^3]^2)/x^6 + 6*b^2*c^2*Log[x] - b*(a + b)*c^2*Lo
g[1 - c*x^3] + (a - b)*b*c^2*Log[1 + c*x^3])/6
```

fricas [A] time = 1.36, size = 151, normalized size = 1.72

$$\frac{24\,b^{2}c^{2}x^{6}\log(x) + 4\left(ab - b^{2}\right)c^{2}x^{6}\log\left(cx^{3}+1\right) - 4\left(ab + b^{2}\right)c^{2}x^{6}\log\left(cx^{3}-1\right) - 8\,abcx^{3} + \left(b^{2}c^{2}x^{6} - b^{2}\right)\log\left(\right.}{24\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^2*c^2*x^6*log(x) + 4*(a*b - b^2)*c^2*x^6*log(c*x^3 + 1) - 4*(a*b
 + b^2)*c^2*x^6*log(c*x^3 - 1) - 8*a*b*c*x^3 + (b^2*c^2*x^6 - b^2)*log(-(c*
x^3 + 1)/(c*x^3 - 1))^2 - 4*a^2 - 4*(b^2*c*x^3 + a*b)*log(-(c*x^3 + 1)/(c*x
^3 - 1)))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(cx^{3}\right) + a\right)^{2}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x^7, x)
```

maple [B] time = 0.28, size = 257, normalized size = 2.92

$$\frac{b^{2}\left(c^{2}x^{6}-1\right)\ln\left(c\,x^{3}+1\right)^{2}}{24x^{6}} - \frac{b\left(x^{6}b\ln\left(-c\,x^{3}+1\right)c^{2} + 2bc\,x^{3} - b\ln\left(-c\,x^{3}+1\right) + 2a\right)\ln\left(c\,x^{3}+1\right)}{12x^{6}} + \frac{b^{2}c^{2}x^{6}\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^2/x^7,x)
```

```
[Out] 1/24*b^2*(c^2*x^6-1)/x^6*ln(c*x^3+1)^2-1/12*b*(x^6*b*ln(-c*x^3+1)*c^2+2*b*c
*x^3-b*ln(-c*x^3+1)+2*a)/x^6*ln(c*x^3+1)+1/24*(b^2*c^2*x^6*ln(-c*x^3+1)^2+2
4*b^2*c^2*ln(x)*x^6+4*b*c^2*ln(c*x^3+1)*x^6*a-4*b^2*c^2*ln(c*x^3+1)*x^6-4*b
*c^2*ln(c*x^3-1)*x^6*a-4*b^2*c^2*ln(c*x^3-1)*x^6+4*b^2*c*x^3*ln(-c*x^3+1)-8
*a*b*c*x^3-b^2*ln(-c*x^3+1)^2+4*b*ln(-c*x^3+1)*a-4*a^2)/x^6
```

maxima [B] time = 0.33, size = 175, normalized size = 1.99

$$\frac{1}{6}\left(\left(c\log\left(cx^{3}+1\right) - c\log\left(cx^{3}-1\right) - \frac{2}{x^{3}}\right)c - \frac{2\,\text{artanh}\left(cx^{3}\right)}{x^{6}}\right)ab + \frac{1}{24}\left(\left(2\left(\log\left(cx^{3}-1\right) - 2\right)\log\left(cx^{3}+1\right) - \text{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^6
)*a*b + 1/24*((2*(log(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 - l
og(c*x^3 - 1)^2 - 4*log(c*x^3 - 1) + 24*log(x))*c^2 + 4*(c*log(c*x^3 + 1) -
 c*log(c*x^3 - 1) - 2/x^3)*c*arctanh(c*x^3))*b^2 - 1/6*b^2*arctanh(c*x^3)^2
/x^6 - 1/6*a^2/x^6
```

mupad [B] time = 1.54, size = 278, normalized size = 3.16

$$\frac{b^{2}\,c^{2}\ln\left(c\,x^{3}+1\right)^{2}}{24} - \frac{b^{2}\,c^{2}\,\ln\left(c\,x^{3}-1\right)}{6} - \frac{b^{2}\,c^{2}\,\ln\left(c\,x^{3}+1\right)}{6} - \frac{a^{2}}{6\,x^{6}} + \frac{b^{2}\,c^{2}\ln\left(1-c\,x^{3}\right)^{2}}{24} - \frac{b^{2}\ln\left(c\,x^{3}+1\right)^{2}}{24\,x^{6}} - \frac{b^{2}\ln\left(\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^2/x^7,x)
```

```
[Out] (b^2*c^2*log(c*x^3 + 1)^2)/24 - (b^2*c^2*log(c*x^3 - 1))/6 - (b^2*c^2*log(c
*x^3 + 1))/6 - a^2/(6*x^6) + (b^2*c^2*log(1 - c*x^3)^2)/24 - (b^2*log(c*x^3
 + 1)^2)/(24*x^6) - (b^2*log(1 - c*x^3)^2)/(24*x^6) + b^2*c^2*log(x) - (a*b
*c^2*log(c*x^3 - 1))/6 + (a*b*c^2*log(c*x^3 + 1))/6 - (a*b*c)/(3*x^3) - (a*
b*log(c*x^3 + 1))/(6*x^6) + (a*b*log(1 - c*x^3))/(6*x^6) - (b^2*c^2*log(c*x
^3 + 1)*log(1 - c*x^3))/12 - (b^2*c*log(c*x^3 + 1))/(6*x^3) + (b^2*c*log(1
- c*x^3))/(6*x^3) + (b^2*log(c*x^3 + 1)*log(1 - c*x^3))/(12*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x**7,x)
```

```
[Out] Timed out
```

3.123 $\int \frac{\left(a+b\tanh^{-1}(cx^3)\right)^2}{x^{10}}\,dx$

Optimal. Leaf size=144

$$\frac{1}{9}c^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{2}{9}bc^3\log\left(2-\frac{2}{cx^3+1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{9x^9}-\frac{bc\left(a+b\tanh\right.}{9x^6}$$

```
[Out] -1/9*b^2*c^2/x^3+1/9*b^2*c^3*arctanh(c*x^3)-1/9*b*c*(a+b*arctanh(c*x^3))/x^
6+1/9*c^3*(a+b*arctanh(c*x^3))^2-1/9*(a+b*arctanh(c*x^3))^2/x^9+2/9*b*c^3*(
a+b*arctanh(c*x^3))*ln(2-2/(c*x^3+1))-1/9*b^2*c^3*polylog(2,-1+2/(c*x^3+1))
```

Rubi [B] time = 1.31, antiderivative size = 420, normalized size of antiderivative = 2.92, number of steps used = 59, number of rules used = 24, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2395, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{9}b^2c^3\text{PolyLog}\left(2,-cx^3\right)+\frac{1}{9}b^2c^3\text{PolyLog}\left(2,cx^3\right)+\frac{1}{18}b^2c^3\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)-\frac{1}{18}b^2c^3\text{PolyLog}\left(2,\frac{1}{2}\left(c\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c*x^3])^2/x^10,x]
```

```
[Out] -(b^2*c^2)/(9*x^3) + (2*a*b*c^3*Log[x])/3 - (b*c*(2*a - b*Log[1 - c*x^3]))/
(18*x^6) + (b*c^2*(2*a - b*Log[1 - c*x^3]))/(18*x^3) - (b*c^2*(1 - c*x^3)*(
2*a - b*Log[1 - c*x^3]))/(18*x^3) + (c^3*(2*a - b*Log[1 - c*x^3])^2)/36 - (
2*a - b*Log[1 - c*x^3])^2/(36*x^9) - (b*c^3*(2*a - b*Log[1 - c*x^3])*Log[(1
 + c*x^3)/2])/18 + (b^2*c^3*Log[1 + c*x^3])/18 - (b^2*c*Log[1 + c*x^3])/(18
*x^6) - (b^2*c^3*Log[(1 - c*x^3)/2]*Log[1 + c*x^3])/18 - (b*(2*a - b*Log[1
- c*x^3])*Log[1 + c*x^3])/(18*x^9) - (b^2*c^3*Log[1 + c*x^3]^2)/36 - (b^2*L
og[1 + c*x^3]^2)/(36*x^9) - (b^2*c^3*PolyLog[2, -(c*x^3)])/9 + (b^2*c^3*Pol
yLog[2, c*x^3])/9 + (b^2*c^3*PolyLog[2, (1 - c*x^3)/2])/18 - (b^2*c^3*PolyL
og[2, (1 + c*x^3)/2])/18
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
```

```
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^{10}}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{4x^{10}} - \frac{b\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{2x^{10}} + \frac{b^2\log^2(\ldots}{4\ldots}$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{x^{10}}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{x^{10}}\,d\ldots$$

$$= \frac{1}{12}\,\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^4}\,dx, x, x^3\right) - \frac{1}{6}b\,\text{Subst}\left(\int \frac{(-2a+b\log(1-c\ldots}{x^4}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{36x^9} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{18x^9} - \frac{b^2\log^2\left(1+\ldots\right.}{36x^9}$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{36x^9} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{18x^9} - \frac{b^2\log^2\left(1+\ldots\right.}{36x^9}$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{36x^9} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{18x^9} - \frac{b^2\log^2\left(1+\ldots\right.}{36x^9}$$

$$= \frac{1}{3}abc^3\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{18x^6} + \frac{bc^2\left(2a-b\log\left(1-cx^3\right)\right)}{18x^3} - \frac{\left(2a-b\ldots\right.}{}$$

$$= \frac{1}{3}abc^3\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{18x^6} + \frac{bc^2\left(2a-b\log\left(1-cx^3\right)\right)}{18x^3} - \frac{bc^2\left(1-\ldots\right.}{}$$

$$= -\frac{b^2c^2}{18x^3} + \frac{2}{3}abc^3\log(x) + \frac{1}{6}b^2c^3\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{18x^6} + \frac{bc^2\left(2a-b\ldots\right.}{1\ldots}$$

$$= -\frac{b^2c^2}{18x^3} + \frac{2}{3}abc^3\log(x) + \frac{1}{6}b^2c^3\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{18x^6} + \frac{bc^2\left(2a-b\ldots\right.}{1\ldots}$$

Mathematica [A] time = 0.38, size = 159, normalized size = 1.10

$$-\frac{a^2-2abc^3x^9\log\left(cx^3\right)+b\tanh^{-1}\left(cx^3\right)\left(2a-bc^3x^9-2bc^3x^9\log\left(1-e^{-2\tanh^{-1}\left(cx^3\right)}\right)+bcx^3\right)+abc^3x^9\log\left(1\ldots\right.}{9x^9}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^2/x^10,x]
```

```
[Out] -1/9*(a^2 + a*b*c*x^3 + b^2*c^2*x^6 + b^2*(1 - c^3*x^9)*ArcTanh[c*x^3]^2 +
b*ArcTanh[c*x^3]*(2*a + b*c*x^3 - b*c^3*x^9 - 2*b*c^3*x^9*Log[1 - E^(-2*Arc
Tanh[c*x^3])]) - 2*a*b*c^3*x^9*Log[c*x^3] + a*b*c^3*x^9*Log[1 - c^2*x^6] +
b^2*c^3*x^9*PolyLog[2, E^(-2*ArcTanh[c*x^3])])/x^9
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(cx^3\right)^2+2\,ab\,\text{artanh}\left(cx^3\right)+a^2}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x^10, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(cx^3\right) + a\right)^2}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x^10, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{arctanh}\left(c\, x^3\right)\right)^2}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^2/x^10,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^2/x^10,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{9}\left(\left(c^2 \log\left(c^2x^6 - 1\right) - c^2 \log\left(x^6\right) + \frac{1}{x^6}\right)c + \frac{2 \operatorname{artanh}\left(cx^3\right)}{x^9}\right)ab - \frac{1}{36}\, b^2\left(\frac{\log\left(-cx^3 + 1\right)^2}{x^9} + 9 \int -\frac{3\left(cx^3 - 1\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/9*((c^2*log(c^2*x^6 - 1) - c^2*log(x^6) + 1/x^6)*c + 2*arctanh(c*x^3)/x^
9)*a*b - 1/36*b^2*(log(-c*x^3 + 1)^2/x^9 + 9*integrate(-1/3*(3*(c*x^3 - 1)*
log(c*x^3 + 1)^2 + 2*(c*x^3 - 3*(c*x^3 - 1)*log(c*x^3 + 1))*log(-c*x^3 + 1)
)/(c*x^13 - x^10), x)) - 1/9*a^2/x^9
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{atanh}\left(c\, x^3\right)\right)^2}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^2/x^10,x)
```

```
[Out] int((a + b*atanh(c*x^3))^2/x^10, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x**10,x)
```

```
[Out] Timed out
```

3.124 $\int x^8 \left(a + b \tanh^{-1}\left(cx^3\right)\right)^3 \, dx$

Optimal. Leaf size=231

$$-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{3c^3}+\frac{ab^2x^3}{3c^2}+\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{9c^3}-\frac{b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6c^3}-\frac{b\log\left(\frac{2}{1-cx^3}\right)}{}$$

```
[Out] 1/3*a*b^2*x^3/c^2+1/3*b^3*x^3*arctanh(c*x^3)/c^2-1/6*b*(a+b*arctanh(c*x^3))
^2/c^3+1/6*b*x^6*(a+b*arctanh(c*x^3))^2/c+1/9*(a+b*arctanh(c*x^3))^3/c^3+1/
9*x^9*(a+b*arctanh(c*x^3))^3-1/3*b*(a+b*arctanh(c*x^3))^2*ln(2/(-c*x^3+1))/
c^3+1/6*b^3*ln(-c^2*x^6+1)/c^3-1/3*b^2*(a+b*arctanh(c*x^3))*polylog(2,1-2/(
-c*x^3+1))/c^3+1/6*b^3*polylog(3,1-2/(-c*x^3+1))/c^3
```

Rubi [B] time = 6.35, antiderivative size = 1421, normalized size of antiderivative = 6.15, number of steps used = 239, number of rules used = 32, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2398, 2411, 43, 2334, 12, 14, 2301, 2439, 2416, 2396, 2433, 2374, 6589, 6742, 2430, 2394, 2393, 2391, 2395, 2375, 2317, 2410, 2425}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[x^8*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (2*a*b^2*x^3)/(3*c^2) - (7*b^3*x^3)/(216*c^2) - (23*b^3*x^6)/(432*c) + (b^3
*x^9)/324 + (b^3*(1 - c*x^3)^2)/(48*c^3) + (b^3*(1 + c*x^3)^2)/(24*c^3) - (
b^3*(1 + c*x^3)^3)/(324*c^3) - (b^3*Log[1 - c*x^3])/(24*c^3) + (b^3*(1 - c*
x^3)*Log[1 - c*x^3])/(3*c^3) - (b^3*Log[1 - c*x^3]^2)/(72*c^3) - (b^2*x^6*(
2*a - b*Log[1 - c*x^3]))/(24*c) + (b^2*(1 - c*x^3)^2*(2*a - b*Log[1 - c*x^3
]))/(12*c^3) - (b^2*(1 - c*x^3)^3*(2*a - b*Log[1 - c*x^3]))/(108*c^3) - (b*
x^9*(2*a - b*Log[1 - c*x^3])^2)/72 - (b*(1 - c*x^3)*(2*a - b*Log[1 - c*x^3]
)^2)/(8*c^3) + (b*(1 - c*x^3)^2*(2*a - b*Log[1 - c*x^3])^2)/(12*c^3) - (b*(
1 - c*x^3)^3*(2*a - b*Log[1 - c*x^3])^2)/(72*c^3) - ((1 - c*x^3)*(2*a - b*L
og[1 - c*x^3])^3)/(24*c^3) + ((1 - c*x^3)^2*(2*a - b*Log[1 - c*x^3])^3)/(24
*c^3) - ((1 - c*x^3)^3*(2*a - b*Log[1 - c*x^3])^3)/(72*c^3) + (b^2*(2*a - b
*Log[1 - c*x^3])*((18*(1 - c*x^3))/c^3 - (9*(1 - c*x^3)^2)/c^3 + (2*(1 - c*
x^3)^3)/c^3 - (6*Log[1 - c*x^3])/c^3))/216 - (b^2*(2*a - b*Log[1 - c*x^3])*
Log[(1 + c*x^3)/2])/(12*c^3) + (b*(2*a - b*Log[1 - c*x^3])^2*Log[(1 + c*x^3
)/2])/(12*c^3) - (7*b^3*Log[1 + c*x^3])/(108*c^3) + (b^3*x^6*Log[1 + c*x^3]
)/(18*c) - (b^3*x^9*Log[1 + c*x^3])/108 + (11*b^3*(1 + c*x^3)*Log[1 + c*x^3
])/(36*c^3) - (b^3*(1 + c*x^3)^2*Log[1 + c*x^3])/(12*c^3) + (b^3*(1 + c*x^3
)^3*Log[1 + c*x^3])/(108*c^3) + (b^3*Log[(1 - c*x^3)/2]*Log[1 + c*x^3])/(12
*c^3) + (b^2*x^6*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3])/(12*c) - (b*(2*a
- b*Log[1 - c*x^3])^2*Log[1 + c*x^3])/(24*c^3) + (b*x^9*(2*a - b*Log[1 - c*
x^3])^2*Log[1 + c*x^3])/24 + (b^3*Log[1 + c*x^3]^2)/(72*c^3) + (b^3*x^9*Log
[1 + c*x^3]^2)/72 - (b^3*(1 + c*x^3)*Log[1 + c*x^3]^2)/(8*c^3) + (b^3*(1 +
c*x^3)^2*Log[1 + c*x^3]^2)/(12*c^3) - (b^3*(1 + c*x^3)^3*Log[1 + c*x^3]^2)/
(72*c^3) + (b^3*Log[(1 - c*x^3)/2]*Log[1 + c*x^3]^2)/(12*c^3) + (b^2*(2*a -
 b*Log[1 - c*x^3])*Log[1 + c*x^3]^2)/(24*c^3) + (b^2*x^9*(2*a - b*Log[1 - c
*x^3])*Log[1 + c*x^3]^2)/24 + (b^3*(1 + c*x^3)*Log[1 + c*x^3]^3)/(24*c^3) -
 (b^3*(1 + c*x^3)^2*Log[1 + c*x^3]^3)/(24*c^3) + (b^3*(1 + c*x^3)^3*Log[1 +
 c*x^3]^3)/(72*c^3) + (b^3*PolyLog[2, (1 - c*x^3)/2])/(12*c^3) - (b^2*(2*a
- b*Log[1 - c*x^3])*PolyLog[2, (1 - c*x^3)/2])/(6*c^3) + (b^3*PolyLog[2, (1
 + c*x^3)/2])/(12*c^3) + (b^3*Log[1 + c*x^3]*PolyLog[2, (1 + c*x^3)/2])/(6*
c^3) - (b^3*PolyLog[3, (1 - c*x^3)/2])/(6*c^3) - (b^3*PolyLog[3, (1 + c*x^3
)/2])/(6*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
```

```
] && EqQ[m, -1])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^8 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 dx = \int \left(\frac{1}{8}x^8\left(2a-b\log\left(1-cx^3\right)\right)^3+\frac{3}{8}bx^8\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)-\right.$$

$$= \frac{1}{8}\int x^8\left(2a-b\log\left(1-cx^3\right)\right)^3 dx+\frac{1}{8}(3b)\int x^8\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1\right.$$

$$= \frac{1}{24}\text{Subst}\left(\int x^2(2a-b\log(1-cx))^3\,dx,x,x^3\right)+\frac{1}{8}b\,\text{Subst}\left(\int x^2(-2a+b\log(1\right.$$

$$= \frac{1}{24}bx^9\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{24}b^2x^9\left(2a-b\log\left(1-cx^3\right)\right)\log^2$$

$$= \frac{1}{24}bx^9\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{24}b^2x^9\left(2a-b\log\left(1-cx^3\right)\right)\log^2$$

$$= \frac{1}{24}bx^9\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{24}b^2x^9\left(2a-b\log\left(1-cx^3\right)\right)\log^2$$

$$= -\frac{1}{72}bx^9\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c^3}+\frac{\left(1-cx^3\right)^2}{}$$

$$= -\frac{1}{72}bx^9\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c^3}+\frac{b\left(1-cx^3\right.}{}$$

$$= \frac{ab^2x^3}{3c^2}-\frac{b^3x^3}{6c^2}+\frac{b^3\left(1-cx^3\right)^2}{32c^3}-\frac{b^3\left(1-cx^3\right)^3}{324c^3}+\frac{b^3\left(1+cx^3\right)^2}{32c^3}-\frac{b^3\left(1+cx^3\right)^3}{324c^3}+$$

$$= \frac{ab^2x^3}{3c^2}+\frac{b^3\left(1-cx^3\right)^2}{32c^3}-\frac{b^3\left(1-cx^3\right)^3}{324c^3}+\frac{b^3\left(1+cx^3\right)^2}{32c^3}-\frac{b^3\left(1+cx^3\right)^3}{324c^3}+\frac{b^3\left(1-\right.}{}$$

$$= \frac{ab^2x^3}{2c^2}-\frac{b^3x^3}{18c^2}+\frac{b^3\left(1-cx^3\right)^2}{24c^3}-\frac{b^3\left(1-cx^3\right)^3}{324c^3}+\frac{b^3\left(1+cx^3\right)^2}{24c^3}-\frac{b^3\left(1+cx^3\right)^3}{324c^3}+$$

$$= \frac{ab^2x^3}{2c^2}-\frac{7b^3x^3}{216c^2}-\frac{5b^3x^6}{432c}+\frac{b^3x^9}{324}+\frac{b^3\left(1-cx^3\right)^2}{48c^3}+\frac{b^3\left(1+cx^3\right)^2}{24c^3}-\frac{b^3\left(1+cx^3\right)^3}{324c^3}$$

$$= \frac{ab^2x^3}{2c^2}-\frac{7b^3x^3}{216c^2}-\frac{5b^3x^6}{432c}+\frac{b^3x^9}{324}+\frac{b^3\left(1-cx^3\right)^2}{48c^3}+\frac{b^3\left(1+cx^3\right)^2}{24c^3}-\frac{b^3\left(1+cx^3\right)^3}{324c^3}$$

Mathematica [A] time = 0.50, size = 334, normalized size = 1.45

$$2a^3c^3x^9+6a^2bc^3x^9\tanh^{-1}\left(cx^3\right)+3a^2bc^2x^6+3a^2b\log\left(1-c^2x^6\right)+6ab^2c^3x^9\tanh^{-1}\left(cx^3\right)^2+6ab^2c^2x^6\tanh^{-1}\left(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (6*a*b^2*c*x^3 + 3*a^2*b*c^2*x^6 + 2*a^3*c^3*x^9 - 6*a*b^2*ArcTanh[c*x^3] +
 6*b^3*c*x^3*ArcTanh[c*x^3] + 6*a*b^2*c^2*x^6*ArcTanh[c*x^3] + 6*a^2*b*c^3*
x^9*ArcTanh[c*x^3] - 6*a*b^2*ArcTanh[c*x^3]^2 - 3*b^3*ArcTanh[c*x^3]^2 + 3*
b^3*c^2*x^6*ArcTanh[c*x^3]^2 + 6*a*b^2*c^3*x^9*ArcTanh[c*x^3]^2 - 2*b^3*Arc
Tanh[c*x^3]^3 + 2*b^3*c^3*x^9*ArcTanh[c*x^3]^3 - 12*a*b^2*ArcTanh[c*x^3]*Lo
g[1 + E^(-2*ArcTanh[c*x^3])] - 6*b^3*ArcTanh[c*x^3]^2*Log[1 + E^(-2*ArcTanh
[c*x^3])] + 3*a^2*b*Log[1 - c^2*x^6] + 3*b^3*Log[1 - c^2*x^6] + 6*b^2*(a +
b*ArcTanh[c*x^3])*PolyLog[2, -E^(-2*ArcTanh[c*x^3])] + 3*b^3*PolyLog[3, -E^
(-2*ArcTanh[c*x^3])])/(18*c^3)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^8 \operatorname{artanh}\left(cx^3\right)^3 + 3\,ab^2x^8 \operatorname{artanh}\left(cx^3\right)^2 + 3\,a^2bx^8 \operatorname{artanh}\left(cx^3\right) + a^3x^8, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^8*arctanh(c*x^3)^3 + 3*a*b^2*x^8*arctanh(c*x^3)^2 + 3*a^2*b*
x^8*arctanh(c*x^3) + a^3*x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(cx^3\right) + a\right)^3 x^8\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*x^8, x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int x^8 \left(a + b \operatorname{arctanh}\left(c\,x^3\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctanh(c*x^3))^3,x)
```

```
[Out] int(x^8*(a+b*arctanh(c*x^3))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{9}\,a^3x^9 + \frac{1}{6}\left(2\,x^9 \operatorname{artanh}\left(cx^3\right) + \left(\frac{x^6}{c^2} + \frac{\log\left(c^2x^6 - 1\right)}{c^4}\right)c\right)a^2b - \frac{\left(b^3c^3x^9 - b^3\right)\log\left(-cx^3 + 1\right)^3 - 3\left(2\,ab^2c^3x^9 + b^3c\right.}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/9*a^3*x^9 + 1/6*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)*
c)*a^2*b - 1/72*((b^3*c^3*x^9 - b^3)*log(-c*x^3 + 1)^3 - 3*(2*a*b^2*c^3*x^9
 + b^3*c^2*x^6 + (b^3*c^3*x^9 + b^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/c^3
 - integrate(-1/8*((b^3*c^3*x^11 - b^3*c^2*x^8)*log(c*x^3 + 1)^3 + 6*(a*b^2
*c^3*x^11 - a*b^2*c^2*x^8)*log(c*x^3 + 1)^2 - (4*a*b^2*c^3*x^11 + 2*b^3*c^2
*x^8 + 3*(b^3*c^3*x^11 - b^3*c^2*x^8)*log(c*x^3 + 1)^2 - 2*(6*a*b^2*c^2*x^8
 - (6*a*b^2*c^3 + b^3*c^3)*x^11 - b^3*x^2)*log(c*x^3 + 1))*log(-c*x^3 + 1))
/(c^3*x^3 - c^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^8 \left(a + b \operatorname{atanh}\left(c x^3\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*atanh(c*x^3))^3,x)
[Out] int(x^8*(a + b*atanh(c*x^3))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atanh(c*x**3))**3,x)
[Out] Timed out
```

3.125 $\int x^5 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 \, dx$

Optimal. Leaf size=139

$$-\frac{b^2\log\left(\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{c^2}-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{6c^2}+\frac{b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{2c^2}+\frac{bx^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{2c}$$

```
[Out] 1/2*b*(a+b*arctanh(c*x^3))^2/c^2+1/2*b*x^3*(a+b*arctanh(c*x^3))^2/c-1/6*(a+
b*arctanh(c*x^3))^3/c^2+1/6*x^6*(a+b*arctanh(c*x^3))^3-b^2*(a+b*arctanh(c*x
^3))*ln(2/(-c*x^3+1))/c^2-1/2*b^3*polylog(2,1-2/(-c*x^3+1))/c^2
```

Rubi [B] time = 4.20, antiderivative size = 479, normalized size of antiderivative = 3.45, number of steps used = 155, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2439, 2416, 2396, 2433, 2374, 6589, 2411, 43, 2334, 12, 14, 2301, 6742, 2430, 2394, 2393, 2391, 2395, 2375, 2317, 2425}

$$-\frac{b^3\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)}{4c^2}+\frac{b^3\text{PolyLog}\left(2,\frac{1}{2}\left(cx^3+1\right)\right)}{4c^2}-\frac{b^2\log^2\left(cx^3+1\right)\left(2a-b\log\left(1-cx^3\right)\right)}{16c^2}+\frac{b^2\log\left(\frac{1}{2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] -(b*(1 - c*x^3)*(2*a - b*Log[1 - c*x^3])^2)/(8*c^2) - ((1 - c*x^3)*(2*a - b
*Log[1 - c*x^3])^3)/(24*c^2) + ((1 - c*x^3)^2*(2*a - b*Log[1 - c*x^3])^3)/(
48*c^2) + (b^2*(2*a - b*Log[1 - c*x^3])*Log[(1 + c*x^3)/2])/(4*c^2) + (b^3*
Log[(1 - c*x^3)/2]*Log[1 + c*x^3])/(4*c^2) + (b^2*x^3*(2*a - b*Log[1 - c*x^
3])*Log[1 + c*x^3])/(4*c) - (b*(2*a - b*Log[1 - c*x^3])^2*Log[1 + c*x^3])/(
16*c^2) + (b*x^6*(2*a - b*Log[1 - c*x^3])^2*Log[1 + c*x^3])/16 + (b^3*(1 +
c*x^3)*Log[1 + c*x^3]^2)/(8*c^2) - (b^2*(2*a - b*Log[1 - c*x^3])*Log[1 + c*
x^3]^2)/(16*c^2) + (b^2*x^6*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3]^2)/16 -
 (b^3*(1 + c*x^3)*Log[1 + c*x^3]^3)/(24*c^2) + (b^3*(1 + c*x^3)^2*Log[1 + c
*x^3]^3)/(48*c^2) - (b^3*PolyLog[2, (1 - c*x^3)/2])/(4*c^2) + (b^3*PolyLog[
2, (1 + c*x^3)/2])/(4*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\,dx=\int\left(\frac{1}{8}x^5\left(2a-b\log\left(1-cx^3\right)\right)^3+\frac{3}{8}bx^5\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)\right.$$

$$=\frac{1}{8}\int x^5\left(2a-b\log\left(1-cx^3\right)\right)^3\,dx+\frac{1}{8}(3b)\int x^5\left(-2a+b\log\left(1-cx^3\right)\right)^2\log$$

$$=\frac{1}{24}\mathrm{Subst}\left(\int x(2a-b\log(1-cx))^3\,dx,x,x^3\right)+\frac{1}{8}b\,\mathrm{Subst}\left(\int x(-2a+b\log(\right.\right.$$

$$=\frac{1}{16}bx^6\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{16}b^2x^6\left(2a-b\log\left(1-cx^3\right)\right)\mathrm{lo}$$

$$=\frac{1}{16}bx^6\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{16}b^2x^6\left(2a-b\log\left(1-cx^3\right)\right)\mathrm{lo}$$

$$=\frac{1}{16}bx^6\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{16}b^2x^6\left(2a-b\log\left(1-cx^3\right)\right)\mathrm{lo}$$

$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c^2}+\frac{\left(1-cx^3\right)^2\left(2a-b\log\left(1-cx^3\right)\right)^3}{48c^2}-\frac{b\left(2\right.}{}$$

$$=-\frac{3b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16c^2}+\frac{b\left(1-cx^3\right)^2\left(2a-b\log\left(1-cx^3\right)\right)^2}{32c^2}-$$

$$=\frac{3ab^2x^3}{4c}+\frac{3b^3x^3}{8c}+\frac{b^3\left(1-cx^3\right)^2}{64c^2}-\frac{b^3\left(1+cx^3\right)^2}{64c^2}+\frac{b^2\left(1-cx^3\right)^2\left(2a-b\log\left(\right.\right.}{32c^2}$$

$$=\frac{3ab^2x^3}{4c}+\frac{3b^3x^3}{4c}+\frac{b^3\left(1-cx^3\right)^2}{64c^2}-\frac{b^3\left(1+cx^3\right)^2}{64c^2}+\frac{3b^3\left(1-cx^3\right)\log\left(1-cx^3\right)}{8c^2}$$

$$=\frac{ab^2x^3}{2c}+\frac{5b^3x^3}{8c}+\frac{3b^3\left(1-cx^3\right)\log\left(1-cx^3\right)}{8c^2}-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-cx\right.\right.}{8c^2}$$

$$=\frac{ab^2x^3}{2c}+\frac{b^3x^3}{2c}+\frac{b^3\left(1-cx^3\right)\log\left(1-cx^3\right)}{4c^2}-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right.}{8c^2}$$

Mathematica [A] time = 0.32, size = 185, normalized size = 1.33

$$a\left(2a^2c^2x^6+6abcx^3+3ab\log\left(1-cx^3\right)-3ab\log\left(cx^3+1\right)+6b^2\log\left(1-c^2x^6\right)\right)+6b^2\left(cx^3-1\right)\tanh^{-1}\left(cx^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (6*b^2*(-1 + c*x^3)*(a + b + a*c*x^3)*ArcTanh[c*x^3]^2 + 2*b^3*(-1 + c^2*x^
6)*ArcTanh[c*x^3]^3 + 6*b*ArcTanh[c*x^3]*(a*c*x^3*(2*b + a*c*x^3) - 2*b^2*L
og[1 + E^(-2*ArcTanh[c*x^3])]) + a*(6*a*b*c*x^3 + 2*a^2*c^2*x^6 + 3*a*b*Log
[1 - c*x^3] - 3*a*b*Log[1 + c*x^3] + 6*b^2*Log[1 - c^2*x^6]) + 6*b^3*PolyLo
g[2, -E^(-2*ArcTanh[c*x^3])])/(12*c^2)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^5\operatorname{artanh}\left(cx^3\right)^3+3\,ab^2x^5\operatorname{artanh}\left(cx^3\right)^2+3\,a^2bx^5\operatorname{artanh}\left(cx^3\right)+a^3x^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^5*arctanh(c*x^3)^3 + 3*a*b^2*x^5*arctanh(c*x^3)^2 + 3*a^2*b*
x^5*arctanh(c*x^3) + a^3*x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\operatorname{artanh}\left(cx^3\right)+a\right)^3x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*x^5, x)
```

maple [C] time = 0.44, size = 750, normalized size = 5.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^3))^3,x)
```

```
[Out] 1/48*b^3*(c^2*x^6-1)/c^2*ln(c*x^3+1)^3+1/16*b^2*(-x^6*b*ln(-c*x^3+1)*c^2+2*
a*c^2*x^6+2*b*c*x^3+b*ln(-c*x^3+1)-2*a+2*b)/c^2*ln(c*x^3+1)^2+(1/16*b^3*(c^
2*x^6-1)/c^2*ln(-c*x^3+1)^2-1/4*b^2*x^3*(a*c*x^3+b)/c*ln(-c*x^3+1)+1/4*b*(a
^2*c^2*x^6+2*a*b*c*x^3+b*ln(-c*x^3+1)*a+b^2*ln(-c*x^3+1))/c^2)*ln(c*x^3+1)+
1/8*b^2/c^2*a*ln(c*x^3-1)-1/4*a^2*b/c^2*ln(c*x^3+1)+1/2*a*b^2/c^2*ln(c*x^3+
1)+1/4*a^2*b/c^2*ln(c*x^3-1)+1/8*a*b^2*x^6*ln(-c*x^3+1)^2+3/8*a*b^2/c^2*ln(
-c*x^3+1)-1/8*a*b^2/c^2*ln(-c*x^3+1)^2+1/8*b^3/c*x^3*ln(-c*x^3+1)^2-1/4*a^2
*b*x^6*ln(-c*x^3+1)+1/6*x^6*a^3+1/4/c^2*b^3*ln(-c*x^3+1)-1/4/c^2*b^3*ln(c*x
^3-1)-1/48*b^3*x^6*ln(-c*x^3+1)^3-1/8*b^3/c^2*ln(-c*x^3+1)^2+1/48*b^3/c^2*l
n(-c*x^3+1)^3+3/4*b^2/c*Sum(-2/3*(ln(x-_alpha)*ln(-c*x^3+1)+3*c*(-1/3*ln(x-
_alpha)*(ln((RootOf(_Z^2+_Z*_alpha+_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+
_Z*_alpha+_alpha^2,index=1))+ln((RootOf(_Z^2+_Z*_alpha+_alpha^2,index=2)-x+
_alpha)/RootOf(_Z^2+_Z*_alpha+_alpha^2,index=2))+ln(1/2*(x+_alpha)/_alpha))
/c-1/3*(dilog((RootOf(_Z^2+_Z*_alpha+_alpha^2,index=1)-x+_alpha)/RootOf(_Z^
2+_Z*_alpha+_alpha^2,index=1))+dilog((RootOf(_Z^2+_Z*_alpha+_alpha^2,index=
2)-x+_alpha)/RootOf(_Z^2+_Z*_alpha+_alpha^2,index=2))+dilog(1/2*(x+_alpha)/
_alpha))/c))*b/c,_alpha=RootOf(_Z^3*c+1))-1/2*a*b^2/c*x^3*ln(-c*x^3+1)-1/8*
b^3/c^2+1/2/c*a^2*b*x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}ab^2x^6\operatorname{artanh}\left(cx^3\right)^2+\frac{1}{6}a^3x^6+\frac{1}{4}\left(2\,x^6\operatorname{artanh}\left(cx^3\right)+c\left(\frac{2\,x^3}{c^2}-\frac{\log\left(cx^3+1\right)}{c^3}+\frac{\log\left(cx^3-1\right)}{c^3}\right)\right)a^2b+\frac{1}{8}\left(4\,c\left(\frac{2\,x^3}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b^2*x^6*arctanh(c*x^3)^2 + 1/6*a^3*x^6 + 1/4*(2*x^6*arctanh(c*x^3) +
c*(2*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3))*a^2*b + 1/8*(4*c*(
2*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3)*arctanh(c*x^3) - (2*(l
og(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 - log(c*x^3 - 1)^2 - 4
*log(c*x^3 - 1))/c^2)*a*b^2 - 1/192*(4*x^6*log(-c*x^3 + 1)^3 + 3*(x^6/c^3 +
 log(c^2*x^6 - 1)/c^5)*c^3 - 6*c*((c*x^6 + 2*x^3)/c^2 + 2*log(c*x^3 - 1)/c^
3)*log(-c*x^3 + 1)^2 + 21*c^2*(2*x^3/c^3 - log(c*x^3 + 1)/c^4 + log(c*x^3 -
 1)/c^4) + c*(6*(c^2*x^6 + 6*c*x^3 + 2*log(c*x^3 - 1)^2 + 6*log(c*x^3 - 1))
*log(-c*x^3 + 1)/c^3 - (3*c^2*x^6 + 42*c*x^3 + 4*log(c*x^3 - 1)^3 + 18*log(
c*x^3 - 1)^2 + 42*log(c*x^3 - 1))/c^3) - 1728*c*integrate(1/4*x^5*log(c*x^3
 + 1)/(c^3*x^6 - c), x) - 2*(12*c*x^3*log(c*x^3 + 1)^2 + 2*(c^2*x^6 - 1)*lo
g(c*x^3 + 1)^3 - 3*(c^2*x^6 - 2*c*x^3 - 2*(c^2*x^6 - 1)*log(c*x^3 + 1) + 1)
*log(-c*x^3 + 1)^2 + 3*(c^2*x^6 + 6*c*x^3 - 2*(c^2*x^6 - 1)*log(c*x^3 + 1)^
2 - 8*(c*x^3 + 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/c^2 + 18*log(4*c^3*x^6 -
 4*c)/c^2 - 576*integrate(1/4*x^2*log(c*x^3 + 1)/(c^3*x^6 - c), x))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(a + b \operatorname{atanh}\left(c\, x^3\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c*x^3))^3,x)
```

```
[Out] int(x^5*(a + b*atanh(c*x^3))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

3.126 $\int x^2 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 dx$

Optimal. Leaf size=130

$$-\frac{b^2\text{Li}_2\left(1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3+\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{3c}-\frac{b\log\left(\frac{2}{1-cx^3}\right)\left(a+b\tanh\right.}{c}$$

```
[Out] 1/3*(a+b*arctanh(c*x^3))^3/c+1/3*x^3*(a+b*arctanh(c*x^3))^3-b*(a+b*arctanh(
c*x^3))^2*ln(2/(-c*x^3+1))/c-b^2*(a+b*arctanh(c*x^3))*polylog(2,1-2/(-c*x^3
+1))/c+1/2*b^3*polylog(3,1-2/(-c*x^3+1))/c
```

Rubi [B] time = 2.47, antiderivative size = 390, normalized size of antiderivative = 3.00, number of steps used = 82, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)\left(2a-b\log\left(1-cx^3\right)\right)}{2c}-\frac{b^3\text{PolyLog}\left(3,\frac{1}{2}\left(1-cx^3\right)\right)}{2c}-\frac{b^3\text{PolyLog}\left(3,\frac{1}{2}\left(cx^3+1\right)\right)}{2c}+\frac{b^3}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] -((1 - c*x^3)*(2*a - b*Log[1 - c*x^3])^3)/(24*c) + (b*(2*a - b*Log[1 - c*x^
3])^2*Log[(1 + c*x^3)/2])/(4*c) - (b*(2*a - b*Log[1 - c*x^3])^2*Log[1 + c*x
^3])/(8*c) + (b*x^3*(2*a - b*Log[1 - c*x^3])^2*Log[1 + c*x^3])/8 + (b^3*Log
[(1 - c*x^3)/2]*Log[1 + c*x^3]^2)/(4*c) + (b^2*(2*a - b*Log[1 - c*x^3])*Log
[1 + c*x^3]^2)/(8*c) + (b^2*x^3*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3]^2)/
8 + (b^3*(1 + c*x^3)*Log[1 + c*x^3]^3)/(24*c) - (b^2*(2*a - b*Log[1 - c*x^3
])*PolyLog[2, (1 - c*x^3)/2])/(2*c) + (b^3*Log[1 + c*x^3]*PolyLog[2, (1 + c
*x^3)/2])/(2*c) - (b^3*PolyLog[3, (1 - c*x^3)/2])/(2*c) - (b^3*PolyLog[3, (
1 + c*x^3)/2])/(2*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
```

```
Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\,dx=\int\left(\frac{1}{8}x^2\left(2a-b\log\left(1-cx^3\right)\right)^3+\frac{3}{8}bx^2\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)\right.$$

$$=\frac{1}{8}\int x^2\left(2a-b\log\left(1-cx^3\right)\right)^3\,dx+\frac{1}{8}(3b)\int x^2\left(-2a+b\log\left(1-cx^3\right)\right)^2\log$$

$$=\frac{1}{24}\operatorname{Subst}\left(\int(2a-b\log(1-cx))^3\,dx,x,x^3\right)+\frac{1}{8}b\operatorname{Subst}\left(\int(-2a+b\log(1-\right.$$

$$=\frac{1}{8}bx^3\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{8}b^2x^3\left(2a-b\log\left(1-cx^3\right)\right)\log$$

$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{1}{8}bx^3\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx\right.$$

$$=-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{8c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{1}{8}b$$

$$=\frac{1}{2}ab^2x^3-\frac{b^3x^3}{4}-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{8c}-\frac{\left(1-cx^3\right)\left(2a-b\log\right.}{24c}$$

$$=\frac{1}{2}ab^2x^3+\frac{b^3\left(1-cx^3\right)\log\left(1-cx^3\right)}{4c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{b\left(\right.}{}$$

$$=\frac{b^3x^3}{4}+\frac{b^3\left(1-cx^3\right)\log\left(1-cx^3\right)}{4c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{b\left(2a\right.}{}$$

$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{b\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(\frac{1}{2}\left(1+cx^3\right.\right.}{4c}$$

$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{b\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(\frac{1}{2}\left(1+cx^3\right.\right.}{4c}$$

Mathematica [A] time = 0.26, size = 208, normalized size = 1.60

$$\frac{a^3x^3}{3}+\frac{a^2b\left(cx^3\tanh^{-1}\left(cx^3\right)-\log\left(\frac{1}{\sqrt{1-c^2x^6}}\right)\right)}{c}+\frac{ab^2\left(\operatorname{Li}_2\left(-e^{-2\tanh^{-1}\left(cx^3\right)}\right)+\tanh^{-1}\left(cx^3\right)\left(cx^3\tanh^{-1}\left(cx^3\right)-\text{ta}\right.\right.}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (a^3*x^3)/3 + (a^2*b*(c*x^3*ArcTanh[c*x^3] - Log[1/Sqrt[1 - c^2*x^6]]))/c +
 (a*b^2*(ArcTanh[c*x^3]*(-ArcTanh[c*x^3] + c*x^3*ArcTanh[c*x^3] - 2*Log[1 +
```

```
 E^(-2*ArcTanh[c*x^3])]) + PolyLog[2, -E^(-2*ArcTanh[c*x^3])]))/c + (b^3*(A
rcTanh[c*x^3]^2*(-ArcTanh[c*x^3] + c*x^3*ArcTanh[c*x^3] - 3*Log[1 + E^(-2*A
rcTanh[c*x^3])]) + 3*ArcTanh[c*x^3]*PolyLog[2, -E^(-2*ArcTanh[c*x^3])] + (3
*PolyLog[3, -E^(-2*ArcTanh[c*x^3])])/2))/(3*c)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^2\,\text{artanh}\left(cx^3\right)^3+3\,ab^2x^2\,\text{artanh}\left(cx^3\right)^2+3\,a^2bx^2\,\text{artanh}\left(cx^3\right)+a^3x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctanh(c*x^3)^3 + 3*a*b^2*x^2*arctanh(c*x^3)^2 + 3*a^2*b*
x^2*arctanh(c*x^3) + a^3*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\,\text{artanh}\left(cx^3\right)+a\right)^3x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*x^2, x)
```

maple [B] time = 0.20, size = 295, normalized size = 2.27

$$\frac{x^3a^3}{3}+\frac{b^3x^3\,\text{arctanh}\left(c\,x^3\right)^3}{3}+\frac{b^3\,\text{arctanh}\left(c\,x^3\right)^3}{3c}-\frac{b^3\,\text{arctanh}\left(c\,x^3\right)^2\ln\left(1+\frac{\left(c\,x^3+1\right)^2}{-c^2x^6+1}\right)}{c}-\frac{b^3\,\text{arctanh}\left(c\,x^3\right)\text{polylog}\left(\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^3))^3,x)
```

```
[Out] 1/3*x^3*a^3+1/3*b^3*x^3*arctanh(c*x^3)^3+1/3/c*b^3*arctanh(c*x^3)^3-1/c*b^3
*arctanh(c*x^3)^2*ln(1+(c*x^3+1)^2/(-c^2*x^6+1))-1/c*b^3*arctanh(c*x^3)*pol
ylog(2,-(c*x^3+1)^2/(-c^2*x^6+1))+1/2/c*b^3*polylog(3,-(c*x^3+1)^2/(-c^2*x^
6+1))+x^3*a*b^2*arctanh(c*x^3)^2+1/c*a*b^2*arctanh(c*x^3)^2-2/c*arctanh(c*x
^3)*ln(1+(c*x^3+1)^2/(-c^2*x^6+1))*a*b^2-1/c*polylog(2,-(c*x^3+1)^2/(-c^2*x
^6+1))*a*b^2+x^3*a^2*b*arctanh(c*x^3)+1/2/c*a^2*b*ln(-c^2*x^6+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^3x^3+\frac{\left(2\,cx^3\,\text{artanh}\left(cx^3\right)+\log\left(-c^2x^6+1\right)\right)a^2b}{2\,c}-\frac{\left(b^3cx^3-b^3\right)\log\left(-cx^3+1\right)^3-3\left(2\,ab^2cx^3+\left(b^3cx^3+b^3\right)\log\right.}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 + 1/2*(2*c*x^3*arctanh(c*x^3) + log(-c^2*x^6 + 1))*a^2*b/c - 1/
24*((b^3*c*x^3 - b^3)*log(-c*x^3 + 1)^3 - 3*(2*a*b^2*c*x^3 + (b^3*c*x^3 + b
^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/c - integrate(-1/8*((b^3*c*x^5 - b^3
*x^2)*log(c*x^3 + 1)^3 + 6*(a*b^2*c*x^5 - a*b^2*x^2)*log(c*x^3 + 1)^2 - 3*(
4*a*b^2*c*x^5 + (b^3*c*x^5 - b^3*x^2)*log(c*x^3 + 1)^2 + 2*((2*a*b^2*c + b^
3*c)*x^5 - (2*a*b^2 - b^3)*x^2)*log(c*x^3 + 1))*log(-c*x^3 + 1))/(c*x^3 - 1
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a + b \operatorname{atanh}\left(c x^3\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^3))^3,x)
```

```
[Out] int(x^2*(a + b*atanh(c*x^3))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

3.127 $\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x}\,dx$

Optimal. Leaf size=210

$$\frac{1}{2}b^2\text{Li}_3\left(1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}b^2\text{Li}_3\left(\frac{2}{1-cx^3}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}b\text{Li}_2\left(1-\frac{2}{1-cx^3}\right)\left(a+b\right.$$

```
[Out] -2/3*(a+b*arctanh(c*x^3))^3*arctanh(-1+2/(-c*x^3+1))-1/2*b*(a+b*arctanh(c*x
^3))^2*polylog(2,1-2/(-c*x^3+1))+1/2*b*(a+b*arctanh(c*x^3))^2*polylog(2,-1+
2/(-c*x^3+1))+1/2*b^2*(a+b*arctanh(c*x^3))*polylog(3,1-2/(-c*x^3+1))-1/2*b^
2*(a+b*arctanh(c*x^3))*polylog(3,-1+2/(-c*x^3+1))-1/4*b^3*polylog(4,1-2/(-c
*x^3+1))+1/4*b^3*polylog(4,-1+2/(-c*x^3+1))
```

Rubi [A] time = 0.56, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6095, 5914, 6052, 5948, 6058, 6062, 6610}

$$\frac{1}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}b^2\text{PolyLog}\left(3,\frac{2}{1-cx^3}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}b\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^3])^3*ArcTanh[1 - 2/(1 - c*x^3)])/3 - (b*(a + b*ArcTa
nh[c*x^3])^2*PolyLog[2, 1 - 2/(1 - c*x^3)])/2 + (b*(a + b*ArcTanh[c*x^3])^2
*PolyLog[2, -1 + 2/(1 - c*x^3)])/2 + (b^2*(a + b*ArcTanh[c*x^3])*PolyLog[3,
 1 - 2/(1 - c*x^3)])/2 - (b^2*(a + b*ArcTanh[c*x^3])*PolyLog[3, -1 + 2/(1 -
 c*x^3)])/2 - (b^3*PolyLog[4, 1 - 2/(1 - c*x^3)])/4 + (b^3*PolyLog[4, -1 +
2/(1 - c*x^3)])/4
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx, x, x^3\right)$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) - (2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(c\right.}{1}\right.$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) + (bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx\right.}{1-c^2}\right.$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) - \frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\text{Li}_2\left(1\right.$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) - \frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\text{Li}_2\left(1\right.$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right) - \frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\text{Li}_2\left(1\right.$$

Mathematica [A] time = 0.19, size = 214, normalized size = 1.02

$$\frac{1}{4}b\left(2\text{Li}_2\left(\frac{cx^3+1}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 - 2\text{Li}_2\left(\frac{cx^3+1}{cx^3-1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 + b\left(-2\text{Li}_3\left(\frac{cx^3+1}{1-cx^3}\right)\left(a+b\,\text{t}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^3])^3*ArcTanh[1 + 2/(-1 + c*x^3)])/3 + (b*(2*(a + b*A
rcTanh[c*x^3])^2*PolyLog[2, (1 + c*x^3)/(1 - c*x^3)] - 2*(a + b*ArcTanh[c*x
^3])^2*PolyLog[2, (1 + c*x^3)/(-1 + c*x^3)] + b*(-2*(a + b*ArcTanh[c*x^3])*
PolyLog[3, (1 + c*x^3)/(1 - c*x^3)] + 2*(a + b*ArcTanh[c*x^3])*PolyLog[3, (
1 + c*x^3)/(-1 + c*x^3)] + b*(PolyLog[4, (1 + c*x^3)/(1 - c*x^3)] - PolyLog
[4, (1 + c*x^3)/(-1 + c*x^3)]))))/4
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}\left(cx^3\right)^3 + 3\,ab^2 \operatorname{artanh}\left(cx^3\right)^2 + 3\,a^2 b \operatorname{artanh}\left(cx^3\right) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(cx^3\right) + a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3/x, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{arctanh}\left(c\,x^3\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^3/x,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \log(x) + \int \frac{b^3\left(\log\left(cx^3+1\right) - \log\left(-cx^3+1\right)\right)^3}{8\,x} + \frac{3\,ab^2\left(\log\left(cx^3+1\right) - \log\left(-cx^3+1\right)\right)^2}{4\,x} + \frac{3\,a^2 b\left(\log\left(cx^3+1\right) - \right.}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c*x^3 + 1) - log(-c*x^3 + 1))^3/x + 3/4
*a*b^2*(log(c*x^3 + 1) - log(-c*x^3 + 1))^2/x + 3/2*a^2*b*(log(c*x^3 + 1) -
 log(-c*x^3 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(c\,x^3\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^3/x,x)
```

```
[Out] int((a + b*atanh(c*x^3))^3/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**3/x,x)
```

```
[Out] Timed out
```

3.128 $\int \frac{(a+b\tanh^{-1}(cx^3))^3}{x^4}\,dx$

Optimal. Leaf size=120

$$-b^2c\mathrm{Li}_2\left(\frac{2}{cx^3+1}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{3}c\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{3x^3}+bc\log\left(2-\frac{2}{cx^3+1}\right.$$

```
[Out] 1/3*c*(a+b*arctanh(c*x^3))^3-1/3*(a+b*arctanh(c*x^3))^3/x^3+b*c*(a+b*arctan
h(c*x^3))^2*ln(2-2/(c*x^3+1))-b^2*c*(a+b*arctanh(c*x^3))*polylog(2,-1+2/(c*
x^3+1))-1/2*b^3*c*polylog(3,-1+2/(c*x^3+1))
```

Rubi [F] time = 0.78, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^3])^3/x^4,x]
```

```
[Out] (b*c*Log[c*x^3]*(2*a - b*Log[1 - c*x^3])^2)/8 - ((1 - c*x^3)*(2*a - b*Log[1
 - c*x^3])^3)/(24*x^3) + (b^3*c*Log[-(c*x^3)]*Log[1 + c*x^3]^2)/8 - (b^3*(1
 + c*x^3)*Log[1 + c*x^3]^3)/(24*x^3) - (b^2*c*(2*a - b*Log[1 - c*x^3])*Poly
Log[2, 1 - c*x^3])/4 + (b^3*c*Log[1 + c*x^3]*PolyLog[2, 1 + c*x^3])/4 - (b^
3*c*PolyLog[3, 1 - c*x^3])/4 - (b^3*c*PolyLog[3, 1 + c*x^3])/4 + (b*Defer[S
ubst][Defer[Int][((-2*a + b*Log[1 - c*x])^2*Log[1 + c*x])/x^2, x], x, x^3])
/8 - (b^2*Defer[Subst][Defer[Int][((-2*a + b*Log[1 - c*x])*Log[1 + c*x]^2)/
x^2, x], x, x^3])/8
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^4}\,dx &= \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{8x^4}+\frac{3b\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)}{8x^4}-\frac{3b^2\left(-2a\right.}{}\right.\\
&= \frac{1}{8}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{x^4}\,dx+\frac{1}{8}(3b)\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)}{x^4}\,d\\
&= \frac{1}{24}\mathrm{Subst}\left(\int \frac{(2a-b\log(1-cx))^3}{x^2}\,dx,x,x^3\right)+\frac{1}{8}b\,\mathrm{Subst}\left(\int \frac{(-2a+b\log(1-cx))}{x^2}\right.\\
&= -\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3}-\frac{b^3\left(1+cx^3\right)\log^3\left(1+cx^3\right)}{24x^3}+\frac{1}{8}b\,\mathrm{Subst}\left(\int \frac{(-}{}\right.\\
&= \frac{1}{8}bc\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3}+\frac{1}{8}b^3c\log\\
&= \frac{1}{8}bc\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3}+\frac{1}{8}b^3c\log\\
&= \frac{1}{8}bc\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3}+\frac{1}{8}b^3c\log\\
&= \frac{1}{8}bc\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3}+\frac{1}{8}b^3c\log
\end{aligned}$$

Mathematica [C] time = 0.41, size = 223, normalized size = 1.86

$$-\frac{a^3}{3x^3}-\frac{1}{2}a^2bc\log\left(1-c^2x^6\right)-\frac{a^2b\tanh^{-1}\left(cx^3\right)}{x^3}+3a^2bc\log(x)+ab^2c\left(\tanh^{-1}\left(cx^3\right)\left(\left(1-\frac{1}{cx^3}\right)\tanh^{-1}\left(cx^3\right)+2\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (a^2*b*ArcTanh[c*x^3])/x^3 + 3*a^2*b*c*Log[x] - (a^2*b*c*Log
[1 - c^2*x^6])/2 + a*b^2*c*(ArcTanh[c*x^3]*((1 - 1/(c*x^3))*ArcTanh[c*x^3]
+ 2*Log[1 - E^(-2*ArcTanh[c*x^3])]) - PolyLog[2, E^(-2*ArcTanh[c*x^3])]) +
(b^3*c*((I/8)*Pi^3 - ArcTanh[c*x^3]^3 - ArcTanh[c*x^3]^3/(c*x^3) + 3*ArcTan
h[c*x^3]^2*Log[1 - E^(2*ArcTanh[c*x^3])] + 3*ArcTanh[c*x^3]*PolyLog[2, E^(2
*ArcTanh[c*x^3])] - (3*PolyLog[3, E^(2*ArcTanh[c*x^3])])/2))/3
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{artanh}\left(cx^3\right)^3+3\,ab^2\,\text{artanh}\left(cx^3\right)^2+3\,a^2b\,\text{artanh}\left(cx^3\right)+a^3}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,\text{artanh}\left(cx^3\right)+a\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3/x^4, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\text{arctanh}\left(c\,x^3\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^3/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^3/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^6-1\right)-\log\left(x^6\right)\right)+\frac{2\,\text{artanh}\left(cx^3\right)}{x^3}\right)a^2b-\frac{a^3}{3\,x^3}-\frac{\left(b^3cx^3-b^3\right)\log\left(-cx^3+1\right)^3+3\left(2\,ab^2+\left(b^3cx^3\right.\right.}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^6 - 1) - log(x^6)) + 2*arctanh(c*x^3)/x^3)*a^2*b - 1/3*a
^3/x^3 - 1/24*((b^3*c*x^3 - b^3)*log(-c*x^3 + 1)^3 + 3*(2*a*b^2 + (b^3*c*x^
3 + b^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/x^3 - integrate(-1/8*((b^3*c*x^
3 - b^3)*log(c*x^3 + 1)^3 + 6*(a*b^2*c*x^3 - a*b^2)*log(c*x^3 + 1)^2 + 3*(4
*a*b^2*c*x^3 - (b^3*c*x^3 - b^3)*log(c*x^3 + 1)^2 + 2*(b^3*c^2*x^6 - (2*a*b
^2*c - b^3*c)*x^3 + 2*a*b^2)*log(c*x^3 + 1))*log(-c*x^3 + 1))/(c*x^7 - x^4)
, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{atanh}\left(c\,x^3\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^3/x^4,x)
```

```
[Out] int((a + b*atanh(c*x^3))^3/x^4, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**3/x**4,x)
```

```
[Out] Timed out
```

3.129 $\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^7}\,dx$

Optimal. Leaf size=136

$$b^2c^2\log\left(2-\frac{2}{cx^3+1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{2}bc^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{1}{6}c^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3-\frac{bc\left(a+b\right.}{}$$

```
[Out] 1/2*b*c^2*(a+b*arctanh(c*x^3))^2-1/2*b*c*(a+b*arctanh(c*x^3))^2/x^3+1/6*c^2
*(a+b*arctanh(c*x^3))^3-1/6*(a+b*arctanh(c*x^3))^3/x^6+b^2*c^2*(a+b*arctanh
(c*x^3))*ln(2-2/(c*x^3+1))-1/2*b^3*c^2*polylog(2,-1+2/(c*x^3+1))
```

Rubi [F] time = 1.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^3])^3/x^7,x]
```

```
[Out] (3*a*b^2*c^2*Log[x])/4 - (b*c*(1 - c*x^3)*(2*a - b*Log[1 - c*x^3])^2)/(16*x
^3) + (b*c^2*Log[c*x^3]*(2*a - b*Log[1 - c*x^3])^2)/16 + (c^2*(2*a - b*Log[
1 - c*x^3])^3)/48 - (2*a - b*Log[1 - c*x^3])^3/(48*x^6) - (b^3*c*(1 + c*x^3
)*Log[1 + c*x^3]^2)/(16*x^3) - (b^3*c^2*Log[-(c*x^3)]*Log[1 + c*x^3]^2)/16
+ (b^3*c^2*Log[1 + c*x^3]^3)/48 - (b^3*Log[1 + c*x^3]^3)/(48*x^6) - (b^3*c^
2*PolyLog[2, -(c*x^3)])/8 + (b^3*c^2*PolyLog[2, c*x^3])/8 - (b^2*c^2*(2*a -
 b*Log[1 - c*x^3])*PolyLog[2, 1 - c*x^3])/8 - (b^3*c^2*Log[1 + c*x^3]*PolyL
og[2, 1 + c*x^3])/8 - (b^3*c^2*PolyLog[3, 1 - c*x^3])/8 + (b^3*c^2*PolyLog[
3, 1 + c*x^3])/8 + (b*Defer[Subst][Defer[Int][((-2*a + b*Log[1 - c*x])^2*Lo
g[1 + c*x])/x^3, x], x, x^3])/8 - (b^2*Defer[Subst][Defer[Int][((-2*a + b*L
og[1 - c*x])*Log[1 + c*x]^2)/x^3, x], x, x^3])/8
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^7}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{8x^7} + \frac{3b\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)}{8x^7} - \frac{3b^2\left(-2a\right.}{}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{x^7}\,dx + \frac{1}{8}(3b)\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)}{x^7}\,d$$

$$= \frac{1}{24}\,\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^3}{x^3}\,dx, x, x^3\right) + \frac{1}{8}b\,\text{Subst}\left(\int \frac{(-2a+b\log(1-cx))}{x^3}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{48x^6} - \frac{b^3\log^3\left(1+cx^3\right)}{48x^6} + \frac{1}{8}b\,\text{Subst}\left(\int \frac{(-2a+b\log(1-cx))}{x^3}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{48x^6} - \frac{b^3\log^3\left(1+cx^3\right)}{48x^6} - \frac{1}{16}b\,\text{Subst}\left(\int \frac{(2a-b\log(x))^2}{x\left(\frac{1}{c}-\frac{x}{c}\right)^2}\,dx,\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{48x^6} - \frac{b^3\log^3\left(1+cx^3\right)}{48x^6} - \frac{1}{16}b\,\text{Subst}\left(\int \frac{(2a-b\log(x))^2}{\left(\frac{1}{c}-\frac{x}{c}\right)^2}\,dx,\right.$$

$$= -\frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16x^3} - \frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{48x^6} - \frac{b^3c\left(1+cx^3\right)\log^2}{16x^3}$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16x^3} + \frac{1}{16}bc^2\log\left(cx^3\right)\left(2a-b\log\right.$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16x^3} + \frac{1}{16}bc^2\log\left(cx^3\right)\left(2a-b\log\right.$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16x^3} + \frac{1}{16}bc^2\log\left(cx^3\right)\left(2a-b\log\right.$$

Mathematica [A] time = 0.30, size = 218, normalized size = 1.60

$$\frac{a\left(-2a^2-3abc^2x^6\log\left(1-cx^3\right)+3abc^2x^6\log\left(cx^3+1\right)-6abcx^3+12b^2c^2x^6\log\left(\frac{cx^3}{\sqrt{1-c^2x^6}}\right)\right)-6b\tanh^{-1}\left(cx^3\right)\left(a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^3/x^7,x]
```

```
[Out] (6*b^2*(-1 + c*x^3)*(a + a*c*x^3 + b*c*x^3)*ArcTanh[c*x^3]^2 + 2*b^3*(-1 +
c^2*x^6)*ArcTanh[c*x^3]^3 - 6*b*ArcTanh[c*x^3]*(a^2 + 2*a*b*c*x^3 - 2*b^2*c
^2*x^6*Log[1 - E^(-2*ArcTanh[c*x^3])]) + a*(-2*a^2 - 6*a*b*c*x^3 - 3*a*b*c^
2*x^6*Log[1 - c*x^3] + 3*a*b*c^2*x^6*Log[1 + c*x^3] + 12*b^2*c^2*x^6*Log[(c
*x^3)/Sqrt[1 - c^2*x^6]]) - 6*b^3*c^2*x^6*PolyLog[2, E^(-2*ArcTanh[c*x^3])]
)/(12*x^6)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{artanh}\left(cx^3\right)^3+3\,ab^2\,\text{artanh}\left(cx^3\right)^2+3\,a^2b\,\text{artanh}\left(cx^3\right)+a^3}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(cx^3\right) + a\right)^3}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3/x^7, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{arctanh}\left(c\, x^3\right)\right)^3}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^3/x^7,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^3/x^7,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(\left(c \log\left(cx^3+1\right) - c \log\left(cx^3-1\right) - \frac{2}{x^3}\right)c - \frac{2 \operatorname{artanh}\left(cx^3\right)}{x^6}\right)a^2b+\frac{1}{8}\left(\left(2\left(\log\left(cx^3-1\right)-2\right)\log\left(cx^3+1\right) - \mathrm{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^6
)*a^2*b + 1/8*((2*(log(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 -
log(c*x^3 - 1)^2 - 4*log(c*x^3 - 1) + 24*log(x))*c^2 + 4*(c*log(c*x^3 + 1)
- c*log(c*x^3 - 1) - 2/x^3)*c*arctanh(c*x^3))*a*b^2 - 1/48*b^3*(((c^2*x^6 -
 1)*log(-c*x^3 + 1)^3 + 3*(2*c*x^3 - (c^2*x^6 - 1)*log(c*x^3 + 1))*log(-c*x
^3 + 1)^2)/x^6 + 6*integrate(-((c*x^3 - 1)*log(c*x^3 + 1)^3 + 3*(2*c^2*x^6
- (c*x^3 - 1)*log(c*x^3 + 1)^2 - (c^3*x^9 - c*x^3)*log(c*x^3 + 1))*log(-c*x
^3 + 1))/(c*x^10 - x^7), x)) - 1/2*a*b^2*arctanh(c*x^3)^2/x^6 - 1/6*a^3/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{atanh}\left(c\, x^3\right)\right)^3}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^3))^3/x^7,x)
```

```
[Out] int((a + b*atanh(c*x^3))^3/x^7, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**3/x**7,x)
```

```
[Out] Timed out
```

3.130 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 \, dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x^3))^3,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^3])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^3])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 \, dx = \int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 \, dx$$

Mathematica [A] time = 1.88, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^3, x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3 \operatorname{artanh}\left(cx^3\right)^3 + 3\,ab^2 \operatorname{artanh}\left(cx^3\right)^2 + 3\,a^2 b \operatorname{artanh}\left(cx^3\right) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(cx^3\right) + a\right)^3 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^3) + a)^3*(d*x)^m, x)
```

maple [A] time = 0.19, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b \arctan\text{h}\left(c\,x^3\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^3))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^3))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3 d^m x x^m \log\left(-cx^3+1\right)^3}{8\,(m+1)}+\frac{(dx)^{m+1}\,a^3}{d(m+1)}+\int\frac{\left(b^3 c d^m (m+1)x^3-b^3 d^m (m+1)\right)x^m \log\left(cx^3+1\right)^3+6\left(ab^2 c d^m (m\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*d^m*x*x^m*log(-c*x^3 + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1)
) + integrate(1/8*((b^3*c*d^m*(m + 1)*x^3 - b^3*d^m*(m + 1))*x^m*log(c*x^3
+ 1)^3 + 6*(a*b^2*c*d^m*(m + 1)*x^3 - a*b^2*d^m*(m + 1))*x^m*log(c*x^3 + 1)
^2 + 12*(a^2*b*c*d^m*(m + 1)*x^3 - a^2*b*d^m*(m + 1))*x^m*log(c*x^3 + 1) +
3*((b^3*c*d^m*(m + 1)*x^3 - b^3*d^m*(m + 1))*x^m*log(c*x^3 + 1) - (2*a*b^2*
d^m*(m + 1) - (2*a*b^2*c*d^m*(m + 1) + 3*b^3*c*d^m)*x^3)*x^m)*log(-c*x^3 +
1)^2 - 3*((b^3*c*d^m*(m + 1)*x^3 - b^3*d^m*(m + 1))*x^m*log(c*x^3 + 1)^2 +
4*(a*b^2*c*d^m*(m + 1)*x^3 - a*b^2*d^m*(m + 1))*x^m*log(c*x^3 + 1) + 4*(a^2
*b*c*d^m*(m + 1)*x^3 - a^2*b*d^m*(m + 1))*x^m)*log(-c*x^3 + 1))/(c*(m + 1)*
x^3 - m - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^3))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^3))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

3.131 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x^3))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^3])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^3])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx = \int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$$

Mathematica [A] time = 1.25, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^2, x]
```

fricas [A] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \operatorname{artanh}\left(cx^3\right)^2 + 2\,ab \operatorname{artanh}\left(cx^3\right) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(cx^3\right) + a\right)^2 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^3) + a)^2*(d*x)^m, x)
```

maple [A] time = 0.19, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b \arctan\text{h}\left(c\,x^3\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^3))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2d^m x x^m \log\left(-cx^3+1\right)^2}{4\,(m+1)}+\frac{(dx)^{m+1}\,a^2}{d(m+1)}-\int -\frac{\left(b^2cd^m(m+1)x^3-b^2d^m(m+1)\right)x^m\log\left(cx^3+1\right)^2+4\left(abcd^m(m-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d^m*x*x^m*log(-c*x^3 + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
 - integrate(-1/4*((b^2*c*d^m*(m + 1)*x^3 - b^2*d^m*(m + 1))*x^m*log(c*x^3
+ 1)^2 + 4*(a*b*c*d^m*(m + 1)*x^3 - a*b*d^m*(m + 1))*x^m*log(c*x^3 + 1) - 2
*((b^2*c*d^m*(m + 1)*x^3 - b^2*d^m*(m + 1))*x^m*log(c*x^3 + 1) - (2*a*b*d^m
*(m + 1) - (2*a*b*c*d^m*(m + 1) + 3*b^2*c*d^m)*x^3)*x^m)*log(-c*x^3 + 1))/(
c*(m + 1)*x^3 - m - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^3))^2,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^3))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

3.132 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=74

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc(dx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{6};\frac{m+10}{6};c^2x^6\right)}{d^4(m+1)(m+4)}$$

[Out] (d*x)^(1+m)*(a+b*arctanh(c*x^3))/d/(1+m)-3*b*c*(d*x)^(4+m)*hypergeom([1, 2/3+1/6*m],[5/3+1/6*m],c^2*x^6)/d^4/(1+m)/(4+m)

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6097, 16, 364}

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc(dx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{6};\frac{m+10}{6};c^2x^6\right)}{d^4(m+1)(m+4)}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^3]),x]

[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x^3]))/(d*(1 + m)) - (3*b*c*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/6, (10 + m)/6, c^2*x^6])/(d^4*(1 + m)*(4 + m))

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m\left(a+b\tanh^{-1}\left(cx^3\right)\right)dx &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{(3bc)\int\frac{x^2(dx)^{1+m}}{1-c^2x^6}\,dx}{d(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{(3bc)\int\frac{(dx)^{3+m}}{1-c^2x^6}\,dx}{d^3(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{3bc(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{6};\frac{10+m}{6};c^2x^6\right)}{d^4(1+m)(4+m)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.86

$$-\frac{x(dx)^m \left(3bcx^3 \, _2F_1\left(1,\frac{m+4}{6};\frac{m+10}{6};c^2x^6\right)-(m+4)\left(a+b\tanh^{-1}\left(cx^3\right)\right)\right)}{(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] -((x*(d*x)^m*(-((4 + m)*(a + b*ArcTanh[c*x^3])) + 3*b*c*x^3*Hypergeometric2
F1[1, (4 + m)/6, (10 + m)/6, c^2*x^6]))/((1 + m)*(4 + m)))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\,\text{artanh}\left(cx^3\right)+a\right)(dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^3) + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\text{artanh}\left(cx^3\right)+a\right)(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)*(d*x)^m, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\text{arctanh}\left(c\,x^3\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^3)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^3)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(6\,cd^m\int\frac{x^3x^m}{c^2(m+1)x^6-m-1}\,dx+\frac{d^mxx^m\log\left(cx^3+1\right)-d^mxx^m\log\left(-cx^3+1\right)}{m+1}\right)b+\frac{(dx)^{m+1}\,a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/2*(6*c*d^m*integrate(x^3*x^m/(c^2*(m + 1)*x^6 - m - 1), x) + (d^m*x*x^m*l
og(c*x^3 + 1) - d^m*x*x^m*log(-c*x^3 + 1))/(m + 1))*b + (d*x)^(m + 1)*a/(d*
(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {(d\,x)}^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^3)),x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

3.133 $\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^3)}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x^3)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^3]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^3]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)}\,dx$$

Mathematica [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3]), x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(dx)^m}{b\,\mathrm{artanh}\left(cx^3\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctanh(c*x^3) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\,\mathrm{artanh}\left(cx^3\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctanh(c*x^3) + a), x)
```

maple [A] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\operatorname{arctanh}\left(c\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^3)),x)
[Out] int((d*x)^m/(a+b*arctanh(c*x^3)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\operatorname{artanh}\left(cx^3\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctanh(c*x^3) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a+b\operatorname{atanh}\left(c\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x^3)),x)
[Out] int((d*x)^m/(a + b*atanh(c*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**3)),x)
[Out] Timed out
```

3.134 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x^3))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^3])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx$$

Mathematica [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3])^2, x]
```

fricas [A] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b^2\,\text{artanh}\left(cx^3\right)^2+2\,ab\,\text{artanh}\left(cx^3\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b\,\text{artanh}\left(cx^3\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^3) + a)^2, x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\,\mathrm{arctanh}\left(c\,x^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x^3))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(c^2d^mx^6-d^m\right)x^m}{3\left(b^2cx^2\log\left(cx^3+1\right)-b^2cx^2\log\left(-cx^3+1\right)+2\,abcx^2\right)}+\int-\frac{2\left(c^2d^m(m+4)x^6-d^m(m-2)\right)x^m}{3\left(b^2cx^3\log\left(cx^3+1\right)-b^2cx^3\log\left(-cx^3+1\right)+2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^m*x^6 - d^m)*x^m/(b^2*c*x^2*log(c*x^3 + 1) - b^2*c*x^2*log(-c*x^
3 + 1) + 2*a*b*c*x^2) + integrate(-2/3*(c^2*d^m*(m + 4)*x^6 - d^m*(m - 2))*
x^m/(b^2*c*x^3*log(c*x^3 + 1) - b^2*c*x^3*log(-c*x^3 + 1) + 2*a*b*c*x^3), x
)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x^3))^2,x)
```

```
[Out] int((d*x)^m/(a + b*atanh(c*x^3))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

3.135 $\int x^3 \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right) dx$

Optimal. Leaf size=50

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{4}bc^4\tanh^{-1}\left(\frac{x}{c}\right)+\frac{1}{4}bc^3x+\frac{1}{12}bcx^3$$

```
[Out] 1/4*b*c^3*x+1/12*b*c*x^3+1/4*x^4*(a+b*arctanh(c/x))-1/4*b*c^4*arctanh(x/c)
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 263, 302, 207}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc^3x-\frac{1}{4}bc^4\tanh^{-1}\left(\frac{x}{c}\right)+\frac{1}{12}bcx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c^3*x)/4 + (b*c*x^3)/12 + (x^4*(a + b*ArcTanh[c/x]))/4 - (b*c^4*ArcTanh[
x/c])/4
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)dx &= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^2}{1-\frac{c^2}{x^2}}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^4}{-c^2+x^2}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\left(c^2+x^2+\frac{c^4}{-c^2+x^2}\right)dx\\
&= \frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}\left(bc^5\right)\int\frac{1}{-c^2+x^2}\,dx\\
&= \frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{4}bc^4\tanh^{-1}\left(\frac{x}{c}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 67, normalized size = 1.34

$$\frac{ax^4}{4}+\frac{1}{8}bc^4\log(x-c)-\frac{1}{8}bc^4\log(c+x)+\frac{1}{4}bc^3x+\frac{1}{4}bx^4\tanh^{-1}\left(\frac{c}{x}\right)+\frac{1}{12}bcx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c^3*x)/4 + (b*c*x^3)/12 + (a*x^4)/4 + (b*x^4*ArcTanh[c/x])/4 + (b*c^4*Lo
g[-c + x])/8 - (b*c^4*Log[c + x])/8
```

fricas [A] time = 0.67, size = 48, normalized size = 0.96

$$\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}ax^4-\frac{1}{8}\left(bc^4-bx^4\right)\log\left(-\frac{c+x}{c-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="fricas")
```

```
[Out] 1/4*b*c^3*x + 1/12*b*c*x^3 + 1/4*a*x^4 - 1/8*(b*c^4 - b*x^4)*log(-(c + x)/(
c - x))
```

giac [B] time = 0.17, size = 262, normalized size = 5.24

$$-\frac{\frac{3\left(\frac{b(c+x)^3c^5}{(c-x)^3}+\frac{b(c+x)c^5}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^4}{(c-x)^4}+\frac{4(c+x)^3}{(c-x)^3}+\frac{6(c+x)^2}{(c-x)^2}+\frac{4(c+x)}{c-x}+1}+\frac{2bc^5+\frac{6a(c+x)^3c^5}{(c-x)^3}+\frac{3b(c+x)^3c^5}{(c-x)^3}+\frac{6b(c+x)^2c^5}{(c-x)^2}+\frac{6a(c+x)c^5}{c-x}+\frac{5b(c+x)c^5}{c-x}}{\frac{(c+x)^4}{(c-x)^4}+\frac{4(c+x)^3}{(c-x)^3}+\frac{6(c+x)^2}{(c-x)^2}+\frac{4(c+x)}{c-x}+1}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="giac")
```

```
[Out] -1/3*(3*(b*(c + x)^3*c^5/(c - x)^3 + b*(c + x)*c^5/(c - x))*log(-(c + x)/(c
 - x))/((c + x)^4/(c - x)^4 + 4*(c + x)^3/(c - x)^3 + 6*(c + x)^2/(c - x)^2
 + 4*(c + x)/(c - x) + 1) + (2*b*c^5 + 6*a*(c + x)^3*c^5/(c - x)^3 + 3*b*(c
 + x)^3*c^5/(c - x)^3 + 6*b*(c + x)^2*c^5/(c - x)^2 + 6*a*(c + x)*c^5/(c -
x) + 5*b*(c + x)*c^5/(c - x))/((c + x)^4/(c - x)^4 + 4*(c + x)^3/(c - x)^3
+ 6*(c + x)^2/(c - x)^2 + 4*(c + x)/(c - x) + 1))/c
```

maple [A] time = 0.04, size = 62, normalized size = 1.24

$$\frac{x^4a}{4}+\frac{b\,x^4\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{4}+\frac{bc\,x^3}{12}+\frac{b\,c^3x}{4}+\frac{c^4b\ln\left(\frac{c}{x}-1\right)}{8}-\frac{c^4b\ln\left(1+\frac{c}{x}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c/x)+1/12*b*c*x^3+1/4*b*c^3*x+1/8*c^4*b*ln(c/x-
1)-1/8*c^4*b*ln(1+c/x)
```

maxima [A] time = 0.31, size = 57, normalized size = 1.14

$$\frac{1}{4}ax^4+\frac{1}{24}\left(6\,x^4\,\mathrm{artanh}\left(\frac{c}{x}\right)-\left(3\,c^3\log\left(c+x\right)-3\,c^3\log\left(-c+x\right)-6\,c^2x-2\,x^3\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/24*(6*x^4*arctanh(c/x) - (3*c^3*log(c + x) - 3*c^3*log(-c + x
) - 6*c^2*x - 2*x^3)*c)*b
```

mupad [B] time = 0.78, size = 45, normalized size = 0.90

$$\frac{a\,x^4}{4}-\frac{b\,c^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{4}+\frac{b\,x^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{4}+\frac{b\,c\,x^3}{12}+\frac{b\,c^3\,x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c/x)),x)
```

```
[Out] (a*x^4)/4 - (b*c^4*atanh(c/x))/4 + (b*x^4*atanh(c/x))/4 + (b*c*x^3)/12 + (b
*c^3*x)/4
```

sympy [A] time = 0.56, size = 46, normalized size = 0.92

$$\frac{ax^4}{4}-\frac{bc^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{4}+\frac{bc^3x}{4}+\frac{bcx^3}{12}+\frac{bx^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x)),x)
```

```
[Out] a*x**4/4 - b*c**4*atanh(c/x)/4 + b*c**3*x/4 + b*c*x**3/12 + b*x**4*atanh(c/
x)/4
```

3.136 $\int x^2 \left(a + b \tanh^{-1}\left(\frac{c}{x}\right)\right) dx$

Optimal. Leaf size=45

$$\frac{1}{3}x^3\left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right) + \frac{1}{6}bc^3\log\left(c^2 - x^2\right) + \frac{1}{6}bcx^2$$

```
[Out] 1/6*b*c*x^2+1/3*x^3*(a+b*arctanh(c/x))+1/6*b*c^3*ln(c^2-x^2)
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 263, 266, 43}

$$\frac{1}{3}x^3\left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right) + \frac{1}{6}bc^3\log\left(c^2 - x^2\right) + \frac{1}{6}bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (x^3*(a + b*ArcTanh[c/x]))/3 + (b*c^3*Log[c^2 - x^2])/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right) dx &= \frac{1}{3}x^3\left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right) + \frac{1}{3}(bc)\int \frac{x}{1 - \frac{c^2}{x^2}} dx \\
&= \frac{1}{3}x^3\left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right) + \frac{1}{3}(bc)\int \frac{x^3}{-c^2 + x^2} dx \\
&= \frac{1}{3}x^3\left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right) + \frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{x}{-c^2 + x} dx, x, x^2\right) \\
&= \frac{1}{3}x^3\left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right) + \frac{1}{6}(bc)\,\text{Subst}\left(\int \left(1 - \frac{c^2}{c^2 - x}\right) dx, x, x^2\right) \\
&= \frac{1}{6}bcx^2 + \frac{1}{3}x^3\left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right) + \frac{1}{6}bc^3\log\left(c^2 - x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.11

$$\frac{ax^3}{3} + \frac{1}{6}bc^3 \log\left(x^2 - c^2\right) + \frac{1}{3}bx^3 \tanh^{-1}\left(\frac{c}{x}\right) + \frac{1}{6}bcx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (a*x^3)/3 + (b*x^3*ArcTanh[c/x])/3 + (b*c^3*Log[-c^2 + x^2])/
6
```

fricas [A] time = 0.68, size = 49, normalized size = 1.09

$$\frac{1}{6}\,bc^3\log\left(-c^2 + x^2\right) + \frac{1}{6}\,bx^3\log\left(-\frac{c+x}{c-x}\right) + \frac{1}{6}\,bcx^2 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="fricas")
```

```
[Out] 1/6*b*c^3*log(-c^2 + x^2) + 1/6*b*x^3*log(-(c + x)/(c - x)) + 1/6*b*c*x^2 +
 1/3*a*x^3
```

giac [B] time = 0.20, size = 227, normalized size = 5.04

$$-\frac{bc^4\log\left(-\frac{c+x}{c-x} - 1\right) - bc^4\log\left(-\frac{c+x}{c-x}\right) + \frac{\left(bc^4 + \frac{3\,b(c+x)^2c^4}{(c-x)^2}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^3}{(c-x)^3} + \frac{3\,(c+x)^2}{(c-x)^2} + \frac{3\,(c+x)}{c-x} + 1} + \frac{2\left(ac^4 + \frac{3\,a(c+x)^2c^4}{(c-x)^2} + \frac{b(c+x)^2c^4}{(c-x)^2} + \frac{b(c+x)c^4}{c-x}\right)}{\frac{(c+x)^3}{(c-x)^3} + \frac{3\,(c+x)^2}{(c-x)^2} + \frac{3\,(c+x)}{c-x} + 1}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="giac")
```

```
[Out] -1/3*(b*c^4*log(-(c + x)/(c - x) - 1) - b*c^4*log(-(c + x)/(c - x)) + (b*c^
4 + 3*b*(c + x)^2*c^4/(c - x)^2)*log(-(c + x)/(c - x))/((c + x)^3/(c - x)^3
 + 3*(c + x)^2/(c - x)^2 + 3*(c + x)/(c - x) + 1) + 2*(a*c^4 + 3*a*(c + x)^
2*c^4/(c - x)^2 + b*(c + x)^2*c^4/(c - x)^2 + b*(c + x)*c^4/(c - x))/((c +
x)^3/(c - x)^3 + 3*(c + x)^2/(c - x)^2 + 3*(c + x)/(c - x) + 1))/c
```

maple [A] time = 0.04, size = 67, normalized size = 1.49

$$\frac{x^3a}{3} + \frac{b\,x^3\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{3} + \frac{bc\,x^2}{6} - \frac{c^3b\ln\left(\frac{c}{x}\right)}{3} + \frac{c^3b\ln\left(\frac{c}{x} - 1\right)}{6} + \frac{c^3b\ln\left(1 + \frac{c}{x}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c/x)+1/6*b*c*x^2-1/3*c^3*b*ln(c/x)+1/6*c^3*b*ln
(c/x-1)+1/6*c^3*b*ln(1+c/x)
```

maxima [A] time = 0.32, size = 42, normalized size = 0.93

$$\frac{1}{3}\,ax^3 + \frac{1}{6}\left(2\,x^3\,\mathrm{artanh}\left(\frac{c}{x}\right) + \left(c^2\log\left(-c^2 + x^2\right) + x^2\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c/x) + (c^2*log(-c^2 + x^2) + x^2)*c)*b
```

mupad [B] time = 0.72, size = 42, normalized size = 0.93

$$\frac{a\,x^3}{3}+\frac{b\,c^3\,\ln\left(x^2-c^2\right)}{6}+\frac{b\,x^3\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3}+\frac{b\,c\,x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c/x)),x)
```

```
[Out] (a*x^3)/3 + (b*c^3*log(x^2 - c^2))/6 + (b*x^3*atanh(c/x))/3 + (b*c*x^2)/6
```

sympy [A] time = 0.45, size = 49, normalized size = 1.09

$$\frac{ax^3}{3}+\frac{bc^3\log\left(-c+x\right)}{3}+\frac{bc^3\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3}+\frac{bcx^2}{6}+\frac{bx^3\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x)),x)
```

```
[Out] a*x**3/3 + b*c**3*log(-c + x)/3 + b*c**3*atanh(c/x)/3 + b*c*x**2/6 + b*x**3
*atanh(c/x)/3
```

3.137 $\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=39

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tanh^{-1}\left(\frac{x}{c}\right)+\frac{bcx}{2}$$

```
[Out] 1/2*b*c*x+1/2*x^2*(a+b*arctanh(c/x))-1/2*b*c^2*arctanh(x/c)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6097, 193, 321, 207}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tanh^{-1}\left(\frac{x}{c}\right)+\frac{bcx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c*x)/2 + (x^2*(a + b*ArcTanh[c/x]))/2 - (b*c^2*ArcTanh[x/c])/2
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\,dx&=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{1}{1-\frac{c^2}{x^2}}\,dx\\&=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{x^2}{-c^2+x^2}\,dx\\&=\frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}\left(bc^3\right)\int\frac{1}{-c^2+x^2}\,dx\\&=\frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tanh^{-1}\left(\frac{x}{c}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.44

$$\frac{ax^2}{2}+\frac{1}{4}bc^2\log(x-c)-\frac{1}{4}bc^2\log(c+x)+\frac{1}{2}bx^2\tanh^{-1}\left(\frac{c}{x}\right)+\frac{bcx}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c*x)/2 + (a*x^2)/2 + (b*x^2*ArcTanh[c/x])/2 + (b*c^2*Log[-c + x])/4 - (b
*c^2*Log[c + x])/4
```

fricas [A] time = 0.62, size = 39, normalized size = 1.00

$$\frac{1}{2}\,bcx+\frac{1}{2}\,ax^2-\frac{1}{4}\left(bc^2-bx^2\right)\log\left(-\frac{c+x}{c-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x)),x, algorithm="fricas")
```

```
[Out] 1/2*b*c*x + 1/2*a*x^2 - 1/4*(b*c^2 - b*x^2)*log(-(c + x)/(c - x))
```

giac [B] time = 0.38, size = 130, normalized size = 3.33

$$-\frac{\frac{b(c+x)c^3\log\left(-\frac{c+x}{c-x}\right)}{(c-x)\left(\frac{(c+x)^2}{(c-x)^2}+\frac{2(c+x)}{c-x}+1\right)}+\frac{bc^3+\frac{2a(c+x)c^3}{c-x}+\frac{b(c+x)c^3}{c-x}}{\frac{(c+x)^2}{(c-x)^2}+\frac{2(c+x)}{c-x}+1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x)),x, algorithm="giac")
```

```
[Out] -(b*(c + x)*c^3*log(-(c + x)/(c - x))/((c - x)*((c + x)^2/(c - x)^2 + 2*(c
+ x)/(c - x) + 1)) + (b*c^3 + 2*a*(c + x)*c^3/(c - x) + b*(c + x)*c^3/(c -
x))/((c + x)^2/(c - x)^2 + 2*(c + x)/(c - x) + 1))/c
```

maple [A] time = 0.04, size = 53, normalized size = 1.36

$$\frac{a\,x^2}{2}+\frac{\operatorname{arctanh}\left(\frac{c}{x}\right)b\,x^2}{2}+\frac{xbc}{2}+\frac{c^2b\ln\left(\frac{c}{x}-1\right)}{4}-\frac{c^2b\ln\left(1+\frac{c}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x)),x)
```

```
[Out] 1/2*a*x^2+1/2*arctanh(c/x)*b*x^2+1/2*x*b*c+1/4*c^2*b*ln(c/x-1)-1/4*c^2*b*ln
(1+c/x)
```

maxima [A] time = 0.32, size = 44, normalized size = 1.13

$$\frac{1}{2}\,ax^2+\frac{1}{4}\left(2\,x^2\operatorname{artanh}\left(\frac{c}{x}\right)-\left(c\log\left(c+x\right)-c\log\left(-c+x\right)-2\,x\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arctanh(c/x) - (c*log(c + x) - c*log(-c + x) - 2*x)*
c)*b
```

mupad [B] time = 0.74, size = 36, normalized size = 0.92

$$\frac{a\,x^2}{2}-\frac{b\,c^2\operatorname{atanh}\left(\frac{c}{x}\right)}{2}+\frac{b\,x^2\operatorname{atanh}\left(\frac{c}{x}\right)}{2}+\frac{b\,c\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c/x)),x)
[Out] (a*x^2)/2 - (b*c^2*atanh(c/x))/2 + (b*x^2*atanh(c/x))/2 + (b*c*x)/2
```

sympy [**A**] time = 0.34, size = 36, normalized size = 0.92

$$\frac{ax^2}{2} - \frac{bc^2 \operatorname{atanh}\left(\frac{c}{x}\right)}{2} + \frac{bcx}{2} + \frac{bx^2 \operatorname{atanh}\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x)),x)
[Out] a*x**2/2 - b*c**2*atanh(c/x)/2 + b*c*x/2 + b*x**2*atanh(c/x)/2
```

3.138 $\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=29

$$ax+\frac{1}{2}bc\log\left(c^2-x^2\right)+bx\tanh^{-1}\left(\frac{c}{x}\right)$$

```
[Out] a*x+b*x*arctanh(c/x)+1/2*b*c*ln(c^2-x^2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6091, 263, 260}

$$ax+\frac{1}{2}bc\log\left(c^2-x^2\right)+bx\tanh^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c/x],x]
```

```
[Out] a*x + b*x*ArcTanh[c/x] + (b*c*Log[c^2 - x^2])/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\,dx &= ax+b\int \tanh^{-1}\left(\frac{c}{x}\right)\,dx\\ &= ax+bx\tanh^{-1}\left(\frac{c}{x}\right)+(bc)\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x}\,dx\\ &= ax+bx\tanh^{-1}\left(\frac{c}{x}\right)+(bc)\int \frac{x}{-c^2+x^2}\,dx\\ &= ax+bx\tanh^{-1}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 29, normalized size = 1.00

$$ax+\frac{1}{2}bc\log\left(c^2-x^2\right)+bx\tanh^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c/x],x]
```

```
[Out] a*x + b*x*ArcTanh[c/x] + (b*c*Log[c^2 - x^2])/2
```

fricas [A] time = 0.56, size = 35, normalized size = 1.21

$$\frac{1}{2}\,bc\log\left(-c^2+x^2\right)+\frac{1}{2}\,bx\log\left(-\frac{c+x}{c-x}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x),x, algorithm="fricas")
```

```
[Out] 1/2*b*c*log(-c^2 + x^2) + 1/2*b*x*log(-(c + x)/(c - x)) + a*x
```

giac [B] time = 0.21, size = 150, normalized size = 5.17

$$ax+\frac{\left(c^2\left(\log\left(\frac{|-c-x|}{|c-x|}\right)-\log\left(\left|-\frac{c+x}{c-x}-1\right|\right)\right)-\frac{c^2\log\left(-\frac{\frac{c\left(\frac{c+x}{(c-x)c}+\frac{1}{c}\right)}{\frac{c+x}{c-x}-1}+1}{\frac{c\left(\frac{c+x}{(c-x)c}+\frac{1}{c}\right)}{\frac{c+x}{c-x}-1}-1}\right)}{\frac{c+x}{c-x}+1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x),x, algorithm="giac")
```

```
[Out] a*x + (c^2*(log(abs(-c - x)/abs(c - x)) - log(abs(-(c + x)/(c - x) - 1))) -
 c^2*log(-(c*((c + x)/((c - x)*c) + 1/c)/((c + x)/(c - x) - 1) + 1)/(c*((c
+ x)/((c - x)*c) + 1/c)/((c + x)/(c - x) - 1) - 1))/((c + x)/(c - x) + 1))*
b/c
```

maple [A] time = 0.03, size = 48, normalized size = 1.66

$$ax+bx\,\mathrm{arctanh}\left(\frac{c}{x}\right)-bc\ln\left(\frac{c}{x}\right)+\frac{bc\ln\left(\frac{c}{x}-1\right)}{2}+\frac{bc\ln\left(1+\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c/x),x)
```

```
[Out] a*x+b*x*arctanh(c/x)-b*c*ln(c/x)+1/2*b*c*ln(c/x-1)+1/2*b*c*ln(1+c/x)
```

maxima [A] time = 0.31, size = 29, normalized size = 1.00

$$\frac{1}{2}\left(2\,x\,\mathrm{artanh}\left(\frac{c}{x}\right)+c\log\left(-c^2+x^2\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*x*arctanh(c/x) + c*log(-c^2 + x^2))*b + a*x
```

mupad [B] time = 0.68, size = 27, normalized size = 0.93

$$a\,x+b\,x\,\mathrm{atanh}\left(\frac{c}{x}\right)+\frac{b\,c\,\ln\left(x^2-c^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c/x),x)
```

```
[Out] a*x + b*x*atanh(c/x) + (b*c*log(x^2 - c^2))/2
```

sympy [A] time = 0.25, size = 24, normalized size = 0.83

$$ax + b\left(c\log\left(c - x\right) + c\operatorname{atanh}\left(\frac{c}{x}\right) + x\operatorname{atanh}\left(\frac{c}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c/x),x)
[Out] a*x + b*(c*log(c - x) + c*atanh(c/x) + x*atanh(c/x))
```

3.139 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}\,dx$

Optimal. Leaf size=30

$$a\log(x)+\frac{1}{2}b\text{Li}_2\left(-\frac{c}{x}\right)-\frac{1}{2}b\text{Li}_2\left(\frac{c}{x}\right)$$

```
[Out] a*ln(x)+1/2*b*polylog(2,-c/x)-1/2*b*polylog(2,c/x)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6095, 5912}

$$\frac{1}{2}b\text{PolyLog}\left(2,-\frac{c}{x}\right)-\frac{1}{2}b\text{PolyLog}\left(2,\frac{c}{x}\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])/x,x]
```

```
[Out] a*Log[x] + (b*PolyLog[2, -(c/x)])/2 - (b*PolyLog[2, c/x])/2
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}\,dx &= -\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,\frac{1}{x}\right)\\ &= a\log(x)+\frac{1}{2}b\text{Li}_2\left(-\frac{c}{x}\right)-\frac{1}{2}b\text{Li}_2\left(\frac{c}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.93

$$a\log(x)+\frac{1}{2}b\left(\text{Li}_2\left(-\frac{c}{x}\right)-\text{Li}_2\left(\frac{c}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])/x,x]
```

```
[Out] a*Log[x] + (b*(PolyLog[2, -(c/x)] - PolyLog[2, c/x]))/2
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(\frac{c}{x}\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c/x) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{artanh}\left(\frac{c}{x}\right) + a}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)/x, x)
```

maple [B] time = 0.04, size = 63, normalized size = 2.10

$$-a \ln\left(\frac{c}{x}\right) - b \ln\left(\frac{c}{x}\right) \operatorname{arctanh}\left(\frac{c}{x}\right) + \frac{b \operatorname{dilog}\left(\frac{c}{x}\right)}{2} + \frac{b \operatorname{dilog}\left(1+\frac{c}{x}\right)}{2} + \frac{b \ln\left(\frac{c}{x}\right) \ln\left(1+\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))/x,x)
```

```
[Out] -a*ln(c/x)-b*ln(c/x)*arctanh(c/x)+1/2*b*dilog(c/x)+1/2*b*dilog(1+c/x)+1/2*b
*ln(c/x)*ln(1+c/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} b \int \frac{\log\left(\frac{c}{x}+1\right) - \log\left(-\frac{c}{x}+1\right)}{x} \, dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c/x + 1) - log(-c/x + 1))/x, x) + a*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{atanh}\left(\frac{c}{x}\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))/x,x)
```

```
[Out] int((a + b*atanh(c/x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}\left(\frac{c}{x}\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))/x,x)
```

```
[Out] Integral((a + b*atanh(c/x))/x, x)
```

3.140 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}$$

```
[Out] (-a-b*arctanh(c/x))/x-1/2*b*ln(1-c^2/x^2)/c
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 260}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])/x^2,x]
```

```
[Out] -((a + b*ArcTanh[c/x])/x) - (b*Log[1 - c^2/x^2])/(2*c)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-(bc)\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x^3}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.09

$$-\frac{a}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}-\frac{b\tanh^{-1}\left(\frac{c}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTanh[c/x])/x - (b*Log[1 - c^2/x^2])/(2*c)
```

fricas [A] time = 0.66, size = 48, normalized size = 1.37

$$-\frac{bx\log\left(-c^2+x^2\right)-2\,bx\log(x)+bc\log\left(-\frac{c+x}{c-x}\right)+2\,ac}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*log(-c^2 + x^2) - 2*b*x*log(x) + b*c*log(-(c + x)/(c - x)) + 2*a*
c)/(c*x)
```

giac [B] time = 0.20, size = 87, normalized size = 2.49

$$\frac{b\log\left(-\frac{c+x}{c-x}+1\right)-b\log\left(-\frac{c+x}{c-x}\right)-\frac{b\log\left(-\frac{c+x}{c-x}\right)}{\frac{c+x}{c-x}-1}-\frac{2\,a}{\frac{c+x}{c-x}-1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^2,x, algorithm="giac")
```

```
[Out] (b*log(-(c + x)/(c - x) + 1) - b*log(-(c + x)/(c - x)) - b*log(-(c + x)/(c
- x))/((c + x)/(c - x) - 1) - 2*a/((c + x)/(c - x) - 1))/c
```

maple [A] time = 0.02, size = 37, normalized size = 1.06

$$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{x}-\frac{b\ln\left(1-\frac{c^2}{x^2}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c/x)-1/2*b*ln(1-c^2/x^2)/c
```

maxima [A] time = 0.31, size = 37, normalized size = 1.06

$$-\frac{b\left(\frac{2\,c\,\mathrm{artanh}\left(\frac{c}{x}\right)}{x}+\log\left(-\frac{c^2}{x^2}+1\right)\right)}{2\,c}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a/x
```

mupad [B] time = 0.74, size = 43, normalized size = 1.23

$$\frac{b\,x\,\ln(x)-\frac{b\,x\,\ln\left(x^2-c^2\right)}{2}}{c\,x}-\frac{a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))/x^2,x)
```

```
[Out] (b*x*log(x) - (b*x*log(x^2 - c^2))/2)/(c*x) - (a + b*atanh(c/x))/x
```

sympy [A] time = 0.79, size = 39, normalized size = 1.11

$$\begin{cases} -\frac{a}{x}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{x}+\frac{b\log(x)}{c}-\frac{b\log(-c+x)}{c}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{c} & \text{for}\ c\neq 0\\ -\frac{a}{x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))/x**2,x)
```

```
[Out] Piecewise((-a/x - b*atanh(c/x)/x + b*log(x)/c - b*log(-c + x)/c - b*atanh(c
/x)/c, Ne(c, 0)), (-a/x, True))
```

3.141 $\int \frac{a+b \tanh^{-1}\left(\frac{c}{x}\right)}{x^3} dx$

Optimal. Leaf size=43

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tanh^{-1}\left(\frac{x}{c}\right)}{2c^2}-\frac{b}{2cx}$$

```
[Out] -1/2*b/c/x+1/2*(-a-b*arctanh(c/x))/x^2+1/2*b*arctanh(x/c)/c^2
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 263, 325, 207}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tanh^{-1}\left(\frac{x}{c}\right)}{2c^2}-\frac{b}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])/x^3,x]
```

```
[Out] -b/(2*c*x) - (a + b*ArcTanh[c/x])/(2*x^2) + (b*ArcTanh[x/c])/(2*c^2)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}-\frac{1}{2}(bc)\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x^4}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}-\frac{1}{2}(bc)\int \frac{1}{x^2\left(-c^2+x^2\right)}\,dx\\ &= -\frac{b}{2cx}-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}-\frac{b\int \frac{1}{-c^2+x^2}\,dx}{2c}\\ &= -\frac{b}{2cx}-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tanh^{-1}\left(\frac{x}{c}\right)}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.40

$$-\frac{a}{2x^2}-\frac{b\log(x-c)}{4c^2}+\frac{b\log(c+x)}{4c^2}-\frac{b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}-\frac{b}{2cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])/x^3,x]
[Out] -1/2*a/x^2 - b/(2*c*x) - (b*ArcTanh[c/x])/(2*x^2) - (b*Log[-c + x])/(4*c^2)
 + (b*Log[c + x])/(4*c^2)
```

fricas [A] time = 0.51, size = 46, normalized size = 1.07

$$-\frac{2\,ac^2+2\,bcx+\left(bc^2-bx^2\right)\log\left(-\frac{c+x}{c-x}\right)}{4\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^3,x, algorithm="fricas")
[Out] -1/4*(2*a*c^2 + 2*b*c*x + (b*c^2 - b*x^2)*log(-(c + x)/(c - x)))/(c^2*x^2)
```

giac [B] time = 0.17, size = 123, normalized size = 2.86

$$-\frac{\frac{b(c+x)\log\left(-\frac{c+x}{c-x}\right)}{\left(\frac{(c+x)^2c}{(c-x)^2}-\frac{2\,(c+x)c}{c-x}+c\right)(c-x)}-\frac{b-\frac{2\,a(c+x)}{c-x}-\frac{b(c+x)}{c-x}}{\frac{(c+x)^2c}{(c-x)^2}-\frac{2\,(c+x)c}{c-x}+c}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^3,x, algorithm="giac")
[Out] -(b*(c + x)*log(-(c + x)/(c - x))/(((c + x)^2*c/(c - x)^2 - 2*(c + x)*c/(c
- x) + c)*(c - x)) - (b - 2*a*(c + x)/(c - x) - b*(c + x)/(c - x))/((c + x)
^2*c/(c - x)^2 - 2*(c + x)*c/(c - x) + c))/c
```

maple [A] time = 0.03, size = 57, normalized size = 1.33

$$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{2x^2}-\frac{b}{2cx}-\frac{b\ln\left(\frac{c}{x}-1\right)}{4c^2}+\frac{b\ln\left(1+\frac{c}{x}\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c/x)-1/2*b/c/x-1/4/c^2*b*ln(c/x-1)+1/4/c^2*b*l
n(1+c/x)
```

maxima [A] time = 0.32, size = 52, normalized size = 1.21

$$\frac{1}{4}\left(c\left(\frac{\log\left(c+x\right)}{c^3}-\frac{\log\left(-c+x\right)}{c^3}-\frac{2}{c^2x}\right)-\frac{2\,\operatorname{artanh}\left(\frac{c}{x}\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2)
*b - 1/2*a/x^2
```

mupad [B] time = 0.71, size = 49, normalized size = 1.14

$$\frac{b\,c\,\operatorname{atan}\left(\frac{x}{\sqrt{-c^2}}\right)}{2\left(-c^2\right)^{3/2}}-\frac{b}{2\,c\,x}-\frac{b\,\operatorname{atanh}\left(\frac{c}{x}\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))/x^3,x)
```

```
[Out] (b*c*atan(x/(-c^2)^(1/2)))/(2*(-c^2)^(3/2)) - b/(2*c*x) - (b*atanh(c/x))/(2
*x^2) - a/(2*x^2)
```

sympy [A] time = 0.95, size = 44, normalized size = 1.02

$$\begin{cases} -\frac{a}{2x^2}-\frac{b\operatorname{atanh}\left(\frac{c}{x}\right)}{2x^2}-\frac{b}{2cx}+\frac{b\operatorname{atanh}\left(\frac{c}{x}\right)}{2c^2} & \text{for } c \neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - b*atanh(c/x)/(2*x**2) - b/(2*c*x) + b*atanh(c/x)/(
2*c**2), Ne(c, 0)), (-a/(2*x**2), True))
```

3.142 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^4}\,dx$

Optimal. Leaf size=57

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2-x^2\right)}{6c^3}-\frac{b}{6cx^2}$$

```
[Out] -1/6*b/c/x^2+1/3*(-a-b*arctanh(c/x))/x^3+1/3*b*ln(x)/c^3-1/6*b*ln(c^2-x^2)/
c^3
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 263, 266, 44}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{b\log\left(c^2-x^2\right)}{6c^3}+\frac{b\log(x)}{3c^3}-\frac{b}{6cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])/x^4,x]
```

```
[Out] -b/(6*c*x^2) - (a + b*ArcTanh[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (b*Log[c
^2 - x^2])/(6*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{3}(bc)\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x^5}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{3}(bc)\int \frac{1}{x^3\left(-c^2+x^2\right)}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(-c^2+x\right)}\,dx,x,x^2\right)\\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}(bc)\,\text{Subst}\left(\int \left(-\frac{1}{c^4\left(c^2-x\right)}-\frac{1}{c^2x^2}-\frac{1}{c^4x}\right)dx,x,x^2\right)\\
&= -\frac{b}{6cx^2}-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2-x^2\right)}{6c^3}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 62, normalized size = 1.09

$$
-\frac{a}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(x^2-c^2\right)}{6c^3}-\frac{b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{b}{6cx^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/(6*c*x^2) - (b*ArcTanh[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) -
(b*Log[-c^2 + x^2])/(6*c^3)
```

fricas [A] time = 1.12, size = 62, normalized size = 1.09

$$
-\frac{bx^3\log\left(-c^2+x^2\right)-2\,bx^3\log(x)+bc^3\log\left(-\frac{c+x}{c-x}\right)+2\,ac^3+bc^2x}{6\,c^3x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*x^3*log(-c^2 + x^2) - 2*b*x^3*log(x) + b*c^3*log(-(c + x)/(c - x))
+ 2*a*c^3 + b*c^2*x)/(c^3*x^3)
```

giac [B] time = 0.31, size = 234, normalized size = 4.11

$$
-\frac{\frac{\left(b+\frac{3\,b(c+x)^2}{(c-x)^2}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^3c^2}{(c-x)^3}-\frac{3\,(c+x)^2c^2}{(c-x)^2}+\frac{3\,(c+x)c^2}{c-x}-c^2}+\frac{2\left(a+\frac{3\,a(c+x)^2}{(c-x)^2}+\frac{b(c+x)^2}{(c-x)^2}-\frac{b(c+x)}{c-x}\right)}{\frac{(c+x)^3c^2}{(c-x)^3}-\frac{3\,(c+x)^2c^2}{(c-x)^2}+\frac{3\,(c+x)c^2}{c-x}-c^2}-\frac{b\log\left(-\frac{c+x}{c-x}+1\right)}{c^2}+\frac{b\log\left(-\frac{c+x}{c-x}\right)}{c^2}}{3\,c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*((b + 3*b*(c + x)^2/(c - x)^2)*log(-(c + x)/(c - x))/((c + x)^3*c^2/(c
 - x)^3 - 3*(c + x)^2*c^2/(c - x)^2 + 3*(c + x)*c^2/(c - x) - c^2) + 2*(a +
 3*a*(c + x)^2/(c - x)^2 + b*(c + x)^2/(c - x)^2 - b*(c + x)/(c - x))/((c +
 x)^3*c^2/(c - x)^3 - 3*(c + x)^2*c^2/(c - x)^2 + 3*(c + x)*c^2/(c - x) - c
^2) - b*log(-(c + x)/(c - x) + 1)/c^2 + b*log(-(c + x)/(c - x))/c^2)/c
```

maple [A] time = 0.03, size = 57, normalized size = 1.00

$$
-\frac{a}{3x^3}-\frac{b\,\text{arctanh}\left(\frac{c}{x}\right)}{3x^3}-\frac{b}{6c\,x^2}-\frac{b\ln\left(\frac{c}{x}-1\right)}{6c^3}-\frac{b\ln\left(1+\frac{c}{x}\right)}{6c^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c/x)-1/6*b/c/x^2-1/6/c^3*b*ln(c/x-1)-1/6/c^3*b
*ln(1+c/x)
```

maxima [A] time = 0.32, size = 55, normalized size = 0.96

$$-\frac{1}{6}\left(c\left(\frac{\log\left(-c^2+x^2\right)}{c^4}-\frac{\log\left(x^2\right)}{c^4}+\frac{1}{c^2x^2}\right)+\frac{2\ \mathrm{artanh}\left(\frac{c}{x}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*(log(-c^2 + x^2)/c^4 - log(x^2)/c^4 + 1/(c^2*x^2)) + 2*arctanh(c/x)
/x^3)*b - 1/3*a/x^3
```

mupad [B] time = 0.75, size = 59, normalized size = 1.04

$$-\frac{\frac{a}{3}+\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3}}{x^3}-\frac{\frac{b\,x^3\,\ln\left(x^2-c^2\right)}{6}-\frac{b\,x^3\,\ln(x)}{3}+\frac{b\,c^2\,x}{6}}{c^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))/x^4,x)
```

```
[Out] - (a/3 + (b*atanh(c/x))/3)/x^3 - ((b*x^3*log(x^2 - c^2))/6 - (b*x^3*log(x))
/3 + (b*c^2*x)/6)/(c^3*x^3)
```

sympy [A] time = 1.29, size = 68, normalized size = 1.19

$$\begin{cases} -\frac{a}{3x^3}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3x^3}-\frac{b}{6cx^2}+\frac{b\log\left(x\right)}{3c^3}-\frac{b\log\left(-c+x\right)}{3c^3}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3c^3} & \text{for}\ c\neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) - b*atanh(c/x)/(3*x**3) - b/(6*c*x**2) + b*log(x)/(3
*c**3) - b*log(-c + x)/(3*c**3) - b*atanh(c/x)/(3*c**3), Ne(c, 0)), (-a/(3*
x**3), True))
```

3.143 $\int x^3 \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx$

Optimal. Leaf size=123

$$-\frac{1}{4}c^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}bc^3x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{4}x^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{6}bcx^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{2}{3}b^2c^4$$

```
[Out] 1/12*b^2*c^2*x^2+1/2*b*c^3*x*(a+b*arccoth(x/c))+1/6*b*c*x^3*(a+b*arccoth(x/
c))-1/4*c^4*(a+b*arccoth(x/c))^2+1/4*x^4*(a+b*arccoth(x/c))^2+1/3*b^2*c^4*l
n(1-c^2/x^2)+2/3*b^2*c^4*ln(x)
```

Rubi [C] time = 1.70, antiderivative size = 812, normalized size of antiderivative = 6.60, number of steps used = 88, number of rules used = 34, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.125, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2455, 263, 43, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 193, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{16}\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2c^4-\frac{1}{16}b^2\log^2\left(\frac{c+x}{x}\right)c^4+\frac{5}{48}b^2\log\left(1-\frac{c}{x}\right)c^4+\frac{5}{48}b^2\log(c-x)c^4+\frac{1}{8}b^2\log\left(\frac{c}{x}+1\right)\log(c$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (a*b*c^3*x)/4 - (a*b*c^2*x^2)/8 + (b^2*c^2*x^2)/12 + (a*b*c*x^3)/12 + (5*b^
2*c^4*Log[1 - c/x])/48 - (b^2*c^3*x*Log[1 - c/x])/8 + (b^2*c^2*x^2*Log[1 -
c/x])/16 - (b^2*c*x^3*Log[1 - c/x])/24 + (b*c^3*(1 - c/x)*x*(2*a - b*Log[1
- c/x]))/8 + (b*c^2*x^2*(2*a - b*Log[1 - c/x]))/16 + (b*c*x^3*(2*a - b*Log[
1 - c/x]))/24 - (c^4*(2*a - b*Log[1 - c/x])^2)/16 + (x^4*(2*a - b*Log[1 - c
/x])^2)/16 + (b^2*c^3*x*Log[1 + c/x])/8 + (b^2*c^2*x^2*Log[1 + c/x])/16 + (
b^2*c*x^3*Log[1 + c/x])/24 + (a*b*x^4*Log[1 + c/x])/4 - (b^2*x^4*Log[1 - c/
x]*Log[1 + c/x])/8 + (5*b^2*c^4*Log[c - x])/48 + (b^2*c^4*Log[1 + c/x]*Log[
c - x])/8 + (a*b*c^4*Log[x])/4 + (11*b^2*c^4*Log[x])/24 + (b^2*c^4*Log[c -
x]*Log[x/c])/8 - (a*b*c^4*Log[c + x])/4 + (5*b^2*c^4*Log[c + x])/48 + (b^2*
c^4*Log[1 - c/x]*Log[c + x])/8 - (b^2*c^4*Log[(c - x)/(2*c)]*Log[c + x])/8
+ (b^2*c^4*Log[-(x/c)]*Log[c + x])/8 - (b^2*c^4*Log[c - x]*Log[(c + x)/(2*c
)])/8 + (11*b^2*c^4*Log[(c + x)/x])/48 + (b^2*c^3*x*Log[(c + x)/x])/8 - (b^
2*c^2*x^2*Log[(c + x)/x])/16 + (b^2*c*x^3*Log[(c + x)/x])/24 - (b^2*c^4*Log
[(c + x)/x]^2)/16 + (b^2*x^4*Log[(c + x)/x]^2)/16 - (b^2*c^4*PolyLog[2, (c
- x)/(2*c)])/8 - (b^2*c^4*PolyLog[2, -(c/x)])/8 - (b^2*c^4*PolyLog[2, c/x])
/8 - (b^2*c^4*PolyLog[2, (c + x)/(2*c)])/8 + (b^2*c^4*PolyLog[2, 1 - x/c])/
8 + (b^2*c^4*PolyLog[2, 1 + x/c])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
```

```
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int\left(\frac{1}{4}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}bx^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)+\frac{1}{4}b^2x\right.$$

$$=\frac{1}{4}\int x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2 dx+\frac{1}{2}b\int x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right) d$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^2}{x^5}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(2ax^3\log\left(1+\frac{c}{x}\right)-bx\right.$$

$$=\frac{1}{16}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{16}b^2x^4\log^2\left(\frac{c+x}{x}\right)+(ab)\int x^3\log\left(1+\frac{c}{x}\right) dx$$

$$=\frac{1}{16}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{4}abx^4\log\left(1+\frac{c}{x}\right)-\frac{1}{8}b^2x^4\log\left(1-\frac{c}{x}\right)\log\left(1+\right.$$

$$=\frac{1}{16}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{4}abx^4\log\left(1+\frac{c}{x}\right)-\frac{1}{8}b^2x^4\log\left(1-\frac{c}{x}\right)\log\left(1+\right.$$

$$=\frac{1}{24}bcx^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{16}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{4}abx^4\log\left(1+\frac{c}{x}\right.$$

$$=\frac{1}{4}abc^3x-\frac{1}{8}abc^2x^2+\frac{1}{12}abcx^3+\frac{1}{16}bc^2x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{24}bcx^3\left(2a-\right.$$

$$=\frac{1}{4}abc^3x-\frac{1}{16}b^2c^3x-\frac{1}{8}abc^2x^2+\frac{1}{24}b^2c^2x^2+\frac{1}{12}abcx^3+\frac{1}{24}b^2c^4\log\left(1-\frac{c}{x}\right)-$$

$$=\frac{1}{4}abc^3x-\frac{1}{8}abc^2x^2+\frac{1}{24}b^2c^2x^2+\frac{1}{12}abcx^3+\frac{5}{48}b^2c^4\log\left(1-\frac{c}{x}\right)-\frac{1}{8}b^2c^3x\log$$

$$=\frac{1}{4}abc^3x-\frac{1}{8}abc^2x^2+\frac{1}{24}b^2c^2x^2+\frac{1}{12}abcx^3+\frac{5}{48}b^2c^4\log\left(1-\frac{c}{x}\right)-\frac{1}{8}b^2c^3x\log$$

$$=\frac{1}{4}abc^3x-\frac{1}{8}abc^2x^2+\frac{1}{12}b^2c^2x^2+\frac{1}{12}abcx^3+\frac{5}{48}b^2c^4\log\left(1-\frac{c}{x}\right)-\frac{1}{8}b^2c^3x\log$$

$$=\frac{1}{4}abc^3x-\frac{1}{8}abc^2x^2+\frac{1}{12}b^2c^2x^2+\frac{1}{12}abcx^3+\frac{5}{48}b^2c^4\log\left(1-\frac{c}{x}\right)-\frac{1}{8}b^2c^3x\log$$

$$=\frac{1}{4}abc^3x-\frac{1}{8}abc^2x^2+\frac{1}{12}b^2c^2x^2+\frac{1}{12}abcx^3+\frac{5}{48}b^2c^4\log\left(1-\frac{c}{x}\right)-\frac{1}{8}b^2c^3x\log$$

Mathematica [A] time = 0.06, size = 131, normalized size = 1.07

$$\frac{1}{12}\left(3a^2x^4+bc^4(3a+4b)\log(x-c)-3abc^4\log(c+x)+6abc^3x+2bx\tanh^{-1}\left(\frac{c}{x}\right)\left(3ax^3+bc\left(3c^2+x^2\right)\right)+2a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (6*a*b*c^3*x + b^2*c^2*x^2 + 2*a*b*c*x^3 + 3*a^2*x^4 + 2*b*x*(3*a*x^3 + b*c
*(3*c^2 + x^2))*ArcTanh[c/x] + 3*b^2*(-c^4 + x^4)*ArcTanh[c/x]^2 + b*(3*a +
 4*b)*c^4*Log[-c + x] - 3*a*b*c^4*Log[c + x] + 4*b^2*c^4*Log[c + x])/12
```

fricas [A] time = 0.59, size = 149, normalized size = 1.21

$$\frac{1}{2}\,abc^3x+\frac{1}{12}\,b^2c^2x^2+\frac{1}{6}\,abcx^3+\frac{1}{4}\,a^2x^4-\frac{1}{12}\left(3\,ab-4\,b^2\right)c^4\log\left(c+x\right)+\frac{1}{12}\left(3\,ab+4\,b^2\right)c^4\log\left(-c+x\right)-\frac{1}{16}\left(b^2c^4-b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*b*c^3*x + 1/12*b^2*c^2*x^2 + 1/6*a*b*c*x^3 + 1/4*a^2*x^4 - 1/12*(3*a*
b - 4*b^2)*c^4*log(c + x) + 1/12*(3*a*b + 4*b^2)*c^4*log(-c + x) - 1/16*(b^
2*c^4 - b^2*x^4)*log(-(c + x)/(c - x))^2 + 1/12*(3*b^2*c^3*x + b^2*c*x^3 +
3*a*b*x^4)*log(-(c + x)/(c - x))
```

giac [B] time = 0.17, size = 552, normalized size = 4.49

$$-\frac{4\,b^2c^5\log\left(-\frac{c+x}{c-x}-1\right)-4\,b^2c^5\log\left(-\frac{c+x}{c-x}\right)+\frac{3\left(\frac{b^2(c+x)^3c^5}{(c-x)^3}+\frac{b^2(c+x)c^5}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)^2}{\frac{(c+x)^4}{(c-x)^4}+\frac{4\,(c+x)^3}{(c-x)^3}+\frac{6\,(c+x)^2}{(c-x)^2}+\frac{4\,(c+x)}{c-x}+1}+\frac{2\left(2\,b^2c^5+\frac{6\,ab(c+x)^3c^5}{(c-x)^3}+\frac{3\,b^2(c+x)^3c^5}{(c-x)^3}+\frac{6\,b^2(c+x)^2c^5}{(c-x)^2}+\right.}{\frac{(c+x)^4}{(c-x)^4}+\frac{4\,(c+x)^3}{(c-x)^3}+\frac{6\,(c+x)^2}{(c-x)^2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] -1/6*(4*b^2*c^5*log(-(c + x)/(c - x) - 1) - 4*b^2*c^5*log(-(c + x)/(c - x))
 + 3*(b^2*(c + x)^3*c^5/(c - x)^3 + b^2*(c + x)*c^5/(c - x))*log(-(c + x)/(
c - x))^2/((c + x)^4/(c - x)^4 + 4*(c + x)^3/(c - x)^3 + 6*(c + x)^2/(c - x
)^2 + 4*(c + x)/(c - x) + 1) + 2*(2*b^2*c^5 + 6*a*b*(c + x)^3*c^5/(c - x)^3
 + 3*b^2*(c + x)^3*c^5/(c - x)^3 + 6*b^2*(c + x)^2*c^5/(c - x)^2 + 6*a*b*(c
 + x)*c^5/(c - x) + 5*b^2*(c + x)*c^5/(c - x))*log(-(c + x)/(c - x))/((c +
x)^4/(c - x)^4 + 4*(c + x)^3/(c - x)^3 + 6*(c + x)^2/(c - x)^2 + 4*(c + x)/
(c - x) + 1) + 2*(4*a*b*c^5 + 6*a^2*(c + x)^3*c^5/(c - x)^3 + 6*a*b*(c + x)
^3*c^5/(c - x)^3 + b^2*(c + x)^3*c^5/(c - x)^3 + 12*a*b*(c + x)^2*c^5/(c -
x)^2 + 2*b^2*(c + x)^2*c^5/(c - x)^2 + 6*a^2*(c + x)*c^5/(c - x) + 10*a*b*(
c + x)*c^5/(c - x) + b^2*(c + x)*c^5/(c - x))/((c + x)^4/(c - x)^4 + 4*(c +
 x)^3/(c - x)^3 + 6*(c + x)^2/(c - x)^2 + 4*(c + x)/(c - x) + 1))/c
```

maple [B] time = 0.06, size = 328, normalized size = 2.67

$$\frac{a^2x^4}{4}+\frac{b^2x^4\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{4}+\frac{c\,b^2\operatorname{arctanh}\left(\frac{c}{x}\right)x^3}{6}+\frac{c^3b^2\operatorname{arctanh}\left(\frac{c}{x}\right)x}{2}+\frac{c^4b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{4}-\frac{c^4b^2\operatorname{arctanh}\left(\frac{c}{x}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x))^2,x)
```

```
[Out] 1/4*a^2*x^4+1/4*b^2*x^4*arctanh(c/x)^2+1/6*c*b^2*arctanh(c/x)*x^3+1/2*c^3*b
^2*arctanh(c/x)*x+1/4*c^4*b^2*arctanh(c/x)*ln(c/x-1)-1/4*c^4*b^2*arctanh(c/
x)*ln(1+c/x)+1/16*c^4*b^2*ln(c/x-1)^2-1/8*c^4*b^2*ln(c/x-1)*ln(1/2+1/2*c/x)
+1/16*c^4*b^2*ln(1+c/x)^2+1/8*c^4*b^2*ln(-1/2*c/x+1/2)*ln(1/2+1/2*c/x)-1/8*
c^4*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)+1/12*b^2*c^2*x^2-2/3*c^4*b^2*ln(c/x)+1/3
*c^4*b^2*ln(c/x-1)+1/3*c^4*b^2*ln(1+c/x)+1/2*a*b*x^4*arctanh(c/x)+1/6*a*b*c
*x^3+1/2*c^3*x*a*b+1/4*c^4*a*b*ln(c/x-1)-1/4*c^4*a*b*ln(1+c/x)
```

maxima [A] time = 0.34, size = 189, normalized size = 1.54

$$\frac{1}{4}\,b^2x^4\operatorname{artanh}\left(\frac{c}{x}\right)^2+\frac{1}{4}\,a^2x^4+\frac{1}{12}\left(6\,x^4\operatorname{artanh}\left(\frac{c}{x}\right)-\left(3\,c^3\log\left(c+x\right)-3\,c^3\log\left(-c+x\right)-6\,c^2x-2\,x^3\right)c\right)ab+\frac{1}{48}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c/x)^2 + 1/4*a^2*x^4 + 1/12*(6*x^4*arctanh(c/x) - (3*c^
3*log(c + x) - 3*c^3*log(-c + x) - 6*c^2*x - 2*x^3)*c)*a*b + 1/48*((3*c^2*l
og(c + x)^2 + 3*c^2*log(-c + x)^2 + 16*c^2*log(c + x) + 4*x^2 - 2*(3*c^2*lo
g(c + x) - 8*c^2)*log(-c + x))*c^2 - 4*(3*c^3*log(c + x) - 3*c^3*log(-c + x
) - 6*c^2*x - 2*x^3)*c*arctanh(c/x))*b^2
```

mupad [B] time = 0.89, size = 142, normalized size = 1.15

$$\frac{a^2\,x^4}{4}-\frac{b^2\,c^4\,\mathrm{atanh}\left(\frac{c}{x}\right)^2}{4}+\frac{b^2\,x^4\,\mathrm{atanh}\left(\frac{c}{x}\right)^2}{4}+\frac{b^2\,c^4\,\ln\left(x^2-c^2\right)}{3}+\frac{b^2\,c^2\,x^2}{12}+\frac{b^2\,c\,x^3\,\mathrm{atanh}\left(\frac{c}{x}\right)}{6}+\frac{b^2\,c^3\,x\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c/x))^2,x)
```

```
[Out] (a^2*x^4)/4 - (b^2*c^4*atanh(c/x)^2)/4 + (b^2*x^4*atanh(c/x)^2)/4 + (b^2*c^
4*log(x^2 - c^2))/3 + (b^2*c^2*x^2)/12 + (b^2*c*x^3*atanh(c/x))/6 + (b^2*c^
3*x*atanh(c/x))/2 + (a*b*c*x^3)/6 + (a*b*c^3*x)/2 - (a*b*c^4*atanh(c/x))/2
+ (a*b*x^4*atanh(c/x))/2
```

sympy [A] time = 0.96, size = 158, normalized size = 1.28

$$\frac{a^2x^4}{4}-\frac{abc^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}+\frac{abc^3x}{2}+\frac{abcx^3}{6}+\frac{abx^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}+\frac{2b^2c^4\log\left(-c+x\right)}{3}-\frac{b^2c^4\,\mathrm{atanh}^2\left(\frac{c}{x}\right)}{4}+\frac{2b^2c^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x))**2,x)
```

```
[Out] a**2*x**4/4 - a*b*c**4*atanh(c/x)/2 + a*b*c**3*x/2 + a*b*c*x**3/6 + a*b*x**
4*atanh(c/x)/2 + 2*b**2*c**4*log(-c + x)/3 - b**2*c**4*atanh(c/x)**2/4 + 2*
b**2*c**4*atanh(c/x)/3 + b**2*c**3*x*atanh(c/x)/2 + b**2*c**2*x**2/12 + b**
2*c*x**3*atanh(c/x)/6 + b**2*x**4*atanh(c/x)**2/4
```

3.144 $\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=142

$$-\frac{1}{3}c^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{2}{3}bc^3\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{3}x^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}bcx^2\left(a+b\coth\right.$$

```
[Out] 1/3*b^2*c^2*x-1/3*b^2*c^3*arccoth(x/c)+1/3*b*c*x^2*(a+b*arccoth(x/c))-1/3*c
^3*(a+b*arccoth(x/c))^2+1/3*x^3*(a+b*arccoth(x/c))^2-2/3*b*c^3*(a+b*arccoth
(x/c))*ln(2-2/(1+c/x))+1/3*b^2*c^3*polylog(2,-1+2/(1+c/x))
```

Rubi [B] time = 1.39, antiderivative size = 695, normalized size of antiderivative = 4.89, number of steps used = 73, number of rules used = 34, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.125, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2455, 263, 43, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 193, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{6}b^2c^3\text{PolyLog}\left(2,\frac{c-x}{2c}\right)+\frac{1}{6}b^2c^3\text{PolyLog}\left(2,-\frac{c}{x}\right)-\frac{1}{6}b^2c^3\text{PolyLog}\left(2,\frac{c}{x}\right)+\frac{1}{6}b^2c^3\text{PolyLog}\left(2,\frac{c+x}{2c}\right)+\frac{1}{6}b^2c^3\text{Pol}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] -(a*b*c^2*x)/3 + (b^2*c^2*x)/3 + (a*b*c*x^2)/6 + (b^2*c^3*Log[1 - c/x])/12
+ (b^2*c^2*x*Log[1 - c/x])/6 - (b^2*c*x^2*Log[1 - c/x])/12 + (b*c^2*(1 - c/
x)*x*(2*a - b*Log[1 - c/x]))/6 + (b*c*x^2*(2*a - b*Log[1 - c/x]))/12 - (c^3
*(2*a - b*Log[1 - c/x])^2)/12 + (x^3*(2*a - b*Log[1 - c/x])^2)/12 + (b^2*c^
2*x*Log[1 + c/x])/6 + (b^2*c*x^2*Log[1 + c/x])/12 + (a*b*x^3*Log[1 + c/x])/
3 - (b^2*x^3*Log[1 - c/x]*Log[1 + c/x])/6 - (b^2*c^3*Log[c - x])/12 + (b^2*
c^3*Log[1 + c/x]*Log[c - x])/6 + (a*b*c^3*Log[x])/3 + (b^2*c^3*Log[c - x]*L
og[x/c])/6 + (a*b*c^3*Log[c + x])/3 + (b^2*c^3*Log[c + x])/12 - (b^2*c^3*Lo
g[1 - c/x]*Log[c + x])/6 + (b^2*c^3*Log[(c - x)/(2*c)]*Log[c + x])/6 - (b^2
*c^3*Log[-(x/c)]*Log[c + x])/6 - (b^2*c^3*Log[c - x]*Log[(c + x)/(2*c)])/6
- (b^2*c^3*Log[(c + x)/x])/4 - (b^2*c^2*x*Log[(c + x)/x])/6 + (b^2*c*x^2*Lo
g[(c + x)/x])/12 + (b^2*c^3*Log[(c + x)/x]^2)/12 + (b^2*x^3*Log[(c + x)/x]^
2)/12 - (b^2*c^3*PolyLog[2, (c - x)/(2*c)])/6 + (b^2*c^3*PolyLog[2, -(c/x)]
)/6 - (b^2*c^3*PolyLog[2, c/x])/6 + (b^2*c^3*PolyLog[2, (c + x)/(2*c)])/6 +
 (b^2*c^3*PolyLog[2, 1 - x/c])/6 - (b^2*c^3*PolyLog[2, 1 + x/c])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] && !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int\left(\frac{1}{4}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}bx^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)+\frac{1}{4}b^2x\right.$$

$$=\frac{1}{4}\int x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2dx+\frac{1}{2}b\int x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)d$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^2}{x^4}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(2ax^2\log\left(1+\frac{c}{x}\right)-b\right.$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{12}b^2x^3\log^2\left(\frac{c+x}{x}\right)+(ab)\int x^2\log\left(1+\frac{c}{x}\right)dx$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{3}abx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{6}b^2x^3\log\left(1-\frac{c}{x}\right)\log\left(1+\right.$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{3}abx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{6}b^2x^3\log\left(1-\frac{c}{x}\right)\log\left(1+\right.$$

$$=\frac{1}{12}bcx^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{3}abx^3\log\left(1+\frac{c}{x}\right.$$

$$=-\frac{1}{3}abc^2x+\frac{1}{6}abcx^2+\frac{1}{6}bc^2\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{12}bcx^2\left(2a-b\log\right.$$

$$=-\frac{1}{3}abc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{12}$$

$$=-\frac{1}{3}abc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{12}$$

$$=-\frac{1}{3}abc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{12}$$

$$=-\frac{1}{3}abc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{12}$$

$$=-\frac{1}{3}abc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{12}$$

$$=-\frac{1}{3}abc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{12}$$

Mathematica [A] time = 0.33, size = 145, normalized size = 1.02

$$\frac{1}{3}\left(a^2x^3+b\tanh^{-1}\left(\frac{c}{x}\right)\left(2ax^3-2bc^3\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)-bc^3+bcx^2\right)-2abc^3\log\left(\frac{c}{x}\right)+abc^3\log\left(1-\frac{c^2}{x^2}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (b^2*c^2*x + a*b*c*x^2 + a^2*x^3 + b^2*(-c^3 + x^3)*ArcTanh[c/x]^2 + b*ArcT
anh[c/x]*(-(b*c^3) + b*c*x^2 + 2*a*x^3 - 2*b*c^3*Log[1 - E^(-2*ArcTanh[c/x]
)]) + a*b*c^3*Log[1 - c^2/x^2] - 2*a*b*c^3*Log[c/x] + b^2*c^3*PolyLog[2, E^
(-2*ArcTanh[c/x])])/3
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\,\text{artanh}\left(\frac{c}{x}\right)^2+2\,abx^2\,\text{artanh}\left(\frac{c}{x}\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c/x)^2 + 2*a*b*x^2*arctanh(c/x) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x}\right) + a\right)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2*x^2, x)
```

maple [B] time = 0.07, size = 391, normalized size = 2.75

$$\frac{x^3a^2}{3}+\frac{b^2x^3\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{3}+\frac{c\,b^2\operatorname{arctanh}\left(\frac{c}{x}\right)x^2}{3}-\frac{2c^3b^2\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)}{3}+\frac{c^3b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{3}+\frac{c^3b^2\operatorname{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arctanh(c/x)^2+1/3*c*b^2*arctanh(c/x)*x^2-2/3*c^3*b
^2*ln(c/x)*arctanh(c/x)+1/3*c^3*b^2*arctanh(c/x)*ln(c/x-1)+1/3*c^3*b^2*arct
anh(c/x)*ln(1+c/x)+1/3*b^2*c^2*x+1/6*c^3*b^2*ln(c/x-1)-1/6*c^3*b^2*ln(1+c/x
)+1/12*c^3*b^2*ln(c/x-1)^2-1/3*c^3*b^2*dilog(1/2+1/2*c/x)-1/6*c^3*b^2*ln(c/
x-1)*ln(1/2+1/2*c/x)-1/12*c^3*b^2*ln(1+c/x)^2-1/6*c^3*b^2*ln(-1/2*c/x+1/2)*
ln(1/2+1/2*c/x)+1/6*c^3*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)+1/3*c^3*b^2*dilog(c/
x)+1/3*c^3*b^2*dilog(1+c/x)+1/3*c^3*b^2*ln(c/x)*ln(1+c/x)+2/3*a*b*x^3*arcta
nh(c/x)+1/3*a*b*c*x^2-2/3*c^3*a*b*ln(c/x)+1/3*c^3*a*b*ln(c/x-1)+1/3*c^3*a*b
*ln(1+c/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2x^3+\frac{1}{3}\left(2x^3\operatorname{artanh}\left(\frac{c}{x}\right)+\left(c^2\log\left(-c^2+x^2\right)+x^2\right)c\right)ab+\frac{1}{12}\left(6c^4\int -\frac{\log\left(c+x\right)}{3\left(c^2-x^2\right)}\,dx+x^3\log\left(c+x\right)^2+6c^3\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c/x) + (c^2*log(-c^2 + x^2) + x^2)*c)*a*b
+ 1/12*(6*c^4*integrate(-1/3*log(c + x)/(c^2 - x^2), x) + x^3*log(c + x)^2
+ 6*c^3*integrate(-1/3*x*log(c + x)/(c^2 - x^2), x) - (c*log(c + x) - c*log
(-c + x) - 2*x)*c^2 - (c^3 - x^3)*log(-c + x)^2 + (c^2*log(-c^2 + x^2) + x^
2)*c + 12*c*integrate(-1/3*x^3*log(c + x)/(c^2 - x^2), x) - 2*(c*x^2 + (c^3
 + x^3)*log(c + x))*log(-c + x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c/x))^2,x)
```

```
[Out] int(x^2*(a + b*atanh(c/x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b\,\text{atanh}\left(\frac{c}{x}\right)\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x))**2,x)
[Out] Integral(x**2*(a + b*atanh(c/x))**2, x)
```

3.145 $\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=83

$$-\frac{1}{2}c^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+bcx\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b^2c^2\log\left(1-\frac{c^2}{x^2}\right)+b^2c^2\log(x)$$

```
[Out] b*c*x*(a+b*arccoth(x/c))-1/2*c^2*(a+b*arccoth(x/c))^2+1/2*x^2*(a+b*arccoth(
x/c))^2+1/2*b^2*c^2*ln(1-c^2/x^2)+b^2*c^2*ln(x)
```

Rubi [C] time = 1.04, antiderivative size = 574, normalized size of antiderivative = 6.92, number of steps used = 58, number of rules used = 32, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.286, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2455, 193, 43, 6742, 30, 2557, 12, 2466, 2448, 263, 2462, 260, 2416, 2394, 2393, 2391, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,\frac{c-x}{2c}\right)-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,-\frac{c}{x}\right)-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,\frac{c}{x}\right)-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,\frac{c+x}{2c}\right)+\frac{1}{4}b^2c^2\text{Poly}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (a*b*c*x)/2 - (b^2*c*x*Log[1 - c/x])/4 + (b*c*(1 - c/x)*x*(2*a - b*Log[1 -
c/x]))/4 - (c^2*(2*a - b*Log[1 - c/x])^2)/8 + (x^2*(2*a - b*Log[1 - c/x])^2
)/8 + (b^2*c*x*Log[1 + c/x])/4 + (a*b*x^2*Log[1 + c/x])/2 - (b^2*x^2*Log[1
- c/x]*Log[1 + c/x])/4 + (b^2*c^2*Log[c - x])/4 + (b^2*c^2*Log[1 + c/x]*Log
[c - x])/4 + (a*b*c^2*Log[x])/2 + (b^2*c^2*Log[x])/2 + (b^2*c^2*Log[c - x]*
Log[x/c])/4 - (a*b*c^2*Log[c + x])/2 + (b^2*c^2*Log[c + x])/4 + (b^2*c^2*Lo
g[1 - c/x]*Log[c + x])/4 - (b^2*c^2*Log[(c - x)/(2*c)]*Log[c + x])/4 + (b^2
*c^2*Log[-(x/c)]*Log[c + x])/4 - (b^2*c^2*Log[c - x]*Log[(c + x)/(2*c)])/4
+ (b^2*c^2*Log[(c + x)/x])/4 + (b^2*c*x*Log[(c + x)/x])/4 - (b^2*c^2*Log[(c
 + x)/x]^2)/8 + (b^2*x^2*Log[(c + x)/x]^2)/8 - (b^2*c^2*PolyLog[2, (c - x)/
(2*c)])/4 - (b^2*c^2*PolyLog[2, -(c/x)])/4 - (b^2*c^2*PolyLog[2, c/x])/4 -
(b^2*c^2*PolyLog[2, (c + x)/(2*c)])/4 + (b^2*c^2*PolyLog[2, 1 - x/c])/4 + (
b^2*c^2*PolyLog[2, 1 + x/c])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
```

```
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(\frac{1}{4}x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}bx\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)+\frac{1}{4}b^2x\log^2\right.$$

$$=\frac{1}{4}\int x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2 dx+\frac{1}{2}b\int x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)dx+\frac{1}{4}$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^2}{x^3}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(2ax\log\left(1+\frac{c}{x}\right)-bx\log\right.$$

$$=\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{8}b^2x^2\log^2\left(\frac{c+x}{x}\right)+(ab)\int x\log\left(1+\frac{c}{x}\right)dx-\frac{1}{2}b^2$$

$$=\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)+$$

$$=\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)+$$

$$=\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\right.$$

$$=\frac{1}{2}abcx+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{8}x^2\left(2a\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\right.$$

Mathematica [A] time = 0.05, size = 92, normalized size = 1.11

$$\frac{1}{2}\left(a^2x^2+bc^2(a+b)\log(x-c)-abc^2\log(c+x)+2abcx+2bx\tanh^{-1}\left(\frac{c}{x}\right)(ax+bc)+b^2\left(x^2-c^2\right)\tanh^{-1}\left(\frac{c}{x}\right)^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (2*a*b*c*x + a^2*x^2 + 2*b*x*(b*c + a*x)*ArcTanh[c/x] + b^2*(-c^2 + x^2)*Ar
cTanh[c/x]^2 + b*(a + b)*c^2*Log[-c + x] - a*b*c^2*Log[c + x] + b^2*c^2*Log
[c + x])/2
```

fricas [A] time = 0.64, size = 111, normalized size = 1.34

$$abcx+\frac{1}{2}a^2x^2-\frac{1}{2}\left(ab-b^2\right)c^2\log(c+x)+\frac{1}{2}\left(ab+b^2\right)c^2\log(-c+x)-\frac{1}{8}\left(b^2c^2-b^2x^2\right)\log\left(-\frac{c+x}{c-x}\right)^2+\frac{1}{2}\left(b^2cx+abx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

```
[Out] a*b*c*x + 1/2*a^2*x^2 - 1/2*(a*b - b^2)*c^2*log(c + x) + 1/2*(a*b + b^2)*c^
2*log(-c + x) - 1/8*(b^2*c^2 - b^2*x^2)*log(-(c + x)/(c - x))^2 + 1/2*(b^2*
c*x + a*b*x^2)*log(-(c + x)/(c - x))
```

giac [B] time = 0.58, size = 268, normalized size = 3.23

$$-\frac{2\,b^2c^3\log\left(-\frac{c+x}{c-x}-1\right)-2\,b^2c^3\log\left(-\frac{c+x}{c-x}\right)+\frac{b^2(c+x)c^3\log\left(-\frac{c+x}{c-x}\right)^2}{(c-x)\left(\frac{(c+x)^2}{(c-x)^2}+\frac{2\,(c+x)}{c-x}+1\right)}+\frac{2\left(b^2c^3+\frac{2\,ab(c+x)c^3}{c-x}+\frac{b^2(c+x)c^3}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^2}{(c-x)^2}+\frac{2\,(c+x)}{c-x}+1}+\frac{4\left(abc^3+\frac{a^2}{}\right.}{\frac{(c+x)^2}{(c-x)^2}}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*c^3*log(-(c + x)/(c - x) - 1) - 2*b^2*c^3*log(-(c + x)/(c - x))
 + b^2*(c + x)*c^3*log(-(c + x)/(c - x))^2/((c - x)*((c + x)^2/(c - x)^2 +
2*(c + x)/(c - x) + 1)) + 2*(b^2*c^3 + 2*a*b*(c + x)*c^3/(c - x) + b^2*(c +
 x)*c^3/(c - x))*log(-(c + x)/(c - x))/((c + x)^2/(c - x)^2 + 2*(c + x)/(c
- x) + 1) + 4*(a*b*c^3 + a^2*(c + x)*c^3/(c - x) + a*b*(c + x)*c^3/(c - x))
/((c + x)^2/(c - x)^2 + 2*(c + x)/(c - x) + 1))/c
```

maple [B] time = 0.06, size = 287, normalized size = 3.46

$$\frac{a^2x^2}{2}+\frac{b^2x^2\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{2}+c\,b^2\operatorname{arctanh}\left(\frac{c}{x}\right)x+\frac{c^2b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{2}-\frac{c^2b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}+\frac{c^2b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x))^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*b^2*x^2*arctanh(c/x)^2+c*b^2*arctanh(c/x)*x+1/2*c^2*b^2*arc
tanh(c/x)*ln(c/x-1)-1/2*c^2*b^2*arctanh(c/x)*ln(1+c/x)+1/8*c^2*b^2*ln(c/x-1
)^2-1/4*c^2*b^2*ln(c/x-1)*ln(1/2+1/2*c/x)-c^2*b^2*ln(c/x)+1/2*c^2*b^2*ln(c/
x-1)+1/2*c^2*b^2*ln(1+c/x)+1/8*c^2*b^2*ln(1+c/x)^2-1/4*c^2*b^2*ln(-1/2*c/x+
1/2)*ln(1+c/x)+1/4*c^2*b^2*ln(-1/2*c/x+1/2)*ln(1/2+1/2*c/x)+a*b*x^2*arctanh
(c/x)+a*b*c*x+1/2*c^2*a*b*ln(c/x-1)-1/2*c^2*a*b*ln(1+c/x)
```

maxima [A] time = 0.33, size = 136, normalized size = 1.64

$$\frac{1}{2}\,b^2x^2\operatorname{artanh}\left(\frac{c}{x}\right)^2+\frac{1}{2}\,a^2x^2+\frac{1}{2}\left(2\,x^2\operatorname{artanh}\left(\frac{c}{x}\right)-\left(c\log\left(c+x\right)-c\log\left(-c+x\right)-2\,x\right)c\right)ab+\frac{1}{8}\left(\left(\log\left(c+x\right)^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctanh(c/x)^2 + 1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c/x) - (c*log
(c + x) - c*log(-c + x) - 2*x)*c)*a*b + 1/8*((log(c + x)^2 - 2*(log(c + x)
- 2)*log(-c + x) + log(-c + x)^2 + 4*log(c + x))*c^2 - 4*(c*log(c + x) - c*
log(-c + x) - 2*x)*c*arctanh(c/x))*b^2
```

mupad [B] time = 0.78, size = 101, normalized size = 1.22

$$\frac{a^2\,x^2}{2}-\frac{b^2\,c^2\operatorname{atanh}\left(\frac{c}{x}\right)^2}{2}+\frac{b^2\,x^2\operatorname{atanh}\left(\frac{c}{x}\right)^2}{2}+\frac{b^2\,c^2\,\ln\left(x^2-c^2\right)}{2}-a\,b\,c^2\operatorname{atanh}\left(\frac{c}{x}\right)+a\,b\,x^2\operatorname{atanh}\left(\frac{c}{x}\right)+b^2\,c\,x\operatorname{atanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c/x))^2,x)
```

```
[Out] (a^2*x^2)/2 - (b^2*c^2*atanh(c/x)^2)/2 + (b^2*x^2*atanh(c/x)^2)/2 + (b^2*c^
2*log(x^2 - c^2))/2 - a*b*c^2*atanh(c/x) + a*b*x^2*atanh(c/x) + b^2*c*x*ata
nh(c/x) + a*b*c*x
```

sympy [A] time = 0.52, size = 104, normalized size = 1.25

$$\frac{a^2x^2}{2} - abc^2 \operatorname{atanh}\left(\frac{c}{x}\right) + abcx + abx^2 \operatorname{atanh}\left(\frac{c}{x}\right) + b^2c^2 \log\left(-c + x\right) - \frac{b^2c^2 \operatorname{atanh}^2\left(\frac{c}{x}\right)}{2} + b^2c^2 \operatorname{atanh}\left(\frac{c}{x}\right) + b^2cx \operatorname{atanh}\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x))**2,x)
```

```
[Out] a**2*x**2/2 - a*b*c**2*atanh(c/x) + a*b*c*x + a*b*x**2*atanh(c/x) + b**2*c*
*2*log(-c + x) - b**2*c**2*atanh(c/x)**2/2 + b**2*c**2*atanh(c/x) + b**2*c*
x*atanh(c/x) + b**2*x**2*atanh(c/x)**2/2
```

3.146 $\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx$

Optimal. Leaf size=74

$$c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-2bc\log\left(\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+b^2(-c)\text{Li}_2\left(-\frac{c+x}{c-x}\right)$$

```
[Out] c*(a+b*arccoth(x/c))^2+x*(a+b*arccoth(x/c))^2-2*b*c*(a+b*arccoth(x/c))*ln(2
*c/(c-x))-b^2*c*polylog(2,(-c-x)/(c-x))
```

Rubi [B] time = 0.40, antiderivative size = 370, normalized size of antiderivative = 5.00, number of steps used = 31, number of rules used = 14, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {6093, 2448, 263, 31, 2449, 2391, 2556, 12, 2462, 260, 2416, 2394, 2315, 2393}

$$-\frac{1}{2}b^2c\text{PolyLog}\left(2,\frac{c-x}{2c}\right)+\frac{1}{2}b^2c\text{PolyLog}\left(2,-\frac{c}{x}\right)-\frac{1}{2}b^2c\text{PolyLog}\left(2,\frac{c}{x}\right)+\frac{1}{2}b^2c\text{PolyLog}\left(2,\frac{c+x}{2c}\right)+\frac{1}{2}b^2c\text{PolyI}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] a^2*x - a*b*x*Log[1 - c/x] - (b^2*(c - x)*Log[1 - c/x]^2)/4 + a*b*x*Log[1 +
 c/x] - (b^2*x*Log[1 - c/x]*Log[1 + c/x])/2 + (b^2*(c + x)*Log[1 + c/x]^2)/
4 - (b^2*c*Log[1 - c/x]*Log[-c - x])/2 + a*b*c*Log[c - x] + (b^2*c*Log[-c -
 x]*Log[(c - x)/(2*c)])/2 - (b^2*c*Log[-c - x]*Log[-(x/c)])/2 + (b^2*c*Log[
1 + c/x]*Log[-c + x])/2 + (b^2*c*Log[x/c]*Log[-c + x])/2 + a*b*c*Log[c + x]
 - (b^2*c*Log[-c + x]*Log[(c + x)/(2*c)])/2 - (b^2*c*PolyLog[2, (c - x)/(2*
c)])/2 + (b^2*c*PolyLog[2, -(c/x)])/2 - (b^2*c*PolyLog[2, c/x])/2 + (b^2*c*
PolyLog[2, (c + x)/(2*c)])/2 + (b^2*c*PolyLog[2, 1 - x/c])/2 - (b^2*c*PolyL
og[2, 1 + x/c])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 6093

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Int[Expand
Integrand[(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x] /; Fr
eeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]
```

Rubi steps

$$\int \left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(a^2 - ab\log\left(1 - \frac{c}{x}\right) + \frac{1}{4}b^2\log^2\left(1 - \frac{c}{x}\right) + ab\log\left(1 + \frac{c}{x}\right) - \frac{1}{2}b^2\log\left(1 - \frac{c}{x}\right)\mathrm{lo}\right.$$
$$= a^2x - (ab)\int \log\left(1 - \frac{c}{x}\right)dx + (ab)\int \log\left(1 + \frac{c}{x}\right)dx + \frac{1}{4}b^2\int \log^2\left(1 - \frac{c}{x}\right)dx -$$
$$= a^2x - abx\log\left(1 - \frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1 - \frac{c}{x}\right) + abx\log\left(1 + \frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1\right.$$
$$= a^2x - abx\log\left(1 - \frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1 - \frac{c}{x}\right) + abx\log\left(1 + \frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1\right.$$
$$= a^2x - abx\log\left(1 - \frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1 - \frac{c}{x}\right) + abx\log\left(1 + \frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1\right.$$
$$= a^2x - abx\log\left(1 - \frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1 - \frac{c}{x}\right) + abx\log\left(1 + \frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1\right.$$
$$= a^2x - abx\log\left(1 - \frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1 - \frac{c}{x}\right) + abx\log\left(1 + \frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1\right.$$
$$= a^2x - abx\log\left(1 - \frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1 - \frac{c}{x}\right) + abx\log\left(1 + \frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1\right.$$
$$= a^2x - abx\log\left(1 - \frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1 - \frac{c}{x}\right) + abx\log\left(1 + \frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1\right.$$
$$= a^2x - abx\log\left(1 - \frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1 - \frac{c}{x}\right) + abx\log\left(1 + \frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1\right.$$

Mathematica [A] time = 0.13, size = 97, normalized size = 1.31

$$a\left(ax + bc\log\left(1 - \frac{c^2}{x^2}\right) - 2bc\log\left(\frac{c}{x}\right)\right) + 2b\tanh^{-1}\left(\frac{c}{x}\right)\left(ax - bc\log\left(1 - e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right) + b^2c\mathrm{Li}_2\left(e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right) + b$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] b^2*(-c + x)*ArcTanh[c/x]^2 + 2*b*ArcTanh[c/x]*(a*x - b*c*Log[1 - E^(-2*Arc
Tanh[c/x])]) + a*(a*x + b*c*Log[1 - c^2/x^2] - 2*b*c*Log[c/x]) + b^2*c*Poly
Log[2, E^(-2*ArcTanh[c/x])]
```

fricas [F] time = 1.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^2\operatorname{artanh}\left(\frac{c}{x}\right)^2 + 2\,ab\operatorname{artanh}\left(\frac{c}{x}\right) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\operatorname{artanh}\left(\frac{c}{x}\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2, x)
```

maple [B] time = 0.06, size = 282, normalized size = 3.81

$$a^2x+b^2x\,\mathrm{arctanh}\left(\frac{c}{x}\right)^2-2c\,b^2\ln\left(\frac{c}{x}\right)\mathrm{arctanh}\left(\frac{c}{x}\right)+c\,b^2\,\mathrm{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)+c\,b^2\,\mathrm{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)+\frac{c\,b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^2,x)
```

```
[Out] a^2*x+b^2*x*arctanh(c/x)^2-2*c*b^2*ln(c/x)*arctanh(c/x)+c*b^2*arctanh(c/x)*
ln(c/x-1)+c*b^2*arctanh(c/x)*ln(1+c/x)+1/4*c*b^2*ln(c/x-1)^2-c*b^2*dilog(1/
2+1/2*c/x)-1/2*c*b^2*ln(c/x-1)*ln(1/2+1/2*c/x)-1/4*c*b^2*ln(1+c/x)^2+1/2*c*
b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-1/2*c*b^2*ln(-1/2*c/x+1/2)*ln(1/2+1/2*c/x)+c
*b^2*dilog(c/x)+c*b^2*dilog(1+c/x)+c*b^2*ln(c/x)*ln(1+c/x)+2*a*b*x*arctanh(
c/x)-2*c*a*b*ln(c/x)+c*a*b*ln(c/x-1)+c*a*b*ln(1+c/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(2\,x\,\mathrm{artanh}\left(\frac{c}{x}\right)+c\log\left(-c^2+x^2\right)\right)ab+\frac{1}{4}\left(x\log\left(c+x\right)^2-2\left(c+x\right)\log\left(c+x\right)\log\left(-c+x\right)-\left(c-x\right)\log\left(-c+x\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

```
[Out] (2*x*arctanh(c/x) + c*log(-c^2 + x^2))*a*b + 1/4*(x*log(c + x)^2 - 2*(c + x
)*log(c + x)*log(-c + x) - (c - x)*log(-c + x)^2 + integrate(-2*(c^2 + 3*c*
x)*log(c + x)/(c^2 - x^2), x))*b^2 + a^2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^2,x)
```

```
[Out] int((a + b*atanh(c/x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**2,x)
```

```
[Out] Integral((a + b*atanh(c/x))**2, x)
```

3.147 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=133

$$b\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-b\text{Li}_2\left(\frac{2}{1-\frac{c}{x}}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\right.\right.$$

```
[Out] 2*(a+b*arccoth(x/c))^2*arctanh(-1+2/(1-c/x))+b*(a+b*arccoth(x/c))*polylog(2
,1-2/(1-c/x))-b*(a+b*arccoth(x/c))*polylog(2,-1+2/(1-c/x))-1/2*b^2*polylog(
3,1-2/(1-c/x))+1/2*b^2*polylog(3,-1+2/(1-c/x))
```

Rubi [A] time = 0.31, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$b\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-b\text{PolyLog}\left(2,\frac{2}{1-\frac{c}{x}}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{2}b^2\text{PolyLog}\left(3,1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcCoth[x/c])^2*ArcTanh[1 - 2/(1 - c/x)] + b*(a + b*ArcCoth[x/c])
*PolyLog[2, 1 - 2/(1 - c/x)] - b*(a + b*ArcCoth[x/c])*PolyLog[2, -1 + 2/(1
- c/x)] - (b^2*PolyLog[3, 1 - 2/(1 - c/x)])/2 + (b^2*PolyLog[3, -1 + 2/(1 -
 c/x)])/2
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x}\,dx = -\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2 \tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) + (4bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\tan}{1-c^2x^2}\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2 \tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) - (2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log}{1-c^2x^2}\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2 \tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) + b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right) -$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2 \tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right) + b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right) -$$

Mathematica [A] time = 0.09, size = 114, normalized size = 0.86

$$\frac{1}{2}b\left(2\text{Li}_2\left(\frac{c+x}{c-x}\right)\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right) - 2\text{Li}_2\left(\frac{c+x}{x-c}\right)\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right) + b\left(\text{Li}_3\left(\frac{c+x}{x-c}\right) - \text{Li}_3\left(\frac{c+x}{c-x}\right)\right)\right) - 2\tanh$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcTanh[c/x])^2*ArcTanh[(c + x)/(c - x)] + (b*(2*(a + b*ArcTanh[c
/x])*PolyLog[2, (c + x)/(c - x)] - 2*(a + b*ArcTanh[c/x])*PolyLog[2, (c + x
)/(-c + x)] + b*(-PolyLog[3, (c + x)/(c - x)] + PolyLog[3, (c + x)/(-c + x)
])))/2
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(\frac{c}{x}\right)^2 + 2\,ab\,\text{artanh}\left(\frac{c}{x}\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right) + a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2/x, x)
```

maple [C] time = 0.20, size = 780, normalized size = 5.86

$$-a^2\ln\left(\frac{c}{x}\right)-b^2\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)^2+b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)-\frac{b^2\operatorname{polylog}\left(3,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)}{2}+b^2\operatorname{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^2/x,x)
```

```
[Out] -a^2*ln(c/x)-b^2*ln(c/x)*arctanh(c/x)^2+b^2*arctanh(c/x)*polylog(2,-(1+c/x)
^2/(1-c^2/x^2))-1/2*b^2*polylog(3,-(1+c/x)^2/(1-c^2/x^2))+b^2*arctanh(c/x)^
2*ln((1+c/x)^2/(1-c^2/x^2)-1)-b^2*arctanh(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(
1/2))-2*b^2*arctanh(c/x)*polylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))+2*b^2*polylog
(3,(1+c/x)/(1-c^2/x^2)^(1/2))-b^2*arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(
1/2))-2*b^2*arctanh(c/x)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))+2*b^2*polylo
g(3,-(1+c/x)/(1-c^2/x^2)^(1/2))-1/2*I*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-
1))*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(
1+c/x)^2/(1-c^2/x^2)))*arctanh(c/x)^2+1/2*I*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2
/x^2)-1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arc
tanh(c/x)^2+1/2*I*b^2*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*((1+c/x)^
2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arctanh(c/x)^2-1/2*I*b^2*Pi*c
sgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^3*arctanh(c/x)^2
-2*a*b*ln(c/x)*arctanh(c/x)+a*b*ln(c/x)*ln(1+c/x)+a*b*dilog(c/x)+a*b*dilog(
1+c/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\log(x)+\int\frac{b^2\left(\log\left(\frac{c}{x}+1\right)-\log\left(-\frac{c}{x}+1\right)\right)^2}{4x}+\frac{ab\left(\log\left(\frac{c}{x}+1\right)-\log\left(-\frac{c}{x}+1\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c/x + 1) - log(-c/x + 1))^2/x + a*b*(lo
g(c/x + 1) - log(-c/x + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^2/x,x)
```

```
[Out] int((a + b*atanh(c/x))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c/x))**2/x, x)
```

3.148 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=87

$$-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{x}+\frac{2b\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{c}+\frac{b^2\mathrm{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)}{c}$$

```
[Out] -(a+b*arccoth(x/c))^2/c-(a+b*arccoth(x/c))^2/x+2*b*(a+b*arccoth(x/c))*ln(2/
(1-c/x))/c+b^2*polylog(2,1-2/(1-c/x))/c
```

Rubi [B] time = 0.51, antiderivative size = 205, normalized size of antiderivative = 2.36, number of steps used = 28, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,-\frac{c-x}{2x}\right)}{2c}-\frac{b^2\text{PolyLog}\left(2,\frac{c+x}{2x}\right)}{2c}-\frac{b\log\left(\frac{c+x}{2x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)}{2c}-\frac{b\log\left(\frac{c+x}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)}{2x}+$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c/x])^2/x^2,x]
```

```
[Out] ((1 - c/x)*(2*a - b*Log[1 - c/x])^2)/(4*c) - (b*(2*a - b*Log[1 - c/x])*Log[
(c + x)/(2*x)])/(2*c) - (b*(2*a - b*Log[1 - c/x])*Log[(c + x)/x])/(2*x) - (
b^2*Log[-(c - x)/(2*x)]*Log[(c + x)/x])/(2*c) - (b^2*(1 + c/x)*Log[(c + x)/
x]^2)/(4*c) + (b^2*PolyLog[2, -(c - x)/(2*x)])/(2*c) - (b^2*PolyLog[2, (c +
 x)/(2*x)])/(2*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4x^2}+\frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)}{2x^2}+\frac{b^2\log^2\left(1+\frac{c}{x}\right)}{4x^2}\right)$$

$$=\frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{x^2}\,dx+\frac{1}{2}b\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)}{x^2}\,dx+\frac{1}{4}b^2$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int (2a-b\log(1-cx))^2\,dx,x,\frac{1}{x}\right)\right)-\frac{1}{2}b\,\text{Subst}\left(\int (2a-b\log(1-cx))\right.$$

$$=-\frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(\frac{c+x}{x}\right)}{2x}+\frac{\text{Subst}\left(\int (2a-b\log(x))^2\,dx,x,1-\frac{c}{x}\right)}{4c}-\frac{b^2\,\text{Su}}{}$$

$$=\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c}-\frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(\frac{c+x}{x}\right)}{2x}-\frac{b^2\left(1+\frac{c}{x}\right)\log^2}{4c}$$

$$=-\frac{ab}{x}-\frac{b^2}{2x}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c}+\frac{b^2\left(1+\frac{c}{x}\right)\log\left(\frac{c+x}{x}\right)}{2c}-\frac{b\left(2a-b\log\right.}{}$$

$$=-\frac{b^2}{x}-\frac{b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{2c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c}-\frac{b\left(2a-b\log\left(1-\right.\right.}{2c}$$

$$=-\frac{b^2}{2x}-\frac{b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{2c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c}-\frac{b\left(2a-b\log\left(1-\right.\right.}{2c}$$

$$=\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c}-\frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(\frac{c+x}{2x}\right)}{2c}-\frac{b\left(2a-b\log\left(1\right.\right.}{2}$$

Mathematica [A] time = 0.19, size = 101, normalized size = 1.16

$$\frac{a\left(2bx\log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^2}}}\right)-ac\right)+2b\tanh^{-1}\left(\frac{c}{x}\right)\left(bx\log\left(e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}+1\right)-ac\right)-b^2x\text{Li}_2\left(-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)+b^2(x-c)}{cx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^2/x^2,x]
```

```
[Out] (b^2*(-c + x)*ArcTanh[c/x]^2 + 2*b*ArcTanh[c/x]*(-(a*c) + b*x*Log[1 + E^(-2
*ArcTanh[c/x])]) + a*(-(a*c) + 2*b*x*Log[1/Sqrt[1 - c^2/x^2]]) - b^2*x*Poly
Log[2, -E^(-2*ArcTanh[c/x])])/(c*x)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(\frac{c}{x}\right)^2+2\,ab\,\text{artanh}\left(\frac{c}{x}\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2/x^2, x)
```

maple [A] time = 0.18, size = 144, normalized size = 1.66

$$-\frac{a^2}{x}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{x}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{c}+\frac{2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)b^2}{c}+\frac{\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)b^2}{c}-\frac{2ab\operatorname{arctanh}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^2/x^2,x)
```

```
[Out] -a^2/x-1/x*b^2*arctanh(c/x)^2-1/c*b^2*arctanh(c/x)^2+2/c*arctanh(c/x)*ln(1+
(1+c/x)^2/(1-c^2/x^2))*b^2+1/c*polylog(2,-(1+c/x)^2/(1-c^2/x^2))*b^2-2/x*a*
b*arctanh(c/x)-1/c*a*b*ln(1-c^2/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(c^3\int-\frac{\log(x)^2}{c^3x^2-cx^4}\,dx+c^2\left(\frac{\log\left(-c^2+x^2\right)}{c^3}-\frac{\log\left(x^2\right)}{c^3}\right)-4\,c^2\int-\frac{x\log\left(c+x\right)}{c^3x^2-cx^4}\,dx+2\,c^2\int-\frac{x\log(x)}{c^3x^2-cx^4}\,dx+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*integrate(-log(x)^2/(c^3*x^2 - c*x^4), x) + c^2*(log(-c^2 + x^2)/c
^3 - log(x^2)/c^3) - 4*c^2*integrate(-x*log(c + x)/(c^3*x^2 - c*x^4), x) +
2*c^2*integrate(-x*log(x)/(c^3*x^2 - c*x^4), x) + 2*c*(log(-c + x)/c^2 - lo
g(x)/c^2 + 1/(c*x))*log(-c/x + 1) - c*(log(c + x)/c^2 - log(-c + x)/c^2) -
c*integrate(-x^2*log(x)^2/(c^3*x^2 - c*x^4), x) - 2*c*integrate(-x^2*log(c
+ x)/(c^3*x^2 - c*x^4), x) + 4*c*integrate(-x^2*log(x)/(c^3*x^2 - c*x^4), x
) - log(-c/x + 1)^2/x - (c*log(c + x)^2 - 2*((c + x)*log(c + x) - (c + x)*l
og(x) - c)*log(-c + x))/(c*x) - (x*log(-c + x)^2 + x*log(x)^2 - 2*(x*log(x)
 - x)*log(-c + x) - 2*x*log(x) + 2*c)/(c*x) - 2*integrate(-x^3*log(c + x)/(
c^3*x^2 - c*x^4), x) + 2*integrate(-x^3*log(x)/(c^3*x^2 - c*x^4), x))*b^2 -
 a*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^2/x^2,x)
```

```
[Out] int((a + b*atanh(c/x))^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**2/x**2,x)
```

```
[Out] Integral((a + b*atanh(c/x))**2/x**2, x)
```

3.149 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=87

$$\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}-\frac{ab}{cx}-\frac{b^2\log\left(1-\frac{c^2}{x^2}\right)}{2c^2}-\frac{b^2\coth^{-1}\left(\frac{x}{c}\right)}{cx}$$

```
[Out] -a*b/c/x-b^2*arccoth(x/c)/c/x+1/2*(a+b*arccoth(x/c))^2/c^2-1/2*(a+b*arccoth
(x/c))^2/x^2-1/2*b^2*ln(1-c^2/x^2)/c^2
```

Rubi [C] time = 1.25, antiderivative size = 707, normalized size of antiderivative = 8.13, number of steps used = 66, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 6742, 30, 2557, 12, 2466, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,\frac{c-x}{2c}\right)}{4c^2}+\frac{b^2\text{PolyLog}\left(2,-\frac{c}{x}\right)}{4c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{c}{x}\right)}{4c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{c+x}{2c}\right)}{4c^2}-\frac{b^2\text{PolyLog}\left(2,1-\frac{x}{c}\right)}{4c^2}-\frac{b^2\text{P}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c/x])^2/x^3,x]
```

```
[Out] -(b^2*(1 - c/x)^2)/(16*c^2) - (b^2*(1 + c/x)^2)/(16*c^2) + (a*b)/(4*x^2) +
b^2/(8*x^2) - (3*a*b)/(2*c*x) + (b^2*Log[1 - c/x])/(8*c^2) - (3*b^2*(1 - c/
x)*Log[1 - c/x])/(4*c^2) - (b^2*Log[1 - c/x])/(8*x^2) - (b*(1 - c/x)^2*(2*a
 - b*Log[1 - c/x]))/(8*c^2) + ((1 - c/x)*(2*a - b*Log[1 - c/x])^2)/(4*c^2)
- ((1 - c/x)^2*(2*a - b*Log[1 - c/x])^2)/(8*c^2) + (b^2*Log[1 - c/x]*Log[1
+ c/x])/(4*x^2) - (b^2*Log[1 + c/x]*Log[c - x])/(4*c^2) - (b^2*Log[c - x]*L
og[x/c])/(4*c^2) - (b^2*Log[1 - c/x]*Log[c + x])/(4*c^2) + (b^2*Log[(c - x)
/(2*c)]*Log[c + x])/(4*c^2) - (b^2*Log[-(x/c)]*Log[c + x])/(4*c^2) + (b^2*L
og[c - x]*Log[(c + x)/(2*c)])/(4*c^2) + (a*b*Log[(c + x)/x])/(2*c^2) + (b^2
*Log[(c + x)/x])/(8*c^2) - (3*b^2*(1 + c/x)*Log[(c + x)/x])/(4*c^2) + (b^2*
(1 + c/x)^2*Log[(c + x)/x])/(8*c^2) - (a*b*Log[(c + x)/x])/(2*x^2) - (b^2*L
og[(c + x)/x])/(8*x^2) + (b^2*(1 + c/x)*Log[(c + x)/x]^2)/(4*c^2) - (b^2*(1
 + c/x)^2*Log[(c + x)/x]^2)/(8*c^2) + (b^2*PolyLog[2, (c - x)/(2*c)])/(4*c^
2) + (b^2*PolyLog[2, -(c/x)])/(4*c^2) + (b^2*PolyLog[2, c/x])/(4*c^2) + (b^
2*PolyLog[2, (c + x)/(2*c)])/(4*c^2) - (b^2*PolyLog[2, 1 - x/c])/(4*c^2) -
(b^2*PolyLog[2, 1 + x/c])/(4*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
```

```
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4x^3}+\frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)}{2x^3}+\frac{b^2\log^2\left(1+\frac{c}{x}\right)}{4x^3}\right)dx$$

$$=\frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{x^3}\,dx+\frac{1}{2}b\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)}{x^3}\,dx+\frac{1}{4}b^2\int \frac{\mathrm{l}}{-}$$

$$=-\left(\frac{1}{4}\mathrm{Subst}\left(\int x(2a-b\log(1-cx))^2\,dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(\frac{2a\log\left(1+\frac{c}{x}\right)}{x^3}-\frac{b\log\left(1-\right.}{}\right.$$

$$=-\left(\frac{1}{4}\mathrm{Subst}\left(\int\left(\frac{(2a-b\log(1-cx))^2}{c}-\frac{(1-cx)(2a-b\log(1-cx))^2}{c}\right)dx,x,\frac{1}{x}\right)\right)+($$

$$=\frac{b^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2}-(ab)\,\mathrm{Subst}\left(\int x\log(1+cx)\,dx,x,\frac{1}{x}\right)+\frac{1}{2}b^2\int\frac{c\log\left(1-\right.}{2x^3(c+}$$

$$=\frac{b^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2}-\frac{ab\log\left(\frac{c+x}{x}\right)}{2x^2}+\frac{\mathrm{Subst}\left(\int(2a-b\log(x))^2\,dx,x,1-\frac{c}{x}\right)}{4c^2}-$$

$$=\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c^2}-\frac{\left(1-\frac{c}{x}\right)^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8c^2}+\frac{b^2\log\left(1-\frac{c}{x}\right)\log\left(1\right.}{4x^2}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}-\frac{3ab}{2cx}+\frac{b^2}{2cx}-\frac{b\left(1-\frac{c}{x}\right)^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)}{8c^2}+($$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}-\frac{3ab}{2cx}-\frac{b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{2c^2}-\frac{b\left(1-\frac{c}{x}\right)^2\left(2a-\right.}{8c^2}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}-\frac{3ab}{2cx}-\frac{b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{2c^2}-\frac{b^2\log\left(1-\frac{c}{x}\right)}{8x^2}-$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}-\frac{3ab}{2cx}-\frac{3b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{4c^2}-\frac{b^2\log\left(1-\frac{c}{x}\right)}{8x^2}-$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}+\frac{b^2}{8x^2}-\frac{3ab}{2cx}+\frac{b^2\log\left(1-\frac{c}{x}\right)}{8c^2}-\frac{3b^2\left(1-\frac{c}{x}\right)\log\left(1\right.}{4c^2}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}+\frac{b^2}{8x^2}-\frac{3ab}{2cx}+\frac{b^2\log\left(1-\frac{c}{x}\right)}{8c^2}-\frac{3b^2\left(1-\frac{c}{x}\right)\log\left(1\right.}{4c^2}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}+\frac{b^2}{8x^2}-\frac{3ab}{2cx}+\frac{b^2\log\left(1-\frac{c}{x}\right)}{8c^2}-\frac{3b^2\left(1-\frac{c}{x}\right)\log\left(1\right.}{4c^2}$$

Mathematica [A] time = 0.07, size = 119, normalized size = 1.37

$$-\frac{a^2c^2+abx^2\log(x-c)-abx^2\log(c+x)+2abcx+2bc\tanh^{-1}\left(\frac{c}{x}\right)(ac+bx)+b^2\left(c^2-x^2\right)\tanh^{-1}\left(\frac{c}{x}\right)^2+b^2x^2\mathrm{lo}}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])^2/x^3,x]
```

```
[Out] -1/2*(a^2*c^2 + 2*a*b*c*x + 2*b*c*(a*c + b*x)*ArcTanh[c/x] + b^2*(c^2 - x^2
)*ArcTanh[c/x]^2 - 2*b^2*x^2*Log[x] + a*b*x^2*Log[-c + x] + b^2*x^2*Log[-c
+ x] - a*b*x^2*Log[c + x] + b^2*x^2*Log[c + x])/(c^2*x^2)
```

fricas [A] time = 0.70, size = 130, normalized size = 1.49

$$\frac{8\,b^2x^2\log(x) - 4\,a^2c^2 - 8\,abcx + 4\left(ab - b^2\right)x^2\log\left(c + x\right) - 4\left(ab + b^2\right)x^2\log\left(-c + x\right) - \left(b^2c^2 - b^2x^2\right)\log\left(-\frac{c}{c}\right.}{8\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^2*x^2*log(x) - 4*a^2*c^2 - 8*a*b*c*x + 4*(a*b - b^2)*x^2*log(c + x
) - 4*(a*b + b^2)*x^2*log(-c + x) - (b^2*c^2 - b^2*x^2)*log(-(c + x)/(c - x
))^2 - 4*(a*b*c^2 + b^2*c*x)*log(-(c + x)/(c - x)))/(c^2*x^2)
```

giac [B] time = 0.18, size = 255, normalized size = 2.93

$$-\frac{\frac{b^2(c+x)\log\left(-\frac{c+x}{c-x}\right)^2}{\left(\frac{(c+x)^2c}{(c-x)^2} - \frac{2\,(c+x)c}{c-x} + c\right)(c-x)} - \frac{2\,b^2\log\left(-\frac{c+x}{c-x} + 1\right)}{c} + \frac{2\,b^2\log\left(-\frac{c+x}{c-x}\right)}{c} - \frac{2\left(b^2 - \frac{2\,ab(c+x)}{c-x} - \frac{b^2(c+x)}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^2c}{(c-x)^2} - \frac{2\,(c+x)c}{c-x} + c} - \frac{4\left(ab - \frac{a^2(c+x)}{c-x} - \frac{ab(c+x)}{c-x}\right)}{\frac{(c+x)^2c}{(c-x)^2} - \frac{2\,(c+x)c}{c-x} + c}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*(c + x)*log(-(c + x)/(c - x))^2/(((c + x)^2*c/(c - x)^2 - 2*(c +
x)*c/(c - x) + c)*(c - x)) - 2*b^2*log(-(c + x)/(c - x) + 1)/c + 2*b^2*log(
-(c + x)/(c - x))/c - 2*(b^2 - 2*a*b*(c + x)/(c - x) - b^2*(c + x)/(c - x))
*log(-(c + x)/(c - x))/((c + x)^2*c/(c - x)^2 - 2*(c + x)*c/(c - x) + c) -
4*(a*b - a^2*(c + x)/(c - x) - a*b*(c + x)/(c - x))/((c + x)^2*c/(c - x)^2
- 2*(c + x)*c/(c - x) + c))/c
```

maple [B] time = 0.05, size = 284, normalized size = 3.26

$$-\frac{a^2}{2x^2} - \frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{2x^2} - \frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)}{cx} - \frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x} - 1\right)}{2c^2} + \frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1 + \frac{c}{x}\right)}{2c^2} - \frac{b^2\ln\left(\frac{c}{x} - 1\right)^2}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2-1/2/x^2*b^2*arctanh(c/x)^2-1/c*b^2*arctanh(c/x)/x-1/2/c^2*b^2*
arctanh(c/x)*ln(c/x-1)+1/2/c^2*b^2*arctanh(c/x)*ln(1+c/x)-1/8/c^2*b^2*ln(c/
x-1)^2+1/4/c^2*b^2*ln(c/x-1)*ln(1/2+1/2*c/x)-1/2/c^2*b^2*ln(c/x-1)-1/2/c^2*
b^2*ln(1+c/x)-1/8/c^2*b^2*ln(1+c/x)^2-1/4/c^2*b^2*ln(-1/2*c/x+1/2)*ln(1/2+1
/2*c/x)+1/4/c^2*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-a*b/x^2*arctanh(c/x)-a*b/c/x
-1/2/c^2*a*b*ln(c/x-1)+1/2/c^2*a*b*ln(1+c/x)
```

maxima [B] time = 0.33, size = 165, normalized size = 1.90

$$\frac{1}{2}\left(c\left(\frac{\log\left(c + x\right)}{c^3} - \frac{\log\left(-c + x\right)}{c^3} - \frac{2}{c^2x}\right) - \frac{2\,\operatorname{artanh}\left(\frac{c}{x}\right)}{x^2}\right)ab - \frac{1}{8}\left(c^2\left(\frac{\log\left(c + x\right)^2 - 2\left(\log\left(c + x\right) - 2\right)\log\left(-c + \right.}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2)
*a*b - 1/8*(c^2*((log(c + x)^2 - 2*(log(c + x) - 2)*log(-c + x) + log(-c +
x)^2 + 4*log(c + x))/c^4 - 8*log(x)/c^4) - 4*c*(log(c + x)/c^3 - log(-c + x
)/c^3 - 2/(c^2*x))*arctanh(c/x))*b^2 - 1/2*b^2*arctanh(c/x)^2/x^2 - 1/2*a^2
/x^2
```

mupad [B] time = 1.28, size = 235, normalized size = 2.70

$$\ln\left(1-\frac{c}{x}\right)\left(\frac{a\,b}{2\,x^2}-\ln\left(\frac{c}{x}+1\right)\left(\frac{b^2}{4\,c^2}-\frac{b^2}{4\,x^2}\right)+\frac{b^2\,\left(2\,c\,x-c^2\right)}{8\,c^2\,x^2}+\frac{b^2\,\left(2\,c^2+4\,x\,c\right)}{16\,c^2\,x^2}\right)-\frac{\frac{a^2}{2}+\frac{a\,b\,x}{c}}{x^2}+\ln\left(\frac{c}{x}+1\right)^2\,\left(\frac{b^2}{8\,c^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^2/x^3,x)
```

```
[Out] log(1 - c/x)*((a*b)/(2*x^2) - log(c/x + 1)*(b^2/(4*c^2) - b^2/(4*x^2)) + (b
^2*(2*c*x - c^2))/(8*c^2*x^2) + (b^2*(4*c*x + 2*c^2))/(16*c^2*x^2)) - (a^2/
2 + (a*b*x)/c)/x^2 + log(c/x + 1)^2*(b^2/(8*c^2) - b^2/(8*x^2)) + log(1 - c
/x)^2*(b^2/(8*c^2) - b^2/(8*x^2)) - (log(x - c)*(a*b + b^2))/(2*c^2) + (log
(c + x)*(a*b - b^2))/(2*c^2) - (log(c/x + 1)*((a*b)/2 + (b^2*x)/(2*c)))/x^2
 + (b^2*log(x))/c^2
```

sympy [A] time = 1.17, size = 124, normalized size = 1.43

$$\begin{cases} -\frac{a^2}{2x^2}-\frac{ab\operatorname{atanh}\left(\frac{c}{x}\right)}{x^2}-\frac{ab}{cx}+\frac{ab\operatorname{atanh}\left(\frac{c}{x}\right)}{c^2}-\frac{b^2\operatorname{atanh}^2\left(\frac{c}{x}\right)}{2x^2}-\frac{b^2\operatorname{atanh}\left(\frac{c}{x}\right)}{cx}+\frac{b^2\log\left(x\right)}{c^2}-\frac{b^2\log\left(-c+x\right)}{c^2}+\frac{b^2\operatorname{atanh}^2\left(\frac{c}{x}\right)}{2c^2}-\frac{b^2\operatorname{atanh}\left(\frac{c}{x}\right)}{c^2} \\ -\frac{a^2}{2x^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - a*b*atanh(c/x)/x**2 - a*b/(c*x) + a*b*atanh(c/x
)/c**2 - b**2*atanh(c/x)**2/(2*x**2) - b**2*atanh(c/x)/(c*x) + b**2*log(x)/
c**2 - b**2*log(-c + x)/c**2 + b**2*atanh(c/x)**2/(2*c**2) - b**2*atanh(c/x
)/c**2, Ne(c, 0)), (-a**2/(2*x**2), True))
```

3.150 $\int x^3 \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx$

Optimal. Leaf size=203

$$-2b^2c^4\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{4}b^2c^2x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{4}c^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-bc^4\left(a+b\,\mathrm{c}\right.$$

```
[Out] 1/4*b^3*c^3*x-1/4*b^3*c^4*arccoth(x/c)+1/4*b^2*c^2*x^2*(a+b*arccoth(x/c))-b
*c^4*(a+b*arccoth(x/c))^2+3/4*b*c^3*x*(a+b*arccoth(x/c))^2+1/4*b*c*x^3*(a+b
*arccoth(x/c))^2-1/4*c^4*(a+b*arccoth(x/c))^3+1/4*x^4*(a+b*arccoth(x/c))^3-
2*b^2*c^4*(a+b*arccoth(x/c))*ln(2-2/(1+c/x))+b^3*c^4*polylog(2,-1+2/(1+c/x)
)
```

Rubi [F] time = 4.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (3*a^2*b*c^3*x)/8 - (5*a*b^2*c^3*x)/16 + (b^3*c^3*x)/16 - (3*a^2*b*c^2*x^2)
/16 + (3*a*b^2*c^2*x^2)/16 + (a^2*b*c*x^3)/8 + (b^3*c^4*Log[1 - c/x])/32 -
(3*a*b^2*c^3*x*Log[1 - c/x])/8 + (3*a*b^2*c^2*x^2*Log[1 - c/x])/16 - (a*b^2
*c*x^3*Log[1 - c/x])/8 + (5*b^2*c^3*(1 - c/x)*x*(2*a - b*Log[1 - c/x]))/32
+ (b^2*c^2*x^2*(2*a - b*Log[1 - c/x]))/32 - (5*b*c^4*(2*a - b*Log[1 - c/x])
^2)/64 + (3*b*c^3*(1 - c/x)*x*(2*a - b*Log[1 - c/x])^2)/32 + (3*b*c^2*x^2*(
2*a - b*Log[1 - c/x])^2)/64 + (b*c*x^3*(2*a - b*Log[1 - c/x])^2)/32 - (c^4*
(2*a - b*Log[1 - c/x])^3)/32 + (x^4*(2*a - b*Log[1 - c/x])^3)/32 + (3*a*b^2
*c^3*x*Log[1 + c/x])/8 + (3*a*b^2*c^2*x^2*Log[1 + c/x])/16 + (a*b^2*c*x^3*L
og[1 + c/x])/8 + (3*a^2*b*x^4*Log[1 + c/x])/8 - (3*a*b^2*x^4*Log[1 - c/x]*L
og[1 + c/x])/8 + (5*a*b^2*c^4*Log[c - x])/16 + (3*a*b^2*c^4*Log[1 + c/x]*Lo
g[c - x])/8 - (3*b*c^4*(2*a - b*Log[1 - c/x])^2*Log[c/x])/32 + (11*a*b^2*c^
4*Log[x])/8 + (3*a*b^2*c^4*Log[c - x]*Log[x/c])/8 - (3*a^2*b*c^4*Log[c + x]
)/8 + (5*a*b^2*c^4*Log[c + x])/16 + (3*a*b^2*c^4*Log[1 - c/x]*Log[c + x])/8
 - (3*a*b^2*c^4*Log[(c - x)/(2*c)]*Log[c + x])/8 + (3*a*b^2*c^4*Log[-(x/c)]
*Log[c + x])/8 - (3*a*b^2*c^4*Log[c - x]*Log[(c + x)/(2*c)])/8 + (11*a*b^2*
c^4*Log[(c + x)/x])/16 - (b^3*c^4*Log[(c + x)/x])/32 + (3*a*b^2*c^3*x*Log[(
c + x)/x])/8 - (5*b^3*c^3*(1 + c/x)*x*Log[(c + x)/x])/32 - (3*a*b^2*c^2*x^2
*Log[(c + x)/x])/16 + (b^3*c^2*x^2*Log[(c + x)/x])/32 + (a*b^2*c*x^3*Log[(c
 + x)/x])/8 - (3*a*b^2*c^4*Log[(c + x)/x]^2)/16 + (5*b^3*c^4*Log[(c + x)/x]
^2)/64 + (3*b^3*c^3*(1 + c/x)*x*Log[(c + x)/x]^2)/32 - (3*b^3*c^2*x^2*Log[(
c + x)/x]^2)/64 + (b^3*c*x^3*Log[(c + x)/x]^2)/32 + (3*a*b^2*x^4*Log[(c + x
)/x]^2)/16 + (3*b^3*c^4*Log[-(c/x)]*Log[(c + x)/x]^2)/32 - (b^3*c^4*Log[(c
+ x)/x]^3)/32 + (b^3*x^4*Log[(c + x)/x]^3)/32 + (3*b^2*c^4*(2*a - b*Log[1 -
 c/x])*PolyLog[2, 1 - c/x])/16 - (3*a*b^2*c^4*PolyLog[2, (c - x)/(2*c)])/8
- (3*a*b^2*c^4*PolyLog[2, -(c/x)])/8 + (11*b^3*c^4*PolyLog[2, -(c/x)])/32 -
 (11*b^3*c^4*PolyLog[2, c/x])/32 - (3*a*b^2*c^4*PolyLog[2, (c + x)/(2*c)])/
8 + (3*b^3*c^4*Log[(c + x)/x]*PolyLog[2, (c + x)/x])/16 + (3*a*b^2*c^4*Poly
Log[2, 1 - x/c])/8 + (3*a*b^2*c^4*PolyLog[2, 1 + x/c])/8 + (3*b^3*c^4*PolyL
og[3, 1 - c/x])/16 - (3*b^3*c^4*PolyLog[3, (c + x)/x])/16 + (3*b^3*Defer[In
t][x^3*Log[1 - c/x]^2*Log[1 + c/x], x])/8 - (3*b^3*Defer[Int][x^3*Log[1 - c
/x]*Log[1 + c/x]^2, x])/8
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int\left(\frac{1}{8}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}bx^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)+\frac{3}{8}b^2x^3$$

$$=\frac{1}{8}\int x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3 dx+\frac{1}{8}(3b)\int x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)$$

$$=-\left(\frac{1}{8}\,\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^3}{x^5}dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3b)\int\left(4a^2x^3\log\left(1+\frac{c}{x}\right)-\right.$$

$$=\frac{1}{32}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{32}b^3x^4\log^3\left(\frac{c+x}{x}\right)+\frac{1}{2}\left(3a^2b\right)\int x^3\log\left(1+\frac{c}{x}\right)$$

$$=\frac{1}{32}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}a^2bx^4\log\left(1+\frac{c}{x}\right)-\frac{3}{8}ab^2x^4\log\left(1-\frac{c}{x}\right)\log\left(1+\right.$$

$$=\frac{1}{32}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}a^2bx^4\log\left(1+\frac{c}{x}\right)-\frac{3}{8}ab^2x^4\log\left(1-\frac{c}{x}\right)\log\left(1+\right.$$

$$=\frac{1}{32}bcx^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{32}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}a^2bx^4\log\left(1+\frac{c}{x}\right)$$

$$=\frac{3}{8}a^2bc^3x-\frac{3}{16}a^2bc^2x^2+\frac{1}{8}a^2bcx^3+\frac{3}{64}bc^2x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{32}bcx^3\left(2a-\right.$$

$$=\frac{3}{8}a^2bc^3x-\frac{3}{16}a^2bc^2x^2+\frac{1}{8}a^2bcx^3-\frac{3}{8}ab^2c^3x\log\left(1-\frac{c}{x}\right)+\frac{3}{16}ab^2c^2x^2\log\left(1-\frac{c}{x}\right)$$

$$=\frac{3}{8}a^2bc^3x-\frac{3}{16}a^2bc^2x^2+\frac{1}{8}a^2bcx^3-\frac{3}{8}ab^2c^3x\log\left(1-\frac{c}{x}\right)+\frac{3}{16}ab^2c^2x^2\log\left(1-\frac{c}{x}\right)$$

$$=\frac{3}{8}a^2bc^3x-\frac{5}{16}ab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}a^2bc^2x^2+\frac{1}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3+\frac{1}{32}b^3c^4\,\text{lo}$$

$$=\frac{3}{8}a^2bc^3x-\frac{5}{16}ab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}a^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3+\frac{1}{32}b^3c^4\,\text{lo}$$

$$=\frac{3}{8}a^2bc^3x-\frac{5}{16}ab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}a^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3+\frac{1}{32}b^3c^4\,\text{lo}$$

$$=\frac{3}{8}a^2bc^3x-\frac{5}{16}ab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}a^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3+\frac{1}{32}b^3c^4\,\text{lo}$$

Mathematica [A] time = 0.68, size = 286, normalized size = 1.41

$$\frac{1}{8}\left(2a^3x^4+2b\tanh^{-1}\left(\frac{c}{x}\right)\left(3a^2x^4+2abcx\left(3c^2+x^2\right)-8b^2c^4\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)+b^2\left(c^2x^2-c^4\right)\right)+3a^2bc^4\log\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (-2*a*b^2*c^4 + 6*a^2*b*c^3*x + 2*b^3*c^3*x + 2*a*b^2*c^2*x^2 + 2*a^2*b*c*x
^3 + 2*a^3*x^4 + 2*b^2*(b*c*(-4*c^3 + 3*c^2*x + x^3) + 3*a*(-c^4 + x^4))*Ar
```

```
cTanh[c/x]^2 + 2*b^3*(-c^4 + x^4)*ArcTanh[c/x]^3 + 2*b*ArcTanh[c/x]*(3*a^2*
x^4 + 2*a*b*c*x*(3*c^2 + x^2) + b^2*(-c^4 + c^2*x^2) - 8*b^2*c^4*Log[1 - E^
(-2*ArcTanh[c/x])]) + 3*a^2*b*c^4*Log[1 - c/x] - 16*a*b^2*c^4*Log[c/(Sqrt[1
 - c^2/x^2]*x)] - 3*a^2*b*c^4*Log[(c + x)/x] + 8*b^3*c^4*PolyLog[2, E^(-2*A
rcTanh[c/x])])/8
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^3\,\text{artanh}\left(\frac{c}{x}\right)^3+3\,ab^2x^3\,\text{artanh}\left(\frac{c}{x}\right)^2+3\,a^2bx^3\,\text{artanh}\left(\frac{c}{x}\right)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctanh(c/x)^3 + 3*a*b^2*x^3*arctanh(c/x)^2 + 3*a^2*b*x^3*
arctanh(c/x) + a^3*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3*x^3, x)
```

maple [C] time = 0.71, size = 1410, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x))^3,x)
```

```
[Out] 3/16*I*c^4*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))^
3*arctanh(c/x)^2+3/16*I*c^4*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2))^3*arctanh
(c/x)^2+c^4*b^3*arctanh(c/x)^2-3/16*I*c^4*b^3*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2
/x^2)))*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+
c/x)^2/(1-c^2/x^2)))*arctanh(c/x)^2+3/4*a*b^2*x^4*arctanh(c/x)^2+3/8*c^4*b^
3*arctanh(c/x)^2*ln(c/x-1)+3/8*c^4*a^2*b*ln(c/x-1)-3/8*c^4*a^2*b*ln(1+c/x)-
2*c^4*b^3*arctanh(c/x)*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))+3/4*c^4*b^3*arctanh(
c/x)^2*ln((1+c/x)/(1-c^2/x^2)^(1/2))+3/16*c^4*a*b^2*ln(c/x-1)^2+1/4*c^4*b^3
/(c/x+1-(1-c^2/x^2)^(1/2))*(1-c^2/x^2)^(1/2)-2*c^4*a*b^2*ln(c/x)+3/16*c^4*a
*b^2*ln(1+c/x)^2+1/4*c*b^3*arctanh(c/x)^2*x^3+3/4*c^3*b^3*arctanh(c/x)^2*x+
1/4*c^2*b^3*arctanh(c/x)*x^2-3/8*c^4*b^3*arctanh(c/x)^2*ln(1+c/x)-1/4*c^4*b
^3/((1-c^2/x^2)^(1/2)+c/x+1)*(1-c^2/x^2)^(1/2)+c^4*a*b^2*ln(c/x-1)+c^4*a*b^
2*ln(1+c/x)+3/4*c^3*x*a^2*b+1/4*c*a^2*b*x^3+1/4*c^2*a*b^2*x^2+3/4*a^2*b*x^4
*arctanh(c/x)+3/16*I*c^4*b^3*Pi*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))^2*csgn(I*
(1+c/x)^2/(-1+c^2/x^2))*arctanh(c/x)^2-3/8*I*c^4*b^3*Pi*csgn(I/(1+(1+c/x)^2
/(1-c^2/x^2)))^3*arctanh(c/x)^2+3/8*I*c^4*b^3*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2
/x^2)))^2*arctanh(c/x)^2+3/16*I*c^4*b^3*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2))
)*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arctanh(c/x)^2
+3/8*I*c^4*b^3*Pi*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))*csgn(I*(1+c/x)^2/(-1+c^
2/x^2))^2*arctanh(c/x)^2+1/4*b^3*x^4*arctanh(c/x)^3-2*c^4*b^3*dilog(1+(1+c/
x)/(1-c^2/x^2)^(1/2))-1/4*c^4*b^3*arctanh(c/x)^3-1/4*c^4*b^3*arctanh(c/x)-3
/16*I*c^4*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^2/(-1+c^2/x^
2)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arctanh(c/x)^2+1/4*x^4*a^3-3/4*c^4*a*b^2*ar
ctanh(c/x)*ln(1+c/x)-3/8*I*c^4*b^3*Pi*arctanh(c/x)^2+2*c^4*b^3*dilog((1+c/x
)/(1-c^2/x^2)^(1/2))-3/8*c^4*a*b^2*ln(c/x-1)*ln(1/2+1/2*c/x)+1/2*c*a*b^2*ar
```

```
ctanh(c/x)*x^3+3/2*c^3*a*b^2*arctanh(c/x)*x-3/8*c^4*a*b^2*ln(-1/2*c/x+1/2)*
ln(1+c/x)+3/8*c^4*a*b^2*ln(-1/2*c/x+1/2)*ln(1/2+1/2*c/x)+3/4*c^4*a*b^2*arct
anh(c/x)*ln(c/x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{4}\,ab^2x^4\,\operatorname{artanh}\left(\frac{c}{x}\right)^2+\frac{1}{4}\,a^3x^4+\frac{1}{8}\left(6\,x^4\,\operatorname{artanh}\left(\frac{c}{x}\right)-\left(3\,c^3\log\left(c+x\right)-3\,c^3\log\left(-c+x\right)-6\,c^2x-2\,x^3\right)c\right)a^2b+\frac{1}{16}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctanh(c/x)^2 + 1/4*a^3*x^4 + 1/8*(6*x^4*arctanh(c/x) - (3*c
^3*log(c + x) - 3*c^3*log(-c + x) - 6*c^2*x - 2*x^3)*c)*a^2*b + 1/16*((3*c^
2*log(c + x)^2 + 3*c^2*log(-c + x)^2 + 16*c^2*log(c + x) + 4*x^2 - 2*(3*c^2
*log(c + x) - 8*c^2)*log(-c + x))*c^2 - 4*(3*c^3*log(c + x) - 3*c^3*log(-c
+ x) - 6*c^2*x - 2*x^3)*c*arctanh(c/x))*a*b^2 + 1/32*(16*c^5*integrate(-log
(c + x)/(c^2 - x^2), x) + 40*c^4*integrate(-x*log(c + x)/(c^2 - x^2), x) -
2*(c*log(c + x) - c*log(-c + x) - 2*x)*c^3 - (c^4 - x^4)*log(c + x)^3 + (c^
4 - x^4)*log(-c + x)^3 + 2*(c^2*log(-c^2 + x^2) + x^2)*c^2 + 8*c^2*integrat
e(-x^3*log(c + x)/(c^2 - x^2), x) + 2*(3*c^3*x + c*x^3)*log(c + x)^2 - (8*c
^4 - 6*c^3*x - 2*c*x^3 + 3*(c^4 - x^4)*log(c + x))*log(-c + x)^2 - (4*c^2*x
^2 - 3*(c^4 - x^4)*log(c + x)^2 + 4*(4*c^4 + 3*c^3*x + c*x^3)*log(c + x))*l
og(-c + x))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c/x))^3,x)
```

```
[Out] int(x^3*(a + b*atanh(c/x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c/x))**3, x)
```

3.151 $\int x^2 \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx$

Optimal. Leaf size=217

$$b^2c^3\text{Li}_2\left(\frac{2}{\frac{c}{x}+1}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+b^2c^2x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{3}c^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-\frac{1}{2}bc^3\left(a+b\coth^{-1}\right.$$

```
[Out] b^2*c^2*x*(a+b*arccoth(x/c))-1/2*b*c^3*(a+b*arccoth(x/c))^2+1/2*b*c*x^2*(a+
b*arccoth(x/c))^2-1/3*c^3*(a+b*arccoth(x/c))^3+1/3*x^3*(a+b*arccoth(x/c))^3
-b*c^3*(a+b*arccoth(x/c))^2*ln(2-2/(1+c/x))+1/2*b^3*c^3*ln(1-c^2/x^2)+b^3*c
^3*ln(x)+b^2*c^3*(a+b*arccoth(x/c))*polylog(2,-1+2/(1+c/x))+1/2*b^3*c^3*pol
ylog(3,-1+2/(1+c/x))
```

Rubi [F] time = 3.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] -(a^2*b*c^2*x)/2 + (3*a*b^2*c^2*x)/4 + (a^2*b*c*x^2)/4 + (a*b^2*c^2*x*Log[1
 - c/x])/2 - (a*b^2*c*x^2*Log[1 - c/x])/4 + (b^2*c^2*(1 - c/x)*x*(2*a - b*L
og[1 - c/x]))/8 - (b*c^3*(2*a - b*Log[1 - c/x])^2)/16 + (b*c^2*(1 - c/x)*x*
(2*a - b*Log[1 - c/x])^2)/8 + (b*c*x^2*(2*a - b*Log[1 - c/x])^2)/16 - (c^3*
(2*a - b*Log[1 - c/x])^3)/24 + (x^3*(2*a - b*Log[1 - c/x])^3)/24 + (a*b^2*c
^2*x*Log[1 + c/x])/2 + (a*b^2*c*x^2*Log[1 + c/x])/4 + (a^2*b*x^3*Log[1 + c/
x])/2 - (a*b^2*x^3*Log[1 - c/x]*Log[1 + c/x])/2 - (a*b^2*c^3*Log[c - x])/4
+ (a*b^2*c^3*Log[1 + c/x]*Log[c - x])/2 - (b*c^3*(2*a - b*Log[1 - c/x])^2*L
og[c/x])/8 + (b^3*c^3*Log[x])/4 + (a*b^2*c^3*Log[c - x]*Log[x/c])/2 + (a^2*
b*c^3*Log[c + x])/2 + (a*b^2*c^3*Log[c + x])/4 - (a*b^2*c^3*Log[1 - c/x]*Lo
g[c + x])/2 + (a*b^2*c^3*Log[(c - x)/(2*c)]*Log[c + x])/2 - (a*b^2*c^3*Log[
-(x/c)]*Log[c + x])/2 - (a*b^2*c^3*Log[c - x]*Log[(c + x)/(2*c)])/2 - (3*a*
b^2*c^3*Log[(c + x)/x])/4 - (a*b^2*c^2*x*Log[(c + x)/x])/2 + (b^3*c^2*(1 +
c/x)*x*Log[(c + x)/x])/8 + (a*b^2*c*x^2*Log[(c + x)/x])/4 + (a*b^2*c^3*Log[
(c + x)/x]^2)/4 - (b^3*c^3*Log[(c + x)/x]^2)/16 - (b^3*c^2*(1 + c/x)*x*Log[
(c + x)/x]^2)/8 + (b^3*c*x^2*Log[(c + x)/x]^2)/16 + (a*b^2*x^3*Log[(c + x)/
x]^2)/4 - (b^3*c^3*Log[-(c/x)]*Log[(c + x)/x]^2)/8 + (b^3*c^3*Log[(c + x)/x
]^3)/24 + (b^3*x^3*Log[(c + x)/x]^3)/24 + (b^2*c^3*(2*a - b*Log[1 - c/x])*P
olyLog[2, 1 - c/x])/4 - (a*b^2*c^3*PolyLog[2, (c - x)/(2*c)])/2 + (a*b^2*c^
3*PolyLog[2, -(c/x)])/2 - (3*b^3*c^3*PolyLog[2, -(c/x)])/8 - (3*b^3*c^3*Pol
yLog[2, c/x])/8 + (a*b^2*c^3*PolyLog[2, (c + x)/(2*c)])/2 - (b^3*c^3*Log[(c
 + x)/x]*PolyLog[2, (c + x)/x])/4 + (a*b^2*c^3*PolyLog[2, 1 - x/c])/2 - (a*
b^2*c^3*PolyLog[2, 1 + x/c])/2 + (b^3*c^3*PolyLog[3, 1 - c/x])/4 + (b^3*c^3
*PolyLog[3, (c + x)/x])/4 + (3*b^3*Defer[Int][x^2*Log[1 - c/x]^2*Log[1 + c/
x], x])/8 - (3*b^3*Defer[Int][x^2*Log[1 - c/x]*Log[1 + c/x]^2, x])/8
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}bx^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)+\frac{3}{8}b^2x^2\right.$$
$$=\frac{1}{8}\int x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3 dx+\frac{1}{8}(3b)\int x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)$$
$$=-\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^3}{x^4}dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3b)\int\left(4a^2x^2\log\left(1+\frac{c}{x}\right)-\right.$$
$$=\frac{1}{24}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{24}b^3x^3\log^3\left(\frac{c+x}{x}\right)+\frac{1}{2}\left(3a^2b\right)\int x^2\log\left(1+\frac{c}{x}\right)$$
$$=\frac{1}{24}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{2}a^2bx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{2}ab^2x^3\log\left(1-\frac{c}{x}\right)\log\left(1+\right.$$
$$=\frac{1}{24}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{2}a^2bx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{2}ab^2x^3\log\left(1-\frac{c}{x}\right)\log\left(1+\right.$$
$$=\frac{1}{16}bcx^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{24}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{2}a^2bx^3\log\left(1+\frac{c}{x}\right)$$
$$=-\frac{1}{2}a^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{8}bc^2\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{16}bcx^2\left(2a-b\log\right.$$
$$=-\frac{1}{2}a^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+\frac{1}{8}b^2c^2\left(1-\right.$$
$$=-\frac{1}{2}a^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+\frac{1}{8}b^2c^2\left(1-\right.$$
$$=-\frac{1}{2}a^2bc^2x+\frac{1}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+$$
$$=-\frac{1}{2}a^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+$$
$$=-\frac{1}{2}a^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+$$
$$=-\frac{1}{2}a^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+$$

Mathematica [C] time = 0.79, size = 316, normalized size = 1.46

$$\frac{1}{6}\left(2a^3x^3+3a^2bc^3\log\left(x^2-c^2\right)+6a^2bx^3\tanh^{-1}\left(\frac{c}{x}\right)+3a^2bcx^2+6ab^2\left(c^3\text{Li}_2\left(e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)+\left(x^3-c^3\right)\tanh^{-1}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (3*a^2*b*c*x^2 + 2*a^3*x^3 + 6*a^2*b*x^3*ArcTanh[c/x] + 3*a^2*b*c^3*Log[-c^
2 + x^2] + 6*a*b^2*(c^2*x + (-c^3 + x^3)*ArcTanh[c/x]^2 + c*ArcTanh[c/x]*(-
c^2 + x^2 - 2*c^2*Log[1 - E^(-2*ArcTanh[c/x])]) + c^3*PolyLog[2, E^(-2*ArcT
```

```
anh[c/x])]) + (b^3*((-I)*c^3*Pi^3 + 24*c^2*x*ArcTanh[c/x] - 12*c^3*ArcTanh[
c/x]^2 + 12*c*x^2*ArcTanh[c/x]^2 + 8*c^3*ArcTanh[c/x]^3 + 8*x^3*ArcTanh[c/x
]^3 - 24*c^3*ArcTanh[c/x]^2*Log[1 - E^(2*ArcTanh[c/x])] - 24*c^3*Log[c/(Sqr
t[1 - c^2/x^2]*x)] - 24*c^3*ArcTanh[c/x]*PolyLog[2, E^(2*ArcTanh[c/x])] + 1
2*c^3*PolyLog[3, E^(2*ArcTanh[c/x])]))/4)/6
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^2\,\text{artanh}\left(\frac{c}{x}\right)^3 + 3\,ab^2x^2\,\text{artanh}\left(\frac{c}{x}\right)^2 + 3\,a^2bx^2\,\text{artanh}\left(\frac{c}{x}\right) + a^3x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctanh(c/x)^3 + 3*a*b^2*x^2*arctanh(c/x)^2 + 3*a^2*b*x^2*
arctanh(c/x) + a^3*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\text{artanh}\left(\frac{c}{x}\right) + a\right)^3 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3*x^2, x)
```

maple [C] time = 0.87, size = 2033, normalized size = 9.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x))^3,x)
```

```
[Out] 1/3*x^3*a^3-2*c^3*a*b^2*arctanh(c/x)*ln(c/x)+1/2*c^3*a^2*b*ln(1+c/x)+1/2*c*
b^3*arctanh(c/x)^2*x^2+c^2*b^3*arctanh(c/x)*x-c^3*b^3*arctanh(c/x)^2*ln(1+(
1+c/x)/(1-c^2/x^2)^(1/2))-c^3*b^3*arctanh(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(
1/2))-1/2*I*c^3*b^3*arctanh(c/x)^2*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I
*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I/(1+(1+c/x)^2/(
1-c^2/x^2)))*Pi-1/4*I*c^3*b^3*arctanh(c/x)^2*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/
2))^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*Pi-1/2*I*c^3*b^3*arctanh(c/x)^2*csgn(I
*(1+c/x)/(1-c^2/x^2)^(1/2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2))^2*Pi-1/4*I*c^3*b
^3*arctanh(c/x)^2*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*(1+c/x)^2/(-1+c^
2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*Pi+1/4*I*c^3*b^3*arctanh(c/x)^2*csgn(I*
(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x
^2)))^2*Pi+1/4*I*c^3*b^3*arctanh(c/x)^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn
(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1
-c^2/x^2)))*Pi+2*c^3*b^3*polylog(3,-(1+c/x)/(1-c^2/x^2)^(1/2))-c^3*b^3*ln((
1+c/x)/(1-c^2/x^2)^(1/2)-1)-c^3*b^3*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))-1/2*c^3
*b^3*arctanh(c/x)^2+1/3*c^3*b^3*arctanh(c/x)^3+c^3*b^3*arctanh(c/x)+2*c^3*b
^3*polylog(3,(1+c/x)/(1-c^2/x^2)^(1/2))+1/3*b^3*x^3*arctanh(c/x)^3+1/2*I*c^
3*b^3*arctanh(c/x)^2*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))^2*Pi-1/2*I*c^3*b^3*a
rctanh(c/x)^2*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^3
*Pi-1/2*I*c^3*b^3*arctanh(c/x)^2*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))^3*Pi-c^3
*b^3*ln(c/x)*arctanh(c/x)^2-2*c^3*b^3*arctanh(c/x)*polylog(2,(1+c/x)/(1-c^2
/x^2)^(1/2))+c^3*b^3*arctanh(c/x)^2*ln((1+c/x)^2/(1-c^2/x^2)-1)+1/2*c^3*b^3
*arctanh(c/x)^2*ln(c/x-1)+1/2*c^3*b^3*arctanh(c/x)^2*ln(1+c/x)-c^3*b^3*arct
anh(c/x)^2*ln((1+c/x)/(1-c^2/x^2)^(1/2))+1/2*c*a^2*b*x^2+c^2*a*b^2*x+a^2*b*
x^3*arctanh(c/x)+a*b^2*x^3*arctanh(c/x)^2-c^3*b^3*arctanh(c/x)^2*ln(2)-c^3*
```

```
a^2*b*ln(c/x)+c^3*a*b^2*dilog(c/x)+c^3*a*b^2*dilog(1+c/x)-c^3*a*b^2*dilog(1
/2+1/2*c/x)+1/4*c^3*a*b^2*ln(c/x-1)^2-1/4*c^3*a*b^2*ln(1+c/x)^2-2*c^3*b^3*a
rctanh(c/x)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))+c^3*a*b^2*ln(c/x)*ln(1+c/
x)-1/2*c^3*a*b^2*ln(c/x-1)*ln(1/2+1/2*c/x)+c*a*b^2*arctanh(c/x)*x^2+1/2*c^3
*a*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-1/2*c^3*a*b^2*ln(-1/2*c/x+1/2)*ln(1/2+1/2
*c/x)+c^3*a*b^2*arctanh(c/x)*ln(c/x-1)+1/2*I*c^3*b^3*arctanh(c/x)^2*csgn(I*
((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*csgn(I/(1+(1+c/x)^2/
(1-c^2/x^2)))*Pi+1/2*c^3*a^2*b*ln(c/x-1)-1/4*I*c^3*b^3*arctanh(c/x)^2*csgn(
I*(1+c/x)^2/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))^3*Pi+1/2*I*c^3*b^3*arct
anh(c/x)^2*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-
1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*Pi+c^3*a*b^2*arctanh(c/x)*ln(1+c/x)-1/2*I*c
^3*b^3*arctanh(c/x)^2*Pi-1/4*I*c^3*b^3*arctanh(c/x)^2*csgn(I*(1+c/x)^2/(-1+
c^2/x^2))^3*Pi+1/2*c^3*a*b^2*ln(c/x-1)-1/2*c^3*a*b^2*ln(1+c/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^3x^3+\frac{1}{2}\left(2\,x^3\operatorname{artanh}\left(\frac{c}{x}\right)+\left(c^2\log\left(-c^2+x^2\right)+x^2\right)c\right)a^2b+\frac{1}{24}\left(b^3c^3-b^3x^3\right)\log\left(-c+x\right)^3+\frac{1}{8}\left(b^3cx^2+2\,ab^2x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 + 1/2*(2*x^3*arctanh(c/x) + (c^2*log(-c^2 + x^2) + x^2)*c)*a^2*
b + 1/24*(b^3*c^3 - b^3*x^3)*log(-c + x)^3 + 1/8*(b^3*c*x^2 + 2*a*b^2*x^3 +
 (b^3*c^3 + b^3*x^3)*log(c + x))*log(-c + x)^2 - integrate(-1/8*((b^3*c*x^2
 - b^3*x^3)*log(c + x)^3 + 6*(a*b^2*c*x^2 - a*b^2*x^3)*log(c + x)^2 + (2*b^
3*c*x^2 + 4*a*b^2*x^3 - 3*(b^3*c*x^2 - b^3*x^3)*log(c + x)^2 + 2*(b^3*c^3 -
 6*a*b^2*c*x^2 + (6*a*b^2 + b^3)*x^3)*log(c + x))*log(-c + x))/(c - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a+b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c/x))^3,x)
```

```
[Out] int(x^2*(a + b*atanh(c/x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c/x))**3, x)
```

3.152 $\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3\,dx$

Optimal. Leaf size=135

$$-3b^2c^2\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{3}{2}bc^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{2}c^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}x^2\left(a+b\,\mathrm{co}\right.$$

```
[Out] -3/2*b*c^2*(a+b*arccoth(x/c))^2+3/2*b*c*x*(a+b*arccoth(x/c))^2-1/2*c^2*(a+b
*arccoth(x/c))^3+1/2*x^2*(a+b*arccoth(x/c))^3-3*b^2*c^2*(a+b*arccoth(x/c))*
ln(2-2/(1+c/x))+3/2*b^3*c^2*polylog(2,-1+2/(1+c/x))
```

Rubi [F] time = 2.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (3*a^2*b*c*x)/4 - (3*a*b^2*c*x*Log[1 - c/x])/4 + (3*b*c*(1 - c/x)*x*(2*a -
b*Log[1 - c/x])^2)/16 - (c^2*(2*a - b*Log[1 - c/x])^3)/16 + (x^2*(2*a - b*L
og[1 - c/x])^3)/16 + (3*a*b^2*c*x*Log[1 + c/x])/4 + (3*a^2*b*x^2*Log[1 + c/
x])/4 - (3*a*b^2*x^2*Log[1 - c/x]*Log[1 + c/x])/4 + (3*a*b^2*c^2*Log[c - x]
)/4 + (3*a*b^2*c^2*Log[1 + c/x]*Log[c - x])/4 - (3*b*c^2*(2*a - b*Log[1 - c
/x])^2*Log[c/x])/16 + (3*a*b^2*c^2*Log[x])/2 + (3*a*b^2*c^2*Log[c - x]*Log[
x/c])/4 - (3*a^2*b*c^2*Log[c + x])/4 + (3*a*b^2*c^2*Log[c + x])/4 + (3*a*b^
2*c^2*Log[1 - c/x]*Log[c + x])/4 - (3*a*b^2*c^2*Log[(c - x)/(2*c)]*Log[c +
x])/4 + (3*a*b^2*c^2*Log[-(x/c)]*Log[c + x])/4 - (3*a*b^2*c^2*Log[c - x]*Lo
g[(c + x)/(2*c)])/4 + (3*a*b^2*c^2*Log[(c + x)/x])/4 + (3*a*b^2*c*x*Log[(c
+ x)/x])/4 - (3*a*b^2*c^2*Log[(c + x)/x]^2)/8 + (3*b^3*c*(1 + c/x)*x*Log[(c
 + x)/x]^2)/16 + (3*a*b^2*x^2*Log[(c + x)/x]^2)/8 + (3*b^3*c^2*Log[-(c/x)]*
Log[(c + x)/x]^2)/16 - (b^3*c^2*Log[(c + x)/x]^3)/16 + (b^3*x^2*Log[(c + x)
/x]^3)/16 + (3*b^2*c^2*(2*a - b*Log[1 - c/x])*PolyLog[2, 1 - c/x])/8 - (3*a
*b^2*c^2*PolyLog[2, (c - x)/(2*c)])/4 - (3*a*b^2*c^2*PolyLog[2, -(c/x)])/4
+ (3*b^3*c^2*PolyLog[2, -(c/x)])/8 - (3*b^3*c^2*PolyLog[2, c/x])/8 - (3*a*b
^2*c^2*PolyLog[2, (c + x)/(2*c)])/4 + (3*b^3*c^2*Log[(c + x)/x]*PolyLog[2,
(c + x)/x])/8 + (3*a*b^2*c^2*PolyLog[2, 1 - x/c])/4 + (3*a*b^2*c^2*PolyLog[
2, 1 + x/c])/4 + (3*b^3*c^2*PolyLog[3, 1 - c/x])/8 - (3*b^3*c^2*PolyLog[3,
(c + x)/x])/8 + (3*b^3*Defer[Int][x*Log[1 - c/x]^2*Log[1 + c/x], x])/8 - (3
*b^3*Defer[Int][x*Log[1 - c/x]*Log[1 + c/x]^2, x])/8
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(\frac{1}{8}x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}bx\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)+\frac{3}{8}b^2x\left(2a\right.\right.$$

$$=\frac{1}{8}\int x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3 dx+\frac{1}{8}(3b)\int x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)dx$$

$$=-\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^3}{x^3}dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3b)\int\left(4a^2x\log\left(1+\frac{c}{x}\right)-4a\right.$$

$$=\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{16}b^3x^2\log^3\left(\frac{c+x}{x}\right)+\frac{1}{2}\left(3a^2b\right)\int x\log\left(1+\frac{c}{x}\right)dx$$

$$=\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{4}a^2bx^2\log\left(1+\frac{c}{x}\right)-\frac{3}{4}ab^2x^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right.$$

$$=\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{4}a^2bx^2\log\left(1+\frac{c}{x}\right)-\frac{3}{4}ab^2x^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right.$$

$$=\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{4}a^2bx^2\log\left(\right.$$

$$=\frac{3}{4}a^2bcx+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{4}a$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\frac{1}{16}c^2\left(2a-\right.$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\frac{1}{16}c^2\left(2a-\right.$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\frac{1}{16}c^2\left(2a-\right.$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\frac{1}{16}c^2\left(2a-\right.$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\frac{1}{16}c^2\left(2a-\right.$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\frac{1}{16}c^2\left(2a-\right.$$

Mathematica [A] time = 0.34, size = 193, normalized size = 1.43

$$\frac{1}{4}\left(a\left(a\left(2ax^2-3bc^2\log\left(\frac{c+x}{x}\right)+6bcx\right)+3abc^2\log\left(1-\frac{c}{x}\right)-12b^2c^2\log\left(\frac{c}{x\sqrt{1-\frac{c^2}{x^2}}}\right)\right)+6b\tanh^{-1}\left(\frac{c}{x}\right)\left(ax(ax\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (6*b^2*(-c + x)*(b*c + a*(c + x))*ArcTanh[c/x]^2 + 2*b^3*(-c^2 + x^2)*ArcTa
nh[c/x]^3 + 6*b*ArcTanh[c/x]*(a*x*(2*b*c + a*x) - 2*b^2*c^2*Log[1 - E^(-2*A
rcTanh[c/x])]) + a*(3*a*b*c^2*Log[1 - c/x] - 12*b^2*c^2*Log[c/(Sqrt[1 - c^2
/x^2]*x)] + a*(6*b*c*x + 2*a*x^2 - 3*b*c^2*Log[(c + x)/x])) + 6*b^3*c^2*Pol
yLog[2, E^(-2*ArcTanh[c/x])])/4
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x\,\text{artanh}\left(\frac{c}{x}\right)^3+3\,ab^2x\,\text{artanh}\left(\frac{c}{x}\right)^2+3\,a^2bx\,\text{artanh}\left(\frac{c}{x}\right)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctanh(c/x)^3 + 3*a*b^2*x*arctanh(c/x)^2 + 3*a^2*b*x*arctan
h(c/x) + a^3*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3*x, x)
```

maple [C] time = 0.60, size = 5536, normalized size = 41.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2}\,ab^2x^2\,\text{artanh}\left(\frac{c}{x}\right)^2+\frac{1}{2}\,a^3x^2+\frac{3}{4}\left(2\,x^2\,\text{artanh}\left(\frac{c}{x}\right)-\left(c\log\left(c+x\right)-c\log\left(-c+x\right)-2\,x\right)c\right)a^2b+\frac{3}{8}\left(\left(\log\left(c+x\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arctanh(c/x)^2 + 1/2*a^3*x^2 + 3/4*(2*x^2*arctanh(c/x) - (c*l
og(c + x) - c*log(-c + x) - 2*x)*c)*a^2*b + 3/8*((log(c + x)^2 - 2*(log(c +
 x) - 2)*log(-c + x) + log(-c + x)^2 + 4*log(c + x))*c^2 - 4*(c*log(c + x)
- c*log(-c + x) - 2*x)*c*arctanh(c/x))*a*b^2 + 1/16*(6*c*x*log(c + x)^2 - (
c^2 - x^2)*log(c + x)^3 + (c^2 - x^2)*log(-c + x)^3 - 3*(2*c^2 - 2*c*x + (c
^2 - x^2)*log(c + x))*log(-c + x)^2 + 3*((c^2 - x^2)*log(c + x)^2 - 4*(c^2
+ c*x)*log(c + x))*log(-c + x) + 2*integrate(-6*(c^3 + 3*c^2*x)*log(c + x)/
(c^2 - x^2), x))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c/x))^3,x)
```

```
[Out] int(x*(a + b*atanh(c/x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x))**3,x)
```

```
[Out] Integral(x*(a + b*atanh(c/x))**3, x)
```

3.153 $\int \left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx$

Optimal. Leaf size=108

$$-3b^2c\text{Li}_2\left(1-\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-3bc\log\left(\frac{2c}{c-x}\right)\left(a+b\right.$$

```
[Out] c*(a+b*arccoth(x/c))^3+x*(a+b*arccoth(x/c))^3-3*b*c*(a+b*arccoth(x/c))^2*ln
(2*c/(c-x))-3*b^2*c*(a+b*arccoth(x/c))*polylog(2,1-2*c/(c-x))+3/2*b^3*c*pol
ylog(3,1-2*c/(c-x))
```

Rubi [F] time = 0.70, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] a^3*x - (3*a^2*b*x*Log[1 - c/x])/2 - (3*a*b^2*(c - x)*Log[1 - c/x]^2)/4 + (
b^3*(c - x)*Log[1 - c/x]^3)/8 + (3*a^2*b*x*Log[1 + c/x])/2 - (3*a*b^2*x*Log
[1 - c/x]*Log[1 + c/x])/2 + (3*a*b^2*(c + x)*Log[1 + c/x]^2)/4 + (b^3*(c +
x)*Log[1 + c/x]^3)/8 - (3*a*b^2*c*Log[1 - c/x]*Log[-c - x])/2 + (3*a^2*b*c*
Log[c - x])/2 + (3*a*b^2*c*Log[-c - x]*Log[(c - x)/(2*c)])/2 - (3*b^3*c*Log
[1 - c/x]^2*Log[c/x])/8 - (3*a*b^2*c*Log[-c - x]*Log[-(x/c)])/2 + (3*a*b^2*
c*Log[1 + c/x]*Log[-c + x])/2 + (3*a*b^2*c*Log[x/c]*Log[-c + x])/2 + (3*a^2
*b*c*Log[c + x])/2 - (3*a*b^2*c*Log[-c + x]*Log[(c + x)/(2*c)])/2 - (3*b^3*
c*Log[-(c/x)]*Log[(c + x)/x]^2)/8 - (3*b^3*c*Log[1 - c/x]*PolyLog[2, 1 - c/
x])/4 - (3*a*b^2*c*PolyLog[2, (c - x)/(2*c)])/2 + (3*a*b^2*c*PolyLog[2, -(c
/x)])/2 - (3*a*b^2*c*PolyLog[2, c/x])/2 + (3*a*b^2*c*PolyLog[2, (c + x)/(2*
c)])/2 - (3*b^3*c*Log[(c + x)/x]*PolyLog[2, (c + x)/x])/4 + (3*a*b^2*c*Poly
Log[2, 1 - x/c])/2 - (3*a*b^2*c*PolyLog[2, 1 + x/c])/2 + (3*b^3*c*PolyLog[3
, 1 - c/x])/4 + (3*b^3*c*PolyLog[3, (c + x)/x])/4 + (3*b^3*Defer[Int][Log[1
 - c/x]^2*Log[1 + c/x], x])/8 - (3*b^3*Defer[Int][Log[1 - c/x]*Log[1 + c/x]
^2, x])/8
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(a^3-\frac{3}{2}a^2b\log\left(1-\frac{c}{x}\right)+\frac{3}{4}ab^2\log^2\left(1-\frac{c}{x}\right)-\frac{1}{8}b^3\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a^2b\log\left(1+\right.\right.$$
$$= a^3x-\frac{1}{2}\left(3a^2b\right)\int \log\left(1-\frac{c}{x}\right)dx+\frac{1}{2}\left(3a^2b\right)\int \log\left(1+\frac{c}{x}\right)dx+\frac{1}{4}\left(3ab^2\right)\int \log^2\left(\right.$$
$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a$$
$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a$$
$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a$$
$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a$$
$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a$$
$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a$$
$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a$$
$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a$$

Mathematica [C] time = 0.28, size = 198, normalized size = 1.83

$$a^3x+\frac{3}{2}a^2bc\log\left(x^2-c^2\right)+3a^2bx\tanh^{-1}\left(\frac{c}{x}\right)-3ab^2\left(\tanh^{-1}\left(\frac{c}{x}\right)\left((c-x)\tanh^{-1}\left(\frac{c}{x}\right)+2c\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcTanh[c/x] + (3*a^2*b*c*Log[-c^2 + x^2])/2 - 3*a*b^2*(A
rcTanh[c/x]*((c - x)*ArcTanh[c/x] + 2*c*Log[1 - E^(-2*ArcTanh[c/x])]) - c*P
olyLog[2, E^(-2*ArcTanh[c/x])]) + (b^3*((-I)*c*Pi^3 + 8*c*ArcTanh[c/x]^3 +
8*x*ArcTanh[c/x]^3 - 24*c*ArcTanh[c/x]^2*Log[1 - E^(2*ArcTanh[c/x])] - 24*c
*ArcTanh[c/x]*PolyLog[2, E^(2*ArcTanh[c/x])] + 12*c*PolyLog[3, E^(2*ArcTanh
[c/x])]))/8
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3\,\text{artanh}\left(\frac{c}{x}\right)^3+3\,ab^2\,\text{artanh}\left(\frac{c}{x}\right)^2+3\,a^2b\,\text{artanh}\left(\frac{c}{x}\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x)
 + a^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3, x)
```

maple [C] time = 0.36, size = 1756, normalized size = 16.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3,x)
```

```
[Out] 3/4*I*c*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)
/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arctanh(c/x)^2-3/4*I*c*b^3*Pi*csgn(I*(1+c/x)^
2/(-1+c^2/x^2))^3*arctanh(c/x)^2-3/4*I*c*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^
2)/(1+(1+c/x)^2/(1-c^2/x^2)))^3*arctanh(c/x)^2+3/2*I*c*b^3*Pi*csgn(I/(1+(1+
c/x)^2/(1-c^2/x^2)))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x
^2)))^2*arctanh(c/x)^2+3/2*I*c*b^3*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csg
n(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arctanh(c/x)^2+3
/2*c*a^2*b*ln(1+c/x)-3*c*b^3*arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))
+3*a*b^2*x*arctanh(c/x)^2+3*a^2*b*x*arctanh(c/x)-3*c*b^3*arctanh(c/x)^2*ln(
2)-3*c*a^2*b*ln(c/x)+3*c*a*b^2*dilog(c/x)+3*c*a*b^2*dilog(1+c/x)-3*c*a*b^2*
dilog(1/2+1/2*c/x)-3*c*b^3*arctanh(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(1/2))-6
*c*b^3*arctanh(c/x)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))-3*c*b^3*ln(c/x)*a
rctanh(c/x)^2-6*c*b^3*arctanh(c/x)*polylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))+3*c
*b^3*arctanh(c/x)^2*ln((1+c/x)^2/(1-c^2/x^2)-1)+3/2*c*b^3*arctanh(c/x)^2*ln
(c/x-1)+3/2*c*b^3*arctanh(c/x)^2*ln(1+c/x)-3*c*b^3*arctanh(c/x)^2*ln((1+c/x
)/(1-c^2/x^2)^(1/2))+3/4*c*a*b^2*ln(c/x-1)^2-3/4*c*a*b^2*ln(1+c/x)^2+3/2*c*
a^2*b*ln(c/x-1)-3/2*c*a*b^2*ln(c/x-1)*ln(1/2+1/2*c/x)+3/4*I*c*b^3*Pi*csgn(I
/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^2
/(-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))*arctanh(c/x)^2+b^3*x*arctanh(c/x)^
3+6*c*b^3*polylog(3,(1+c/x)/(1-c^2/x^2)^(1/2))+6*c*b^3*polylog(3,-(1+c/x)/(
1-c^2/x^2)^(1/2))+c*b^3*arctanh(c/x)^3-3/4*I*c*b^3*Pi*csgn(I*(1+c/x)/(1-c^2
/x^2)^(1/2))^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*arctanh(c/x)^2-3/2*I*c*b^3*Pi
*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2))^2*arctanh
(c/x)^2-3/4*I*c*b^3*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*(1+c/x)^2/(
-1+c^2/x^2)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arctanh(c/x)^2-3/2*I*c*b^3*Pi*csgn
(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*((1+
c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))*arctanh(c/x)^2+3/2*c*a*b^2
*ln(-1/2*c/x+1/2)*ln(1+c/x)-3/2*c*a*b^2*ln(-1/2*c/x+1/2)*ln(1/2+1/2*c/x)-3/
2*I*c*b^3*Pi*arctanh(c/x)^2-3/2*I*c*b^3*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2))
)^3*arctanh(c/x)^2+3/2*I*c*b^3*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arcta
nh(c/x)^2-3/2*I*c*b^3*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c
^2/x^2)))^3*arctanh(c/x)^2+3*c*a*b^2*arctanh(c/x)*ln(c/x-1)+3*c*a*b^2*arcta
nh(c/x)*ln(1+c/x)-6*c*a*b^2*arctanh(c/x)*ln(c/x)+3*c*a*b^2*ln(c/x)*ln(1+c/x
)+x*a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2}\left(2\,x\,\text{artanh}\left(\frac{c}{x}\right)+c\log\left(-c^2+x^2\right)\right)a^2b+a^3x+\frac{1}{8}\left(b^3c-b^3x\right)\log\left(-c+x\right)^3+\frac{3}{8}\left(2\,ab^2x+\left(b^3c+b^3x\right)\log\left(c+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*(2*x*arctanh(c/x) + c*log(-c^2 + x^2))*a^2*b + a^3*x + 1/8*(b^3*c - b^3
*x)*log(-c + x)^3 + 3/8*(2*a*b^2*x + (b^3*c + b^3*x)*log(c + x))*log(-c + x
)^2 - integrate(-1/8*((b^3*c - b^3*x)*log(c + x)^3 + 6*(a*b^2*c - a*b^2*x)*
```

```
log(c + x)^2 + 3*(4*a*b^2*x - (b^3*c - b^3*x)*log(c + x)^2 - 2*(2*a*b^2*c -
 b^3*c - (2*a*b^2 + b^3)*x)*log(c + x))*log(-c + x))/(c - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^3,x)
```

```
[Out] int((a + b*atanh(c/x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**3,x)
```

```
[Out] Integral((a + b*atanh(c/x))**3, x)
```

3.154 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

Optimal. Leaf size=208

$$-\frac{3}{2}b^2\mathrm{Li}_3\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}b^2\mathrm{Li}_3\left(\frac{2}{1-\frac{c}{x}}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}b\mathrm{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\cot\right.$$

```
[Out] 2*(a+b*arccoth(x/c))^3*arctanh(-1+2/(1-c/x))+3/2*b*(a+b*arccoth(x/c))^2*pol
ylog(2,1-2/(1-c/x))-3/2*b*(a+b*arccoth(x/c))^2*polylog(2,-1+2/(1-c/x))-3/2*
b^2*(a+b*arccoth(x/c))*polylog(3,1-2/(1-c/x))+3/2*b^2*(a+b*arccoth(x/c))*po
lylog(3,-1+2/(1-c/x))+3/4*b^3*polylog(4,1-2/(1-c/x))-3/4*b^3*polylog(4,-1+2
/(1-c/x))
```

Rubi [A] time = 0.51, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6095, 5914, 6052, 5948, 6058, 6062, 6610}

$$-\frac{3}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}b^2\text{PolyLog}\left(3,\frac{2}{1-\frac{c}{x}}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}b\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^3/x,x]
```

```
[Out] -2*(a + b*ArcCoth[x/c])^3*ArcTanh[1 - 2/(1 - c/x)] + (3*b*(a + b*ArcCoth[x/
c])^2*PolyLog[2, 1 - 2/(1 - c/x)])/2 - (3*b*(a + b*ArcCoth[x/c])^2*PolyLog[
2, -1 + 2/(1 - c/x)])/2 - (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[3, 1 - 2/(1 -
 c/x)])/2 + (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[3, -1 + 2/(1 - c/x)])/2 + (
3*b^3*PolyLog[4, 1 - 2/(1 - c/x)])/4 - (3*b^3*PolyLog[4, -1 + 2/(1 - c/x)])
/4
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
```

```
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x}\,dx = -\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+(6bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\tan}{1-c^2x^2}\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)-(3bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log}{1-c^2x^2}\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+\frac{3}{2}b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+\frac{3}{2}b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+\frac{3}{2}b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)$$

Mathematica [A] time = 0.20, size = 171, normalized size = 0.82

$$\frac{3}{4}b\left(2\text{Li}_2\left(\frac{c+x}{c-x}\right)\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2-2\text{Li}_2\left(\frac{c+x}{x-c}\right)\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2+b\left(-2\text{Li}_3\left(\frac{c+x}{c-x}\right)\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3/x,x]
```

```
[Out] -2*(a + b*ArcTanh[c/x])^3*ArcTanh[(c + x)/(c - x)] + (3*b*(2*(a + b*ArcTanh
[c/x])^2*PolyLog[2, (c + x)/(c - x)] - 2*(a + b*ArcTanh[c/x])^2*PolyLog[2,
(c + x)/(-c + x)] + b*(-2*(a + b*ArcTanh[c/x])*PolyLog[3, (c + x)/(c - x)]
```

```
+ 2*(a + b*ArcTanh[c/x])*PolyLog[3, (c + x)/(-c + x)] + b*(PolyLog[4, (c +
x)/(c - x)] - PolyLog[4, (c + x)/(-c + x)]))))/4
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{artanh}\left(\frac{c}{x}\right)^3+3\,ab^2\,\text{artanh}\left(\frac{c}{x}\right)^2+3\,a^2b\,\text{artanh}\left(\frac{c}{x}\right)+a^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x
) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3/x, x)
```

maple [C] time = 0.13, size = 1631, normalized size = 7.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3/x,x)
```

```
[Out] -3*a*b^2*arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))-6*a*b^2*arctanh(c/x
)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))-3*a*b^2*ln(c/x)*arctanh(c/x)^2-3*a^
2*b*ln(c/x)*arctanh(c/x)+3/2*a^2*b*ln(c/x)*ln(1+c/x)+3*a*b^2*arctanh(c/x)*p
olylog(2,-(1+c/x)^2/(1-c^2/x^2))+3*a*b^2*arctanh(c/x)^2*ln((1+c/x)^2/(1-c^2
/x^2)-1)-3*a*b^2*arctanh(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(1/2))-6*a*b^2*arc
tanh(c/x)*polylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))-3/2*I*a*b^2*Pi*csgn(I*((1+c/
x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^3*arctanh(c/x)^2+1/2*I*b^3*P
i*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+
c/x)^2/(1-c^2/x^2)))^2*arctanh(c/x)^3+1/2*I*b^3*Pi*csgn(I*((1+c/x)^2/(1-c^2
/x^2)-1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arc
tanh(c/x)^3+3/2*I*a*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I*((1+c/x
)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arctanh(c/x)^2-1/2*I*b^3*Pi
*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I
*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))*arctanh(c/x)^3+3/2*I*
a*b^2*Pi*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)
/(1+(1+c/x)^2/(1-c^2/x^2)))^2*arctanh(c/x)^2+6*a*b^2*polylog(3,(1+c/x)/(1-c
^2/x^2)^(1/2))-b^3*ln(c/x)*arctanh(c/x)^3+b^3*arctanh(c/x)^3*ln((1+c/x)^2/(
1-c^2/x^2)-1)+3/2*b^3*arctanh(c/x)^2*polylog(2,-(1+c/x)^2/(1-c^2/x^2))-3/2*
b^3*arctanh(c/x)*polylog(3,-(1+c/x)^2/(1-c^2/x^2))-b^3*arctanh(c/x)^3*ln(1-
(1+c/x)/(1-c^2/x^2)^(1/2))-3*b^3*arctanh(c/x)^2*polylog(2,(1+c/x)/(1-c^2/x^
2)^(1/2))+6*b^3*arctanh(c/x)*polylog(3,(1+c/x)/(1-c^2/x^2)^(1/2))-b^3*arcta
nh(c/x)^3*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))-3*b^3*arctanh(c/x)^2*polylog(2,-(
1+c/x)/(1-c^2/x^2)^(1/2))+6*b^3*arctanh(c/x)*polylog(3,-(1+c/x)/(1-c^2/x^2)
^(1/2))+3/2*a^2*b*dilog(c/x)+3/2*a^2*b*dilog(1+c/x)+6*a*b^2*polylog(3,-(1+c
/x)/(1-c^2/x^2)^(1/2))-3/2*a*b^2*polylog(3,-(1+c/x)^2/(1-c^2/x^2))-3/2*I*a*
b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I/(1+(1+c/x)^2/(1-c^2/x^2)))*
csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))*arctanh(c/x)^2-
```

```
1/2*I*b^3*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/(1+(1+c/x)^2/(1-c^2/x^2)))^3*
arctanh(c/x)^3-6*b^3*polylog(4,-(1+c/x)/(1-c^2/x^2)^(1/2))-a^3*ln(c/x)+3/4*
b^3*polylog(4,-(1+c/x)^2/(1-c^2/x^2))-6*b^3*polylog(4,(1+c/x)/(1-c^2/x^2)^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\log(x)+\int\frac{b^3\left(\log\left(\frac{c}{x}+1\right)-\log\left(-\frac{c}{x}+1\right)\right)^3}{8x}+\frac{3\,ab^2\left(\log\left(\frac{c}{x}+1\right)-\log\left(-\frac{c}{x}+1\right)\right)^2}{4x}+\frac{3\,a^2b\left(\log\left(\frac{c}{x}+1\right)-\log\left(-\frac{c}{x}\right.\right.}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c/x + 1) - log(-c/x + 1))^3/x + 3/4*a*b
^2*(log(c/x + 1) - log(-c/x + 1))^2/x + 3/2*a^2*b*(log(c/x + 1) - log(-c/x
+ 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^3/x,x)
```

```
[Out] int((a + b*atanh(c/x))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**3/x,x)
```

```
[Out] Integral((a + b*atanh(c/x))**3/x, x)
```

3.155 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=126

$$\frac{3b^2\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{x}+\frac{3b\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right.}{c}$$

```
[Out] -(a+b*arccoth(x/c))^3/c-(a+b*arccoth(x/c))^3/x+3*b*(a+b*arccoth(x/c))^2*ln(
2/(1-c/x))/c+3*b^2*(a+b*arccoth(x/c))*polylog(2,1-2/(1-c/x))/c-3/2*b^3*poly
log(3,1-2/(1-c/x))/c
```

Rubi [B] time = 2.24, antiderivative size = 387, normalized size of antiderivative = 3.07, number of steps used = 82, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$\frac{3b^2\text{PolyLog}\left(2,-\frac{c-x}{2x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,-\frac{c-x}{2x}\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,\frac{c+x}{2x}\right)}{2c}-\frac{3b^3\log\left(\frac{c+x}{x}\right)\text{Poly}}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c/x])^3/x^2,x]
```

```
[Out] ((1 - c/x)*(2*a - b*Log[1 - c/x])^3)/(8*c) - (3*b*(2*a - b*Log[1 - c/x])^2*
Log[(c + x)/(2*x)])/(4*c) + (3*b*(2*a - b*Log[1 - c/x])^2*Log[(c + x)/x])/(
8*c) - (3*b*(2*a - b*Log[1 - c/x])^2*Log[(c + x)/x])/(8*x) - (3*b^2*(2*a -
b*Log[1 - c/x])*Log[(c + x)/x]^2)/(8*c) - (3*b^2*(2*a - b*Log[1 - c/x])*Log
[(c + x)/x]^2)/(8*x) - (3*b^3*Log[-(c - x)/(2*x)]*Log[(c + x)/x]^2)/(4*c) -
 (b^3*(1 + c/x)*Log[(c + x)/x]^3)/(8*c) + (3*b^2*(2*a - b*Log[1 - c/x])*Pol
yLog[2, -(c - x)/(2*x)])/(2*c) - (3*b^3*Log[(c + x)/x]*PolyLog[2, (c + x)/(
2*x)])/(2*c) + (3*b^3*PolyLog[3, -(c - x)/(2*x)])/(2*c) + (3*b^3*PolyLog[3,
 (c + x)/(2*x)])/(2*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8x^2}+\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)}{8x^2}+\frac{3b^2\left(2a-b\log\left(1\right.\right.}{8}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{x^2}\,dx+\frac{1}{8}(3b)\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)}{x^2}\,dx+\frac{1}{8}\left(3\right.$$

$$= -\left(\frac{1}{8}\operatorname{Subst}\left(\int (2a-b\log(1-cx))^3\,dx,x,\frac{1}{x}\right)\right)-\frac{1}{8}(3b)\operatorname{Subst}\left(\int (2a-b\log(1-cx))\right.$$

$$= -\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(\frac{c+x}{x}\right)}{8x}-\frac{3b^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log^2\left(\frac{c+x}{x}\right)}{8x}+\frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(\frac{c+x}{x}\right)}{8x}-\frac{3b^2\left(2a-b\log\left(\right.\right.}{8}$$

$$= \frac{3b\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\right.\right.}{8x}$$

$$= -\frac{3ab^2}{2x}+\frac{3b^3}{4x}+\frac{3b\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b}{}$$

$$= -\frac{3ab^2}{2x}-\frac{3b^3\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{4c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1\right.\right.}{8c}$$

$$= -\frac{3b^3}{4x}-\frac{3b^3\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{4c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\right.\right.}{4c}$$

$$= \frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(\frac{c+x}{2x}\right)}{4c}+\frac{3b\left(2a-b\log\left(1\right.\right.}{8}$$

$$= \frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(\frac{c+x}{2x}\right)}{4c}+\frac{3b\left(2a-b\log\left(1\right.\right.}{8}$$

Mathematica [A] time = 0.12, size = 206, normalized size = 1.63

$$-\frac{a^3}{x}-\frac{3a^2b\left(\frac{c\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^2}}}\right)\right)}{c}-\frac{3ab^2\left(\operatorname{Li}_2\left(-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)+\tanh^{-1}\left(\frac{c}{x}\right)\left(\frac{c\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\tanh^{-1}\left(\frac{c}{x}\right)-2\log\left(\right.\right.\right.}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*((c*ArcTanh[c/x])/x - Log[1/Sqrt[1 - c^2/x^2]]))/c - (3
*a*b^2*(ArcTanh[c/x]*(-ArcTanh[c/x] + (c*ArcTanh[c/x])/x - 2*Log[1 + E^(-2*
ArcTanh[c/x])]) + PolyLog[2, -E^(-2*ArcTanh[c/x])]))/c - (b^3*(ArcTanh[c/x]
^2*(-ArcTanh[c/x] + (c*ArcTanh[c/x])/x - 3*Log[1 + E^(-2*ArcTanh[c/x])]) +
3*ArcTanh[c/x]*PolyLog[2, -E^(-2*ArcTanh[c/x])] + (3*PolyLog[3, -E^(-2*ArcT
anh[c/x])])/2))/c
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\operatorname{artanh}\left(\frac{c}{x}\right)^3+3\,ab^2\operatorname{artanh}\left(\frac{c}{x}\right)^2+3\,a^2b\operatorname{artanh}\left(\frac{c}{x}\right)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x
) + a^3)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3/x^2, x)
```

maple [B] time = 0.21, size = 298, normalized size = 2.37

$$-\frac{a^3}{x}-\frac{b^3\operatorname{arctanh}\left(\frac{c}{x}\right)^3}{x}-\frac{b^3\operatorname{arctanh}\left(\frac{c}{x}\right)^3}{c}+\frac{3b^3\operatorname{arctanh}\left(\frac{c}{x}\right)^2\ln\left(1+\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)}{c}+\frac{3b^3\operatorname{arctanh}\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3/x^2,x)
```

```
[Out] -a^3/x-b^3/x*arctanh(c/x)^3-1/c*b^3*arctanh(c/x)^3+3/c*b^3*arctanh(c/x)^2*l
n(1+(1+c/x)^2/(1-c^2/x^2))+3/c*b^3*arctanh(c/x)*polylog(2,-(1+c/x)^2/(1-c^2
/x^2))-3/2/c*b^3*polylog(3,-(1+c/x)^2/(1-c^2/x^2))-3/x*a*b^2*arctanh(c/x)^2
-3/c*a*b^2*arctanh(c/x)^2+6/c*arctanh(c/x)*ln(1+(1+c/x)^2/(1-c^2/x^2))*a*b^
2+3/c*polylog(2,-(1+c/x)^2/(1-c^2/x^2))*a*b^2-3/x*a^2*b*arctanh(c/x)-3/2/c*
a^2*b*ln(1-c^2/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3\,a^2b\left(\frac{2\,c\operatorname{artanh}\left(\frac{c}{x}\right)}{x}+\log\left(-\frac{c^2}{x^2}+1\right)\right)}{2\,c}-\frac{a^3}{x}+\frac{\left(b^3c-b^3x\right)\log\left(-c+x\right)^3-3\left(2\,ab^2c+\left(b^3c+b^3x\right)\log\left(c+x\right)\right)\log(}{8\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*a^2*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a^3/x + 1/8*((b^3*c
 - b^3*x)*log(-c + x)^3 - 3*(2*a*b^2*c + (b^3*c + b^3*x)*log(c + x))*log(-c
 + x)^2)/(c*x) - integrate(-1/8*((b^3*c^2 - b^3*c*x)*log(c + x)^3 + 6*(a*b^
2*c^2 - a*b^2*c*x)*log(c + x)^2 - 3*(4*a*b^2*c*x + (b^3*c^2 - b^3*c*x)*log(
c + x)^2 + 2*(2*a*b^2*c^2 + b^3*x^2 - (2*a*b^2*c - b^3*c)*x)*log(c + x))*lo
g(-c + x))/(c^2*x^2 - c*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^3/x^2,x)
```

```
[Out] int((a + b*atanh(c/x))^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**3/x**2,x)
```

```
[Out] Integral((a + b*atanh(c/x))**3/x**2, x)
```

3.156 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=139

$$\frac{3b^2\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{c^2}-\frac{3b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}+\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2}-\frac{3b\left(a+b\right.}{}$$

```
[Out] -3/2*b*(a+b*arccoth(x/c))^2/c^2-3/2*b*(a+b*arccoth(x/c))^2/c/x+1/2*(a+b*arc
coth(x/c))^3/c^2-1/2*(a+b*arccoth(x/c))^3/x^2+3*b^2*(a+b*arccoth(x/c))*ln(2
/(1-c/x))/c^2+3/2*b^3*polylog(2,1-2/(1-c/x))/c^2
```

Rubi [F] time = 2.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c/x])^3/x^3,x]
```

```
[Out] (-3*b^3*(1 - c/x)^2)/(64*c^2) - (3*a*b^2*(1 + c/x)^2)/(16*c^2) + (3*b^3*(1
 + c/x)^2)/(64*c^2) + (3*a^2*b)/(8*x^2) + (3*a*b^2)/(8*x^2) - (3*a^2*b)/(4*c
*x) - (3*b^3)/(2*c*x) + (3*a*b^2*Log[1 - c/x])/(8*c^2) - (3*a*b^2*(1 - c/x)
*Log[1 - c/x])/(4*c^2) - (3*b^3*(1 - c/x)*Log[1 - c/x])/(4*c^2) - (3*a*b^2*
Log[1 - c/x])/(8*x^2) - (3*b^2*(1 - c/x)^2*(2*a - b*Log[1 - c/x]))/(32*c^2)
 + (3*b*(1 - c/x)*(2*a - b*Log[1 - c/x])^2)/(8*c^2) - (3*b*(1 - c/x)^2*(2*a
 - b*Log[1 - c/x])^2)/(32*c^2) + ((1 - c/x)*(2*a - b*Log[1 - c/x])^3)/(8*c^
2) - ((1 - c/x)^2*(2*a - b*Log[1 - c/x])^3)/(16*c^2) + (3*a*b^2*Log[1 - c/x
]*Log[1 + c/x])/(4*x^2) - (3*a*b^2*Log[1 + c/x]*Log[c - x])/(4*c^2) - (3*a*
b^2*Log[c - x]*Log[x/c])/(4*c^2) - (3*a*b^2*Log[1 - c/x]*Log[c + x])/(4*c^2
) + (3*a*b^2*Log[(c - x)/(2*c)]*Log[c + x])/(4*c^2) - (3*a*b^2*Log[-(x/c)]*
Log[c + x])/(4*c^2) + (3*a*b^2*Log[c - x]*Log[(c + x)/(2*c)])/(4*c^2) + (3*
a^2*b*Log[(c + x)/x])/(4*c^2) + (3*a*b^2*Log[(c + x)/x])/(8*c^2) - (9*a*b^2
*(1 + c/x)*Log[(c + x)/x])/(4*c^2) + (3*b^3*(1 + c/x)*Log[(c + x)/x])/(4*c^
2) + (3*a*b^2*(1 + c/x)^2*Log[(c + x)/x])/(8*c^2) - (3*b^3*(1 + c/x)^2*Log[
(c + x)/x])/(32*c^2) - (3*a^2*b*Log[(c + x)/x])/(4*x^2) - (3*a*b^2*Log[(c +
 x)/x])/(8*x^2) + (3*a*b^2*(1 + c/x)*Log[(c + x)/x]^2)/(4*c^2) - (3*b^3*(1
+ c/x)*Log[(c + x)/x]^2)/(8*c^2) - (3*a*b^2*(1 + c/x)^2*Log[(c + x)/x]^2)/(
8*c^2) + (3*b^3*(1 + c/x)^2*Log[(c + x)/x]^2)/(32*c^2) + (b^3*(1 + c/x)*Log
[(c + x)/x]^3)/(8*c^2) - (b^3*(1 + c/x)^2*Log[(c + x)/x]^3)/(16*c^2) + (3*a
*b^2*PolyLog[2, (c - x)/(2*c)])/(4*c^2) + (3*a*b^2*PolyLog[2, -(c/x)])/(4*c
^2) + (3*a*b^2*PolyLog[2, c/x])/(4*c^2) + (3*a*b^2*PolyLog[2, (c + x)/(2*c)
])/(4*c^2) - (3*a*b^2*PolyLog[2, 1 - x/c])/(4*c^2) - (3*a*b^2*PolyLog[2, 1
+ x/c])/(4*c^2) + (3*b^3*Defer[Int][(Log[1 - c/x]^2*Log[1 + c/x])/x^3, x])/
8 - (3*b^3*Defer[Int][(Log[1 - c/x]*Log[1 + c/x]^2)/x^3, x])/8
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int\left(\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8x^3}+\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)}{8x^3}+\frac{3b^2\left(2a-b\log\left(1\right.\right.}{8}\right.$$

$$=\frac{1}{8}\int\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{x^3}dx+\frac{1}{8}(3b)\int\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)}{x^3}dx+\frac{1}{8}(3$$

$$=-\left(\frac{1}{8}\text{Subst}\left(\int x(2a-b\log(1-cx))^3\,dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3b)\int\left(\frac{4a^2\log\left(1+\frac{c}{x}\right)}{x^3}-\frac{4ab\log}{}\right.$$

$$=-\left(\frac{1}{8}\text{Subst}\left(\int\left(\frac{(2a-b\log(1-cx))^3}{c}-\frac{(1-cx)(2a-b\log(1-cx))^3}{c}\right)dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}$$

$$=\frac{3ab^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2}-\frac{1}{2}\left(3a^2b\right)\text{Subst}\left(\int x\log(1+cx)\,dx,x,\frac{1}{x}\right)-\frac{1}{4}\left(3ab^2\right)$$

$$=\frac{3ab^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2}-\frac{3a^2b\log\left(\frac{c+x}{x}\right)}{4x^2}-\frac{1}{4}\left(3ab^2\right)\text{Subst}\left(\int\left(-\frac{\log^2(1+cx)}{c}\right.\right.$$

$$=\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c^2}-\frac{\left(1-\frac{c}{x}\right)^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{16c^2}+\frac{3ab^2\log\left(1-\frac{c}{x}\right)\log}{4x^2}$$

$$=\frac{3a^2b}{8x^2}-\frac{3a^2b}{4cx}+\frac{3b\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8c^2}-\frac{3b\left(1-\frac{c}{x}\right)^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{32c^2}+$$

$$=-\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2}+\frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2}+\frac{3a^2b}{8x^2}-\frac{3a^2b}{4cx}-\frac{3ab^2}{2cx}-\frac{3b^3}{4cx}-\frac{3b^2\left(1-\frac{c}{x}\right)^2\left(2a-b\log\right.}{32c^2}$$

$$=-\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2}-\frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2}+\frac{3a^2b}{8x^2}-\frac{3a^2b}{4cx}-\frac{3b^3}{2cx}-\frac{3b^3\left(1-\frac{c}{x}\right)\log}{4c^2}$$

$$=-\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2}-\frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2}+\frac{3a^2b}{8x^2}-\frac{3a^2b}{4cx}-\frac{3b^3}{2cx}-\frac{3ab^2\left(1-\frac{c}{x}\right)}{4c^2}$$

$$=-\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2}-\frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2}+\frac{3a^2b}{8x^2}+\frac{3ab^2}{8x^2}-\frac{3a^2b}{4cx}-\frac{3b^3}{2cx}+\frac{3ab^2}{}$$

$$=-\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2}-\frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2}+\frac{3a^2b}{8x^2}+\frac{3ab^2}{8x^2}-\frac{3a^2b}{4cx}-\frac{3b^3}{2cx}+\frac{3ab^2}{}$$

$$=-\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2}-\frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2}+\frac{3a^2b}{8x^2}+\frac{3ab^2}{8x^2}-\frac{3a^2b}{4cx}-\frac{3b^3}{2cx}+\frac{3ab^2}{}$$

Mathematica [A] time = 0.35, size = 195, normalized size = 1.40

$$\frac{a\left(12b^2x^2\log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^2}}}\right)-a\left(2ac^2+3bx^2\log\left(1-\frac{c}{x}\right)-3bx^2\log\left(\frac{c+x}{x}\right)+6bcx\right)\right)+6b\tanh^{-1}\left(\frac{c}{x}\right)\left(2b^2x^2\log\left(e^{-2\tanh}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3/x^3,x]
```

```
[Out] (6*b^2*(-c + x)*(b*x + a*(c + x))*ArcTanh[c/x]^2 + 2*b^3*(-c^2 + x^2)*ArcTa
nh[c/x]^3 + 6*b*ArcTanh[c/x]*(-(a*c*(a*c + 2*b*x)) + 2*b^2*x^2*Log[1 + E^(-
2*ArcTanh[c/x])]) + a*(12*b^2*x^2*Log[1/Sqrt[1 - c^2/x^2]] - a*(2*a*c^2 + 6
```

```
*b*c*x + 3*b*x^2*Log[1 - c/x] - 3*b*x^2*Log[(c + x)/x])) - 6*b^3*x^2*PolyLo
g[2, -E^(-2*ArcTanh[c/x])])/(4*c^2*x^2)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^3 \operatorname{artanh}\left(\frac{c}{x}\right)^3 + 3\,ab^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + 3\,a^2 b \operatorname{artanh}\left(\frac{c}{x}\right) + a^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x
) + a^3)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x}\right) + a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3/x^3, x)
```

maple [C] time = 0.56, size = 6645, normalized size = 47.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3/x^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2)
*a^2*b - 3/8*(c^2*((log(c + x)^2 - 2*(log(c + x) - 2)*log(-c + x) + log(-c
+ x)^2 + 4*log(c + x))/c^4 - 8*log(x)/c^4) - 4*c*(log(c + x)/c^3 - log(-c +
 x)/c^3 - 2/(c^2*x))*arctanh(c/x))*a*b^2 + 1/64*(32*c^4*integrate(-1/4*log(
x)^3/(c^4*x^3 - c^2*x^5), x) - 3*c^3*(log(c + x)/c^5 - log(-c + x)/c^5 - 2/
(c^4*x)) + 48*c^3*integrate(-1/4*x*log(x)^2/(c^4*x^3 - c^2*x^5), x) + 48*c^
3*integrate(-1/4*x*log(x)/(c^4*x^3 - c^2*x^5), x) - 6*c*(2*log(-c + x)/c^3
- 2*log(x)/c^3 + (c + 2*x)/(c^2*x^2))*log(-c/x + 1)^2 + 21*c^2*(log(c + x)/
c^4 + log(-c + x)/c^4 - 2*log(x)/c^4) - 32*c^2*integrate(-1/4*x^2*log(x)^3/
(c^4*x^3 - c^2*x^5), x) + 48*c^2*integrate(-1/4*x^2*log(x)^2/(c^4*x^3 - c^2
*x^5), x) - 384*c^2*integrate(-1/4*x^2*log(c + x)/(c^4*x^3 - c^2*x^5), x) +
 144*c^2*integrate(-1/4*x^2*log(x)/(c^4*x^3 - c^2*x^5), x) - 18*c*(log(c +
x)/c^3 - log(-c + x)/c^3) + c*(6*(2*x^2*log(-c + x)^2 + 2*x^2*log(x)^2 - 6*
x^2*log(x) + c^2 + 6*c*x - 2*(2*x^2*log(x) - 3*x^2)*log(-c + x))*log(-c/x +
 1)/(c^3*x^2) - (4*x^2*log(-c + x)^3 - 4*x^2*log(x)^3 + 18*x^2*log(x)^2 - 6
*(2*x^2*log(x) - 3*x^2)*log(-c + x)^2 - 42*x^2*log(x) + 3*c^2 + 42*c*x + 6*
(2*x^2*log(x)^2 - 6*x^2*log(x) + 7*x^2)*log(-c + x))/(c^3*x^2)) - 48*c*inte
grate(-1/4*x^3*log(x)^2/(c^4*x^3 - c^2*x^5), x) - 192*c*integrate(-1/4*x^3*
```

```
log(c + x)/(c^4*x^3 - c^2*x^5), x) + 336*c*integrate(-1/4*x^3*log(x)/(c^4*x
^3 - c^2*x^5), x) + 4*log(-c/x + 1)^3/x^2 - 2*(12*c*x*log(c + x)^2 + 2*(c^2
 - x^2)*log(c + x)^3 - 3*(c^2 - 2*c*x + x^2 - 2*(c^2 - x^2)*log(c + x) + 2*
(c^2 - x^2)*log(x))*log(-c + x)^2 - 3*(2*(c^2 - x^2)*log(c + x)^2 - 2*(c^2
- x^2)*log(x)^2 - c^2 - 6*c*x + 8*(c*x + x^2)*log(c + x) - 2*(c^2 + 2*c*x +
 5*x^2)*log(x))*log(-c + x))/(c^2*x^2) - 48*integrate(-1/4*x^4*log(x)^2/(c^
4*x^3 - c^2*x^5), x) - 192*integrate(-1/4*x^4*log(c + x)/(c^4*x^3 - c^2*x^5
), x) + 240*integrate(-1/4*x^4*log(x)/(c^4*x^3 - c^2*x^5), x))*b^3 - 3/2*a*
b^2*arctanh(c/x)^2/x^2 - 1/2*a^3/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x))^3/x^3,x)
```

```
[Out] int((a + b*atanh(c/x))^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**3/x**3,x)
```

```
[Out] Integral((a + b*atanh(c/x))**3/x**3, x)
```

3.157 $\int x^7 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx$

Optimal. Leaf size=54

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{8}bc^4\tanh^{-1}\left(\frac{x^2}{c}\right)+\frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6$$

```
[Out] 1/8*b*c^3*x^2+1/24*b*c*x^6+1/8*x^8*(a+b*arctanh(c/x^2))-1/8*b*c^4*arctanh(x
^2/c)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 263, 275, 302, 207}

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8}bc^3x^2-\frac{1}{8}bc^4\tanh^{-1}\left(\frac{x^2}{c}\right)+\frac{1}{24}bcx^6$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c^3*x^2)/8 + (b*c*x^6)/24 + (x^8*(a + b*ArcTanh[c/x^2]))/8 - (b*c^4*ArcT
anh[x^2/c])/8
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)dx &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}(bc)\int\frac{x^5}{1-\frac{c^2}{x^4}}\,dx\\ &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}(bc)\int\frac{x^9}{-c^2+x^4}\,dx\\ &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8}(bc)\,\mathrm{Subst}\left(\int\frac{x^4}{-c^2+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8}(bc)\,\mathrm{Subst}\left(\int\left(c^2+x^2+\frac{c^4}{-c^2+x^2}\right)dx,x,x^2\right)\\ &= \frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8}\left(bc^5\right)\mathrm{Subst}\left(\int\frac{1}{-c^2+x^2}\,dx,x,\right.\\ &= \frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{8}bc^4\tanh^{-1}\left(\frac{x^2}{c}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 73, normalized size = 1.35

$$\frac{ax^8}{8}+\frac{1}{16}bc^4\log\left(x^2-c\right)-\frac{1}{16}bc^4\log\left(c+x^2\right)+\frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6+\frac{1}{8}bx^8\tanh^{-1}\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c^3*x^2)/8 + (b*c*x^6)/24 + (a*x^8)/8 + (b*x^8*ArcTanh[c/x^2])/8 + (b*c^
4*Log[-c + x^2])/16 - (b*c^4*Log[c + x^2])/16
```

fricas [A] time = 0.53, size = 53, normalized size = 0.98

$$\frac{1}{8}\,ax^8+\frac{1}{24}\,bcx^6+\frac{1}{8}\,bc^3x^2+\frac{1}{16}\left(bx^8-bc^4\right)\log\left(\frac{x^2+c}{x^2-c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] 1/8*a*x^8 + 1/24*b*c*x^6 + 1/8*b*c^3*x^2 + 1/16*(b*x^8 - b*c^4)*log((x^2 +
c)/(x^2 - c))
```

giac [A] time = 0.15, size = 71, normalized size = 1.31

$$\frac{1}{16}\,bx^8\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{8}\,ax^8+\frac{1}{24}\,bcx^6+\frac{1}{8}\,bc^3x^2-\frac{1}{16}\,bc^4\log\left(x^2+c\right)+\frac{1}{16}\,bc^4\log\left(-x^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/16*b*x^8*log((x^2 + c)/(x^2 - c)) + 1/8*a*x^8 + 1/24*b*c*x^6 + 1/8*b*c^3*
x^2 - 1/16*b*c^4*log(x^2 + c) + 1/16*b*c^4*log(-x^2 + c)
```

maple [A] time = 0.04, size = 64, normalized size = 1.19

$$\frac{x^8a}{8}+\frac{b\,x^8\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{8}+\frac{bc\,x^6}{24}+\frac{b\,c^3x^2}{8}-\frac{b\,c^4\ln\left(1+\frac{c}{x^2}\right)}{16}+\frac{b\,c^4\ln\left(\frac{c}{x^2}-1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctanh(c/x^2)),x)
```

```
[Out] 1/8*x^8*a+1/8*b*x^8*arctanh(c/x^2)+1/24*b*c*x^6+1/8*b*c^3*x^2-1/16*b*c^4*ln
(1+c/x^2)+1/16*b*c^4*ln(c/x^2-1)
```

maxima [A] time = 0.32, size = 62, normalized size = 1.15

$$\frac{1}{8}ax^8 + \frac{1}{48}\left(6x^8 \operatorname{artanh}\left(\frac{c}{x^2}\right) + \left(2x^6 + 6c^2x^2 - 3c^3\log\left(x^2+c\right) + 3c^3\log\left(x^2-c\right)\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/48*(6*x^8*arctanh(c/x^2) + (2*x^6 + 6*c^2*x^2 - 3*c^3*log(x^2
 + c) + 3*c^3*log(x^2 - c))*c)*b
```

mupad [B] time = 1.01, size = 66, normalized size = 1.22

$$\frac{a\,x^8}{8} + \frac{b\,c^3\,x^2}{8} + \frac{b\,x^8\,\ln\left(x^2+c\right)}{16} + \frac{b\,c\,x^6}{24} - \frac{b\,x^8\,\ln\left(x^2-c\right)}{16} + \frac{b\,c^4\operatorname{atan}\left(\frac{x^2\,1\mathrm{i}}{c}\right)\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atanh(c/x^2)),x)
```

```
[Out] (a*x^8)/8 + (b*c^3*x^2)/8 + (b*x^8*log(c + x^2))/16 + (b*c^4*atan((x^2*1i)/
c)*1i)/8 + (b*c*x^6)/24 - (b*x^8*log(x^2 - c))/16
```

sympy [A] time = 9.89, size = 51, normalized size = 0.94

$$\frac{ax^8}{8} - \frac{bc^4\operatorname{atanh}\left(\frac{c}{x^2}\right)}{8} + \frac{bc^3x^2}{8} + \frac{bcx^6}{24} + \frac{bx^8\operatorname{atanh}\left(\frac{c}{x^2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atanh(c/x**2)),x)
```

```
[Out] a*x**8/8 - b*c**4*atanh(c/x**2)/8 + b*c**3*x**2/8 + b*c*x**6/24 + b*x**8*at
anh(c/x**2)/8
```

3.158 $\int x^5 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=45

$$\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}bc^3\log\left(c^2-x^4\right)+\frac{1}{12}bcx^4$$

```
[Out] 1/12*b*c*x^4+1/6*x^6*(a+b*arctanh(c/x^2))+1/12*b*c^3*ln(-x^4+c^2)
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 263, 266, 43}

$$\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}bc^3\log\left(c^2-x^4\right)+\frac{1}{12}bcx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^4)/12 + (x^6*(a + b*ArcTanh[c/x^2]))/6 + (b*c^3*Log[c^2 - x^4])/12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)dx &= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}(bc)\int\frac{x^3}{1-\frac{c^2}{x^4}}\,dx\\
&= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}(bc)\int\frac{x^7}{-c^2+x^4}\,dx\\
&= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}(bc)\,\mathrm{Subst}\left(\int\frac{x}{-c^2+x}\,dx,x,x^4\right)\\
&= \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}(bc)\,\mathrm{Subst}\left(\int\left(1-\frac{c^2}{c^2-x}\right)dx,x,x^4\right)\\
&= \frac{1}{12}bcx^4+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}bc^3\log\left(c^2-x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.11

$$\frac{ax^6}{6}+\frac{1}{12}bc^3\log\left(x^4-c^2\right)+\frac{1}{12}bcx^4+\frac{1}{6}bx^6\tanh^{-1}\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^4)/12 + (a*x^6)/6 + (b*x^6*ArcTanh[c/x^2])/6 + (b*c^3*Log[-c^2 + x^4
])/12
```

fricas [A] time = 0.57, size = 52, normalized size = 1.16

$$\frac{1}{12}\,bx^6\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{6}\,ax^6+\frac{1}{12}\,bcx^4+\frac{1}{12}\,bc^3\log\left(x^4-c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] 1/12*b*x^6*log((x^2 + c)/(x^2 - c)) + 1/6*a*x^6 + 1/12*b*c*x^4 + 1/12*b*c^3
*log(x^4 - c^2)
```

giac [A] time = 0.15, size = 52, normalized size = 1.16

$$\frac{1}{12}\,bx^6\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{6}\,ax^6+\frac{1}{12}\,bcx^4+\frac{1}{12}\,bc^3\log\left(x^4-c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/12*b*x^6*log((x^2 + c)/(x^2 - c)) + 1/6*a*x^6 + 1/12*b*c*x^4 + 1/12*b*c^3
*log(x^4 - c^2)
```

maple [A] time = 0.06, size = 65, normalized size = 1.44

$$\frac{x^6a}{6}+\frac{b\,x^6\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{6}+\frac{bc\,x^4}{12}-\frac{b\,c^3\ln\left(\frac{1}{x}\right)}{3}+\frac{b\,c^3\ln\left(1+\frac{c}{x^2}\right)}{12}+\frac{b\,c^3\ln\left(\frac{c}{x^2}-1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c/x^2)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctanh(c/x^2)+1/12*b*c*x^4-1/3*b*c^3*ln(1/x)+1/12*b*c^
3*ln(1+c/x^2)+1/12*b*c^3*ln(c/x^2-1)
```

maxima [A] time = 0.32, size = 42, normalized size = 0.93

$$\frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\operatorname{artanh}\left(\frac{c}{x^2}\right)+\left(x^4+c^2\log\left(x^4-c^2\right)\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctanh(c/x^2) + (x^4 + c^2*log(x^4 - c^2))*c)*b
```

mupad [B] time = 0.81, size = 56, normalized size = 1.24

$$\frac{a\,x^6}{6}+\frac{b\,c^3\,\ln\left(x^4-c^2\right)}{12}+\frac{b\,x^6\,\ln\left(x^2+c\right)}{12}+\frac{b\,c\,x^4}{12}-\frac{b\,x^6\,\ln\left(x^2-c\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atanh(c/x^2)),x)
```

```
[Out] (a*x^6)/6 + (b*c^3*log(x^4 - c^2))/12 + (b*x^6*log(c + x^2))/12 + (b*c*x^4)
/12 - (b*x^6*log(x^2 - c))/12
```

sympy [C] time = 6.84, size = 75, normalized size = 1.67

$$\frac{ax^6}{6} + \frac{bc^3 \log\left(-i\sqrt{c} + x\right)}{6} + \frac{bc^3 \log\left(i\sqrt{c} + x\right)}{6} - \frac{bc^3 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{6} + \frac{bcx^4}{12} + \frac{bx^6 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c/x**2)),x)
```

```
[Out] a*x**6/6 + b*c**3*log(-I*sqrt(c) + x)/6 + b*c**3*log(I*sqrt(c) + x)/6 - b*c
**3*atanh(c/x**2)/6 + b*c*x**4/12 + b*x**6*atanh(c/x**2)/6
```

3.159 $\int x^3 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\,dx$

Optimal. Leaf size=43

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{4}bc^2\tanh^{-1}\left(\frac{x^2}{c}\right)+\frac{1}{4}bcx^2$$

```
[Out] 1/4*b*c*x^2+1/4*x^4*(a+b*arctanh(c/x^2))-1/4*b*c^2*arctanh(x^2/c)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 263, 275, 321, 207}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{4}bc^2\tanh^{-1}\left(\frac{x^2}{c}\right)+\frac{1}{4}bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^2)/4 + (x^4*(a + b*ArcTanh[c/x^2]))/4 - (b*c^2*ArcTanh[x^2/c])/4
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)dx &= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{2}(bc)\int\frac{x}{1-\frac{c^2}{x^4}}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{2}(bc)\int\frac{x^5}{-c^2+x^4}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}(bc)\,\text{Subst}\left(\int\frac{x^2}{-c^2+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{4}bcx^2+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int\frac{1}{-c^2+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{4}bcx^2+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{4}bc^2\tanh^{-1}\left(\frac{x^2}{c}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 62, normalized size = 1.44

$$\frac{ax^4}{4}+\frac{1}{8}bc^2\log\left(x^2-c\right)-\frac{1}{8}bc^2\log\left(c+x^2\right)+\frac{1}{4}bcx^2+\frac{1}{4}bx^4\tanh^{-1}\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^2)/4 + (a*x^4)/4 + (b*x^4*ArcTanh[c/x^2])/4 + (b*c^2*Log[-c + x^2])/
8 - (b*c^2*Log[c + x^2])/8
```

fricas [A] time = 0.55, size = 44, normalized size = 1.02

$$\frac{1}{4}\,ax^4+\frac{1}{4}\,bcx^2+\frac{1}{8}\left(bx^4-bc^2\right)\log\left(\frac{x^2+c}{x^2-c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*a*x^4 + 1/4*b*c*x^2 + 1/8*(b*x^4 - b*c^2)*log((x^2 + c)/(x^2 - c))
```

giac [B] time = 0.18, size = 162, normalized size = 3.77

$$\frac{\frac{\left(x^2+c\right)bc^3\log\left(\frac{x^2+c}{x^2-c}\right)}{\left(x^2-c\right)\left(\frac{\left(x^2+c\right)^2}{\left(x^2-c\right)^2}-\frac{2\left(x^2+c\right)}{x^2-c}+1\right)}+\frac{\frac{2\left(x^2+c\right)ac^3}{x^2-c}+\frac{\left(x^2+c\right)bc^3}{x^2-c}-bc^3}{\frac{\left(x^2+c\right)^2}{\left(x^2-c\right)^2}-\frac{2\left(x^2+c\right)}{x^2-c}+1}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/2*((x^2 + c)*b*c^3*log((x^2 + c)/(x^2 - c))/((x^2 - c)*((x^2 + c)^2/(x^2
- c)^2 - 2*(x^2 + c)/(x^2 - c) + 1)) + (2*(x^2 + c)*a*c^3/(x^2 - c) + (x^2
+ c)*b*c^3/(x^2 - c) - b*c^3)/((x^2 + c)^2/(x^2 - c)^2 - 2*(x^2 + c)/(x^2 -
 c) + 1))/c
```

maple [A] time = 0.04, size = 55, normalized size = 1.28

$$\frac{x^4a}{4}+\frac{\operatorname{arctanh}\left(\frac{c}{x^2}\right)b\,x^4}{4}+\frac{bc\,x^2}{4}-\frac{b\,c^2\ln\left(1+\frac{c}{x^2}\right)}{8}+\frac{b\,c^2\ln\left(\frac{c}{x^2}-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x^2)),x)
```

```
[Out] 1/4*x^4*a+1/4*arctanh(c/x^2)*b*x^4+1/4*b*c*x^2-1/8*b*c^2*ln(1+c/x^2)+1/8*b*
c^2*ln(c/x^2-1)
```

maxima [A] time = 0.31, size = 49, normalized size = 1.14

$$\frac{1}{4}\,ax^4 + \frac{1}{8}\left(2\,x^4\,\operatorname{artanh}\left(\frac{c}{x^2}\right) + \left(2\,x^2 - c\log\left(x^2+c\right) + c\log\left(x^2-c\right)\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/8*(2*x^4*arctanh(c/x^2) + (2*x^2 - c*log(x^2 + c) + c*log(x^2
 - c))*c)*b
```

mupad [B] time = 0.86, size = 57, normalized size = 1.33

$$\frac{a\,x^4}{4} + \frac{b\,x^4\,\ln\left(x^2+c\right)}{8} + \frac{b\,c\,x^2}{4} - \frac{b\,x^4\,\ln\left(x^2-c\right)}{8} + \frac{b\,c^2\,\operatorname{atan}\left(\frac{x^2\,1i}{c}\right)\,1i}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c/x^2)),x)
```

```
[Out] (a*x^4)/4 + (b*x^4*log(c + x^2))/8 + (b*c^2*atan((x^2*1i)/c)*1i)/4 + (b*c*x
^2)/4 - (b*x^4*log(x^2 - c))/8
```

sympy [A] time = 4.60, size = 41, normalized size = 0.95

$$\frac{ax^4}{4} - \frac{bc^2\operatorname{atanh}\left(\frac{c}{x^2}\right)}{4} + \frac{bcx^2}{4} + \frac{bx^4\operatorname{atanh}\left(\frac{c}{x^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x**2)),x)
```

```
[Out] a*x**4/4 - b*c**2*atanh(c/x**2)/4 + b*c*x**2/4 + b*x**4*atanh(c/x**2)/4
```

3.160 $\int x \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=34

$$\frac{1}{2}x^2 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) + \frac{1}{4}bc \log\left(c^2 - x^4\right)$$

```
[Out] 1/2*x^2*(a+b*arctanh(c/x^2))+1/4*b*c*ln(-x^4+c^2)
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{6097, 263, 260\}$

$$\frac{1}{2}x^2 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) + \frac{1}{4}bc \log\left(c^2 - x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (x^2*(a + b*ArcTanh[c/x^2]))/2 + (b*c*Log[c^2 - x^4])/4
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx &= \frac{1}{2}x^2 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) + (bc) \int \frac{1}{\left(1 - \frac{c^2}{x^4}\right) x} dx \\
&= \frac{1}{2}x^2 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) + (bc) \int \frac{x^3}{-c^2 + x^4} dx \\
&= \frac{1}{2}x^2 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) + \frac{1}{4}bc \log\left(c^2 - x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.15

$$\frac{ax^2}{2} + \frac{1}{4}bc \log\left(x^4 - c^2\right) + \frac{1}{2}bx^2 \tanh^{-1}\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcTanh[c/x^2])/2 + (b*c*Log[-c^2 + x^4])/4
```

fricas [A] time = 0.68, size = 43, normalized size = 1.26

$$\frac{1}{4}bx^2\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{2}ax^2+\frac{1}{4}bc\log\left(x^4-c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
[Out] 1/4*b*x^2*log((x^2 + c)/(x^2 - c)) + 1/2*a*x^2 + 1/4*b*c*log(x^4 - c^2)
```

giac [B] time = 0.17, size = 184, normalized size = 5.41

$$\frac{1}{2}ax^2+\frac{\left(c^2\left(\log\left(\frac{|-x^2-c|}{|-x^2+c|}\right)-\log\left(\left|\frac{x^2+c}{x^2-c}-1\right|\right)\right)+\frac{c^2\log\left(-\frac{\frac{c\left(\frac{x^2+c}{(x^2-c)c}-\frac{1}{c}\right)}{\frac{x^2+c}{x^2-c}+1}+1}{\frac{c\left(\frac{x^2+c}{(x^2-c)c}-\frac{1}{c}\right)}{\frac{x^2+c}{x^2-c}+1}-1}\right)}{\frac{x^2+c}{x^2-c}-1}\right)b}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2)),x, algorithm="giac")
[Out] 1/2*a*x^2 + 1/2*(c^2*(log(abs(-x^2 - c)/abs(-x^2 + c)) - log(abs((x^2 + c)/
(x^2 - c) - 1))) + c^2*log(-(c*((x^2 + c)/((x^2 - c)*c) - 1/c)/((x^2 + c)/(
x^2 - c) + 1) + 1)/(c*((x^2 + c)/((x^2 - c)*c) - 1/c)/((x^2 + c)/(x^2 - c)
+ 1) - 1))/((x^2 + c)/(x^2 - c) - 1))*b/c
```

maple [A] time = 0.06, size = 52, normalized size = 1.53

$$\frac{a\,x^2}{2}+\frac{b\,x^2\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{2}-bc\ln\left(\frac{1}{x}\right)+\frac{bc\ln\left(1+\frac{c}{x^2}\right)}{4}+\frac{bc\ln\left(\frac{c}{x^2}-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x^2)),x)
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c/x^2)-b*c*ln(1/x)+1/4*b*c*ln(1+c/x^2)+1/4*b*c*
ln(c/x^2-1)
```

maxima [A] time = 0.32, size = 34, normalized size = 1.00

$$\frac{1}{2}ax^2+\frac{1}{4}\left(2\,x^2\operatorname{artanh}\left(\frac{c}{x^2}\right)+c\log\left(x^4-c^2\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arctanh(c/x^2) + c*log(x^4 - c^2))*b
```

mupad [B] time = 0.79, size = 47, normalized size = 1.38

$$\frac{a\,x^2}{2}+\frac{b\,x^2\,\ln\left(x^2+c\right)}{4}+\frac{b\,c\,\ln\left(x^4-c^2\right)}{4}-\frac{b\,x^2\,\ln\left(x^2-c\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c/x^2)),x)
```

```
[Out] (a*x^2)/2 + (b*x^2*log(c + x^2))/4 + (b*c*log(x^4 - c^2))/4 - (b*x^2*log(x^
2 - c))/4
```

sympy [C] time = 3.20, size = 61, normalized size = 1.79

$$\frac{ax^2}{2} + \frac{bc\log\left(-i\sqrt{c} + x\right)}{2} + \frac{bc\log\left(i\sqrt{c} + x\right)}{2} - \frac{bc\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2} + \frac{bx^2\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x**2)),x)
```

```
[Out] a*x**2/2 + b*c*log(-I*sqrt(c) + x)/2 + b*c*log(I*sqrt(c) + x)/2 - b*c*atanh
(c/x**2)/2 + b*x**2*atanh(c/x**2)/2
```

3.161 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=30

$$a\log(x)+\frac{1}{4}b\text{Li}_2\left(-\frac{c}{x^2}\right)-\frac{1}{4}b\text{Li}_2\left(\frac{c}{x^2}\right)$$

```
[Out] a*ln(x)+1/4*b*polylog(2,-c/x^2)-1/4*b*polylog(2,c/x^2)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6095, 5912}

$$\frac{1}{4}b\text{PolyLog}\left(2,-\frac{c}{x^2}\right)-\frac{1}{4}b\text{PolyLog}\left(2,\frac{c}{x^2}\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x,x]
```

```
[Out] a*Log[x] + (b*PolyLog[2, -(c/x^2)])/4 - (b*PolyLog[2, c/x^2])/4
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,\frac{1}{x^2}\right)\right)\\ &= a\log(x)+\frac{1}{4}b\text{Li}_2\left(-\frac{c}{x^2}\right)-\frac{1}{4}b\text{Li}_2\left(\frac{c}{x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.93

$$a\log(x)+\frac{1}{4}b\left(\text{Li}_2\left(-\frac{c}{x^2}\right)-\text{Li}_2\left(\frac{c}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x,x]
```

```
[Out] a*Log[x] + (b*(PolyLog[2, -(c/x^2)] - PolyLog[2, c/x^2]))/4
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(\frac{c}{x^2}\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c/x^2) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)/x, x)
```

maple [B] time = 0.06, size = 154, normalized size = 5.13

$$-a \ln\left(\frac{1}{x}\right) - b \ln\left(\frac{1}{x}\right) \operatorname{arctanh}\left(\frac{c}{x^2}\right) + \frac{b \ln\left(\frac{1}{x}\right) \ln\left(1 + \frac{\sqrt{-c}}{x}\right)}{2} + \frac{b \ln\left(\frac{1}{x}\right) \ln\left(1 - \frac{\sqrt{-c}}{x}\right)}{2} + \frac{b \operatorname{dilog}\left(1 + \frac{\sqrt{-c}}{x}\right)}{2} + \frac{b \operatorname{dilog}\left(1 - \right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x,x)
```

```
[Out] -a*ln(1/x)-b*ln(1/x)*arctanh(c/x^2)+1/2*b*ln(1/x)*ln(1+(-c)^(1/2)/x)+1/2*b*
ln(1/x)*ln(1-(-c)^(1/2)/x)+1/2*b*dilog(1+(-c)^(1/2)/x)+1/2*b*dilog(1-(-c)^(
1/2)/x)-1/2*b*ln(1/x)*ln(1-1/x*c^(1/2))-1/2*b*ln(1/x)*ln(1+1/x*c^(1/2))-1/2
*b*dilog(1-1/x*c^(1/2))-1/2*b*dilog(1+1/x*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} b \int \frac{\log\left(\frac{c}{x^2} + 1\right) - \log\left(-\frac{c}{x^2} + 1\right)}{x}\, dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c/x^2 + 1) - log(-c/x^2 + 1))/x, x) + a*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x,x)
```

```
[Out] int((a + b*atanh(c/x^2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))/x, x)
```

3.162 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^3}\,dx$

Optimal. Leaf size=37

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}$$

```
[Out] 1/2*(-a-b*arctanh(c/x^2))/x^2-1/4*b*ln(1-c^2/x^4)/c
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 260}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^3,x]
```

```
[Out] -(a + b*ArcTanh[c/x^2])/(2*x^2) - (b*Log[1 - c^2/x^4])/(4*c)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-(bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^5}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.14

$$-\frac{a}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*ArcTanh[c/x^2])/(2*x^2) - (b*Log[1 - c^2/x^4])/(4*c)
```

fricas [A] time = 0.55, size = 55, normalized size = 1.49

$$-\frac{bx^2\log\left(x^4-c^2\right)-4\,bx^2\log(x)+bc\log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac}{4\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*x^2*log(x^4 - c^2) - 4*b*x^2*log(x) + b*c*log((x^2 + c)/(x^2 - c))
+ 2*a*c)/(c*x^2)
```

giac [A] time = 0.19, size = 52, normalized size = 1.41

$$-\frac{b\log\left(x^4-c^2\right)}{4\,c}+\frac{b\log(x)}{c}-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{4\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*b*log(x^4 - c^2)/c + b*log(x)/c - 1/4*b*log((x^2 + c)/(x^2 - c))/x^2 -
 1/2*a/x^2
```

maple [A] time = 0.02, size = 37, normalized size = 1.00

$$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\ln\left(1-\frac{c^2}{x^4}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c/x^2)-1/4*b*ln(1-c^2/x^4)/c
```

maxima [A] time = 0.32, size = 37, normalized size = 1.00

$$-\frac{b\left(\frac{2\,c\,\mathrm{artanh}\left(\frac{c}{x^2}\right)}{x^2}+\log\left(-\frac{c^2}{x^4}+1\right)\right)}{4\,c}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*c*arctanh(c/x^2)/x^2 + log(-c^2/x^4 + 1))/c - 1/2*a/x^2
```

mupad [B] time = 0.84, size = 56, normalized size = 1.51

$$\frac{b\,\ln(x)}{c}-\frac{b\,\ln\left(x^4-c^2\right)}{4\,c}-\frac{a}{2\,x^2}-\frac{b\,\ln\left(x^2+c\right)}{4\,x^2}+\frac{b\,\ln\left(x^2-c\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x^3,x)
```

```
[Out] (b*log(x))/c - (b*log(x^4 - c^2))/(4*c) - a/(2*x^2) - (b*log(c + x^2))/(4*x
^2) + (b*log(x^2 - c))/(4*x^2)
```

sympy [A] time = 11.52, size = 76, normalized size = 2.05

$$\begin{cases} -\frac{a}{2x^2} - \frac{b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{2x^2} + \frac{b\log(x)}{c} - \frac{b\log\left(-i\sqrt{c}+x\right)}{2c} - \frac{b\log\left(i\sqrt{c}+x\right)}{2c} + \frac{b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{2c} & \text{for } c \neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - b*atanh(c/x**2)/(2*x**2) + b*log(x)/c - b*log(-I*s
qrt(c) + x)/(2*c) - b*log(I*sqrt(c) + x)/(2*c) + b*atanh(c/x**2)/(2*c), Ne(
c, 0)), (-a/(2*x**2), True))
```

3.163 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^5}\,dx$

Optimal. Leaf size=45

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}+\frac{b\tanh^{-1}\left(\frac{x^2}{c}\right)}{4c^2}-\frac{b}{4cx^2}$$

```
[Out] -1/4*b/c/x^2+1/4*(-a-b*arctanh(c/x^2))/x^4+1/4*b*arctanh(x^2/c)/c^2
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 263, 275, 325, 207}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}+\frac{b\tanh^{-1}\left(\frac{x^2}{c}\right)}{4c^2}-\frac{b}{4cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^5,x]
```

```
[Out] -b/(4*c*x^2) - (a + b*ArcTanh[c/x^2])/(4*x^4) + (b*ArcTanh[x^2/c])/(4*c^2)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^5}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{1}{2}(bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^7}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{1}{2}(bc)\int \frac{1}{x^3\left(-c^2+x^4\right)}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(-c^2+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{b}{4cx^2}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b\,\text{Subst}\left(\int \frac{1}{-c^2+x^2}\,dx,x,x^2\right)}{4c}\\
&= -\frac{b}{4cx^2}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}+\frac{b\tanh^{-1}\left(\frac{x^2}{c}\right)}{4c^2}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 64, normalized size = 1.42

$$
-\frac{a}{4x^4}-\frac{b\log\left(x^2-c\right)}{8c^2}+\frac{b\log\left(c+x^2\right)}{8c^2}-\frac{b}{4cx^2}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(4*c*x^2) - (b*ArcTanh[c/x^2])/(4*x^4) - (b*Log[-c + x^2])/(
8*c^2) + (b*Log[c + x^2])/(8*c^2)
```

fricas [A] time = 0.69, size = 52, normalized size = 1.16

$$
-\frac{2\,bcx^2+2\,ac^2-\left(bx^4-bc^2\right)\log\left(\frac{x^2+c}{x^2-c}\right)}{8\,c^2x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*c*x^2 + 2*a*c^2 - (b*x^4 - b*c^2)*log((x^2 + c)/(x^2 - c)))/(c^2*
x^4)
```

giac [A] time = 0.70, size = 66, normalized size = 1.47

$$
\frac{b\log\left(x^2+c\right)}{8\,c^2}-\frac{b\log\left(-x^2+c\right)}{8\,c^2}-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{8\,x^4}-\frac{bx^2+ac}{4\,cx^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b*log(x^2 + c)/c^2 - 1/8*b*log(-x^2 + c)/c^2 - 1/8*b*log((x^2 + c)/(x^2
 - c))/x^4 - 1/4*(b*x^2 + a*c)/(c*x^4)
```

maple [A] time = 0.04, size = 57, normalized size = 1.27

$$
-\frac{a}{4x^4}-\frac{b\,\text{arctanh}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b}{4c\,x^2}+\frac{b\ln\left(1+\frac{c}{x^2}\right)}{8c^2}-\frac{b\ln\left(\frac{c}{x^2}-1\right)}{8c^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctanh(c/x^2)-1/4*b/c/x^2+1/8*b/c^2*ln(1+c/x^2)-1/8*b
/c^2*ln(c/x^2-1)
```

maxima [A] time = 0.30, size = 56, normalized size = 1.24

$$\frac{1}{8}\left(c\left(\frac{\log\left(x^2+c\right)}{c^3}-\frac{\log\left(x^2-c\right)}{c^3}-\frac{2}{c^2x^2}\right)-\frac{2\,\mathrm{artanh}\left(\frac{c}{x^2}\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(c*(log(x^2 + c)/c^3 - log(x^2 - c)/c^3 - 2/(c^2*x^2)) - 2*arctanh(c/x^
2)/x^4)*b - 1/4*a/x^4
```

mupad [B] time = 1.00, size = 59, normalized size = 1.31

$$\frac{\frac{b\,x^4\,\mathrm{atanh}\left(\frac{x^2}{c}\right)}{4}-\frac{b\,c\,x^2}{4}}{c^2\,x^4}-\frac{\frac{a}{4}-\frac{b\,\ln\left(x^2-c\right)}{8}+\frac{b\,\ln\left(x^2+c\right)}{8}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x^5,x)
```

```
[Out] ((b*x^4*atanh(x^2/c))/4 - (b*c*x^2)/4)/(c^2*x^4) - (a/4 - (b*log(x^2 - c))/
8 + (b*log(c + x^2))/8)/x^4
```

sympy [A] time = 15.09, size = 49, normalized size = 1.09

$$\begin{cases} -\frac{a}{4x^4}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b}{4cx^2}+\frac{b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{4c^2} & \text{for}\ c\neq 0\\ -\frac{a}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**5,x)
```

```
[Out] Piecewise((-a/(4*x**4) - b*atanh(c/x**2)/(4*x**4) - b/(4*c*x**2) + b*atanh(
c/x**2)/(4*c**2), Ne(c, 0)), (-a/(4*x**4), True))
```

3.164 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^7}\,dx$

Optimal. Leaf size=57

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2-x^4\right)}{12c^3}-\frac{b}{12cx^4}$$

```
[Out] -1/12*b/c/x^4+1/6*(-a-b*arctanh(c/x^2))/x^6+1/3*b*ln(x)/c^3-1/12*b*ln(-x^4+
c^2)/c^3
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 263, 266, 44}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{b\log\left(c^2-x^4\right)}{12c^3}+\frac{b\log(x)}{3c^3}-\frac{b}{12cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^7,x]
```

```
[Out] -b/(12*c*x^4) - (a + b*ArcTanh[c/x^2])/(6*x^6) + (b*Log[x])/(3*c^3) - (b*Lo
g[c^2 - x^4])/(12*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^7}\,dx = -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{1}{3}(bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^9}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{1}{3}(bc)\int \frac{1}{x^5\left(-c^2+x^4\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{1}{12}(bc)\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(-c^2+x\right)}\,dx, x, x^4\right)$$
$$= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{1}{12}(bc)\,\mathrm{Subst}\left(\int \left(-\frac{1}{c^4\left(c^2-x\right)} - \frac{1}{c^2x^2} - \frac{1}{c^4x}\right)dx, x, x^4\right)$$
$$= -\frac{b}{12cx^4} - \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} + \frac{b\log(x)}{3c^3} - \frac{b\log\left(c^2-x^4\right)}{12c^3}$$

Mathematica [A] time = 0.01, size = 62, normalized size = 1.09

$$-\frac{a}{6x^6} + \frac{b\log(x)}{3c^3} - \frac{b\log\left(x^4-c^2\right)}{12c^3} - \frac{b}{12cx^4} - \frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^7,x]
```

```
[Out] -1/6*a/x^6 - b/(12*c*x^4) - (b*ArcTanh[c/x^2])/(6*x^6) + (b*Log[x])/(3*c^3)
 - (b*Log[-c^2 + x^4])/(12*c^3)
```

fricas [A] time = 0.64, size = 67, normalized size = 1.18

$$-\frac{bx^6\log\left(x^4-c^2\right) - 4\,bx^6\log(x) + bc^2x^2 + bc^3\log\left(\frac{x^2+c}{x^2-c}\right) + 2\,ac^3}{12\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(b*x^6*log(x^4 - c^2) - 4*b*x^6*log(x) + b*c^2*x^2 + b*c^3*log((x^2 +
 c)/(x^2 - c)) + 2*a*c^3)/(c^3*x^6)
```

giac [A] time = 0.24, size = 65, normalized size = 1.14

$$-\frac{b\log\left(x^4-c^2\right)}{12\,c^3} + \frac{b\log(x)}{3\,c^3} - \frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{12\,x^6} - \frac{bx^2+2\,ac}{12\,cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="giac")
```

```
[Out] -1/12*b*log(x^4 - c^2)/c^3 + 1/3*b*log(x)/c^3 - 1/12*b*log((x^2 + c)/(x^2 -
 c))/x^6 - 1/12*(b*x^2 + 2*a*c)/(c*x^6)
```

maple [A] time = 0.04, size = 45, normalized size = 0.79

$$-\frac{a}{6x^6} - \frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{b}{12c\,x^4} - \frac{b\ln\left(\frac{c^2}{x^4}-1\right)}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^7,x)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctanh(c/x^2)-1/12*b/c/x^4-1/12*b/c^3*ln(c^2/x^4-1)
```

maxima [A] time = 0.32, size = 55, normalized size = 0.96

$$-\frac{1}{12}\left(c\left(\frac{\log\left(x^4-c^2\right)}{c^4}-\frac{\log\left(x^4\right)}{c^4}+\frac{1}{c^2x^4}\right)+\frac{2\,\operatorname{artanh}\left(\frac{c}{x^2}\right)}{x^6}\right)b-\frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(c*(log(x^4 - c^2)/c^4 - log(x^4)/c^4 + 1/(c^2*x^4)) + 2*arctanh(c/x^
2)/x^6)*b - 1/6*a/x^6
```

mupad [B] time = 0.89, size = 66, normalized size = 1.16

$$\frac{b\,\ln(x)}{3\,c^3}-\frac{b\,\ln\left(x^4-c^2\right)}{12\,c^3}-\frac{b}{12\,c\,x^4}-\frac{a}{6\,x^6}-\frac{b\,\ln\left(x^2+c\right)}{12\,x^6}+\frac{b\,\ln\left(x^2-c\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x^7,x)
```

```
[Out] (b*log(x))/(3*c^3) - (b*log(x^4 - c^2))/(12*c^3) - b/(12*c*x^4) - a/(6*x^6)
 - (b*log(c + x^2))/(12*x^6) + (b*log(x^2 - c))/(12*x^6)
```

sympy [A] time = 23.62, size = 94, normalized size = 1.65

$$\begin{cases} -\frac{a}{6x^6}-\frac{b\operatorname{atanh}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{b}{12cx^4}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(-i\sqrt{c}+x\right)}{6c^3}-\frac{b\log\left(i\sqrt{c}+x\right)}{6c^3}+\frac{b\operatorname{atanh}\left(\frac{c}{x^2}\right)}{6c^3} & \text{for}\ c\neq 0\\ -\frac{a}{6x^6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**7,x)
```

```
[Out] Piecewise((-a/(6*x**6) - b*atanh(c/x**2)/(6*x**6) - b/(12*c*x**4) + b*log(x
)/(3*c**3) - b*log(-I*sqrt(c) + x)/(6*c**3) - b*log(I*sqrt(c) + x)/(6*c**3)
 + b*atanh(c/x**2)/(6*c**3), Ne(c, 0)), (-a/(6*x**6), True))
```

3.165 $\int x^4 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=63

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2}{15}bcx^3$$

```
[Out] 2/15*b*c*x^3+1/5*b*c^(5/2)*arctan(x/c^(1/2))+1/5*x^5*(a+b*arctanh(c/x^2))-1
/5*b*c^(5/2)*arctanh(x/c^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6097, 263, 321, 298, 203, 206}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2}{15}bcx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (2*b*c*x^3)/15 + (b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 + (x^5*(a + b*ArcTanh[c/x^
2]))/5 - (b*c^(5/2)*ArcTanh[x/Sqrt[c]])/5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
```

```
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)dx&=\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}(2bc)\int\frac{x^2}{1-\frac{c^2}{x^4}}\,dx\\&=\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}(2bc)\int\frac{x^6}{-c^2+x^4}\,dx\\&=\frac{2}{15}bcx^3+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}\left(2bc^3\right)\int\frac{x^2}{-c^2+x^4}\,dx\\&=\frac{2}{15}bcx^3+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{5}\left(bc^3\right)\int\frac{1}{c-x^2}\,dx+\frac{1}{5}\left(bc^3\right)\int\frac{1}{c+x^2}\\&=\frac{2}{15}bcx^3+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 88, normalized size = 1.40

$$\frac{ax^5}{5}+\frac{1}{10}bc^{5/2}\log\left(\sqrt{c}-x\right)-\frac{1}{10}bc^{5/2}\log\left(\sqrt{c}+x\right)+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2}{15}bcx^3+\frac{1}{5}bx^5\tanh^{-1}\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (2*b*c*x^3)/15 + (a*x^5)/5 + (b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 + (b*x^5*ArcTa
nh[c/x^2])/5 + (b*c^(5/2)*Log[Sqrt[c] - x])/10 - (b*c^(5/2)*Log[Sqrt[c] + x
])/10
```

fricas [A] time = 0.63, size = 170, normalized size = 2.70

$$\left[\frac{1}{10}bx^5\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{5}ax^5+\frac{2}{15}bcx^3+\frac{1}{5}bc^{\frac{5}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{10}bc^{\frac{5}{2}}\log\left(\frac{x^2-2\sqrt{c}\,x+c}{x^2-c}\right),\frac{1}{10}bx^5\log\left(\frac{x^2}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] [1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b*c^(
5/2)*arctan(x/sqrt(c)) + 1/10*b*c^(5/2)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 -
c)), 1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b
*sqrt(-c)*c^2*arctan(sqrt(-c)*x/c) + 1/10*b*sqrt(-c)*c^2*log((x^2 + 2*sqrt(
-c)*x - c)/(x^2 + c))]
```

giac [A] time = 0.30, size = 67, normalized size = 1.06

$$\frac{1}{10}bx^5\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{5}ax^5+\frac{2}{15}bcx^3+\frac{bc^3\arctan\left(\frac{x}{\sqrt{-c}}\right)}{5\sqrt{-c}}+\frac{1}{5}bc^{\frac{5}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b*c^3*
arctan(x/sqrt(-c))/sqrt(-c) + 1/5*b*c^(5/2)*arctan(x/sqrt(c))
```

maple [A] time = 0.04, size = 53, normalized size = 0.84

$$\frac{a\,x^5}{5}+\frac{b\,x^5\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{5}+\frac{2bc\,x^3}{15}+\frac{b\,c^{\frac{5}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)}{5}-\frac{b\,c^{\frac{5}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c/x^2)),x)
```

```
[Out] 1/5*a*x^5+1/5*b*x^5*arctanh(c/x^2)+2/15*b*c*x^3+1/5*b*c^(5/2)*arctan(x/c^(1
/2))-1/5*b*c^(5/2)*arctanh(1/x*c^(1/2))
```

maxima [A] time = 0.41, size = 62, normalized size = 0.98

$$\frac{1}{5}\,ax^5+\frac{1}{30}\left(6\,x^5\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+\left(4\,x^3+6\,c^{\frac{3}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)+3\,c^{\frac{3}{2}}\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/30*(6*x^5*arctanh(c/x^2) + (4*x^3 + 6*c^(3/2)*arctan(x/sqrt(c
)) + 3*c^(3/2)*log((x - sqrt(c))/(x + sqrt(c))))*c)*b
```

mupad [B] time = 0.98, size = 67, normalized size = 1.06

$$\frac{a\,x^5}{5}+\frac{b\,c^{5/2}\,\mathrm{atan}\left(\frac{x}{\sqrt{c}}\right)}{5}+\frac{b\,x^5\,\ln\left(x^2+c\right)}{10}+\frac{2\,b\,c\,x^3}{15}-\frac{b\,x^5\,\ln\left(x^2-c\right)}{10}+\frac{b\,c^{5/2}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c/x^2)),x)
```

```
[Out] (a*x^5)/5 + (b*c^(5/2)*atan(x/c^(1/2)))/5 + (b*c^(5/2)*atan((x*1i)/c^(1/2))
*1i)/5 + (b*x^5*log(c + x^2))/10 + (2*b*c*x^3)/15 - (b*x^5*log(x^2 - c))/10
```

sympy [A] time = 11.52, size = 717, normalized size = 11.38

$$\begin{cases}\frac{ax^5}{5}\\ \frac{x^5(a-\infty b)}{5}\\ \frac{x^5(a+\infty b)}{5}\\ -\frac{6iac^{\frac{5}{2}}x^5}{-30ic^{\frac{5}{2}}+30i\sqrt{c}\,x^4}+\frac{6ia\sqrt{c}\,x^9}{-30ic^{\frac{5}{2}}+30i\sqrt{c}\,x^4}-\frac{4ibc^{\frac{7}{2}}x^3}{-30ic^{\frac{5}{2}}+30i\sqrt{c}\,x^4}-\frac{6ibc^{\frac{5}{2}}x^5\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{-30ic^{\frac{5}{2}}+30i\sqrt{c}\,x^4}+\frac{4ibc^{\frac{3}{2}}x^7}{-30ic^{\frac{5}{2}}+30i\sqrt{c}\,x^4}+\frac{6ib\sqrt{c}\,x^9\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{-30ic^{\frac{5}{2}}+30i\sqrt{c}\,x^4}-\frac{6ibc^5\log}{-30ic^{\frac{5}{2}}+}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c/x**2)),x)
```

```
[Out] Piecewise((a*x**5/5, Eq(c, 0)), (x**5*(a - oo*b)/5, Eq(c, -x**2)), (x**5*(a
 + oo*b)/5, Eq(c, x**2)), (-6*I*a*c**(5/2)*x**5/(-30*I*c**(5/2) + 30*I*sqrt
(c)*x**4) + 6*I*a*sqrt(c)*x**9/(-30*I*c**(5/2) + 30*I*sqrt(c)*x**4) - 4*I*b
*c**(7/2)*x**3/(-30*I*c**(5/2) + 30*I*sqrt(c)*x**4) - 6*I*b*c**(5/2)*x**5*a
tanh(c/x**2)/(-30*I*c**(5/2) + 30*I*sqrt(c)*x**4) + 4*I*b*c**(3/2)*x**7/(-3
0*I*c**(5/2) + 30*I*sqrt(c)*x**4) + 6*I*b*sqrt(c)*x**9*atanh(c/x**2)/(-30*I
*c**(5/2) + 30*I*sqrt(c)*x**4) - 6*I*b*c**5*log(-sqrt(c) + x)/(-30*I*c**(5/
2) + 30*I*sqrt(c)*x**4) - 3*b*c**5*log(-I*sqrt(c) + x)/(-30*I*c**(5/2) + 30
```

```
*I*sqrt(c)*x**4) + 3*I*b*c**5*log(-I*sqrt(c) + x)/(-30*I*c**(5/2) + 30*I*sq
rt(c)*x**4) + 3*b*c**5*log(I*sqrt(c) + x)/(-30*I*c**(5/2) + 30*I*sqrt(c)*x*
*4) + 3*I*b*c**5*log(I*sqrt(c) + x)/(-30*I*c**(5/2) + 30*I*sqrt(c)*x**4) -
6*I*b*c**5*atanh(c/x**2)/(-30*I*c**(5/2) + 30*I*sqrt(c)*x**4) + 6*I*b*c**3*
x**4*log(-sqrt(c) + x)/(-30*I*c**(5/2) + 30*I*sqrt(c)*x**4) + 3*b*c**3*x**4
*log(-I*sqrt(c) + x)/(-30*I*c**(5/2) + 30*I*sqrt(c)*x**4) - 3*I*b*c**3*x**4
*log(-I*sqrt(c) + x)/(-30*I*c**(5/2) + 30*I*sqrt(c)*x**4) - 3*b*c**3*x**4*l
og(I*sqrt(c) + x)/(-30*I*c**(5/2) + 30*I*sqrt(c)*x**4) - 3*I*b*c**3*x**4*lo
g(I*sqrt(c) + x)/(-30*I*c**(5/2) + 30*I*sqrt(c)*x**4) + 6*I*b*c**3*x**4*ata
nh(c/x**2)/(-30*I*c**(5/2) + 30*I*sqrt(c)*x**4), True))
```

3.166 $\int x^2 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=61

$$\frac{1}{3}x^3\left(a + b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) - \frac{1}{3}bc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{3}bc^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{2bcx}{3}$$

```
[Out] 2/3*b*c*x-1/3*b*c^(3/2)*arctan(x/c^(1/2))+1/3*x^3*(a+b*arctanh(c/x^2))-1/3*
b*c^(3/2)*arctanh(x/c^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6097, 193, 321, 212, 206, 203}

$$\frac{1}{3}x^3\left(a + b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) - \frac{1}{3}bc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{3}bc^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{2bcx}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (2*b*c*x)/3 - (b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (x^3*(a + b*ArcTanh[c/x^2])
)/3 - (b*c^(3/2)*ArcTanh[x/Sqrt[c]])/3
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
```

```
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}(2bc)\int\frac{1}{1-\frac{c^2}{x^4}}\,dx\\
&= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}(2bc)\int\frac{x^4}{-c^2+x^4}\,dx\\
&= \frac{2bcx}{3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}\left(2bc^3\right)\int\frac{1}{-c^2+x^4}\,dx\\
&= \frac{2bcx}{3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{3}\left(bc^2\right)\int\frac{1}{c-x^2}\,dx-\frac{1}{3}\left(bc^2\right)\int\frac{1}{c+x^2}\,dx\\
&= \frac{2bcx}{3}-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 86, normalized size = 1.41

$$\frac{ax^3}{3}+\frac{1}{6}bc^{3/2}\log\left(\sqrt{c}-x\right)-\frac{1}{6}bc^{3/2}\log\left(\sqrt{c}+x\right)-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}bx^3\tanh^{-1}\left(\frac{c}{x^2}\right)+\frac{2bcx}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (2*b*c*x)/3 + (a*x^3)/3 - (b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (b*x^3*ArcTanh[
c/x^2])/3 + (b*c^(3/2)*Log[Sqrt[c] - x])/6 - (b*c^(3/2)*Log[Sqrt[c] + x])/6
```

fricas [A] time = 0.65, size = 162, normalized size = 2.66

$$\left[\frac{1}{6}\,bx^3\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{3}\,ax^3-\frac{1}{3}\,bc^{\frac{3}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{6}\,bc^{\frac{3}{2}}\log\left(\frac{x^2-2\sqrt{c}\,x+c}{x^2-c}\right)+\frac{2}{3}\,bcx,\frac{1}{6}\,bx^3\log\left(\frac{x^2+c}{x^2-c}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] [1/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 - 1/3*b*c^(3/2)*arctan(x/sq
rt(c)) + 1/6*b*c^(3/2)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 - c)) + 2/3*b*c*x,
1/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 + 1/3*b*sqrt(-c)*c*arctan(sq
rt(-c)*x/c) + 1/6*b*sqrt(-c)*c*log((x^2 - 2*sqrt(-c)*x - c)/(x^2 + c)) + 2/
3*b*c*x]
```

giac [A] time = 0.21, size = 69, normalized size = 1.13

$$\frac{1}{3}\,bc^3\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)+\frac{1}{6}\,bx^3\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{3}\,ax^3+\frac{2}{3}\,bcx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/3*b*c^3*(arctan(x/sqrt(-c))/(sqrt(-c)*c) - arctan(x/sqrt(c))/c^(3/2)) + 1
/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 + 2/3*b*c*x
```

maple [A] time = 0.04, size = 51, normalized size = 0.84

$$\frac{x^3 a}{3}+\frac{b\,x^3\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{3}+\frac{2xbc}{3}-\frac{b\,c^{\frac{3}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)}{3}-\frac{b\,c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x^2)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c/x^2)+2/3*x*b*c-1/3*b*c^(3/2)*arctan(x/c^(1/2)
)-1/3*b*c^(3/2)*arctanh(1/x*c^(1/2))
```

maxima [A] time = 0.42, size = 61, normalized size = 1.00

$$\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\operatorname{artanh}\left(\frac{c}{x^2}\right)-\left(2\,\sqrt{c}\,\arctan\left(\frac{x}{\sqrt{c}}\right)-\sqrt{c}\,\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)-4\,x\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c/x^2) - (2*sqrt(c)*arctan(x/sqrt(c)) - sqrt
(c)*log((x - sqrt(c))/(x + sqrt(c))) - 4*x)*c)*b
```

mupad [B] time = 0.90, size = 65, normalized size = 1.07

$$\frac{a\,x^3}{3}-\frac{b\,c^{3/2}\,\mathrm{atan}\left(\frac{x}{\sqrt{c}}\right)}{3}+\frac{2\,b\,c\,x}{3}+\frac{b\,x^3\,\ln\left(x^2+c\right)}{6}-\frac{b\,x^3\,\ln\left(x^2-c\right)}{6}+\frac{b\,c^{3/2}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c/x^2)),x)
```

```
[Out] (a*x^3)/3 - (b*c^(3/2)*atan(x/c^(1/2)))/3 + (b*c^(3/2)*atan((x*1i)/c^(1/2))
*1i)/3 + (2*b*c*x)/3 + (b*x^3*log(c + x^2))/6 - (b*x^3*log(x^2 - c))/6
```

sympy [A] time = 7.79, size = 702, normalized size = 11.51

$$\begin{cases} \frac{ax^3}{3} \\ \frac{x^3(a-\infty b)}{3} \\ \frac{x^3(a+\infty b)}{3} \\ -\frac{2iac^{\frac{5}{2}}x^3}{-6ic^{\frac{5}{2}}+6i\sqrt{c}x^4}+\frac{2ia\sqrt{c}x^7}{-6ic^{\frac{5}{2}}+6i\sqrt{c}x^4}-\frac{4ibc^{\frac{7}{2}}x}{-6ic^{\frac{5}{2}}+6i\sqrt{c}x^4}-\frac{2ibc^{\frac{5}{2}}x^3\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-6ic^{\frac{5}{2}}+6i\sqrt{c}x^4}+\frac{4ibc^{\frac{3}{2}}x^5}{-6ic^{\frac{5}{2}}+6i\sqrt{c}x^4}+\frac{2ib\sqrt{c}x^7\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-6ic^{\frac{5}{2}}+6i\sqrt{c}x^4}-\frac{2ibc^4\log\left(-\sqrt{c}+x\right)}{-6ic^{\frac{5}{2}}+6i\sqrt{c}x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x**2)),x)
```

```
[Out] Piecewise((a*x**3/3, Eq(c, 0)), (x**3*(a - oo*b)/3, Eq(c, -x**2)), (x**3*(a
 + oo*b)/3, Eq(c, x**2)), (-2*I*a*c**(5/2)*x**3/(-6*I*c**(5/2) + 6*I*sqrt(c
)*x**4) + 2*I*a*sqrt(c)*x**7/(-6*I*c**(5/2) + 6*I*sqrt(c)*x**4) - 4*I*b*c**
(7/2)*x/(-6*I*c**(5/2) + 6*I*sqrt(c)*x**4) - 2*I*b*c**(5/2)*x**3*atanh(c/x*
*2)/(-6*I*c**(5/2) + 6*I*sqrt(c)*x**4) + 4*I*b*c**(3/2)*x**5/(-6*I*c**(5/2)
 + 6*I*sqrt(c)*x**4) + 2*I*b*sqrt(c)*x**7*atanh(c/x**2)/(-6*I*c**(5/2) + 6*
I*sqrt(c)*x**4) - 2*I*b*c**4*log(-sqrt(c) + x)/(-6*I*c**(5/2) + 6*I*sqrt(c)
*x**4) + b*c**4*log(-I*sqrt(c) + x)/(-6*I*c**(5/2) + 6*I*sqrt(c)*x**4) + I*
```

```
b*c**4*log(-I*sqrt(c) + x)/(-6*I*c**(5/2) + 6*I*sqrt(c)*x**4) - b*c**4*log(
I*sqrt(c) + x)/(-6*I*c**(5/2) + 6*I*sqrt(c)*x**4) + I*b*c**4*log(I*sqrt(c)
+ x)/(-6*I*c**(5/2) + 6*I*sqrt(c)*x**4) - 2*I*b*c**4*atanh(c/x**2)/(-6*I*c*
*(5/2) + 6*I*sqrt(c)*x**4) + 2*I*b*c**2*x**4*log(-sqrt(c) + x)/(-6*I*c**(5/
2) + 6*I*sqrt(c)*x**4) - b*c**2*x**4*log(-I*sqrt(c) + x)/(-6*I*c**(5/2) + 6
*I*sqrt(c)*x**4) - I*b*c**2*x**4*log(-I*sqrt(c) + x)/(-6*I*c**(5/2) + 6*I*s
qrt(c)*x**4) + b*c**2*x**4*log(I*sqrt(c) + x)/(-6*I*c**(5/2) + 6*I*sqrt(c)*
x**4) - I*b*c**2*x**4*log(I*sqrt(c) + x)/(-6*I*c**(5/2) + 6*I*sqrt(c)*x**4)
 + 2*I*b*c**2*x**4*atanh(c/x**2)/(-6*I*c**(5/2) + 6*I*sqrt(c)*x**4), True))
```

3.167 $\int \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=44

$$ax+bx\tanh^{-1}\left(\frac{c}{x^2}\right)+b\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-b\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

```
[Out] a*x+b*x*arctanh(c/x^2)+b*arctan(x/c^(1/2))*c^(1/2)-b*arctanh(x/c^(1/2))*c^(
1/2)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6091, 263, 298, 203, 206}

$$ax+bx\tanh^{-1}\left(\frac{c}{x^2}\right)+b\sqrt{c}\,\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-b\sqrt{c}\,\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c/x^2],x]
```

```
[Out] a*x + b*Sqrt[c]*ArcTan[x/Sqrt[c]] + b*x*ArcTanh[c/x^2] - b*Sqrt[c]*ArcTanh[
x/Sqrt[c]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \left(a + b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx = ax + b\int \tanh^{-1}\left(\frac{c}{x^2}\right) dx$$
$$= ax + bx\tanh^{-1}\left(\frac{c}{x^2}\right) + (2bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^2} dx$$
$$= ax + bx\tanh^{-1}\left(\frac{c}{x^2}\right) + (2bc)\int \frac{x^2}{-c^2+x^4} dx$$
$$= ax + bx\tanh^{-1}\left(\frac{c}{x^2}\right) - (bc)\int \frac{1}{c-x^2} dx + (bc)\int \frac{1}{c+x^2} dx$$
$$= ax + b\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + bx\tanh^{-1}\left(\frac{c}{x^2}\right) - b\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.23

$$ax + bx\tanh^{-1}\left(\frac{c}{x^2}\right) + \frac{1}{2}b\sqrt{c}\left(\log\left(\sqrt{c}-x\right) - \log\left(\sqrt{c}+x\right) + 2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c/x^2],x]
```

```
[Out] a*x + b*x*ArcTanh[c/x^2] + (b*Sqrt[c]*(2*ArcTan[x/Sqrt[c]] + Log[Sqrt[c] -
x] - Log[Sqrt[c] + x]))/2
```

fricas [A] time = 0.58, size = 138, normalized size = 3.14

$$\left[\frac{1}{2}bx\log\left(\frac{x^2+c}{x^2-c}\right) + b\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{2}b\sqrt{c}\log\left(\frac{x^2-2\sqrt{c}x+c}{x^2-c}\right) + ax, \frac{1}{2}bx\log\left(\frac{x^2+c}{x^2-c}\right) + b\sqrt{-c}\arcta$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*b*x*log((x^2 + c)/(x^2 - c)) + b*sqrt(c)*arctan(x/sqrt(c)) + 1/2*b*sqr
t(c)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 - c)) + a*x, 1/2*b*x*log((x^2 + c)/(x
^2 - c)) + b*sqrt(-c)*arctan(sqrt(-c)*x/c) + 1/2*b*sqrt(-c)*log((x^2 + 2*sq
rt(-c)*x - c)/(x^2 + c)) + a*x]
```

giac [A] time = 0.36, size = 57, normalized size = 1.30

$$\frac{1}{2}\left(2c\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}\right) + x\log\left(-\frac{\frac{c}{x^2}+1}{\frac{c}{x^2}-1}\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*c*(arctan(x/sqrt(-c))/sqrt(-c) + arctan(x/sqrt(c))/sqrt(c)) + x*log(
-(c/x^2 + 1)/(c/x^2 - 1)))*b + a*x
```

maple [A] time = 0.04, size = 39, normalized size = 0.89

$$ax + bx\,\mathrm{arctanh}\left(\frac{c}{x^2}\right) - \mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)\sqrt{c}\,b + b\arctan\left(\frac{x}{\sqrt{c}}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c/x^2),x)
```

```
[Out] a*x+b*x*arctanh(c/x^2)-arctanh(1/x*c^(1/2))*c^(1/2)*b+b*arctan(x/c^(1/2))*c
^(1/2)
```

maxima [A] time = 0.42, size = 51, normalized size = 1.16

$$\frac{1}{2}\left(c\left(\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{\sqrt{c}}\right)+2\,x\,\mathrm{artanh}\left(\frac{c}{x^2}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*arctan(x/sqrt(c))/sqrt(c) + log((x - sqrt(c))/(x + sqrt(c)))/sqrt
(c)) + 2*x*arctanh(c/x^2))*b + a*x
```

mupad [B] time = 0.82, size = 52, normalized size = 1.18

$$a\,x+\frac{b\,x\,\ln\left(x^2+c\right)}{2}+b\,\sqrt{c}\,\mathrm{atan}\left(\frac{x}{\sqrt{c}}\right)-\frac{b\,x\,\ln\left(x^2-c\right)}{2}+b\,\sqrt{c}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c/x^2),x)
```

```
[Out] a*x + (b*x*log(c + x^2))/2 + b*c^(1/2)*atan(x/c^(1/2)) + b*c^(1/2)*atan((x*
1i)/c^(1/2))*1i - (b*x*log(x^2 - c))/2
```

sympy [A] time = 5.14, size = 520, normalized size = 11.82

$$ax+b\left(\begin{cases}0\\-\infty x\\\infty x\\-\frac{2ic^{\frac{5}{2}}x\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{-2ic^{\frac{5}{2}}+2i\sqrt{c}x^4}+\frac{2i\sqrt{c}x^5\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{-2ic^{\frac{5}{2}}+2i\sqrt{c}x^4}-\frac{2ic^3\log\left(-\sqrt{c}+x\right)}{-2ic^{\frac{5}{2}}+2i\sqrt{c}x^4}-\frac{c^3\log\left(-i\sqrt{c}+x\right)}{-2ic^{\frac{5}{2}}+2i\sqrt{c}x^4}+\frac{ic^3\log\left(-i\sqrt{c}+x\right)}{-2ic^{\frac{5}{2}}+2i\sqrt{c}x^4}+\frac{c^3\log\left(i\sqrt{c}+x\right)}{-2ic^{\frac{5}{2}}+2i\sqrt{c}x^4}+\frac{ic^3\log(}{-2ic^{\frac{5}{2}}+}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c/x**2),x)
```

```
[Out] a*x + b*Piecewise((0, Eq(c, 0)), (-oo*x, Eq(c, -x**2)), (oo*x, Eq(c, x**2))
, (-2*I*c**(5/2)*x*atanh(c/x**2)/(-2*I*c**(5/2) + 2*I*sqrt(c)*x**4) + 2*I*s
qrt(c)*x**5*atanh(c/x**2)/(-2*I*c**(5/2) + 2*I*sqrt(c)*x**4) - 2*I*c**3*log
(-sqrt(c) + x)/(-2*I*c**(5/2) + 2*I*sqrt(c)*x**4) - c**3*log(-I*sqrt(c) + x
)/(-2*I*c**(5/2) + 2*I*sqrt(c)*x**4) + I*c**3*log(-I*sqrt(c) + x)/(-2*I*c**
(5/2) + 2*I*sqrt(c)*x**4) + c**3*log(I*sqrt(c) + x)/(-2*I*c**(5/2) + 2*I*sq
rt(c)*x**4) + I*c**3*log(I*sqrt(c) + x)/(-2*I*c**(5/2) + 2*I*sqrt(c)*x**4)
- 2*I*c**3*atanh(c/x**2)/(-2*I*c**(5/2) + 2*I*sqrt(c)*x**4) + 2*I*c*x**4*lo
g(-sqrt(c) + x)/(-2*I*c**(5/2) + 2*I*sqrt(c)*x**4) + c*x**4*log(-I*sqrt(c)
+ x)/(-2*I*c**(5/2) + 2*I*sqrt(c)*x**4) - I*c*x**4*log(-I*sqrt(c) + x)/(-2*
I*c**(5/2) + 2*I*sqrt(c)*x**4) - c*x**4*log(I*sqrt(c) + x)/(-2*I*c**(5/2) +
 2*I*sqrt(c)*x**4) - I*c*x**4*log(I*sqrt(c) + x)/(-2*I*c**(5/2) + 2*I*sqrt(
c)*x**4) + 2*I*c*x**4*atanh(c/x**2)/(-2*I*c**(5/2) + 2*I*sqrt(c)*x**4), Tru
e))
```

3.168 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2} dx$

Optimal. Leaf size=46

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] (-a-b*arctanh(c/x^2))/x+b*arctan(x/c^(1/2))/c^(1/2)+b*arctanh(x/c^(1/2))/c^
(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 263, 212, 206, 203}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^2,x]
```

```
[Out] (b*ArcTan[x/Sqrt[c]])/Sqrt[c] - (a + b*ArcTanh[c/x^2])/x + (b*ArcTanh[x/Sqr
t[c]])/Sqrt[c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2}\,dx = -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x} - (2bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^4}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x} - (2bc)\int \frac{1}{-c^2+x^4}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x} + b\int \frac{1}{c-x^2}\,dx + b\int \frac{1}{c+x^2}\,dx$$
$$= \frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.57

$$-\frac{a}{x} - \frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x} - \frac{b\log\left(\sqrt{c}-x\right)}{2\sqrt{c}} + \frac{b\log\left(\sqrt{c}+x\right)}{2\sqrt{c}} + \frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^2,x]
```

```
[Out] -(a/x) + (b*ArcTan[x/Sqrt[c]])/Sqrt[c] - (b*ArcTanh[c/x^2])/x - (b*Log[Sqrt
[c] - x])/(2*Sqrt[c]) + (b*Log[Sqrt[c] + x])/(2*Sqrt[c])
```

fricas [A] time = 0.53, size = 159, normalized size = 3.46

$$\left[\frac{2\,b\sqrt{c}\,x\arctan\left(\frac{x}{\sqrt{c}}\right) + b\sqrt{c}\,x\log\left(\frac{x^2+2\,\sqrt{c}\,x+c}{x^2-c}\right) - bc\log\left(\frac{x^2+c}{x^2-c}\right) - 2\,ac}{2\,cx}, -\frac{2\,b\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-c}\,x}{c}\right) + b\sqrt{-c}\,x\log\left(\frac{x^2\cdots}{}\right.}{2\,cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*sqrt(c)*x*arctan(x/sqrt(c)) + b*sqrt(c)*x*log((x^2 + 2*sqrt(c)*x
+ c)/(x^2 - c)) - b*c*log((x^2 + c)/(x^2 - c)) - 2*a*c)/(c*x), -1/2*(2*b*sq
rt(-c)*x*arctan(sqrt(-c)*x/c) + b*sqrt(-c)*x*log((x^2 - 2*sqrt(-c)*x - c)/(
x^2 + c)) + b*c*log((x^2 + c)/(x^2 - c)) + 2*a*c)/(c*x)]
```

giac [A] time = 0.22, size = 62, normalized size = 1.35

$$-bc\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}\,c} - \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) - \frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{2\,x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^2,x, algorithm="giac")
```

```
[Out] -b*c*(arctan(x/sqrt(-c))/(sqrt(-c)*c) - arctan(x/sqrt(c))/c^(3/2)) - 1/2*b*
log((x^2 + c)/(x^2 - c))/x - a/x
```

maple [A] time = 0.04, size = 44, normalized size = 0.96

$$-\frac{a}{x} - \frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{x} + \frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c/x^2)+b*arctan(x/c^(1/2))/c^(1/2)+b/c^(1/2)*arctanh(1/x*c
^(1/2))
```

maxima [A] time = 0.42, size = 57, normalized size = 1.24

$$\frac{1}{2}\left(c\left(\frac{2\,\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{2\,\operatorname{artanh}\left(\frac{c}{x^{2}}\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*arctan(x/sqrt(c))/c^(3/2) - log((x - sqrt(c))/(x + sqrt(c)))/c^(3
/2)) - 2*arctanh(c/x^2)/x)*b - a/x
```

mupad [B] time = 0.96, size = 59, normalized size = 1.28

$$\frac{b\,\mathrm{atan}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{a}{x}-\frac{b\,\ln\left(x^2+c\right)}{2\,x}+\frac{b\,\ln\left(x^2-c\right)}{2\,x}-\frac{b\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x^2,x)
```

```
[Out] (b*atan(x/c^(1/2)))/c^(1/2) - a/x - (b*atan((x*1i)/c^(1/2))*1i)/c^(1/2) - (
b*log(c + x^2))/(2*x) + (b*log(x^2 - c))/(2*x)
```

sympy [A] time = 9.31, size = 620, normalized size = 13.48

$$\begin{cases} -\frac{a}{x} \\ -\frac{a-\infty b}{x} \\ -\frac{a+\infty b}{x} \\ \frac{2iac^4}{-2ic^4x+2ic^2x^5}-\frac{2iac^2x^4}{-2ic^4x+2ic^2x^5}+\frac{2ibc^{\frac{7}{2}}x\log\left(-\sqrt{c}+x\right)}{-2ic^4x+2ic^2x^5}-\frac{bc^{\frac{7}{2}}x\log\left(-i\sqrt{c}+x\right)}{-2ic^4x+2ic^2x^5}-\frac{ibc^{\frac{7}{2}}x\log\left(-i\sqrt{c}+x\right)}{-2ic^4x+2ic^2x^5}+\frac{bc^{\frac{7}{2}}x\log\left(i\sqrt{c}+x\right)}{-2ic^4x+2ic^2x^5}-\frac{ibc^{\frac{7}{2}}x\log(}{-2ic^4x+2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**2,x)
```

```
[Out] Piecewise((-a/x, Eq(c, 0)), (-(a - oo*b)/x, Eq(c, -x**2)), (-(a + oo*b)/x,
Eq(c, x**2)), (2*I*a*c**4/(-2*I*c**4*x + 2*I*c**2*x**5) - 2*I*a*c**2*x**4/(
-2*I*c**4*x + 2*I*c**2*x**5) + 2*I*b*c**(7/2)*x*log(-sqrt(c) + x)/(-2*I*c**
4*x + 2*I*c**2*x**5) - b*c**(7/2)*x*log(-I*sqrt(c) + x)/(-2*I*c**4*x + 2*I*
c**2*x**5) - I*b*c**(7/2)*x*log(-I*sqrt(c) + x)/(-2*I*c**4*x + 2*I*c**2*x**
5) + b*c**(7/2)*x*log(I*sqrt(c) + x)/(-2*I*c**4*x + 2*I*c**2*x**5) - I*b*c*
*(7/2)*x*log(I*sqrt(c) + x)/(-2*I*c**4*x + 2*I*c**2*x**5) + 2*I*b*c**(7/2)*
x*atanh(c/x**2)/(-2*I*c**4*x + 2*I*c**2*x**5) - 2*I*b*c**(3/2)*x**5*log(-sq
rt(c) + x)/(-2*I*c**4*x + 2*I*c**2*x**5) + b*c**(3/2)*x**5*log(-I*sqrt(c) +
 x)/(-2*I*c**4*x + 2*I*c**2*x**5) + I*b*c**(3/2)*x**5*log(-I*sqrt(c) + x)/(
-2*I*c**4*x + 2*I*c**2*x**5) - b*c**(3/2)*x**5*log(I*sqrt(c) + x)/(-2*I*c**
4*x + 2*I*c**2*x**5) + I*b*c**(3/2)*x**5*log(I*sqrt(c) + x)/(-2*I*c**4*x +
2*I*c**2*x**5) - 2*I*b*c**(3/2)*x**5*atanh(c/x**2)/(-2*I*c**4*x + 2*I*c**2*
x**5) + 2*I*b*c**4*atanh(c/x**2)/(-2*I*c**4*x + 2*I*c**2*x**5) - 2*I*b*c**2
*x**4*atanh(c/x**2)/(-2*I*c**4*x + 2*I*c**2*x**5), True))
```

3.169 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^4}\,dx$

Optimal. Leaf size=65

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{2b}{3cx}$$

```
[Out] -2/3*b/c/x-1/3*b*arctan(x/c^(1/2))/c^(3/2)+1/3*(-a-b*arctanh(c/x^2))/x^3+1/
3*b*arctanh(x/c^(1/2))/c^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6097, 263, 325, 298, 203, 206}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{2b}{3cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^4,x]
```

```
[Out] (-2*b)/(3*c*x) - (b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) - (a + b*ArcTanh[c/x^2])
/(3*x^3) + (b*ArcTanh[x/Sqrt[c]])/(3*c^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{1}{3}(2bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^6}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{1}{3}(2bc)\int \frac{1}{x^2\left(-c^2+x^4\right)}\,dx\\
&= -\frac{2b}{3cx}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{(2b)\int \frac{x^2}{-c^2+x^4}\,dx}{3c}\\
&= -\frac{2b}{3cx}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}+\frac{b\int \frac{1}{c-x^2}\,dx}{3c}-\frac{b\int \frac{1}{c+x^2}\,dx}{3c}\\
&= -\frac{2b}{3cx}-\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 90, normalized size = 1.38

$$
-\frac{a}{3x^3}-\frac{b\log\left(\sqrt{c}-x\right)}{6c^{3/2}}+\frac{b\log\left(\sqrt{c}+x\right)}{6c^{3/2}}-\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{2b}{3cx}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (2*b)/(3*c*x) - (b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) - (b*ArcTanh
[c/x^2])/(3*x^3) - (b*Log[Sqrt[c] - x])/(6*c^(3/2)) + (b*Log[Sqrt[c] + x])/
(6*c^(3/2))
```

fricas [A] time = 0.65, size = 189, normalized size = 2.91

$$
\left[-\frac{2\,b\sqrt{c}\,x^3\arctan\left(\frac{x}{\sqrt{c}}\right)-b\sqrt{c}\,x^3\log\left(\frac{x^2+2\,\sqrt{c}\,x+c}{x^2-c}\right)+4\,bcx^2+bc^2\log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac^2}{6\,c^2x^3},-\frac{2\,b\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{-c}\,x}{c}\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*b*sqrt(c)*x^3*arctan(x/sqrt(c)) - b*sqrt(c)*x^3*log((x^2 + 2*sqrt(
c)*x + c)/(x^2 - c)) + 4*b*c*x^2 + b*c^2*log((x^2 + c)/(x^2 - c)) + 2*a*c^2
)/(c^2*x^3), -1/6*(2*b*sqrt(-c)*x^3*arctan(sqrt(-c)*x/c) + b*sqrt(-c)*x^3*l
og((x^2 + 2*sqrt(-c)*x - c)/(x^2 + c)) + 4*b*c*x^2 + b*c^2*log((x^2 + c)/(x
^2 - c)) + 2*a*c^2)/(c^2*x^3)]
```

giac [A] time = 0.25, size = 72, normalized size = 1.11

$$
-\frac{b\arctan\left(\frac{x}{\sqrt{-c}}\right)}{3\,\sqrt{-c}\,c}-\frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{3\,c^{\frac{3}{2}}}-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{6\,x^3}-\frac{2\,bx^2+ac}{3\,cx^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*b*arctan(x/sqrt(-c))/(sqrt(-c)*c) - 1/3*b*arctan(x/sqrt(c))/c^(3/2) -
1/6*b*log((x^2 + c)/(x^2 - c))/x^3 - 1/3*(2*b*x^2 + a*c)/(c*x^3)
```

maple [A] time = 0.04, size = 55, normalized size = 0.85

$$-\frac{a}{3x^3}-\frac{b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{2b}{3cx}-\frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{3c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c/x^2)-2/3*b/c/x-1/3*b*arctan(x/c^(1/2))/c^(3/
2)+1/3*b/c^(3/2)*arctanh(1/x*c^(1/2))
```

maxima [A] time = 0.41, size = 64, normalized size = 0.98

$$-\frac{1}{6}\left(c\left(\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{4}{c^2x}\right)+\frac{2\operatorname{artanh}\left(\frac{c}{x^2}\right)}{x^3}\right)b-\frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*(2*arctan(x/sqrt(c))/c^(5/2) + log((x - sqrt(c))/(x + sqrt(c)))/c^(
5/2) + 4/(c^2*x)) + 2*arctanh(c/x^2)/x^3)*b - 1/3*a/x^3
```

mupad [B] time = 0.99, size = 69, normalized size = 1.06

$$\frac{b\ln\left(x^2-c\right)}{6x^3}-\frac{2b}{3cx}-\frac{b\operatorname{atan}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{b\ln\left(x^2+c\right)}{6x^3}-\frac{a}{3x^3}-\frac{b\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)1\mathrm{i}}{3c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x^4,x)
```

```
[Out] (b*log(x^2 - c))/(6*x^3) - (2*b)/(3*c*x) - (b*atan(x/c^(1/2)))/(3*c^(3/2))
- (b*atan((x*1i)/c^(1/2))*1i)/(3*c^(3/2)) - (b*log(c + x^2))/(6*x^3) - a/(3
*x^3)
```

sympy [A] time = 13.69, size = 733, normalized size = 11.28

$$\begin{cases}-\frac{a}{3x^3}\\-\frac{a-\infty b}{3x^3}\\-\frac{a+\infty b}{3x^3}\\\frac{2iac^9}{-6ic^9x^3+6ic^7x^7}-\frac{2iac^7x^4}{-6ic^9x^3+6ic^7x^7}+\frac{2ibc^{\frac{15}{2}}x^3\log\left(-\sqrt{c}+x\right)}{-6ic^9x^3+6ic^7x^7}+\frac{bc^{\frac{15}{2}}x^3\log\left(-i\sqrt{c}+x\right)}{-6ic^9x^3+6ic^7x^7}-\frac{ibc^{\frac{15}{2}}x^3\log\left(-i\sqrt{c}+x\right)}{-6ic^9x^3+6ic^7x^7}-\frac{bc^{\frac{15}{2}}x^3\log\left(i\sqrt{c}+x\right)}{-6ic^9x^3+6ic^7x^7}-\frac{ibc^{\frac{1}{2}}}{-}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3), Eq(c, 0)), (-(a - oo*b)/(3*x**3), Eq(c, -x**2)), (-
(a + oo*b)/(3*x**3), Eq(c, x**2)), (2*I*a*c**9/(-6*I*c**9*x**3 + 6*I*c**7*x
**7) - 2*I*a*c**7*x**4/(-6*I*c**9*x**3 + 6*I*c**7*x**7) + 2*I*b*c**(15/2)*x
**3*log(-sqrt(c) + x)/(-6*I*c**9*x**3 + 6*I*c**7*x**7) + b*c**(15/2)*x**3*l
og(-I*sqrt(c) + x)/(-6*I*c**9*x**3 + 6*I*c**7*x**7) - I*b*c**(15/2)*x**3*lo
g(-I*sqrt(c) + x)/(-6*I*c**9*x**3 + 6*I*c**7*x**7) - b*c**(15/2)*x**3*log(I
*sqrt(c) + x)/(-6*I*c**9*x**3 + 6*I*c**7*x**7) - I*b*c**(15/2)*x**3*log(I*s
qrt(c) + x)/(-6*I*c**9*x**3 + 6*I*c**7*x**7) + 2*I*b*c**(15/2)*x**3*atanh(c
/x**2)/(-6*I*c**9*x**3 + 6*I*c**7*x**7) - 2*I*b*c**(11/2)*x**7*log(-sqrt(c)
 + x)/(-6*I*c**9*x**3 + 6*I*c**7*x**7) - b*c**(11/2)*x**7*log(-I*sqrt(c) +
x)/(-6*I*c**9*x**3 + 6*I*c**7*x**7) + I*b*c**(11/2)*x**7*log(-I*sqrt(c) + x
)/(-6*I*c**9*x**3 + 6*I*c**7*x**7) + b*c**(11/2)*x**7*log(I*sqrt(c) + x)/(-
6*I*c**9*x**3 + 6*I*c**7*x**7) + I*b*c**(11/2)*x**7*log(I*sqrt(c) + x)/(-6*
I*c**9*x**3 + 6*I*c**7*x**7) - 2*I*b*c**(11/2)*x**7*atanh(c/x**2)/(-6*I*c**
9*x**3 + 6*I*c**7*x**7) + 2*I*b*c**9*atanh(c/x**2)/(-6*I*c**9*x**3 + 6*I*c*
*7*x**7) + 4*I*b*c**8*x**2/(-6*I*c**9*x**3 + 6*I*c**7*x**7) - 2*I*b*c**7*x*
*4*atanh(c/x**2)/(-6*I*c**9*x**3 + 6*I*c**7*x**7) - 4*I*b*c**6*x**6/(-6*I*c
**9*x**3 + 6*I*c**7*x**7), True))
```

3.170 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^6} dx$

Optimal. Leaf size=65

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{2b}{15cx^3}$$

```
[Out] -2/15*b/c/x^3+1/5*b*arctan(x/c^(1/2))/c^(5/2)+1/5*(-a-b*arctanh(c/x^2))/x^5
+1/5*b*arctanh(x/c^(1/2))/c^(5/2)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6097, 263, 325, 212, 206, 203}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{2b}{15cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^6,x]
```

```
[Out] (-2*b)/(15*c*x^3) + (b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (a + b*ArcTanh[c/x^
2])/(5*x^5) + (b*ArcTanh[x/Sqrt[c]])/(5*c^(5/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^6}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{1}{5}(2bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^8}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{1}{5}(2bc)\int \frac{1}{x^4\left(-c^2+x^4\right)}\,dx\\
&= -\frac{2b}{15cx^3}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{(2b)\int \frac{1}{-c^2+x^4}\,dx}{5c}\\
&= -\frac{2b}{15cx^3}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\int \frac{1}{c-x^2}\,dx}{5c^2}+\frac{b\int \frac{1}{c+x^2}\,dx}{5c^2}\\
&= -\frac{2b}{15cx^3}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 90, normalized size = 1.38

$$-\frac{a}{5x^5}-\frac{b\log\left(\sqrt{c}-x\right)}{10c^{5/2}}+\frac{b\log\left(\sqrt{c}+x\right)}{10c^{5/2}}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{2b}{15cx^3}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (2*b)/(15*c*x^3) + (b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (b*ArcT
anh[c/x^2])/(5*x^5) - (b*Log[Sqrt[c] - x])/(10*c^(5/2)) + (b*Log[Sqrt[c] +
x])/(10*c^(5/2))
```

fricas [A] time = 0.53, size = 196, normalized size = 3.02

$$\left[\frac{6\,b\sqrt{c}\,x^5\arctan\left(\frac{x}{\sqrt{c}}\right)+3\,b\sqrt{c}\,x^5\log\left(\frac{x^2+2\,\sqrt{c}\,x+c}{x^2-c}\right)-4\,bc^2x^2-3\,bc^3\log\left(\frac{x^2+c}{x^2-c}\right)-6\,ac^3}{30\,c^3x^5},-\frac{6\,b\sqrt{-c}\,x^5\arctan\left(\frac{\sqrt{-}}{c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(6*b*sqrt(c)*x^5*arctan(x/sqrt(c)) + 3*b*sqrt(c)*x^5*log((x^2 + 2*sqr
t(c)*x + c)/(x^2 - c)) - 4*b*c^2*x^2 - 3*b*c^3*log((x^2 + c)/(x^2 - c)) - 6
*a*c^3)/(c^3*x^5), -1/30*(6*b*sqrt(-c)*x^5*arctan(sqrt(-c)*x/c) + 3*b*sqrt(
-c)*x^5*log((x^2 - 2*sqrt(-c)*x - c)/(x^2 + c)) + 4*b*c^2*x^2 + 3*b*c^3*log
((x^2 + c)/(x^2 - c)) + 6*a*c^3)/(c^3*x^5)]
```

giac [A] time = 0.18, size = 74, normalized size = 1.14

$$-\frac{1}{5}b\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^2}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{10\,x^5}-\frac{2\,bx^2+3\,ac}{15\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^6,x, algorithm="giac")
```

```
[Out] -1/5*b*(arctan(x/sqrt(-c))/(sqrt(-c)*c^2) - arctan(x/sqrt(c))/c^(5/2)) - 1/
10*b*log((x^2 + c)/(x^2 - c))/x^5 - 1/15*(2*b*x^2 + 3*a*c)/(c*x^5)
```

maple [A] time = 0.04, size = 55, normalized size = 0.85

$$-\frac{a}{5x^5}-\frac{b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{2b}{15c\,x^3}+\frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{5c^{\frac{5}{2}}}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{5c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctanh(c/x^2)-2/15*b/c/x^3+1/5*b*arctan(x/c^(1/2))/c^
(5/2)+1/5*b/c^(5/2)*arctanh(1/x*c^(1/2))
```

maxima [A] time = 0.42, size = 65, normalized size = 1.00

$$\frac{1}{30}\left(c\left(\frac{6\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{7}{2}}}-\frac{3\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^{\frac{7}{2}}}-\frac{4}{c^2x^3}\right)-\frac{6\operatorname{artanh}\left(\frac{c}{x^2}\right)}{x^5}\right)b-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^6,x, algorithm="maxima")
```

```
[Out] 1/30*(c*(6*arctan(x/sqrt(c))/c^(7/2) - 3*log((x - sqrt(c))/(x + sqrt(c)))/c
^(7/2) - 4/(c^2*x^3)) - 6*arctanh(c/x^2)/x^5)*b - 1/5*a/x^5
```

mupad [B] time = 1.00, size = 69, normalized size = 1.06

$$\frac{b\operatorname{atan}\left(\frac{x}{\sqrt{c}}\right)}{5\,c^{5/2}}-\frac{2\,b}{15\,c\,x^3}-\frac{a}{5\,x^5}-\frac{b\,\ln\left(x^2+c\right)}{10\,x^5}+\frac{b\,\ln\left(x^2-c\right)}{10\,x^5}-\frac{b\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{5\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))/x^6,x)
```

```
[Out] (b*atan(x/c^(1/2)))/(5*c^(5/2)) - (2*b)/(15*c*x^3) - a/(5*x^5) - (b*atan((x
*1i)/c^(1/2))*1i)/(5*c^(5/2)) - (b*log(c + x^2))/(10*x^5) + (b*log(x^2 - c)
)/(10*x^5)
```

sympy [A] time = 20.04, size = 797, normalized size = 12.26

$$\begin{cases}-\frac{a}{5x^5}\\ -\frac{a-\infty b}{5x^5}\\ -\frac{a+\infty b}{5x^5}\\ \frac{6iac^{\frac{27}{2}}}{-30ic^{\frac{27}{2}}x^5+30ic^{\frac{23}{2}}x^9}-\frac{6iac^{\frac{23}{2}}x^4}{-30ic^{\frac{27}{2}}x^5+30ic^{\frac{23}{2}}x^9}+\frac{6ibc^{\frac{27}{2}}\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-30ic^{\frac{27}{2}}x^5+30ic^{\frac{23}{2}}x^9}+\frac{4ibc^{\frac{25}{2}}x^2}{-30ic^{\frac{27}{2}}x^5+30ic^{\frac{23}{2}}x^9}-\frac{6ibc^{\frac{23}{2}}x^4\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-30ic^{\frac{27}{2}}x^5+30ic^{\frac{23}{2}}x^9}-\frac{4ibc^{\frac{21}{2}}x^6}{-30ic^{\frac{27}{2}}x^5+30ic^{\frac{23}{2}}x}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5), Eq(c, 0)), (-(a - oo*b)/(5*x**5), Eq(c, -x**2)), (-
(a + oo*b)/(5*x**5), Eq(c, x**2)), (6*I*a*c**(27/2)/(-30*I*c**(27/2)*x**5 +
 30*I*c**(23/2)*x**9) - 6*I*a*c**(23/2)*x**4/(-30*I*c**(27/2)*x**5 + 30*I*c
**(23/2)*x**9) + 6*I*b*c**(27/2)*atanh(c/x**2)/(-30*I*c**(27/2)*x**5 + 30*I
*c**(23/2)*x**9) + 4*I*b*c**(25/2)*x**2/(-30*I*c**(27/2)*x**5 + 30*I*c**(23
/2)*x**9) - 6*I*b*c**(23/2)*x**4*atanh(c/x**2)/(-30*I*c**(27/2)*x**5 + 30*I
*c**(23/2)*x**9) - 4*I*b*c**(21/2)*x**6/(-30*I*c**(27/2)*x**5 + 30*I*c**(23
/2)*x**9) + 6*I*b*c**11*x**5*log(-sqrt(c) + x)/(-30*I*c**(27/2)*x**5 + 30*I
*c**(23/2)*x**9) - 3*b*c**11*x**5*log(-I*sqrt(c) + x)/(-30*I*c**(27/2)*x**5
 + 30*I*c**(23/2)*x**9) - 3*I*b*c**11*x**5*log(-I*sqrt(c) + x)/(-30*I*c**(2
7/2)*x**5 + 30*I*c**(23/2)*x**9) + 3*b*c**11*x**5*log(I*sqrt(c) + x)/(-30*I
*c**(27/2)*x**5 + 30*I*c**(23/2)*x**9) - 3*I*b*c**11*x**5*log(I*sqrt(c) + x
)/(-30*I*c**(27/2)*x**5 + 30*I*c**(23/2)*x**9) + 6*I*b*c**11*x**5*atanh(c/x
**2)/(-30*I*c**(27/2)*x**5 + 30*I*c**(23/2)*x**9) - 6*I*b*c**9*x**9*log(-sq
rt(c) + x)/(-30*I*c**(27/2)*x**5 + 30*I*c**(23/2)*x**9) + 3*b*c**9*x**9*log
(-I*sqrt(c) + x)/(-30*I*c**(27/2)*x**5 + 30*I*c**(23/2)*x**9) + 3*I*b*c**9*
x**9*log(-I*sqrt(c) + x)/(-30*I*c**(27/2)*x**5 + 30*I*c**(23/2)*x**9) - 3*b
*c**9*x**9*log(I*sqrt(c) + x)/(-30*I*c**(27/2)*x**5 + 30*I*c**(23/2)*x**9)
+ 3*I*b*c**9*x**9*log(I*sqrt(c) + x)/(-30*I*c**(27/2)*x**5 + 30*I*c**(23/2)
*x**9) - 6*I*b*c**9*x**9*atanh(c/x**2)/(-30*I*c**(27/2)*x**5 + 30*I*c**(23/
2)*x**9), True))
```

3.171 $\int x^3 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=94

$$-\frac{1}{4}c^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2+\frac{1}{2}bcx^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)+\frac{1}{4}x^4\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2+\frac{1}{4}b^2c^2\log\left(1-\frac{c^2}{x^4}\right)+b^2c^2\log($$

```
[Out] 1/2*b*c*x^2*(a+b*arccoth(x^2/c))-1/4*c^2*(a+b*arccoth(x^2/c))^2+1/4*x^4*(a+
b*arccoth(x^2/c))^2+1/4*b^2*c^2*ln(1-c^2/x^4)+b^2*c^2*ln(x)
```

Rubi [C] time = 1.30, antiderivative size = 599, normalized size of antiderivative = 6.37, number of steps used = 59, number of rules used = 33, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.063, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2455, 263, 266, 43, 193, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,-\frac{c}{x^2}\right)-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{c}{x^2}\right)-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{c-x^2}{2c}\right)-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{c+x^2}{2c}\right)+\frac{1}{8}b^2c^2$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (a*b*c*x^2)/4 - (b^2*c*x^2*Log[1 - c/x^2])/8 + (b*c*(1 - c/x^2)*x^2*(2*a -
b*Log[1 - c/x^2]))/8 - (c^2*(2*a - b*Log[1 - c/x^2])^2)/16 + (x^4*(2*a - b*
Log[1 - c/x^2])^2)/16 + (b^2*c^2*Log[1 + c/x^2])/8 + (b^2*c*x^2*Log[1 + c/x
^2])/4 + (a*b*x^4*Log[1 + c/x^2])/4 - (b^2*x^4*Log[1 - c/x^2]*Log[1 + c/x^2
])/8 - (b^2*c^2*Log[1 + c/x^2]^2)/16 + (b^2*x^4*Log[1 + c/x^2]^2)/16 + (a*b
*c^2*Log[x])/2 + (b^2*c^2*Log[x])/2 + (b^2*c^2*Log[c - x^2])/8 + (b^2*c^2*L
og[1 + c/x^2]*Log[c - x^2])/8 + (b^2*c^2*Log[x^2/c]*Log[c - x^2])/8 - (a*b*
c^2*Log[c + x^2])/4 + (b^2*c^2*Log[c + x^2])/8 + (b^2*c^2*Log[1 - c/x^2]*Lo
g[c + x^2])/8 + (b^2*c^2*Log[-(x^2/c)]*Log[c + x^2])/8 - (b^2*c^2*Log[(c -
x^2)/(2*c)]*Log[c + x^2])/8 - (b^2*c^2*Log[c - x^2]*Log[(c + x^2)/(2*c)])/8
 - (b^2*c^2*PolyLog[2, -(c/x^2)])/8 - (b^2*c^2*PolyLog[2, c/x^2])/8 - (b^2*
c^2*PolyLog[2, (c - x^2)/(2*c)])/8 - (b^2*c^2*PolyLog[2, (c + x^2)/(2*c)])/
8 + (b^2*c^2*PolyLog[2, (c + x^2)/c])/8 + (b^2*c^2*PolyLog[2, 1 - x^2/c])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
```

```
GtQ[p, 0]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
```

```
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2dx=\int\left(\frac{1}{4}x^3\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2-\frac{1}{2}bx^3\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)+\frac{1}{4}$$

$$=\frac{1}{4}\int x^3\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2dx-\frac{1}{2}b\int x^3\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right.$$

$$=-\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^2}{x^3}dx,x,\frac{1}{x^2}\right)\right)-\frac{1}{4}b\,\text{Subst}\left(\int x\left(-2a+b\log\right.\right.$$

$$=\frac{1}{16}x^4\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{16}b^2x^4\log^2\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b\,\text{Subst}\left(\int\left(-2ax\log\right.\right.$$

$$=\frac{1}{16}x^4\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{16}b^2x^4\log^2\left(1+\frac{c}{x^2}\right)+\frac{1}{8}b\,\text{Subst}\left(\int\frac{2a-b\log(}{x\left(\frac{1}{c}-\frac{x}{c}\right)}\right.$$

$$=\frac{1}{16}x^4\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{4}abx^4\log\left(1+\frac{c}{x^2}\right)-\frac{1}{8}b^2x^4\log\left(1-\frac{c}{x^2}\right)\log\left(1+\right.$$

$$=\frac{1}{8}bc\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)+\frac{1}{16}x^4\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{8}b^2cx^2\,\text{lo}$$

$$=\frac{1}{8}bc\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)-\frac{1}{16}c^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{16}x^4\left(2a\right.$$

$$=\frac{1}{4}abcx^2+\frac{1}{8}bc\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)-\frac{1}{16}c^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+$$

$$=\frac{1}{4}abcx^2-\frac{1}{8}b^2cx^2\log\left(1-\frac{c}{x^2}\right)+\frac{1}{8}bc\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)-\frac{1}{16}c^2\left(\right.$$

$$=\frac{1}{4}abcx^2-\frac{1}{8}b^2cx^2\log\left(1-\frac{c}{x^2}\right)+\frac{1}{8}bc\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)-\frac{1}{16}c^2\left(\right.$$

$$=\frac{1}{4}abcx^2-\frac{1}{8}b^2cx^2\log\left(1-\frac{c}{x^2}\right)+\frac{1}{8}bc\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)-\frac{1}{16}c^2\left(\right.$$

$$=\frac{1}{4}abcx^2-\frac{1}{8}b^2cx^2\log\left(1-\frac{c}{x^2}\right)+\frac{1}{8}bc\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)-\frac{1}{16}c^2\left(\right.$$

$$=\frac{1}{4}abcx^2-\frac{1}{8}b^2cx^2\log\left(1-\frac{c}{x^2}\right)+\frac{1}{8}bc\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)-\frac{1}{16}c^2\left(\right.$$

$$=\frac{1}{4}abcx^2-\frac{1}{8}b^2cx^2\log\left(1-\frac{c}{x^2}\right)+\frac{1}{8}bc\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)-\frac{1}{16}c^2\left(\right.$$

Mathematica [A] time = 0.06, size = 104, normalized size = 1.11

$$\frac{1}{4}\left(a^2x^4+bc^2(a+b)\log\left(x^2-c\right)-abc^2\log\left(c+x^2\right)+2abcx^2+2bx^2\tanh^{-1}\left(\frac{c}{x^2}\right)\left(ax^2+bc\right)+b^2c^2\log\left(c+x^2\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (2*a*b*c*x^2 + a^2*x^4 + 2*b*x^2*(b*c + a*x^2)*ArcTanh[c/x^2] + b^2*(-c^2 +
 x^4)*ArcTanh[c/x^2]^2 + b*(a + b)*c^2*Log[-c + x^2] - a*b*c^2*Log[c + x^2]
 + b^2*c^2*Log[c + x^2])/4
```

fricas [A] time = 0.42, size = 126, normalized size = 1.34

$$\frac{1}{4}a^2x^4+\frac{1}{2}abcx^2-\frac{1}{4}\left(ab-b^2\right)c^2\log\left(x^2+c\right)+\frac{1}{4}\left(ab+b^2\right)c^2\log\left(x^2-c\right)+\frac{1}{16}\left(b^2x^4-b^2c^2\right)\log\left(\frac{x^2+c}{x^2-c}\right)^2+\frac{1}{4}\left(ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*a^2*x^4 + 1/2*a*b*c*x^2 - 1/4*(a*b - b^2)*c^2*log(x^2 + c) + 1/4*(a*b +
 b^2)*c^2*log(x^2 - c) + 1/16*(b^2*x^4 - b^2*c^2)*log((x^2 + c)/(x^2 - c))^
2 + 1/4*(a*b*x^4 + b^2*c*x^2)*log((x^2 + c)/(x^2 - c))
```

giac [B] time = 0.29, size = 327, normalized size = 3.48

$$-\frac{2\,b^2c^3\log\left(\frac{x^2+c}{x^2-c}-1\right)-2\,b^2c^3\log\left(\frac{x^2+c}{x^2-c}\right)-\frac{\left(x^2+c\right)b^2c^3\log\left(\frac{x^2+c}{x^2-c}\right)^2}{\left(x^2-c\right)\left(\frac{\left(x^2+c\right)^2}{\left(x^2-c\right)^2}-\frac{2\left(x^2+c\right)}{x^2-c}+1\right)}-\frac{2\left(\frac{2\left(x^2+c\right)abc^3}{x^2-c}+\frac{\left(x^2+c\right)b^2c^3}{x^2-c}-b^2c^3\right)\log\left(\frac{x^2+c}{x^2-c}\right)}{\frac{\left(x^2+c\right)^2}{\left(x^2-c\right)^2}-\frac{2\left(x^2+c\right)}{x^2-c}+1}-\frac{4\left(\frac{(x^2+}{x^2}\right.}{}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*b^2*c^3*log((x^2 + c)/(x^2 - c) - 1) - 2*b^2*c^3*log((x^2 + c)/(x^2
 - c)) - (x^2 + c)*b^2*c^3*log((x^2 + c)/(x^2 - c))^2/((x^2 - c)*((x^2 + c)
^2/(x^2 - c)^2 - 2*(x^2 + c)/(x^2 - c) + 1)) - 2*(2*(x^2 + c)*a*b*c^3/(x^2
- c) + (x^2 + c)*b^2*c^3/(x^2 - c) - b^2*c^3)*log((x^2 + c)/(x^2 - c))/((x^
2 + c)^2/(x^2 - c)^2 - 2*(x^2 + c)/(x^2 - c) + 1) - 4*((x^2 + c)*a^2*c^3/(x
^2 - c) + (x^2 + c)*a*b*c^3/(x^2 - c) - a*b*c^3)/((x^2 + c)^2/(x^2 - c)^2 -
 2*(x^2 + c)/(x^2 - c) + 1))/c
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int(x^3*(a+b*arctanh(c/x^2))^2,x)
```

maxima [A] time = 0.33, size = 157, normalized size = 1.67

$$\frac{1}{4}b^2x^4\,\mathrm{artanh}\left(\frac{c}{x^2}\right)^2+\frac{1}{4}a^2x^4+\frac{1}{4}\left(2\,x^4\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+\left(2\,x^2-c\log\left(x^2+c\right)+c\log\left(x^2-c\right)\right)c\right)ab+\frac{1}{16}\left(\left(\log\left(x^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c/x^2)^2 + 1/4*a^2*x^4 + 1/4*(2*x^4*arctanh(c/x^2) + (2
*x^2 - c*log(x^2 + c) + c*log(x^2 - c))*c)*a*b + 1/16*((log(x^2 + c)^2 - 2*
```

```
(log(x^2 + c) - 2)*log(x^2 - c) + log(x^2 - c)^2 + 4*log(x^2 + c))*c^2 + 4*
(2*x^2 - c*log(x^2 + c) + c*log(x^2 - c))*c*arctanh(c/x^2))*b^2
```

mupad [B] time = 1.21, size = 247, normalized size = 2.63

$$\frac{a^2 x^4}{4} - \frac{a b c^2 \ln\left(x^2+c\right)}{4} + \frac{a b c^2 \ln\left(x^2-c\right)}{4} + \frac{a b c x^2}{2} + \frac{a b x^4 \ln\left(x^2+c\right)}{4} - \frac{a b x^4 \ln\left(x^2-c\right)}{4} - \frac{b^2 c^2 \ln\left(x^2+c\right)^2}{16} + \frac{b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c/x^2))^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*c^2*log(x^2 - c))/4 - (b^2*c^2*log(c + x^2)^2)/16 + (b^2
*x^4*log(c + x^2)^2)/16 - (b^2*c^2*log(x^2 - c)^2)/16 + (b^2*x^4*log(x^2 -
c)^2)/16 + (b^2*c^2*log(c + x^2))/4 + (a*b*x^4*log(c + x^2))/4 + (a*b*c^2*l
og(x^2 - c))/4 + (b^2*c^2*log(c + x^2)*log(x^2 - c))/8 + (a*b*c*x^2)/2 - (a
*b*x^4*log(x^2 - c))/4 + (b^2*c*x^2*log(c + x^2))/4 - (b^2*x^4*log(c + x^2)
*log(x^2 - c))/8 - (b^2*c*x^2*log(x^2 - c))/4 - (a*b*c^2*log(c + x^2))/4
```

sympy [C] time = 6.12, size = 151, normalized size = 1.61

$$\frac{a^2x^4}{4} - \frac{abc^2 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2} + \frac{abcx^2}{2} + \frac{abx^4 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2} + \frac{b^2c^2 \log\left(-i\sqrt{c}+x\right)}{2} + \frac{b^2c^2 \log\left(i\sqrt{c}+x\right)}{2} - \frac{b^2c^2 \operatorname{atanh}^2\left(\frac{c}{x^2}\right)}{4} - \frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] a**2*x**4/4 - a*b*c**2*atanh(c/x**2)/2 + a*b*c*x**2/2 + a*b*x**4*atanh(c/x*
*2)/2 + b**2*c**2*log(-I*sqrt(c) + x)/2 + b**2*c**2*log(I*sqrt(c) + x)/2 -
b**2*c**2*atanh(c/x**2)**2/4 - b**2*c**2*atanh(c/x**2)/2 + b**2*c*x**2*atan
h(c/x**2)/2 + b**2*x**4*atanh(c/x**2)**2/4
```

3.172 $\int x \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=94

$$\frac{1}{2}x^2\left(a + b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2 - \frac{1}{2}c\left(a + b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2 - bc\log\left(2 - \frac{2}{\frac{c}{x^2}+1}\right)\left(a + b\coth^{-1}\left(\frac{x^2}{c}\right)\right) + \frac{1}{2}b^2c\text{Li}_2\left(\frac{2}{\frac{c}{x^2}+1}\right.$$

```
[Out] -1/2*c*(a+b*arccoth(x^2/c))^2+1/2*x^2*(a+b*arccoth(x^2/c))^2-b*c*(a+b*arcco
th(x^2/c))*ln(2-2/(1+c/x^2))+1/2*b^2*c*polylog(2,-1+2/(1+c/x^2))
```

Rubi [B] time = 0.70, antiderivative size = 404, normalized size of antiderivative = 4.30, number of steps used = 34, number of rules used = 19, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.357, Rules used = {6099, 2454, 2397, 2392, 2391, 2455, 263, 260, 6715, 2448, 31, 6742, 2556, 12, 2462, 2416, 2394, 2315, 2393}

$$\frac{1}{4}b^2c\text{PolyLog}\left(2,-\frac{c}{x^2}\right) - \frac{1}{4}b^2c\text{PolyLog}\left(2,\frac{c}{x^2}\right) - \frac{1}{4}b^2c\text{PolyLog}\left(2,\frac{c-x^2}{2c}\right) + \frac{1}{4}b^2c\text{PolyLog}\left(2,\frac{c+x^2}{2c}\right) - \frac{1}{4}b^2c\text{Pol}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] ((1 - c/x^2)*x^2*(2*a - b*Log[1 - c/x^2])^2)/8 + (a*b*x^2*Log[1 + c/x^2])/2
 - (b^2*x^2*Log[1 - c/x^2]*Log[1 + c/x^2])/4 + (b^2*(1 + c/x^2)*x^2*Log[1 +
 c/x^2]^2)/8 + a*b*c*Log[x] - (b^2*c*Log[1 - c/x^2]*Log[-c - x^2])/4 - (b^2
*c*Log[-(x^2/c)]*Log[-c - x^2])/4 + (b^2*c*Log[-c - x^2]*Log[(c - x^2)/(2*c
)])/4 + (b^2*c*Log[1 + c/x^2]*Log[-c + x^2])/4 + (b^2*c*Log[x^2/c]*Log[-c +
 x^2])/4 + (a*b*c*Log[c + x^2])/2 - (b^2*c*Log[-c + x^2]*Log[(c + x^2)/(2*c
)])/4 + (b^2*c*PolyLog[2, -(c/x^2)])/4 - (b^2*c*PolyLog[2, c/x^2])/4 - (b^2
*c*PolyLog[2, (c - x^2)/(2*c)])/4 + (b^2*c*PolyLog[2, (c + x^2)/(2*c)])/4 -
 (b^2*c*PolyLog[2, (c + x^2)/c])/4 + (b^2*c*PolyLog[2, 1 - x^2/c])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(\frac{1}{4}x\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2-\frac{1}{2}bx\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)+\frac{1}{4}b^2x\right.$$

$$=\frac{1}{4}\int x\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 dx-\frac{1}{2}b\int x\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right) d$$

$$=-\left(\frac{1}{8}\operatorname{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^2}dx,x,\frac{1}{x^2}\right)\right)-\frac{1}{4}b\operatorname{Subst}\left(\int \left(-2a+b\log\left(1-\right.\right.\right.$$

$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{8}b^2\left(1+\frac{c}{x^2}\right)x^2\log^2\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b\operatorname{Subst}$$

$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{8}b^2\left(1+\frac{c}{x^2}\right)x^2\log^2\left(1+\frac{c}{x^2}\right)+abc\log(x$$

$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)$$

$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)$$

$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)$$

$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)$$

$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)$$

$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)$$

$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)$$

$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)$$

Mathematica [A] time = 0.14, size = 107, normalized size = 1.14

$$\frac{1}{2}\left(a\left(ax^2+bc\log\left(1-\frac{c^2}{x^4}\right)-2bc\log\left(\frac{c}{x^2}\right)\right)+2b\tanh^{-1}\left(\frac{c}{x^2}\right)\left(ax^2-bc\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x^2}\right)}\right)\right)+b^2c\operatorname{Li}_2\left(e^{-2\tanh}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (b^2*(-c + x^2)*ArcTanh[c/x^2]^2 + 2*b*ArcTanh[c/x^2]*(a*x^2 - b*c*Log[1 -
E^(-2*ArcTanh[c/x^2])]) + a*(a*x^2 + b*c*Log[1 - c^2/x^4] - 2*b*c*Log[c/x^2
]) + b^2*c*PolyLog[2, E^(-2*ArcTanh[c/x^2])])/2
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^2x\operatorname{artanh}\left(\frac{c}{x^2}\right)^2+2\,abx\operatorname{artanh}\left(\frac{c}{x^2}\right)+a^2x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*arctanh(c/x^2)^2 + 2*a*b*x*arctanh(c/x^2) + a^2*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2 x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2*x, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x\left(a + b \operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int(x*(a+b*arctanh(c/x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2x^2+\frac{1}{2}\left(2x^2 \operatorname{artanh}\left(\frac{c}{x^2}\right) + c\log\left(x^4 - c^2\right)\right)ab+\frac{1}{8}\left(x^2\log\left(x^2 + c\right)^2 - 2\left(x^2 + c\right)\log\left(x^2 + c\right)\log\left(x^2 - c\right) + \left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c/x^2) + c*log(x^4 - c^2))*a*b + 1/8*(x^2*
log(x^2 + c)^2 - 2*(x^2 + c)*log(x^2 + c)*log(x^2 - c) + (x^2 - c)*log(x^2
- c)^2 + 2*integrate(2*(3*c*x^3 + c^2*x)*log(x^2 + c)/(x^4 - c^2), x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c/x^2))^2,x)
```

```
[Out] int(x*(a + b*atanh(c/x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Integral(x*(a + b*atanh(c/x**2))**2, x)
```

3.173 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x} dx$

Optimal. Leaf size=144

$$\frac{1}{2}b\text{Li}_2\left(1-\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)-\frac{1}{2}b\text{Li}_2\left(\frac{2}{1-\frac{c}{x^2}}-1\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)-\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\coth\right.$$

```
[Out] (a+b*arccoth(x^2/c))^2*arctanh(-1+2/(1-c/x^2))+1/2*b*(a+b*arccoth(x^2/c))*p
olylog(2,1-2/(1-c/x^2))-1/2*b*(a+b*arccoth(x^2/c))*polylog(2,-1+2/(1-c/x^2)
)-1/4*b^2*polylog(3,1-2/(1-c/x^2))+1/4*b^2*polylog(3,-1+2/(1-c/x^2))
```

Rubi [A] time = 0.32, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$\frac{1}{2}b\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)-\frac{1}{2}b\text{PolyLog}\left(2,\frac{2}{1-\frac{c}{x^2}}-1\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)-\frac{1}{4}b^2\text{PolyLog}\left(3\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x,x]
```

```
[Out] -((a + b*ArcCoth[x^2/c])^2*ArcTanh[1 - 2/(1 - c/x^2)]) + (b*(a + b*ArcCoth[
x^2/c])*PolyLog[2, 1 - 2/(1 - c/x^2)])/2 - (b*(a + b*ArcCoth[x^2/c])*PolyLo
g[2, -1 + 2/(1 - c/x^2)])/2 - (b^2*PolyLog[3, 1 - 2/(1 - c/x^2)])/4 + (b^2*
PolyLog[3, -1 + 2/(1 - c/x^2)])/4
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2 \tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right) + (2bc)\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)}{1-}\right.$$

$$= -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2 \tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right) - (bc)\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\mathrm{l}}{1-c^2x^2}\right.$$

$$= -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2 \tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right) + \frac{1}{2}b\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\operatorname{Li}_2\left(1-\frac{}{1}\right.$$

$$= -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2 \tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right) + \frac{1}{2}b\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\operatorname{Li}_2\left(1-\frac{}{1}\right.$$

Mathematica [A] time = 0.07, size = 183, normalized size = 1.27

$$\frac{1}{2}\left(4bc\left(\frac{1}{2}\left(\frac{\operatorname{Li}_2\left(\frac{-\frac{c}{x^2}-1}{\frac{c}{x^2}-1}\right)\left(-a-b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{2c}+\frac{b\operatorname{Li}_3\left(\frac{-\frac{c}{x^2}-1}{\frac{c}{x^2}-1}\right)}{4c}\right)+\frac{1}{2}\left(-\frac{\operatorname{Li}_2\left(\frac{\frac{c}{x^2}+1}{\frac{c}{x^2}-1}\right)\left(-a-b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{2c}-\frac{b\operatorname{Li}_3\left(\right.}{4}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x,x]
```

```
[Out] (-2*ArcTanh[1 - 2/(1 - c/x^2)]*(a + b*ArcTanh[c/x^2])^2 + 4*b*c*((((-a - b*
ArcTanh[c/x^2])*PolyLog[2, (-1 - c/x^2)/(-1 + c/x^2)])/(2*c) + (b*PolyLog[3
, (-1 - c/x^2)/(-1 + c/x^2)])/(4*c))/2 + (-1/2*((-a - b*ArcTanh[c/x^2])*Pol
yLog[2, (1 + c/x^2)/(-1 + c/x^2)])/c - (b*PolyLog[3, (1 + c/x^2)/(-1 + c/x^
2)])/(4*c))/2))/2
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2\operatorname{artanh}\left(\frac{c}{x^2}\right)^2+2ab\operatorname{artanh}\left(\frac{c}{x^2}\right)+a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x, x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \log(x) + \int \frac{b^2\left(\log\left(\frac{c}{x^2} + 1\right) - \log\left(-\frac{c}{x^2} + 1\right)\right)^2}{4\,x} + \frac{ab\left(\log\left(\frac{c}{x^2} + 1\right) - \log\left(-\frac{c}{x^2} + 1\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c/x^2 + 1) - log(-c/x^2 + 1))^2/x + a*b
*(log(c/x^2 + 1) - log(-c/x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2/x,x)
```

```
[Out] int((a + b*atanh(c/x^2))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2/x, x)
```

3.174 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=99

$$-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{2c}-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{2x^2}+\frac{b\log\left(\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)}{c}+\frac{b^2\mathrm{Li}_2\left(1-\frac{2}{1-\frac{c}{x^2}}\right)}{2c}$$

```
[Out] -1/2*(a+b*arccoth(x^2/c))^2/c-1/2*(a+b*arccoth(x^2/c))^2/x^2+b*(a+b*arccoth
(x^2/c))*ln(2/(1-c/x^2))/c+1/2*b^2*polylog(2,1-2/(1-c/x^2))/c
```

Rubi [B] time = 0.53, antiderivative size = 207, normalized size of antiderivative = 2.09, number of steps used = 28, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-\frac{c}{x^2}\right)\right)}{4c}-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(\frac{c}{x^2}+1\right)\right)}{4c}-\frac{b\log\left(\frac{1}{2}\left(\frac{c}{x^2}+1\right)\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{4c}-\frac{b\log\left(\frac{c}{x^2}+1\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^3,x]
```

```
[Out] ((1 - c/x^2)*(2*a - b*Log[1 - c/x^2])^2)/(8*c) - (b*(2*a - b*Log[1 - c/x^2]
)*Log[(1 + c/x^2)/2])/(4*c) - (b^2*Log[(1 - c/x^2)/2]*Log[1 + c/x^2])/(4*c)
 - (b*(2*a - b*Log[1 - c/x^2])*Log[1 + c/x^2])/(4*x^2) - (b^2*(1 + c/x^2)*L
og[1 + c/x^2]^2)/(8*c) + (b^2*PolyLog[2, (1 - c/x^2)/2])/(4*c) - (b^2*PolyL
og[2, (1 + c/x^2)/2])/(4*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx = \int\left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^3} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^3} + \frac{b^2\log^2\left(1+\right.}{4x^3}\right.$$

$$= \frac{1}{4}\int\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^3}\,dx - \frac{1}{2}b\int\frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^3}\,dx +$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int(2a-b\log(1-cx))^2\,dx, x, \frac{1}{x^2}\right)\right) + \frac{1}{4}b\,\text{Subst}\left(\int(-2a+b\log(1-\right.\right.$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{4x^2} + \frac{\text{Subst}\left(\int(2a-b\log(x))^2\,dx, x, 1-\frac{c}{x^2}\right)}{8c} -$$

$$= \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{4x^2} - \frac{b^2\left(1+\frac{c}{x^2}\right.}{}$$

$$= -\frac{ab}{2x^2} - \frac{b^2}{4x^2} + \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} + \frac{b^2\left(1+\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right)}{4c} - \frac{b\left(2a\right.}{}$$

$$= -\frac{b^2}{2x^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c} + \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} - \frac{b\left(2a-b\log\right.}{}$$

$$= -\frac{b^2}{4x^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c} + \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} - \frac{b\left(2a-b\log\right.}{}$$

$$= \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(\frac{1}{2}\left(1+\frac{c}{x^2}\right)\right)}{4c} - \frac{b^2\log}{}$$

Mathematica [A] time = 0.18, size = 114, normalized size = 1.15

$$-\frac{a^2}{2x^2} - \frac{ab\left(\frac{c\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2} - \log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^4}}}\right)\right)}{c} - \frac{b^2\left(\text{Li}_2\left(-e^{-2\tanh^{-1}\left(\frac{c}{x^2}\right)}\right) + \tanh^{-1}\left(\frac{c}{x^2}\right)\left(\frac{c\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2} - \tanh^{-1}\left(\frac{c}{x^2}\right) - 2\,\text{l}\right.\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 - (a*b*((c*ArcTanh[c/x^2])/x^2 - Log[1/Sqrt[1 - c^2/x^4]]))/c
- (b^2*(ArcTanh[c/x^2]*(-ArcTanh[c/x^2] + (c*ArcTanh[c/x^2])/x^2 - 2*Log[1
+ E^(-2*ArcTanh[c/x^2])]) + PolyLog[2, -E^(-2*ArcTanh[c/x^2])]))/(2*c)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(\frac{c}{x^2}\right)^2 + 2\,ab\,\text{artanh}\left(\frac{c}{x^2}\right) + a^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^3, x)
```

maple [A] time = 0.19, size = 144, normalized size = 1.45

$$-\frac{a^2}{2x^2}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2}{2x^2}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2}{2c}+\frac{\operatorname{arctanh}\left(\frac{c}{x^2}\right)\ln\left(1+\frac{\left(1+\frac{c}{x^2}\right)^2}{1-\frac{c^2}{x^4}}\right)b^2}{c}+\frac{\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x^2}\right)^2}{1-\frac{c^2}{x^4}}\right)b^2}{2c}-\frac{ab\operatorname{arcta}}{x^{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2-1/2/x^2*b^2*arctanh(c/x^2)^2-1/2/c*b^2*arctanh(c/x^2)^2+1/c*ar
ctanh(c/x^2)*ln(1+(1+c/x^2)^2/(1-c^2/x^4))*b^2+1/2/c*polylog(2,-(1+c/x^2)^2
/(1-c^2/x^4))*b^2-1/x^2*a*b*arctanh(c/x^2)-1/2/c*a*b*ln(1-c^2/x^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(8\,c^3\int\frac{\log(x)^2}{cx^7-c^3x^3}\,dx+c^2\left(\frac{\log\left(x^2+c\right)}{c^3}+\frac{\log\left(x^2-c\right)}{c^3}-\frac{4\,\log(x)}{c^3}\right)-8\,c^2\int\frac{x^2\log\left(x^2+c\right)}{cx^7-c^3x^3}\,dx+8\,c^2\int\frac{x^2\,\mathrm{lo}}{cx^7-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(8*c^3*integrate(log(x)^2/(c*x^7 - c^3*x^3), x) + c^2*(log(x^2 + c)/c^3
 + log(x^2 - c)/c^3 - 4*log(x)/c^3) - 8*c^2*integrate(x^2*log(x^2 + c)/(c*x
^7 - c^3*x^3), x) + 8*c^2*integrate(x^2*log(x)/(c*x^7 - c^3*x^3), x) + 2*c*
(log(x^2 - c)/c^2 - log(x^2)/c^2 + 1/(c*x^2))*log(-c/x^2 + 1) - c*(log(x^2
+ c)/c^2 - log(x^2 - c)/c^2) - 8*c*integrate(x^4*log(x)^2/(c*x^7 - c^3*x^3)
, x) - 4*c*integrate(x^4*log(x^2 + c)/(c*x^7 - c^3*x^3), x) + 16*c*integrat
e(x^4*log(x)/(c*x^7 - c^3*x^3), x) - log(-c/x^2 + 1)^2/x^2 - (x^2*log(x^2 -
 c)^2 + 4*x^2*log(x)^2 - 4*x^2*log(x) - 2*(2*x^2*log(x) - x^2)*log(x^2 - c)
 + 2*c)/(c*x^2) - (c*log(x^2 + c)^2 - 2*((x^2 + c)*log(x^2 + c) - 2*(x^2 +
c)*log(x) - c)*log(x^2 - c))/(c*x^2) - 4*integrate(x^6*log(x^2 + c)/(c*x^7
- c^3*x^3), x) + 8*integrate(x^6*log(x)/(c*x^7 - c^3*x^3), x))*b^2 - 1/2*a*
b*(2*c*arctanh(c/x^2)/x^2 + log(-c^2/x^4 + 1))/c - 1/2*a^2/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2/x^3,x)
```

```
[Out] int((a + b*atanh(c/x^2))^2/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**3,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2/x**3, x)
```

3.175 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=97

$$\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{4c^2}-\frac{ab}{2cx^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{4x^4}-\frac{b^2\log\left(1-\frac{c^2}{x^4}\right)}{4c^2}-\frac{b^2\coth^{-1}\left(\frac{x^2}{c}\right)}{2cx^2}$$

```
[Out] -1/2*a*b/c/x^2-1/2*b^2*arccoth(x^2/c)/c/x^2+1/4*(a+b*arccoth(x^2/c))^2/c^2-
1/4*(a+b*arccoth(x^2/c))^2/x^4-1/4*b^2*ln(1-c^2/x^4)/c^2
```

Rubi [C] time = 1.53, antiderivative size = 770, normalized size of antiderivative = 7.94, number of steps used = 67, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 6742, 30, 2557, 12, 2466, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,-\frac{c}{x^2}\right)}{8c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{c}{x^2}\right)}{8c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{c-x^2}{2c}\right)}{8c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{c+x^2}{2c}\right)}{8c^2}-\frac{b^2\text{PolyLog}\left(2,\frac{c+x^2}{c}\right)}{8c^2}-\cdots$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^5,x]
```

```
[Out] -(b^2*(1 - c/x^2)^2)/(32*c^2) - (b^2*(1 + c/x^2)^2)/(32*c^2) + (a*b)/(8*x^4
) + b^2/(16*x^4) - (3*a*b)/(4*c*x^2) + (b^2*Log[1 - c/x^2])/(16*c^2) - (3*b
^2*(1 - c/x^2)*Log[1 - c/x^2])/(8*c^2) - (b^2*Log[1 - c/x^2])/(16*x^4) - (b
*(1 - c/x^2)^2*(2*a - b*Log[1 - c/x^2]))/(16*c^2) + ((1 - c/x^2)*(2*a - b*L
og[1 - c/x^2])^2)/(8*c^2) - ((1 - c/x^2)^2*(2*a - b*Log[1 - c/x^2])^2)/(16*
c^2) - (b^2*(1 + c/x^2)*Log[1 + c/x^2])/(4*c^2) + (b^2*(1 + c/x^2)^2*Log[1
+ c/x^2])/(16*c^2) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(8*x^4) + (b^2*(1
+ c/x^2)*Log[1 + c/x^2]^2)/(8*c^2) - (b^2*(1 + c/x^2)^2*Log[1 + c/x^2]^2)/(
16*c^2) - (b^2*Log[1 + c/x^2]*Log[c - x^2])/(8*c^2) - (b^2*Log[x^2/c]*Log[c
 - x^2])/(8*c^2) - (b^2*Log[1 - c/x^2]*Log[c + x^2])/(8*c^2) - (b^2*Log[-(x
^2/c)]*Log[c + x^2])/(8*c^2) + (b^2*Log[(c - x^2)/(2*c)]*Log[c + x^2])/(8*c
^2) + (b^2*Log[c - x^2]*Log[(c + x^2)/(2*c)])/(8*c^2) + (a*b*Log[(c + x^2)/
x^2])/(4*c^2) + (b^2*Log[(c + x^2)/x^2])/(16*c^2) - (b^2*(1 + c/x^2)*Log[(c
 + x^2)/x^2])/(8*c^2) - (a*b*Log[(c + x^2)/x^2])/(4*x^4) - (b^2*Log[(c + x^
2)/x^2])/(16*x^4) + (b^2*PolyLog[2, -(c/x^2)])/(8*c^2) + (b^2*PolyLog[2, c/
x^2])/(8*c^2) + (b^2*PolyLog[2, (c - x^2)/(2*c)])/(8*c^2) + (b^2*PolyLog[2,
 (c + x^2)/(2*c)])/(8*c^2) - (b^2*PolyLog[2, (c + x^2)/c])/(8*c^2) - (b^2*P
olyLog[2, 1 - x^2/c])/(8*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^5} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^5} + \frac{b^2\log^2\left(1+\right.}{4x^5}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^5}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^5}\,dx +$$

$$= -\left(\frac{1}{8}\,\mathrm{Subst}\left(\int x(2a-b\log(1-cx))^2\,dx, x, \frac{1}{x^2}\right)\right) - \frac{1}{4}b\,\mathrm{Subst}\left(\int \frac{(-2a+b\log(1\right.$$

$$= -\left(\frac{1}{8}\,\mathrm{Subst}\left(\int \left(\frac{(2a-b\log(1-cx))^2}{c} - \frac{(1-cx)(2a-b\log(1-cx))^2}{c}\right)dx, x, \frac{1}{x^2}\right)\right.$$

$$= \frac{1}{2}(ab)\,\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{c}{x}\right)}{x^3}\,dx, x, x^2\right) - \frac{1}{4}b^2\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{x^3}\,d\right.$$

$$= \frac{b^2\log\left(1-\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right)}{8x^4} - \frac{1}{2}(ab)\,\mathrm{Subst}\left(\int x\log(1+cx)\,dx, x, \frac{1}{x^2}\right) + \frac{1}{4}b^2\,\mathrm{Sub}$$

$$= \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c^2} - \frac{\left(1-\frac{c}{x^2}\right)^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{16c^2} - \frac{ab\log\left(1+\right.}{4x^4}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} - \frac{ab}{2cx^2} + \frac{b^2}{4cx^2} - \frac{b\left(1-\frac{c}{x^2}\right)^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{16c^2} +$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c^2} - \frac{b\left(1-\frac{c}{x^2}\right.}{}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c^2} - \frac{b\left(1-\frac{c}{x^2}\right.}{}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c^2} - \frac{b^2\log(1}{16x}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{3b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{8c^2} - \frac{b^2\log(}{16}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} + \frac{b^2}{16x^4} - \frac{3ab}{4cx^2} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)}{16c^2} - \frac{3b^2\left(1-\right.}{}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} + \frac{b^2}{16x^4} - \frac{3ab}{4cx^2} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)}{16c^2} - \frac{3b^2\left(1-\right.}{}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} + \frac{b^2}{16x^4} - \frac{3ab}{4cx^2} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)}{16c^2} - \frac{3b^2\left(1-\right.}{}$$

Mathematica [A] time = 0.08, size = 131, normalized size = 1.35

$$-\frac{a^2c^2+2abcx^2+2bc\tanh^{-1}\left(\frac{c}{x^2}\right)\left(ac+bx^2\right)+abx^4\log\left(x^2-c\right)-abx^4\log\left(c+x^2\right)+b^2\left(c^2-x^4\right)\tanh^{-1}\left(\frac{c}{x^2}\right.}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^5,x]
```

```
[Out] -1/4*(a^2*c^2 + 2*a*b*c*x^2 + 2*b*c*(a*c + b*x^2)*ArcTanh[c/x^2] + b^2*(c^2
 - x^4)*ArcTanh[c/x^2]^2 - 4*b^2*x^4*Log[x] + a*b*x^4*Log[-c + x^2] + b^2*x
^4*Log[-c + x^2] - a*b*x^4*Log[c + x^2] + b^2*x^4*Log[c + x^2])/(c^2*x^4)
```

fricas [A] time = 0.50, size = 143, normalized size = 1.47

$$\frac{16\,b^2x^4\log(x) + 4\left(ab - b^2\right)x^4\log\left(x^2 + c\right) - 4\left(ab + b^2\right)x^4\log\left(x^2 - c\right) - 8\,abcx^2 - 4\,a^2c^2 + \left(b^2x^4 - b^2c^2\right)\log\left(\frac{x^2+}{x^2-}\right.}{16\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^2*x^4*log(x) + 4*(a*b - b^2)*x^4*log(x^2 + c) - 4*(a*b + b^2)*x^
4*log(x^2 - c) - 8*a*b*c*x^2 - 4*a^2*c^2 + (b^2*x^4 - b^2*c^2)*log((x^2 + c
)/(x^2 - c))^2 - 4*(b^2*c*x^2 + a*b*c^2)*log((x^2 + c)/(x^2 - c)))/(c^2*x^4
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^5, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^5,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2/x^5,x)
```

maxima [B] time = 0.33, size = 183, normalized size = 1.89

$$\frac{1}{4}\left(c\left(\frac{\log\left(x^2 + c\right)}{c^3} - \frac{\log\left(x^2 - c\right)}{c^3} - \frac{2}{c^2x^2}\right) - \frac{2\,\text{artanh}\left(\frac{c}{x^2}\right)}{x^4}\right)ab - \frac{1}{16}\left(c^2\left(\frac{\log\left(x^2 + c\right)^2 - 2\left(\log\left(x^2 + c\right) - 2\right)\log\left(x^2\right.}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(log(x^2 + c)/c^3 - log(x^2 - c)/c^3 - 2/(c^2*x^2)) - 2*arctanh(c/x^
2)/x^4)*a*b - 1/16*(c^2*((log(x^2 + c)^2 - 2*(log(x^2 + c) - 2)*log(x^2 - c
) + log(x^2 - c)^2 + 4*log(x^2 + c))/c^4 - 16*log(x)/c^4) - 4*c*(log(x^2 +
c)/c^3 - log(x^2 - c)/c^3 - 2/(c^2*x^2))*arctanh(c/x^2))*b^2 - 1/4*b^2*arct
anh(c/x^2)^2/x^4 - 1/4*a^2/x^4
```

mupad [B] time = 1.54, size = 262, normalized size = 2.70

$$\frac{b^2\ln\left(x^2 + c\right)^2}{16\,c^2} - \frac{b^2\,\ln\left(x^2 - c\right)}{4\,c^2} - \frac{a^2}{4\,x^4} - \frac{b^2\ln\left(x^2 + c\right)^2}{16\,x^4} + \frac{b^2\ln\left(x^2 - c\right)^2}{16\,c^2} - \frac{b^2\ln\left(x^2 - c\right)^2}{16\,x^4} + \frac{b^2\,\ln(x)}{c^2} - \frac{b^2\,\ln\left(x^2 + c\right)}{4\,c^2} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2/x^5,x)
```

```
[Out] (b^2*log(c + x^2)^2)/(16*c^2) - (b^2*log(x^2 - c))/(4*c^2) - a^2/(4*x^4) -
(b^2*log(c + x^2)^2)/(16*x^4) + (b^2*log(x^2 - c)^2)/(16*c^2) - (b^2*log(x^
2 - c)^2)/(16*x^4) + (b^2*log(x))/c^2 - (b^2*log(c + x^2))/(4*c^2) - (a*b*l
og(c + x^2))/(4*x^4) + (b^2*log(x^2 - c))/(4*c*x^2) - (a*b*log(x^2 - c))/(4
*c^2) - (b^2*log(c + x^2)*log(x^2 - c))/(8*c^2) + (a*b*log(x^2 - c))/(4*x^4
) + (b^2*log(c + x^2)*log(x^2 - c))/(8*x^4) - (a*b)/(2*c*x^2) - (b^2*log(c
+ x^2))/(4*c*x^2) + (a*b*log(c + x^2))/(4*c^2)
```

sympy [A] time = 16.63, size = 172, normalized size = 1.77

$$\begin{cases} -\frac{a^2}{4x^4} - \frac{ab\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2x^4} - \frac{ab}{2cx^2} + \frac{ab\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2c^2} - \frac{b^2\operatorname{atanh}^2\left(\frac{c}{x^2}\right)}{4x^4} - \frac{b^2\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2cx^2} + \frac{b^2\log(x)}{c^2} - \frac{b^2\log\left(-i\sqrt{c}+x\right)}{2c^2} - \frac{b^2\log\left(i\sqrt{c}+x\right)}{2c^2} + \\ -\frac{a^2}{4x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) - a*b*atanh(c/x**2)/(2*x**4) - a*b/(2*c*x**2) + a
*b*atanh(c/x**2)/(2*c**2) - b**2*atanh(c/x**2)**2/(4*x**4) - b**2*atanh(c/x
**2)/(2*c*x**2) + b**2*log(x)/c**2 - b**2*log(-I*sqrt(c) + x)/(2*c**2) - b*
*2*log(I*sqrt(c) + x)/(2*c**2) + b**2*atanh(c/x**2)**2/(4*c**2) + b**2*atan
h(c/x**2)/(2*c**2), Ne(c, 0)), (-a**2/(4*x**4), True))
```

3.176 $\int x^4 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=1214

$$\frac{1}{20}\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 x^5+\frac{1}{20}b^2\log^2\left(\frac{c}{x^2}+1\right)x^5+\frac{1}{5}ab\log\left(\frac{c}{x^2}+1\right)x^5-\frac{1}{10}b^2\log\left(1-\frac{c}{x^2}\right)\log\left(\frac{c}{x^2}+1\right)x^5+\frac{2}{15}$$

```
[Out] 8/15*b^2*c^2*x+1/5*b^2*c^(5/2)*arctan(x/c^(1/2))*ln(1+c/x^2)-1/5*b^2*c^(5/2
)*arctanh(x/c^(1/2))*ln(1+c/x^2)-1/10*b^2*x^5*ln(1-c/x^2)*ln(1+c/x^2)-2/5*b
^2*c^(5/2)*arctan(x/c^(1/2))*ln(2*c^(1/2)/(-I*x+c^(1/2)))+1/5*b^2*c^(5/2)*a
rctan(x/c^(1/2))*ln((1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))-2/5*b^2*c^(5/2)*arct
anh(x/c^(1/2))*ln(2*c^(1/2)/(x+c^(1/2)))+1/5*b^2*c^(5/2)*arctanh(x/c^(1/2))
*ln(2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))+1/5*b^2*c^(
5/2)*arctan(x/c^(1/2))*ln((1-I)*(x+c^(1/2))/(-I*x+c^(1/2)))+1/5*b^2*c^(5/2)
*arctanh(x/c^(1/2))*ln(2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(
1/2)))+2/5*b^2*c^(5/2)*arctan(x/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2)))+2/5
*b^2*c^(5/2)*arctanh(x/c^(1/2))*ln(2-2*c^(1/2)/(x+c^(1/2)))-1/5*I*b^2*c^(5/
2)*arctan(x/c^(1/2))^2-1/5*I*b^2*c^(5/2)*polylog(2,-I*x/c^(1/2))-1/5*I*b^2*
c^(5/2)*polylog(2,-1+2*c^(1/2)/(-I*x+c^(1/2)))-1/10*I*b^2*c^(5/2)*polylog(2
,1-(1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))-1/10*I*b^2*c^(5/2)*polylog(2,1+(-1+I)
*(x+c^(1/2))/(-I*x+c^(1/2)))+2/5*a*b*c^(5/2)*arctan(x/c^(1/2))-1/15*b^2*c*x
^3*ln(1-c/x^2)-1/5*b^2*c^(5/2)*arctan(x/c^(1/2))*ln(1-c/x^2)+1/15*b*c*x^3*(
2*a-b*ln(1-c/x^2))-1/5*b*c^(5/2)*arctanh(x/c^(1/2))*(2*a-b*ln(1-c/x^2))+2/1
5*b^2*c*x^3*ln(1+c/x^2)+1/5*I*b^2*c^(5/2)*polylog(2,I*x/c^(1/2))+1/5*I*b^2*
c^(5/2)*polylog(2,1-2*c^(1/2)/(-I*x+c^(1/2)))+1/20*x^5*(2*a-b*ln(1-c/x^2))^
2+1/5*a*b*x^5*ln(1+c/x^2)-4/15*b^2*c^(5/2)*arctan(x/c^(1/2))-4/15*b^2*c^(5/
2)*arctanh(x/c^(1/2))+1/5*b^2*c^(5/2)*arctanh(x/c^(1/2))^2+1/20*b^2*x^5*ln(
1+c/x^2)^2+1/5*b^2*c^(5/2)*polylog(2,-x/c^(1/2))-1/5*b^2*c^(5/2)*polylog(2,
x/c^(1/2))+1/5*b^2*c^(5/2)*polylog(2,1-2*c^(1/2)/(x+c^(1/2)))-1/5*b^2*c^(5/
2)*polylog(2,-1+2*c^(1/2)/(x+c^(1/2)))-1/10*b^2*c^(5/2)*polylog(2,1-2*(-x+(
-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))-1/10*b^2*c^(5/2)*polyl
og(2,1-2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))+2/15*a*b*
c*x^3
```

Rubi [A] time = 2.71, antiderivative size = 1214, normalized size of antiderivative = 1.00, number of steps used = 97, number of rules used = 33, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.063, Rules used = {6099, 2457, 2476, 2448, 263, 207, 2455, 193, 321, 2470, 12, 260, 6688, 5988, 5932, 2447, 302, 6742, 203, 30, 2557, 5992, 5912, 5920, 2402, 2315, 204, 4928, 4848, 2391, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (8*b^2*c^2*x)/15 + (2*a*b*c*x^3)/15 + (2*a*b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 -
 (4*b^2*c^(5/2)*ArcTan[x/Sqrt[c]])/15 - (I/5)*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]
^2 - (4*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]])/15 + (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]
]^2)/5 + (2*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*
x)])/5 - (b^2*c*x^3*Log[1 - c/x^2])/15 - (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log
[1 - c/x^2])/5 + (b*c*x^3*(2*a - b*Log[1 - c/x^2]))/15 - (b*c^(5/2)*ArcTanh
[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/5 + (x^5*(2*a - b*Log[1 - c/x^2])^2)/
20 + (2*b^2*c*x^3*Log[1 + c/x^2])/15 + (a*b*x^5*Log[1 + c/x^2])/5 + (b^2*c^
(5/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/5 - (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]
*Log[1 + c/x^2])/5 - (b^2*x^5*Log[1 - c/x^2]*Log[1 + c/x^2])/10 + (b^2*x^5*
Log[1 + c/x^2]^2)/20 - (2*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sq
rt[c] - I*x)])/5 + (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x
))/(Sqrt[c] - I*x)])/5 - (2*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/
```

```
(Sqrt[c] + x)])/5 + (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c
] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/5 + (b^2*c^(5/2)*ArcTanh[x/S
qrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))
])/5 + (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c]
- I*x)])/5 + (2*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c]
 + x)])/5 + (I/5)*b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] -
 (I/5)*b^2*c^(5/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/10)*b^
2*c^(5/2)*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + (b^2*c^
(5/2)*PolyLog[2, -(x/Sqrt[c])])/5 - (I/5)*b^2*c^(5/2)*PolyLog[2, ((-I)*x)/S
qrt[c]] + (I/5)*b^2*c^(5/2)*PolyLog[2, (I*x)/Sqrt[c]] - (b^2*c^(5/2)*PolyLo
g[2, x/Sqrt[c]])/5 + (b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)]
)/5 - (b^2*c^(5/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/5 - (b^2*c^(
5/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[
c] + x))])/10 - (b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sq
rt[-c] + Sqrt[c])*(Sqrt[c] + x))])/10 - (I/10)*b^2*c^(5/2)*PolyLog[2, 1 - (
(1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
```

```
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^4 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(\frac{1}{4}x^4\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 - \frac{1}{2}bx^4\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)+\right.$$

$$= \frac{1}{4}\int x^4\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 dx - \frac{1}{2}b\int x^4\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\right.$$

$$= \frac{1}{20}x^5\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 + \frac{1}{20}b^2x^5\log^2\left(1+\frac{c}{x^2}\right) - \frac{1}{2}b\int\left(-2ax^4\log\left(1+\right.\right.$$

$$= \frac{1}{20}x^5\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 + \frac{1}{20}b^2x^5\log^2\left(1+\frac{c}{x^2}\right) + (ab)\int x^4\log\left(1+\frac{c}{x^2}\right.$$

$$= \frac{1}{20}x^5\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 + \frac{1}{5}abx^5\log\left(1+\frac{c}{x^2}\right) - \frac{1}{10}b^2x^5\log\left(1-\frac{c}{x^2}\right)\log$$

$$= \frac{2}{5}abc^2x + \frac{1}{15}bcx^3\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right) - \frac{1}{5}bc^{5/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1\right.\right.$$

$$= \frac{2}{5}abc^2x - \frac{1}{5}b^2c^2x\log\left(1-\frac{c}{x^2}\right) + \frac{1}{15}bcx^3\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right) - \frac{1}{5}bc^{5/2}\tanh$$

$$= \frac{4}{15}b^2c^2x + \frac{2}{15}abcx^3 - \frac{2}{5}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}b^2c^2x\log\left(1-\frac{c}{x^2}\right) + \frac{1}{15}bcx^3\left(\right.$$

$$= \frac{4}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{8}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}ib^2c^{5/2}$$

$$= \frac{4}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{8}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}ib^2c^{5/2}$$

$$= \frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{2}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}ib^2c^{5/2}$$

$$= \frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}ib^2c^{5/2}$$

$$= \frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}ib^2c^{5/2}$$

$$= \frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}ib^2c^{5/2}$$

$$= \frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}ib^2c^{5/2}$$

$$= \frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}ib^2c^{5/2}$$

$$= \frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}ib^2c^{5/2}$$

$$= \frac{8}{15}b^2c^2x + \frac{2}{15}abcx^3 + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{5}ib^2c^{5/2}$$

Mathematica [F] time = 5.90, size = 0, normalized size = 0.00

$$\int x^4 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] Integrate[x^4*(a + b*ArcTanh[c/x^2])^2, x]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2 x^4 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2\,abx^4 \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2 x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^4*arctanh(c/x^2)^2 + 2*a*b*x^4*arctanh(c/x^2) + a^2*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2 x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2*x^4, x)
```

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int x^4 \left(a + b \operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int(x^4*(a+b*arctanh(c/x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{5}\, a^2 x^5 + \frac{1}{15}\left(6\, x^5 \operatorname{artanh}\left(\frac{c}{x^2}\right) + \left(4\, x^3 + 6\, c^{\frac{3}{2}} \arctan\left(\frac{x}{\sqrt{c}}\right) + 3\, c^{\frac{3}{2}} \log\left(\frac{x - \sqrt{c}}{x + \sqrt{c}}\right)\right) c\right) ab + \frac{1}{20}\left(x^5 \log\left(x^2 - c\right)^2 - 5 \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 + 1/15*(6*x^5*arctanh(c/x^2) + (4*x^3 + 6*c^(3/2)*arctan(x/sqrt
(c)) + 3*c^(3/2)*log((x - sqrt(c))/(x + sqrt(c))))*c)*a*b + 1/20*(x^5*log(x
^2 - c)^2 - 5*integrate(-1/5*(5*(x^6 - c*x^4)*log(x^2 + c)^2 - 2*(2*x^6 + 5
*(x^6 - c*x^4)*log(x^2 + c))*log(x^2 - c))/(x^2 - c), x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(a + b\, \mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atanh(c/x^2))^2,x)
[Out] int(x^4*(a + b*atanh(c/x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c/x**2))**2,x)
[Out] Integral(x**4*(a + b*atanh(c/x**2))**2, x)
```

3.177 $\int x^2 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=1172

$$\frac{1}{12}\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 x^3+\frac{1}{12}b^2\log^2\left(\frac{c}{x^2}+1\right)x^3+\frac{1}{3}ab\log\left(\frac{c}{x^2}+1\right)x^3-\frac{1}{6}b^2\log\left(1-\frac{c}{x^2}\right)\log\left(\frac{c}{x^2}+1\right)x^3+\frac{4}{3}ab$$

```
[Out] 1/12*x^3*(2*a-b*ln(1-c/x^2))^2-2/3*a*b*c^(3/2)*arctan(x/c^(1/2))-2/3*b^2*c*
x*ln(1-c/x^2)+1/3*b^2*c^(3/2)*arctan(x/c^(1/2))*ln(1-c/x^2)-1/3*b*c^(3/2)*a
rctanh(x/c^(1/2))*(2*a-b*ln(1-c/x^2))+2/3*b^2*c*x*ln(1+c/x^2)+1/3*a*b*x^3*l
n(1+c/x^2)-1/3*b^2*c^(3/2)*arctan(x/c^(1/2))*ln(1+c/x^2)-1/3*b^2*c^(3/2)*ar
ctanh(x/c^(1/2))*ln(1+c/x^2)-1/6*b^2*x^3*ln(1-c/x^2)*ln(1+c/x^2)+2/3*b^2*c^
(3/2)*arctan(x/c^(1/2))*ln(2*c^(1/2)/(-I*x+c^(1/2)))-1/3*b^2*c^(3/2)*arctan
(x/c^(1/2))*ln((1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))-2/3*b^2*c^(3/2)*arctanh(x
/c^(1/2))*ln(2*c^(1/2)/(x+c^(1/2)))+1/3*b^2*c^(3/2)*arctanh(x/c^(1/2))*ln(2
*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))-1/3*b^2*c^(3/2)*
arctan(x/c^(1/2))*ln((1-I)*(x+c^(1/2))/(-I*x+c^(1/2)))+1/3*b^2*c^(3/2)*arct
anh(x/c^(1/2))*ln(2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2))
)-2/3*b^2*c^(3/2)*arctan(x/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2)))+2/3*b^2*
c^(3/2)*arctanh(x/c^(1/2))*ln(2-2*c^(1/2)/(x+c^(1/2)))-1/3*I*b^2*c^(3/2)*po
lylog(2,I*x/c^(1/2))-1/3*I*b^2*c^(3/2)*polylog(2,1-2*c^(1/2)/(-I*x+c^(1/2))
)+1/3*I*b^2*c^(3/2)*polylog(2,-I*x/c^(1/2))+1/3*I*b^2*c^(3/2)*polylog(2,-1+
2*c^(1/2)/(-I*x+c^(1/2)))+1/6*I*b^2*c^(3/2)*polylog(2,1-(1+I)*(-x+c^(1/2))/
(-I*x+c^(1/2)))+1/6*I*b^2*c^(3/2)*polylog(2,1+(-1+I)*(x+c^(1/2))/(-I*x+c^(1
/2)))+1/3*I*b^2*c^(3/2)*arctan(x/c^(1/2))^2+4/3*b^2*c^(3/2)*arctan(x/c^(1/2
))-4/3*b^2*c^(3/2)*arctanh(x/c^(1/2))+1/3*b^2*c^(3/2)*arctanh(x/c^(1/2))^2+
1/12*b^2*x^3*ln(1+c/x^2)^2+1/3*b^2*c^(3/2)*polylog(2,-x/c^(1/2))-1/3*b^2*c^
(3/2)*polylog(2,x/c^(1/2))+1/3*b^2*c^(3/2)*polylog(2,1-2*c^(1/2)/(x+c^(1/2)
))-1/3*b^2*c^(3/2)*polylog(2,-1+2*c^(1/2)/(x+c^(1/2)))-1/6*b^2*c^(3/2)*poly
log(2,1-2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))-1/6*b^2
*c^(3/2)*polylog(2,1-2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/
2)))+4/3*a*b*c*x
```

Rubi [A] time = 2.21, antiderivative size = 1172, normalized size of antiderivative = 1.00, number of steps used = 79, number of rules used = 33, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.063, Rules used = {6099, 2457, 2471, 2448, 263, 207, 2470, 12, 260, 6688, 5988, 5932, 2447, 2455, 193, 321, 6742, 203, 30, 2557, 5992, 5912, 5920, 2402, 2315, 2476, 204, 4928, 4848, 2391, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (4*a*b*c*x)/3 - (2*a*b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (4*b^2*c^(3/2)*ArcTan
[x/Sqrt[c]])/3 + (I/3)*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]^2 - (4*b^2*c^(3/2)*Arc
Tanh[x/Sqrt[c]])/3 + (b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]^2)/3 - (2*b^2*c^(3/2)*
ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/3 - (2*b^2*c*x*Log[
1 - c/x^2])/3 + (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/3 - (b*c^(3/
2)*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/3 + (x^3*(2*a - b*Log[1 - c
/x^2])^2)/12 + (2*b^2*c*x*Log[1 + c/x^2])/3 + (a*b*x^3*Log[1 + c/x^2])/3 -
(b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/3 - (b^2*c^(3/2)*ArcTanh[x/S
qrt[c]]*Log[1 + c/x^2])/3 - (b^2*x^3*Log[1 - c/x^2]*Log[1 + c/x^2])/6 + (b^
2*x^3*Log[1 + c/x^2]^2)/12 + (2*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c
])/(Sqrt[c] - I*x)])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[
c] - x))/(Sqrt[c] - I*x)])/3 - (2*b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqr
t[c])/(Sqrt[c] + x)])/3 + (b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(S
qrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/3 + (b^2*c^(3/2)*ArcTa
```

```
nh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c]
 + x))])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sq
rt[c] - I*x)])/3 + (2*b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(S
qrt[c] + x)])/3 - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I
*x)] + (I/3)*b^2*c^(3/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] + (I/
6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + (b
^2*c^(3/2)*PolyLog[2, -(x/Sqrt[c])])/3 + (I/3)*b^2*c^(3/2)*PolyLog[2, ((-I)
*x)/Sqrt[c]] - (I/3)*b^2*c^(3/2)*PolyLog[2, (I*x)/Sqrt[c]] - (b^2*c^(3/2)*P
olyLog[2, x/Sqrt[c]])/3 + (b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c]
+ x)])/3 - (b^2*c^(3/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/3 - (b^
2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(
Sqrt[c] + x))])/6 - (b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/
((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/6 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 -
 ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(\frac{1}{4}x^2 \left(2a - b\log\left(1 - \frac{c}{x^2}\right)\right)^2 - \frac{1}{2}bx^2\left(-2a + b\log\left(1 - \frac{c}{x^2}\right)\right)\log\left(1 + \frac{c}{x^2}\right) + \cdots$$

$$= \frac{1}{4}\int x^2 \left(2a - b\log\left(1 - \frac{c}{x^2}\right)\right)^2 dx - \frac{1}{2}b\int x^2\left(-2a + b\log\left(1 - \frac{c}{x^2}\right)\right)\log\left(1 + \cdots\right.$$

$$= \frac{1}{12}x^3\left(2a - b\log\left(1 - \frac{c}{x^2}\right)\right)^2 + \frac{1}{12}b^2x^3\log^2\left(1 + \frac{c}{x^2}\right) - \frac{1}{2}b\int\left(-2ax^2\log\left(1 + \cdots\right.\right.$$

$$= \frac{1}{12}x^3\left(2a - b\log\left(1 - \frac{c}{x^2}\right)\right)^2 + \frac{1}{12}b^2x^3\log^2\left(1 + \frac{c}{x^2}\right) + (ab)\int x^2\log\left(1 + \frac{c}{x^2}\cdots\right.$$

$$= \frac{2}{3}abcx + \frac{1}{12}x^3\left(2a - b\log\left(1 - \frac{c}{x^2}\right)\right)^2 + \frac{1}{3}abx^3\log\left(1 + \frac{c}{x^2}\right) - \frac{1}{6}b^2x^3\log\left(1 - \cdots\right.$$

$$= \frac{2}{3}abcx - \frac{1}{3}b^2cx\log\left(1 - \frac{c}{x^2}\right) - \frac{1}{3}bc^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a - b\log\left(1 - \frac{c}{x^2}\right)\right) + \cdots$$

$$= \frac{4}{3}abcx - \frac{1}{3}b^2cx\log\left(1 - \frac{c}{x^2}\right) - \frac{1}{3}bc^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a - b\log\left(1 - \frac{c}{x^2}\right)\right) + \cdots$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{2}{3}b^2c^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\cdots$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2\cdots$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2\cdots$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2\cdots$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2\cdots$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2\cdots$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2\cdots$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2\cdots$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2\cdots$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2\cdots$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2\cdots$$

Mathematica [F] time = 3.17, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x^2])^2,x]
[Out] Integrate[x^2*(a + b*ArcTanh[c/x^2])^2, x]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2\,abx^2 \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*arctanh(c/x^2)^2 + 2*a*b*x^2*arctanh(c/x^2) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x^2) + a)^2*x^2, x)
```

maple [F] time = 0.84, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x^2))^2,x)
[Out] int(x^2*(a+b*arctanh(c/x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^2x^3 + \frac{1}{3}\left(2\,x^3 \operatorname{artanh}\left(\frac{c}{x^2}\right) - \left(2\,\sqrt{c}\,\arctan\left(\frac{x}{\sqrt{c}}\right) - \sqrt{c}\,\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right) - 4\,x\right)c\right)ab + \frac{1}{12}\left(x^3 \log\left(x^2 - c\right)^2 - 3\int -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c/x^2) - (2*sqrt(c)*arctan(x/sqrt(c)) - sq
rt(c)*log((x - sqrt(c))/(x + sqrt(c))) - 4*x)*c)*a*b + 1/12*(x^3*log(x^2 -
c)^2 - 3*integrate(-1/3*(3*(x^4 - c*x^2)*log(x^2 + c)^2 - 2*(2*x^4 + 3*(x^4
 - c*x^2)*log(x^2 + c))*log(x^2 - c))/(x^2 - c), x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c/x^2))^2,x)
[Out] int(x^2*(a + b*atanh(c/x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x**2))**2,x)
[Out] Integral(x**2*(a + b*atanh(c/x**2))**2, x)
```

3.178 $\int \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=1549

result too large to display

```
[Out] 1/2*b^2*ln(1-c/x^2)*ln(x+c^(1/2))*c^(1/2)-1/2*b^2*ln(1/2*(-x+c^(1/2))/c^(1/
2))*ln(x+c^(1/2))*c^(1/2)+1/2*b^2*ln(-x+c^(1/2))*ln(1/2*(x+c^(1/2))/c^(1/2)
)*c^(1/2)-I*b^2*polylog(2,-I*x/c^(1/2))*c^(1/2)-1/2*I*b^2*polylog(2,1-(1+I)
*(-x+c^(1/2))/(-I*x+c^(1/2)))*c^(1/2)-1/2*I*b^2*polylog(2,1+(-1+I)*(x+c^(1/
2))/(-I*x+c^(1/2)))*c^(1/2)-1/2*b^2*x*ln(1-c/x^2)*ln(1+c/x^2)-1/2*b^2*ln(1+
c/x^2)*ln(-x+(-c)^(1/2))*(-c)^(1/2)+1/2*b^2*ln(1+c/x^2)*ln(x+(-c)^(1/2))*(-
c)^(1/2)-1/2*b^2*ln(1/2*(-x+(-c)^(1/2))/(-c)^(1/2))*ln(x+(-c)^(1/2))*(-c)^(
1/2)+1/2*b^2*ln(-x+(-c)^(1/2))*ln(1/2*(x+(-c)^(1/2))/(-c)^(1/2))*(-c)^(1/2)
+2*a*b*arctan(x/c^(1/2))*c^(1/2)-2*a*b*arctanh(x/c^(1/2))*c^(1/2)-1/2*b^2*l
n(1-c/x^2)*ln(-x+c^(1/2))*c^(1/2)-2*b^2*arctan(x/c^(1/2))*ln(2*c^(1/2)/(-I*
x+c^(1/2)))*c^(1/2)-2*b^2*arctanh(x/c^(1/2))*ln(2*c^(1/2)/(x+c^(1/2)))*c^(1
/2)+I*b^2*polylog(2,I*x/c^(1/2))*c^(1/2)+I*b^2*polylog(2,1-2*c^(1/2)/(-I*x+
c^(1/2)))*c^(1/2)-a*b*x*ln(1-c/x^2)-b^2*ln(x/(-c)^(1/2))*ln(-x+(-c)^(1/2))*
(-c)^(1/2)+a*b*x*ln(1+c/x^2)+b^2*ln(-x/(-c)^(1/2))*ln(x+(-c)^(1/2))*(-c)^(1
/2)-b^2*arctan(x/c^(1/2))*ln(1-c/x^2)*c^(1/2)-b^2*arctanh(x/c^(1/2))*ln(1+c
/x^2)*c^(1/2)-b^2*ln(x/c^(1/2))*ln(-x+c^(1/2))*c^(1/2)+b^2*arctan(x/c^(1/2)
)*ln((1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))*c^(1/2)+b^2*arctanh(x/c^(1/2))*ln(2
*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))*c^(1/2)+b^2*ln(-
x/c^(1/2))*ln(x+c^(1/2))*c^(1/2)+b^2*arctan(x/c^(1/2))*ln((1-I)*(x+c^(1/2))
/(-I*x+c^(1/2)))*c^(1/2)+b^2*arctanh(x/c^(1/2))*ln(2*(x+(-c)^(1/2))*c^(1/2)
/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))*c^(1/2)-b^2*polylog(2,1-x/c^(1/2))*c^(1/
2)+1/4*b^2*x*ln(1-c/x^2)^2+1/4*b^2*x*ln(1+c/x^2)^2+1/4*b^2*ln(-x+(-c)^(1/2)
)^2*(-c)^(1/2)-1/4*b^2*ln(x+(-c)^(1/2))^2*(-c)^(1/2)+1/2*b^2*polylog(2,1/2-
1/2*x/(-c)^(1/2))*(-c)^(1/2)-1/2*b^2*polylog(2,1/2*(c-x*(-c)^(1/2))/c)*(-c)
^(1/2)+1/4*b^2*ln(-x+c^(1/2))^2*c^(1/2)-1/4*b^2*ln(x+c^(1/2))^2*c^(1/2)+1/2
*b^2*polylog(2,1/2-1/2*x/c^(1/2))*c^(1/2)-1/2*b^2*polylog(2,1/2*(x+c^(1/2))
/c^(1/2))*c^(1/2)-1/2*b^2*polylog(2,1-2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)
-c^(1/2))/(x+c^(1/2)))*c^(1/2)-1/2*b^2*polylog(2,1-2*(x+(-c)^(1/2))*c^(1/2)
/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))*c^(1/2)+b^2*polylog(2,1+x/c^(1/2))*c^(1/
2)+b^2*polylog(2,-x/c^(1/2))*c^(1/2)-b^2*polylog(2,x/c^(1/2))*c^(1/2)+b^2*p
olylog(2,1-2*c^(1/2)/(x+c^(1/2)))*c^(1/2)-b^2*polylog(2,1-x/(-c)^(1/2))*(-c
)^(1/2)+b^2*polylog(2,1+x/(-c)^(1/2))*(-c)^(1/2)+a^2*x
```

Rubi [A] time = 2.25, antiderivative size = 1549, normalized size of antiderivative = 1.00, number of steps used = 99, number of rules used = 29, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.417, Rules used = {6093, 2448, 263, 207, 2450, 2476, 2462, 260, 2416, 2394, 2315, 2390, 2301, 2393, 2391, 203, 2556, 12, 2470, 6688, 5992, 5912, 5920, 2402, 2447, 204, 4928, 4848, 4856}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] a^2*x + 2*a*b*Sqrt[c]*ArcTan[x/Sqrt[c]] - 2*a*b*Sqrt[c]*ArcTanh[x/Sqrt[c]]
- a*b*x*Log[1 - c/x^2] - b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2] + (b^
2*x*Log[1 - c/x^2]^2)/4 + a*b*x*Log[1 + c/x^2] - b^2*Sqrt[c]*ArcTanh[x/Sqrt
[c]]*Log[1 + c/x^2] - (b^2*x*Log[1 - c/x^2]*Log[1 + c/x^2])/2 + (b^2*x*Log[
1 + c/x^2]^2)/4 - (b^2*Sqrt[-c]*Log[1 + c/x^2]*Log[Sqrt[-c] - x])/2 + (b^2*
Sqrt[-c]*Log[Sqrt[-c] - x]^2)/4 - (b^2*Sqrt[c]*Log[1 - c/x^2]*Log[Sqrt[c] -
 x])/2 + (b^2*Sqrt[c]*Log[Sqrt[c] - x]^2)/4 - 2*b^2*Sqrt[c]*ArcTan[x/Sqrt[c
]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)] + b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[((1
 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] - b^2*Sqrt[-c]*Log[Sqrt[-c] - x]*Log[
x/Sqrt[-c]] - b^2*Sqrt[c]*Log[Sqrt[c] - x]*Log[x/Sqrt[c]] + (b^2*Sqrt[-c]*L
```

```
og[1 + c/x^2]*Log[Sqrt[-c] + x])/2 - (b^2*Sqrt[-c]*Log[(Sqrt[-c] - x)/(2*Sq
rt[-c])]*Log[Sqrt[-c] + x])/2 + b^2*Sqrt[-c]*Log[-(x/Sqrt[-c])]*Log[Sqrt[-c
] + x] - (b^2*Sqrt[-c]*Log[Sqrt[-c] + x]^2)/4 + (b^2*Sqrt[-c]*Log[Sqrt[-c]
- x]*Log[(Sqrt[-c] + x)/(2*Sqrt[-c])])/2 - 2*b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]
*Log[(2*Sqrt[c])/(Sqrt[c] + x)] + b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]*Log[(2*Sqr
t[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))] + b^2*Sqrt[c]*Ar
cTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt
[c] + x))] + (b^2*Sqrt[c]*Log[1 - c/x^2]*Log[Sqrt[c] + x])/2 - (b^2*Sqrt[c]
*Log[(Sqrt[c] - x)/(2*Sqrt[c])]*Log[Sqrt[c] + x])/2 + b^2*Sqrt[c]*Log[-(x/S
qrt[c])]*Log[Sqrt[c] + x] - (b^2*Sqrt[c]*Log[Sqrt[c] + x]^2)/4 + (b^2*Sqrt[
c]*Log[Sqrt[c] - x]*Log[(Sqrt[c] + x)/(2*Sqrt[c])])/2 + b^2*Sqrt[c]*ArcTan[
x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)] + I*b^2*Sqrt[c]*Pol
yLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 -
 ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + b^2*Sqrt[c]*PolyLog[2, -(x/Sqrt
[c])] - I*b^2*Sqrt[c]*PolyLog[2, ((-I)*x)/Sqrt[c]] + I*b^2*Sqrt[c]*PolyLog[
2, (I*x)/Sqrt[c]] - b^2*Sqrt[c]*PolyLog[2, x/Sqrt[c]] - (b^2*Sqrt[c]*PolyLo
g[2, (Sqrt[c] + x)/(2*Sqrt[c])])/2 + (b^2*Sqrt[-c]*PolyLog[2, (1 - x/Sqrt[-
c])/2])/2 - b^2*Sqrt[-c]*PolyLog[2, 1 - x/Sqrt[-c]] + b^2*Sqrt[-c]*PolyLog[
2, 1 + x/Sqrt[-c]] - (b^2*Sqrt[-c]*PolyLog[2, (c - Sqrt[-c]*x)/(2*c)])/2 -
b^2*Sqrt[c]*PolyLog[2, 1 - x/Sqrt[c]] + (b^2*Sqrt[c]*PolyLog[2, 1/2 - x/(2*
Sqrt[c])])/2 + b^2*Sqrt[c]*PolyLog[2, 1 + x/Sqrt[c]] + b^2*Sqrt[c]*PolyLog[
2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)] - (b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c]*
(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/2 - (b^2*Sqrt[c]*Pol
yLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))
])/2 - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] -
I*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6093

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Int[Expand
Integrand[(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x] /; Fr
eeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(a^2 - ab\log\left(1-\frac{c}{x^2}\right) + \frac{1}{4}b^2\log^2\left(1-\frac{c}{x^2}\right) + ab\log\left(1+\frac{c}{x^2}\right) - \frac{1}{2}b^2\log\left(1-\right.\right.$$

$$= a^2x - (ab)\int \log\left(1-\frac{c}{x^2}\right)dx + (ab)\int \log\left(1+\frac{c}{x^2}\right)dx + \frac{1}{4}b^2\int \log^2\left(1-\frac{c}{x^2}\right)$$

$$= a^2x - abx\log\left(1-\frac{c}{x^2}\right) + \frac{1}{4}b^2x\log^2\left(1-\frac{c}{x^2}\right) + abx\log\left(1+\frac{c}{x^2}\right) - \frac{1}{2}b^2x\log\left(1-\right.$$

$$= a^2x - abx\log\left(1-\frac{c}{x^2}\right) + \frac{1}{4}b^2x\log^2\left(1-\frac{c}{x^2}\right) + abx\log\left(1+\frac{c}{x^2}\right) - \frac{1}{2}b^2x\log\left(1-\right.$$

$$= a^2x + 2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1-\frac{c}{x^2}\right) - b^2\sqrt{c}\tan$$

$$= a^2x + 2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1-\frac{c}{x^2}\right) - b^2\sqrt{c}\tan$$

$$= a^2x + 2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1-\frac{c}{x^2}\right) - b^2\sqrt{c}\tan$$

$$= a^2x + 2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1-\frac{c}{x^2}\right) - b^2\sqrt{c}\tan$$

$$= a^2x + 2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1-\frac{c}{x^2}\right) - b^2\sqrt{c}\tan$$

$$= a^2x + 2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1-\frac{c}{x^2}\right) - b^2\sqrt{c}\tan$$

$$= a^2x + 2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1-\frac{c}{x^2}\right) - b^2\sqrt{c}\tan$$

$$= a^2x + 2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1-\frac{c}{x^2}\right) - b^2\sqrt{c}\tan$$

$$= a^2x + 2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1-\frac{c}{x^2}\right) - b^2\sqrt{c}\tan$$

$$= a^2x + 2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - 2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - abx\log\left(1-\frac{c}{x^2}\right) - b^2\sqrt{c}\tan$$

Mathematica [A] time = 3.48, size = 565, normalized size = 0.36

$$a^2x+2abx\tanh^{-1}\left(\frac{c}{x^2}\right)-2abx\sqrt{\frac{c}{x^2}}\left(\tan^{-1}\left(\sqrt{\frac{c}{x^2}}\right)+\tanh^{-1}\left(\sqrt{\frac{c}{x^2}}\right)\right)-\frac{1}{2}b^2x\sqrt{\frac{c}{x^2}}\left(-\mathrm{Li}_2\left(\frac{1}{2}\left(1-\sqrt{\frac{c}{x^2}}\right)\right)+\mathrm{Li}_2\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] a^2*x - 2*a*b*Sqrt[c/x^2]*x*(ArcTan[Sqrt[c/x^2]] + ArcTanh[Sqrt[c/x^2]]) +
2*a*b*x*ArcTanh[c/x^2] - (b^2*Sqrt[c/x^2]*x*((-2*I)*ArcTan[Sqrt[c/x^2]]^2 +
 4*ArcTan[Sqrt[c/x^2]]*ArcTanh[c/x^2] - (2*ArcTanh[c/x^2]^2)/Sqrt[c/x^2] +
2*ArcTan[Sqrt[c/x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c/x^2]])] - 2*ArcTanh[c/
x^2]*Log[1 - Sqrt[c/x^2]] + Log[2]*Log[1 - Sqrt[c/x^2]] - Log[1 - Sqrt[c/x^
2]]^2/2 + Log[1 - Sqrt[c/x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c/x^2])] + 2*ArcT
anh[c/x^2]*Log[1 + Sqrt[c/x^2]] - Log[2]*Log[1 + Sqrt[c/x^2]] - Log[((1 + I
) - (1 - I)*Sqrt[c/x^2])/2]*Log[1 + Sqrt[c/x^2]] - Log[(-1/2 - I/2)*(I + Sq
rt[c/x^2])]*Log[1 + Sqrt[c/x^2]] + Log[1 + Sqrt[c/x^2]]^2/2 + Log[1 - Sqrt[
c/x^2]]*Log[((1 + I) + (1 - I)*Sqrt[c/x^2])/2] - (I/2)*PolyLog[2, -E^((4*I)
*ArcTan[Sqrt[c/x^2]])] - PolyLog[2, (1 - Sqrt[c/x^2])/2] + PolyLog[2, (-1/2
 - I/2)*(-1 + Sqrt[c/x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c/x^2])] +
 PolyLog[2, (1 + Sqrt[c/x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c/x^2])
] - PolyLog[2, (1/2 + I/2)*(1 + Sqrt[c/x^2])]))/2
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^2\,\mathrm{artanh}\left(\frac{c}{x^2}\right)^2 + 2\,ab\,\mathrm{artanh}\left(\frac{c}{x^2}\right) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\mathrm{artanh}\left(\frac{c}{x^2}\right) + a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2, x)
```

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(c\left(\frac{2\,\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{\sqrt{c}}\right) + 2\,x\,\mathrm{artanh}\left(\frac{c}{x^2}\right)\right)ab + \frac{1}{4}\left(x\log\left(x^2-c\right)^2 - \int -\frac{\left(x^2-c\right)\log\left(x^2+c\right)^2 - 2\left(2\,x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] (c*(2*arctan(x/sqrt(c))/sqrt(c) + log((x - sqrt(c))/(x + sqrt(c)))/sqrt(c))
 + 2*x*arctanh(c/x^2))*a*b + 1/4*(x*log(x^2 - c)^2 - integrate(-((x^2 - c)*
log(x^2 + c)^2 - 2*(2*x^2 + (x^2 - c)*log(x^2 + c))*log(x^2 - c))/(x^2 - c)
, x))*b^2 + a^2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2,x)
```

```
[Out] int((a + b*atanh(c/x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2, x)
```

3.179 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=1117

$$-\frac{i\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 b^2}{\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 b^2}{\sqrt{c}}-\frac{\log^2\left(\frac{c}{x^2}+1\right)b^2}{4x}-\frac{2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)b^2}{\sqrt{c}}-\frac{2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)b^2}{\sqrt{c}}-\frac{2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)b^2}{\sqrt{c}}+\frac{2\,\text{ta}}{}$$

```
[Out] 2*a*b/x-1/4*(2*a-b*ln(1-c/x^2))^2/x+1/2*b^2*ln(1-c/x^2)*ln(1+c/x^2)/x-2*a*b
*arccot(x/c^(1/2))/c^(1/2)+2*b^2*arccot(x/c^(1/2))*ln(2/(1-I*c^(1/2)/x))/c^
(1/2)+2*b^2*arccoth(x/c^(1/2))*ln(2/(1+1/x*c^(1/2)))/c^(1/2)+2*b^2*arctan(x
/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2)))/c^(1/2)-2*b^2*arctanh(x/c^(1/2))*l
n(2-2*c^(1/2)/(x+c^(1/2)))/c^(1/2)-I*b^2*arctan(x/c^(1/2))^2/c^(1/2)-I*b^2*
polylog(2,-1+2*c^(1/2)/(-I*x+c^(1/2)))/c^(1/2)-I*b^2*polylog(2,1-2/(1-I*c^(
1/2)/x))/c^(1/2)-a*b*ln(1+c/x^2)/x+b^2*arccot(x/c^(1/2))*ln(1-c/x^2)/c^(1/2
)+b*arctanh(x/c^(1/2))*(2*a-b*ln(1-c/x^2))/c^(1/2)+b^2*arccoth(x/c^(1/2))*l
n(1+c/x^2)/c^(1/2)+b^2*arctan(x/c^(1/2))*ln(1+c/x^2)/c^(1/2)-b^2*arccot(x/c
^(1/2))*ln((1+I)*(1-1/x*c^(1/2))/(1-I*c^(1/2)/x))/c^(1/2)-b^2*arccoth(x/c^(
1/2))*ln(-2*(1-(-c)^(1/2)/x)*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+1/x*c^(1/2)))/
c^(1/2)-b^2*arccoth(x/c^(1/2))*ln(2*(1+(-c)^(1/2)/x)*c^(1/2)/((-c)^(1/2)+c^
(1/2))/(1+1/x*c^(1/2)))/c^(1/2)-b^2*arccot(x/c^(1/2))*ln((1-I)*(1+1/x*c^(1/
2))/(1-I*c^(1/2)/x))/c^(1/2)+1/2*I*b^2*polylog(2,1-(1+I)*(1-1/x*c^(1/2))/(1
-I*c^(1/2)/x))/c^(1/2)+1/2*I*b^2*polylog(2,1+(-1+I)*(1+1/x*c^(1/2))/(1-I*c^
(1/2)/x))/c^(1/2)-b^2*polylog(2,1-2/(1+1/x*c^(1/2)))/c^(1/2)-1/4*b^2*ln(1+c
/x^2)^2/x-2*b^2*arccot(x/c^(1/2))/c^(1/2)-2*b^2*arccoth(x/c^(1/2))/c^(1/2)-
2*b^2*arctan(x/c^(1/2))/c^(1/2)+2*b^2*arctanh(x/c^(1/2))/c^(1/2)+1/2*b^2*po
lylog(2,1+2*(1-(-c)^(1/2)/x)*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+1/x*c^(1/2)))/
c^(1/2)+1/2*b^2*polylog(2,1-2*(1+(-c)^(1/2)/x)*c^(1/2)/((-c)^(1/2)+c^(1/2))
/(1+1/x*c^(1/2)))/c^(1/2)-b^2*ln(1-c/x^2)/x-b*(2*a-b*ln(1-c/x^2))/x-b^2*arc
tanh(x/c^(1/2))^2/c^(1/2)+b^2*polylog(2,-1+2*c^(1/2)/(x+c^(1/2)))/c^(1/2)
```

Rubi [A] time = 2.15, antiderivative size = 1117, normalized size of antiderivative = 1.00, number of steps used = 71, number of rules used = 29, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.812, Rules used = {6099, 2457, 2476, 2455, 263, 325, 207, 206, 2470, 12, 260, 6688, 5988, 5932, 2447, 6715, 2448, 321, 6742, 203, 2556, 5992, 5920, 2402, 2315, 4928, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^2,x]
```

```
[Out] (2*a*b)/x - (2*a*b*ArcCot[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcCot[x/Sqrt[c]])/S
qrt[c] - (2*b^2*ArcCoth[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcTan[x/Sqrt[c]])/Sqr
t[c] - (I*b^2*ArcTan[x/Sqrt[c]]^2)/Sqrt[c] + (2*b^2*ArcTanh[x/Sqrt[c]])/Sqr
t[c] - (b^2*ArcTanh[x/Sqrt[c]]^2)/Sqrt[c] + (2*b^2*ArcTan[x/Sqrt[c]]*Log[2
- (2*Sqrt[c])/(Sqrt[c] - I*x)])/Sqrt[c] - (b^2*Log[1 - c/x^2])/x + (b^2*Arc
Cot[x/Sqrt[c]]*Log[1 - c/x^2])/Sqrt[c] - (b*(2*a - b*Log[1 - c/x^2]))/x + (
b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/Sqrt[c] - (2*a - b*Log[1 - c
/x^2])^2/(4*x) - (a*b*Log[1 + c/x^2])/x + (b^2*ArcCoth[x/Sqrt[c]]*Log[1 + c
/x^2])/Sqrt[c] + (b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/Sqrt[c] + (b^2*Log[
1 - c/x^2]*Log[1 + c/x^2])/(2*x) - (b^2*Log[1 + c/x^2]^2)/(4*x) + (2*b^2*Ar
cCot[x/Sqrt[c]]*Log[2/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (b^2*ArcCot[x/Sqrt[c]
]*Log[((1 + I)*(1 - Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] + (2*b^2*ArcC
oth[x/Sqrt[c]]*Log[2/(1 + Sqrt[c]/x)])/Sqrt[c] - (b^2*ArcCoth[x/Sqrt[c]]*Lo
g[(-2*Sqrt[c]*(1 - Sqrt[-c]/x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]/x))])/Sq
rt[c] - (b^2*ArcCoth[x/Sqrt[c]]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]/x))/((Sqrt[-c]
```

```
 + Sqrt[c])*(1 + Sqrt[c]/x))])/Sqrt[c] - (b^2*ArcCot[x/Sqrt[c]]*Log[((1 - I
)*(1 + Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (2*b^2*ArcTanh[x/Sqrt[c]
]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c] - (I*b^2*PolyLog[2, 1 - 2/(1
- (I*Sqrt[c])/x)])/Sqrt[c] + ((I/2)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (b^2*PolyLog[2, 1 - 2/(1 + Sqrt[c]/x)
])/Sqrt[c] + (b^2*PolyLog[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]/x))/((Sqrt[-c] -
Sqrt[c])*(1 + Sqrt[c]/x))])/(2*Sqrt[c]) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1
 + Sqrt[-c]/x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]/x))])/(2*Sqrt[c]) + ((I/
2)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[
c] - (I*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)])/Sqrt[c] + (b^2*Po
lyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx = \int\left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^2} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^2} + \frac{b^2\log^2\left(1+\right.}{4x^2}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^2}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^2}\,dx +$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x} + \frac{1}{2}b\,\mathrm{Subst}\left(\int\left(-2a+b\log\left(1-cx^2\right.\right.\right.$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x} + \frac{1}{2}b\,\mathrm{Subst}\left(\int\left(-2a\log\left(1+cx^2\right)+\right.\right.$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x} + b\int \frac{2a-b\log\left(1-\frac{c}{x^2}\right)}{x^2}\,dx + b\int$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{x} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{\sqrt{c}} - \frac{\left(2a-b\log\left(1-\right.\right.}{4x}$$

$$= \frac{2ab}{x} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{x} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{\sqrt{c}} - \frac{\left(2a-b\log\right.}{}$$

$$= \frac{2ab}{x} - \frac{4b^2}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{x} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\right.}{\sqrt{c}}$$

$$= \frac{2ab}{x} - \frac{4b^2}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}} + \frac{2b^2\tanh^{-1}}{\sqrt{c}}$$

$$= \frac{2ab}{x} - \frac{4b^2}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}} + \frac{2b^2\tanh^{-1}}{\sqrt{c}}$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}} + \frac{2b^2\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} -$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} -$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} -$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} -$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} -$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} -$$

Mathematica [A] time = 3.01, size = 568, normalized size = 0.51

$$\frac{-2a^2-4ab\tanh^{-1}\left(\frac{c}{x^2}\right)-\frac{4ab\left(\tan^{-1}\left(\sqrt{\frac{c}{x^2}}\right)-\tanh^{-1}\left(\sqrt{\frac{c}{x^2}}\right)\right)}{\sqrt{\frac{c}{x^2}}}+\frac{b^2\left(-\text{Li}_2\left(\frac{1}{2}\left(1-\sqrt{\frac{c}{x^2}}\right)\right)+\text{Li}_2\left(\left(-\frac{1}{2}-\frac{i}{2}\right)\left(\sqrt{\frac{c}{x^2}}-1\right)\right)+\text{Li}_2\left(\left(-\frac{1}{2}+\frac{i}{2}\right)\left(\sqrt{\frac{c}{x^2}}-1\right)\right)+\text{L}}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^2,x]
```

```
[Out] (-2*a^2 - (4*a*b*(ArcTan[Sqrt[c/x^2]] - ArcTanh[Sqrt[c/x^2]]))/Sqrt[c/x^2]
- 4*a*b*ArcTanh[c/x^2] + (b^2*((2*I)*ArcTan[Sqrt[c/x^2]]^2 - 4*ArcTan[Sqrt[
c/x^2]]*ArcTanh[c/x^2] - 2*Sqrt[c/x^2]*ArcTanh[c/x^2]^2 - 2*ArcTan[Sqrt[c/x
^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c/x^2]])] - 2*ArcTanh[c/x^2]*Log[1 - Sqrt
[c/x^2]] + Log[2]*Log[1 - Sqrt[c/x^2]] - Log[1 - Sqrt[c/x^2]]^2/2 + Log[1 -
 Sqrt[c/x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c/x^2])] + 2*ArcTanh[c/x^2]*Log[1
+ Sqrt[c/x^2]] - Log[2]*Log[1 + Sqrt[c/x^2]] - Log[((1 + I) - (1 - I)*Sqrt[
c/x^2])/2]*Log[1 + Sqrt[c/x^2]] - Log[(-1/2 - I/2)*(I + Sqrt[c/x^2])]*Log[1
 + Sqrt[c/x^2]] + Log[1 + Sqrt[c/x^2]]^2/2 + Log[1 - Sqrt[c/x^2]]*Log[((1 +
 I) + (1 - I)*Sqrt[c/x^2])/2] + (I/2)*PolyLog[2, -E^((4*I)*ArcTan[Sqrt[c/x^
2]])] - PolyLog[2, (1 - Sqrt[c/x^2])/2] + PolyLog[2, (-1/2 - I/2)*(-1 + Sqr
t[c/x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c/x^2])] + PolyLog[2, (1 +
Sqrt[c/x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c/x^2])] - PolyLog[2, (1
/2 + I/2)*(1 + Sqrt[c/x^2])]))/Sqrt[c/x^2])/(2*x)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(\frac{c}{x^2}\right)^2+2\,ab\,\text{artanh}\left(\frac{c}{x^2}\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^2, x)
```

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^2,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(c\left(\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{2\,\mathrm{artanh}\left(\frac{c}{x^2}\right)}{x}\right)ab-\frac{1}{4}b^2\left(\frac{\log\left(x^2-c\right)^2}{x}+\int-\frac{\left(x^2-c\right)\log\left(x^2+c\right)^2+2\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="maxima")
```

```
[Out] (c*(2*arctan(x/sqrt(c))/c^(3/2) - log((x - sqrt(c))/(x + sqrt(c)))/c^(3/2))
 - 2*arctanh(c/x^2)/x)*a*b - 1/4*b^2*(log(x^2 - c)^2/x + integrate(-((x^2 -
 c)*log(x^2 + c)^2 + 2*(2*x^2 - (x^2 - c)*log(x^2 + c))*log(x^2 - c))/(x^4
- c*x^2), x)) - a^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2/x^2,x)
```

```
[Out] int((a + b*atanh(c/x^2))^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**2,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2/x**2, x)
```

3.180 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=1263

$$\frac{i\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 b^2}{3c^{3/2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 b^2}{3c^{3/2}}-\frac{\log^2\left(\frac{c}{x^2}+1\right)b^2}{12x^3}+\frac{4\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)b^2}{3c^{3/2}}+\frac{4\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)b^2}{3c^{3/2}}-\frac{2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\right.}{3c^{3/2}}$$

```
[Out] 2/9*a*b/x^3-1/12*(2*a-b*ln(1-c/x^2))^2/x^3-1/3*a*b*ln(1+c/x^2)/x^3-2/3*b^2*
ln(1+c/x^2)/c/x-1/3*b^2*arctan(x/c^(1/2))*ln(1+c/x^2)/c^(3/2)+1/3*b^2*arcta
nh(x/c^(1/2))*ln(1+c/x^2)/c^(3/2)+1/6*b^2*ln(1-c/x^2)*ln(1+c/x^2)/x^3+2/3*b
^2*arctan(x/c^(1/2))*ln(2*c^(1/2)/(-I*x+c^(1/2)))/c^(3/2)-1/3*b^2*arctan(x/
c^(1/2))*ln((1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))/c^(3/2)+2/3*b^2*arctanh(x/c^
(1/2))*ln(2*c^(1/2)/(x+c^(1/2)))/c^(3/2)-1/3*b^2*arctanh(x/c^(1/2))*ln(2*(-
x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))/c^(3/2)-1/3*b^2*arc
tan(x/c^(1/2))*ln((1-I)*(x+c^(1/2))/(-I*x+c^(1/2)))/c^(3/2)-1/3*b^2*arctanh
(x/c^(1/2))*ln(2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))/c
^(3/2)-2/3*b^2*arctan(x/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2)))/c^(3/2)-2/3
*b^2*arctanh(x/c^(1/2))*ln(2-2*c^(1/2)/(x+c^(1/2)))/c^(3/2)-1/3*I*b^2*polyl
og(2,I*x/c^(1/2))/c^(3/2)-1/3*I*b^2*polylog(2,1-2*c^(1/2)/(-I*x+c^(1/2)))/c
^(3/2)+1/3*I*b^2*polylog(2,-I*x/c^(1/2))/c^(3/2)+1/3*I*b^2*polylog(2,-1+2*c
^(1/2)/(-I*x+c^(1/2)))/c^(3/2)+1/6*I*b^2*polylog(2,1-(1+I)*(-x+c^(1/2))/(-I
*x+c^(1/2)))/c^(3/2)+1/6*I*b^2*polylog(2,1+(-1+I)*(x+c^(1/2))/(-I*x+c^(1/2)
))/c^(3/2)+1/3*I*b^2*arctan(x/c^(1/2))^2/c^(3/2)-2/3*a*b*arctan(x/c^(1/2))/
c^(3/2)+1/3*b^2*ln(1-c/x^2)/c/x+1/3*b^2*arctan(x/c^(1/2))*ln(1-c/x^2)/c^(3/
2)-1/3*b*(2*a-b*ln(1-c/x^2))/c/x+1/3*b*arctanh(x/c^(1/2))*(2*a-b*ln(1-c/x^2
))/c^(3/2)+4/3*b^2*arctan(x/c^(1/2))/c^(3/2)+4/3*b^2*arctanh(x/c^(1/2))/c^(
3/2)-1/3*b^2*arctanh(x/c^(1/2))^2/c^(3/2)-1/9*b^2*ln(1-c/x^2)/x^3-1/9*b*(2*
a-b*ln(1-c/x^2))/x^3-1/12*b^2*ln(1+c/x^2)^2/x^3-1/3*b^2*polylog(2,-x/c^(1/2
))/c^(3/2)+1/3*b^2*polylog(2,x/c^(1/2))/c^(3/2)-1/3*b^2*polylog(2,1-2*c^(1/
2)/(x+c^(1/2)))/c^(3/2)+1/3*b^2*polylog(2,-1+2*c^(1/2)/(x+c^(1/2)))/c^(3/2)
+1/6*b^2*polylog(2,1-2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1
/2)))/c^(3/2)+1/6*b^2*polylog(2,1-2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c
)^(1/2)+c^(1/2)))/c^(3/2)-2/3*a*b/c/x
```

Rubi [A] time = 2.58, antiderivative size = 1263, normalized size of antiderivative = 1.00, number of steps used = 104, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {6099, 2457, 2476, 2455, 263, 325, 207, 206, 2470, 12, 260, 6688, 5988, 5932, 2447, 6742, 203, 30, 2557, 5992, 5912, 5920, 2402, 2315, 4928, 4848, 2391, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^4,x]
```

```
[Out] (2*a*b)/(9*x^3) - (2*a*b)/(3*c*x) - (2*a*b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) +
 (4*b^2*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) + ((I/3)*b^2*ArcTan[x/Sqrt[c]]^2)/c^
(3/2) + (4*b^2*ArcTanh[x/Sqrt[c]])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]^2)
/(3*c^(3/2)) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)
])/(3*c^(3/2)) - (b^2*Log[1 - c/x^2])/(9*x^3) + (b^2*Log[1 - c/x^2])/(3*c*x
) + (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(3*c^(3/2)) - (b*(2*a - b*Log[1
- c/x^2]))/(9*x^3) - (b*(2*a - b*Log[1 - c/x^2]))/(3*c*x) + (b*ArcTanh[x/Sq
rt[c]]*(2*a - b*Log[1 - c/x^2]))/(3*c^(3/2)) - (2*a - b*Log[1 - c/x^2])^2/(
12*x^3) - (a*b*Log[1 + c/x^2])/(3*x^3) - (2*b^2*Log[1 + c/x^2])/(3*c*x) - (
b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*ArcTanh[x/Sqrt[c]]
*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(6*x^3)
- (b^2*Log[1 + c/x^2]^2)/(12*x^3) + (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c]
```

```
)/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt
[c] - x))/(Sqrt[c] - I*x)])/(3*c^(3/2)) + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*
Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[
c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(3*c^(3/2)) - (b^
2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(
Sqrt[c] + x))])/(3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c]
+ x))/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*
Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) - ((I/3)*b^2*PolyLog[2, 1 - (2*Sqrt[c]
)/(Sqrt[c] - I*x)])/c^(3/2) + ((I/3)*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[
c] - I*x)])/c^(3/2) + ((I/6)*b^2*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sq
rt[c] - I*x)])/c^(3/2) - (b^2*PolyLog[2, -(x/Sqrt[c])])/(3*c^(3/2)) + ((I/3
)*b^2*PolyLog[2, ((-I)*x)/Sqrt[c]])/c^(3/2) - ((I/3)*b^2*PolyLog[2, (I*x)/S
qrt[c]])/c^(3/2) + (b^2*PolyLog[2, x/Sqrt[c]])/(3*c^(3/2)) - (b^2*PolyLog[2
, 1 - (2*Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) + (b^2*PolyLog[2, -1 + (2*Sqr
t[c])/(Sqrt[c] + x)])/(3*c^(3/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c
] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(6*c^(3/2)) + (b^2*PolyLog[2
, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(6*
c^(3/2)) + ((I/6)*b^2*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x
)])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx = \int\left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^4} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^4} + \frac{b^2\log^2\left(1+\cdots\right.}{4x^4}\cdots\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^4}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^4}\,dx + \cdots$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{12x^3} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{12x^3} - \frac{1}{2}b\int\left(-\frac{2a\log\left(1+\frac{c}{x^2}\right)}{x^4} + \frac{b\log\left(1\cdots\right.}{\cdots}\right.$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{12x^3} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{12x^3} + (ab)\int \frac{\log\left(1+\frac{c}{x^2}\right)}{x^4}\,dx - \frac{1}{2}b^2\int \frac{\log\cdots}{\cdots}$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{12x^3} - \frac{ab\log\left(1+\frac{c}{x^2}\right)}{3x^3} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right)}{6x^3} - \frac{b^2\log\cdots}{\cdots}$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{9x^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{3cx} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1\cdots\right.\right.}{3c^{3/2}}\cdots$$

$$= \frac{2ab}{9x^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{9x^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{3cx} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\cdots\right.}{3c^{3/2}}\cdots$$

$$= \frac{2ab}{9x^3} - \frac{4b^2}{27x^3} - \frac{2ab}{3cx} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{9x^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{3cx} + \frac{b\tanh^{-1}\cdots}{\cdots}$$

$$= \frac{2ab}{9x^3} - \frac{4b^2}{27x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{2b^2\cdots}{\cdots}$$

$$= \frac{2ab}{9x^3} - \frac{4b^2}{27x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{8b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{9c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{8b^2\cdots}{\cdots}$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{8b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{9c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{8b^2\tanh^{-1}\cdots}{9c^{3/2}}\cdots$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{14b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{9c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{14b^2\tanh\cdots}{9c^{3/\cdots}}\cdots$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{4b^2\tanh^{-1}\cdots}{3c^{3/2}}\cdots$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{4b^2\tanh^{-1}\cdots}{3c^{3/2}}\cdots$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{4b^2\tanh^{-1}\cdots}{3c^{3/2}}\cdots$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{4b^2\tanh^{-1}\cdots}{3c^{3/2}}\cdots$$

Mathematica [F] time = 2.91, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^4,x]
[Out] Integrate[(a + b*ArcTanh[c/x^2])^2/x^4, x]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(\frac{c}{x^2}\right)^2+2\,ab\,\text{artanh}\left(\frac{c}{x^2}\right)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^4,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^4,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^4, x)
```

maple [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^4,x)
[Out] int((a+b*arctanh(c/x^2))^2/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(c\left(\frac{2\,\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{4}{c^2x}\right)+\frac{2\,\text{artanh}\left(\frac{c}{x^2}\right)}{x^3}\right)ab-\frac{1}{12}\,b^2\left(\frac{\log\left(x^2-c\right)^2}{x^3}+3\int-\frac{3\left(x^2-c\right)\log\left(x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^4,x, algorithm="maxima")
[Out] -1/3*(c*(2*arctan(x/sqrt(c))/c^(5/2) + log((x - sqrt(c))/(x + sqrt(c)))/c^(
5/2) + 4/(c^2*x)) + 2*arctanh(c/x^2)/x^3)*a*b - 1/12*b^2*(log(x^2 - c)^2/x^
3 + 3*integrate(-1/3*(3*(x^2 - c)*log(x^2 + c)^2 + 2*(2*x^2 - 3*(x^2 - c)*l
og(x^2 + c))*log(x^2 - c))/(x^6 - c*x^4), x)) - 1/3*a^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2/x^4,x)
```

```
[Out] int((a + b*atanh(c/x^2))^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**4,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2/x**4, x)
```

3.181 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=1337

$$-\frac{i\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 b^2}{5c^{5/2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 b^2}{5c^{5/2}}-\frac{\log^2\left(\frac{c}{x^2}+1\right)b^2}{20x^5}-\frac{4\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)b^2}{15c^{5/2}}+\frac{4\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)b^2}{15c^{5/2}}+\frac{2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\right.}{5c^{5/2}}$$

```
[Out] 2/25*a*b/x^5-8/15*b^2/c^2/x-1/5*b^2*arctan(x/c^(1/2))*ln(1-c/x^2)/c^(5/2)-1
/15*b*(2*a-b*ln(1-c/x^2))/c/x^3-1/5*b*(2*a-b*ln(1-c/x^2))/c^2/x+1/5*I*b^2*p
olylog(2,I*x/c^(1/2))/c^(5/2)+1/5*I*b^2*polylog(2,1-2*c^(1/2)/(-I*x+c^(1/2)
))/c^(5/2)-1/20*(2*a-b*ln(1-c/x^2))^2/x^5+1/5*b*arctanh(x/c^(1/2))*(2*a-b*l
n(1-c/x^2))/c^(5/2)-1/5*a*b*ln(1+c/x^2)/x^5-2/15*b^2*ln(1+c/x^2)/c/x^3+1/5*
b^2*arctan(x/c^(1/2))*ln(1+c/x^2)/c^(5/2)+1/5*b^2*arctanh(x/c^(1/2))*ln(1+c
/x^2)/c^(5/2)+1/10*b^2*ln(1-c/x^2)*ln(1+c/x^2)/x^5-2/5*b^2*arctan(x/c^(1/2)
)*ln(2*c^(1/2)/(-I*x+c^(1/2)))/c^(5/2)+1/5*b^2*arctan(x/c^(1/2))*ln((1+I)*(
-x+c^(1/2))/(-I*x+c^(1/2)))/c^(5/2)+2/5*b^2*arctanh(x/c^(1/2))*ln(2*c^(1/2)
/(x+c^(1/2)))/c^(5/2)-1/5*b^2*arctanh(x/c^(1/2))*ln(2*(-x+(-c)^(1/2))*c^(1/
2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))/c^(5/2)+1/5*b^2*arctan(x/c^(1/2))*ln((
1-I)*(x+c^(1/2))/(-I*x+c^(1/2)))/c^(5/2)-1/5*b^2*arctanh(x/c^(1/2))*ln(2*(x
+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))/c^(5/2)+2/5*b^2*arct
an(x/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2)))/c^(5/2)-2/5*b^2*arctanh(x/c^(1
/2))*ln(2-2*c^(1/2)/(x+c^(1/2)))/c^(5/2)-1/5*I*b^2*arctan(x/c^(1/2))^2/c^(5
/2)-1/5*I*b^2*polylog(2,-I*x/c^(1/2))/c^(5/2)-1/5*I*b^2*polylog(2,-1+2*c^(1
/2)/(-I*x+c^(1/2)))/c^(5/2)-1/10*I*b^2*polylog(2,1-(1+I)*(-x+c^(1/2))/(-I*x
+c^(1/2)))/c^(5/2)-1/10*I*b^2*polylog(2,1+(-1+I)*(x+c^(1/2))/(-I*x+c^(1/2))
)/c^(5/2)+2/5*a*b*arctan(x/c^(1/2))/c^(5/2)+1/15*b^2*ln(1-c/x^2)/c/x^3-1/5*
b^2*ln(1-c/x^2)/c^2/x-4/15*b^2*arctan(x/c^(1/2))/c^(5/2)+4/15*b^2*arctanh(x
/c^(1/2))/c^(5/2)-1/5*b^2*arctanh(x/c^(1/2))^2/c^(5/2)-1/25*b^2*ln(1-c/x^2)
/x^5-1/25*b*(2*a-b*ln(1-c/x^2))/x^5-1/20*b^2*ln(1+c/x^2)^2/x^5-1/5*b^2*poly
log(2,-x/c^(1/2))/c^(5/2)+1/5*b^2*polylog(2,x/c^(1/2))/c^(5/2)-1/5*b^2*poly
log(2,1-2*c^(1/2)/(x+c^(1/2)))/c^(5/2)+1/5*b^2*polylog(2,-1+2*c^(1/2)/(x+c^
(1/2)))/c^(5/2)+1/10*b^2*polylog(2,1-2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-
c^(1/2))/(x+c^(1/2)))/c^(5/2)+1/10*b^2*polylog(2,1-2*(x+(-c)^(1/2))*c^(1/2)
/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))/c^(5/2)-2/15*a*b/c/x^3+2/5*a*b/c^2/x
```

Rubi [A] time = 2.79, antiderivative size = 1337, normalized size of antiderivative = 1.00, number of steps used = 129, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {6099, 2457, 2476, 2455, 263, 325, 207, 206, 2470, 12, 260, 6688, 5988, 5932, 2447, 6742, 203, 30, 2557, 5992, 5912, 5920, 2402, 2315, 4928, 4848, 2391, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^6,x]
```

```
[Out] (2*a*b)/(25*x^5) - (2*a*b)/(15*c*x^3) + (2*a*b)/(5*c^2*x) - (8*b^2)/(15*c^2
*x) + (2*a*b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (4*b^2*ArcTan[x/Sqrt[c]])/(15
*c^(5/2)) - ((I/5)*b^2*ArcTan[x/Sqrt[c]]^2)/c^(5/2) + (4*b^2*ArcTanh[x/Sqrt
[c]])/(15*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]^2)/(5*c^(5/2)) + (2*b^2*ArcTan
[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/(5*c^(5/2)) - (b^2*Log[1
- c/x^2])/(25*x^5) + (b^2*Log[1 - c/x^2])/(15*c*x^3) - (b^2*Log[1 - c/x^2])
/(5*c^2*x) - (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(5*c^(5/2)) - (b*(2*a -
 b*Log[1 - c/x^2]))/(25*x^5) - (b*(2*a - b*Log[1 - c/x^2]))/(15*c*x^3) - (b
*(2*a - b*Log[1 - c/x^2]))/(5*c^2*x) + (b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1
 - c/x^2]))/(5*c^(5/2)) - (2*a - b*Log[1 - c/x^2])^2/(20*x^5) - (a*b*Log[1
+ c/x^2])/(5*x^5) - (2*b^2*Log[1 + c/x^2])/(15*c*x^3) + (b^2*ArcTan[x/Sqrt[
```

```
c]]*Log[1 + c/x^2])/(5*c^(5/2)) + (b^2*ArcTanh[x/Sqrt[c]]*Log[1 + c/x^2])/(
5*c^(5/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(10*x^5) - (b^2*Log[1 + c/
x^2]^2)/(20*x^5) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)
])/(5*c^(5/2)) + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x))/(Sqrt[c
] - I*x)])/(5*c^(5/2)) + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c]
 + x)])/(5*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] - x)
)/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(5*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[
c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(
5*c^(5/2)) + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] -
I*x)])/(5*c^(5/2)) - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c]
 + x)])/(5*c^(5/2)) + ((I/5)*b^2*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)
])/c^(5/2) - ((I/5)*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)])/c^(5/
2) - ((I/10)*b^2*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/c
^(5/2) - (b^2*PolyLog[2, -(x/Sqrt[c])])/(5*c^(5/2)) - ((I/5)*b^2*PolyLog[2,
 ((-I)*x)/Sqrt[c]])/c^(5/2) + ((I/5)*b^2*PolyLog[2, (I*x)/Sqrt[c]])/c^(5/2)
 + (b^2*PolyLog[2, x/Sqrt[c]])/(5*c^(5/2)) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]
)/(Sqrt[c] + x)])/(5*c^(5/2)) + (b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] +
 x)])/(5*c^(5/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-
c] - Sqrt[c])*(Sqrt[c] + x))])/(10*c^(5/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c
]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(10*c^(5/2)) - ((I
/10)*b^2*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
```

```
& IntegerQ[s]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
```

```
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx = \int\left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^6} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^6} + \frac{b^2\log^2\left(1+\right.}{4x^6}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^6}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^6}\,dx +$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{20x^5} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{20x^5} - \frac{1}{2}b\int\left(-\frac{2a\log\left(1+\frac{c}{x^2}\right)}{x^6} + \frac{b\log\left(1\right.}{}\right.$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{20x^5} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{20x^5} + (ab)\int \frac{\log\left(1+\frac{c}{x^2}\right)}{x^6}\,dx - \frac{1}{2}b^2\int \frac{\log}{}$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{20x^5} - \frac{ab\log\left(1+\frac{c}{x^2}\right)}{5x^5} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right)}{10x^5} - \frac{b^2\log}{}$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{25x^5} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{15cx^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{5c^2x} + \frac{b\,\mathrm{ta}}{}$$

$$= \frac{2ab}{25x^5} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{25x^5} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{15cx^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{5c^2x}$$

$$= \frac{2ab}{25x^5} - \frac{4b^2}{125x^5} - \frac{2ab}{15cx^3} - \frac{4b^2}{5c^2x} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{25x^5} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{15cx^3}$$

$$= \frac{2ab}{25x^5} - \frac{4b^2}{125x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{16b^2}{15c^2x} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}} +$$

$$= \frac{2ab}{25x^5} - \frac{4b^2}{125x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{92b^2}{75c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{8b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{15c^{5/2}} -$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{32b^2}{75c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{46b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{75c^{5/2}} - \frac{ib^2\tan^{-1}}{5c^{5/2}}$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{52b^2}{75c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{16b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{75c^{5/2}} - \frac{ib^2\tan^{-1}}{5c^{5/2}}$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{8b^2}{15c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{26b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{75c^{5/2}} - \frac{ib^2\tan^{-1}}{5c^{5/2}}$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{8b^2}{15c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{15c^{5/2}} - \frac{ib^2\tan^{-1}}{5c^{5/2}}$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{8b^2}{15c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{15c^{5/2}} - \frac{ib^2\tan^{-1}}{5c^{5/2}}$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{8b^2}{15c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{15c^{5/2}} - \frac{ib^2\tan^{-1}}{5c^{5/2}}$$

Mathematica [F] time = 2.77, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^6,x]
[Out] Integrate[(a + b*ArcTanh[c/x^2])^2/x^6, x]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(\frac{c}{x^2}\right)^2+2\,ab\,\text{artanh}\left(\frac{c}{x^2}\right)+a^2}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^6,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^6,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^6, x)
```

maple [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^6,x)
[Out] int((a+b*arctanh(c/x^2))^2/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}\left(c\left(\frac{6\,\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{7}{2}}}-\frac{3\,\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^{\frac{7}{2}}}-\frac{4}{c^2x^3}\right)-\frac{6\,\text{artanh}\left(\frac{c}{x^2}\right)}{x^5}\right)ab-\frac{1}{20}\,b^2\left(\frac{\log\left(x^2-c\right)^2}{x^5}+5\int-\frac{5\left(x^2-c\right)\log\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^6,x, algorithm="maxima")
[Out] 1/15*(c*(6*arctan(x/sqrt(c))/c^(7/2) - 3*log((x - sqrt(c))/(x + sqrt(c)))/c
^(7/2) - 4/(c^2*x^3)) - 6*arctanh(c/x^2)/x^5)*a*b - 1/20*b^2*(log(x^2 - c)^
2/x^5 + 5*integrate(-1/5*(5*(x^2 - c)*log(x^2 + c)^2 + 2*(2*x^2 - 5*(x^2 -
c)*log(x^2 + c))*log(x^2 - c))/(x^8 - c*x^6), x)) - 1/5*a^2/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c/x^2))^2/x^6,x)
```

```
[Out] int((a + b*atanh(c/x^2))^2/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**6,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2/x**6, x)
```

3.182 $\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c/x^2))^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c/x^2])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c/x^2])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 dx = \int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 dx$$

Mathematica [A] time = 2.67, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^3, x]
```

fricas [A] time = 0.97, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^3\,\mathrm{artanh}\left(\frac{c}{x^2}\right)^3 + 3\,ab^2\,\mathrm{artanh}\left(\frac{c}{x^2}\right)^2 + 3\,a^2b\,\mathrm{artanh}\left(\frac{c}{x^2}\right) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="fricas")
[Out] integral((b^3*arctanh(c/x^2)^3 + 3*a*b^2*arctanh(c/x^2)^2 + 3*a^2*b*arctanh
(c/x^2) + a^3)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\mathrm{artanh}\left(\frac{c}{x^2}\right) + a\right)^3 (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x^2) + a)^3*(d*x)^m, x)
```

maple [A] time = 0.92, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c/x^2))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c/x^2))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3 d^m x x^m \log\left(x^2 - c\right)^3}{8\,(m+1)} + \frac{(dx)^{m+1}\,a^3}{d(m+1)} + \int \frac{\left(b^3 d^m (m+1)x^2 - b^3 c d^m (m+1)\right)x^m \log\left(x^2 + c\right)^3 + 6\left(ab^2 d^m (m+1)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*d^m*x*x^m*log(x^2 - c)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1)) +
 integrate(1/8*((b^3*d^m*(m + 1)*x^2 - b^3*c*d^m*(m + 1))*x^m*log(x^2 + c)^
3 + 6*(a*b^2*d^m*(m + 1)*x^2 - a*b^2*c*d^m*(m + 1))*x^m*log(x^2 + c)^2 + 12
*(a^2*b*d^m*(m + 1)*x^2 - a^2*b*c*d^m*(m + 1))*x^m*log(x^2 + c) + 3*((b^3*d
^m*(m + 1)*x^2 - b^3*c*d^m*(m + 1))*x^m*log(x^2 + c) - 2*(a*b^2*c*d^m*(m +
1) - (a*b^2*d^m*(m + 1) + b^3*d^m)*x^2)*x^m)*log(x^2 - c)^2 - 3*((b^3*d^m*(
m + 1)*x^2 - b^3*c*d^m*(m + 1))*x^m*log(x^2 + c)^2 + 4*(a*b^2*d^m*(m + 1)*x
^2 - a*b^2*c*d^m*(m + 1))*x^m*log(x^2 + c) + 4*(a^2*b*d^m*(m + 1)*x^2 - a^2
*b*c*d^m*(m + 1))*x^m)*log(x^2 - c))/((m + 1)*x^2 - c*(m + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m \left(a + b\operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c/x^2))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c/x^2))^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b\operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c/x**2))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c/x**2))**3, x)
```

3.183 $\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c/x^2))^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c/x^2])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c/x^2])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Mathematica [A] time = 1.72, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^2, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\,\text{artanh}\left(\frac{c}{x^2}\right)^2+2\,ab\,\text{artanh}\left(\frac{c}{x^2}\right)+a^2\right)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2 (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c/x^2) + a)^2*(d*x)^m, x)
```

maple [A] time = 0.40, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b \operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c/x^2))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2 d^m x x^m \log\left(x^2 - c\right)^2}{4\,(m+1)} + \frac{(dx)^{m+1}\,a^2}{d(m+1)} - \int -\frac{\left(b^2 d^m (m+1) x^2 - b^2 c d^m (m+1)\right) x^m \log\left(x^2 + c\right)^2 + 4\left(abd^m (m+1)x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d^m*x*x^m*log(x^2 - c)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1)) -
integrate(-1/4*((b^2*d^m*(m + 1)*x^2 - b^2*c*d^m*(m + 1))*x^m*log(x^2 + c)^
2 + 4*(a*b*d^m*(m + 1)*x^2 - a*b*c*d^m*(m + 1))*x^m*log(x^2 + c) - 2*((b^2*
d^m*(m + 1)*x^2 - b^2*c*d^m*(m + 1))*x^m*log(x^2 + c) - 2*(a*b*c*d^m*(m + 1
) - (a*b*d^m*(m + 1) + b^2*d^m)*x^2)*x^m)*log(x^2 - c))/((m + 1)*x^2 - c*(m
 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c/x^2))^2,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c/x^2))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c/x**2))**2, x)
```

3.184 $\int (dx)^m \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=75

$$\frac{(dx)^{m+1}\left(a + b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(m+1)} - \frac{2bcd(dx)^{m-1}\,{}_2F_1\left(1, \frac{1-m}{4}; \frac{5-m}{4}; \frac{c^2}{x^4}\right)}{1-m^2}$$

```
[Out] (d*x)^(1+m)*(a+b*arctanh(c/x^2))/d/(1+m)-2*b*c*d*(d*x)^(-1+m)*hypergeom([1,
 1/4-1/4*m],[5/4-1/4*m],c^2/x^4)/(-m^2+1)
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6097, 16, 339, 364}

$$\frac{(dx)^{m+1}\left(a + b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(m+1)} - \frac{2bcd(dx)^{m-1}\,{}_2F_1\left(1, \frac{1-m}{4}; \frac{5-m}{4}; \frac{c^2}{x^4}\right)}{1-m^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c/x^2]))/(d*(1 + m)) - (2*b*c*d*(d*x)^(-1 + m
)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, c^2/x^4])/(1 - m^2)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx = \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} + \frac{(2bc)\int \frac{(dx)^{1+m}}{\left(1-\frac{c^2}{x^4}\right)x^3}\,dx}{d(1+m)}$$

$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} + \frac{\left(2bcd^2\right)\int \frac{(dx)^{-2+m}}{1-\frac{c^2}{x^4}}\,dx}{1+m}$$

$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} - \frac{\left(2bcd\left(\frac{1}{x}\right)^{-1+m}(dx)^{-1+m}\right)\text{Subst}\left(\int \frac{x^{-m}}{1-c^2x^4}\,dx,\right.}{1+m}$$

$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} - \frac{2bcd(dx)^{-1+m}\,{}_2F_1\left(1,\frac{1-m}{4};\frac{5-m}{4};\frac{c^2}{x^4}\right)}{1-m^2}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 0.91

$$\frac{(dx)^m\left((m-1)x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+2bc\,{}_2F_1\left(1,\frac{1}{4}-\frac{m}{4};\frac{5}{4}-\frac{m}{4};\frac{c^2}{x^4}\right)\right)}{(m-1)(m+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] ((d*x)^m*((-1 + m)*x^2*(a + b*ArcTanh[c/x^2]) + 2*b*c*Hypergeometric2F1[1,
1/4 - m/4, 5/4 - m/4, c^2/x^4]))/((-1 + m)*(1 + m)*x)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c/x^2) + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)*(d*x)^m, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c/x^2)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c/x^2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(4\,cd^{m}\int\frac{x^{2}x^{m}}{(m+1)x^{4}-c^{2}(m+1)}\,dx+\frac{d^{m}xx^{m}\log\left(x^{2}+c\right)-d^{m}xx^{m}\log\left(x^{2}-c\right)}{m+1}\right)b+\frac{(dx)^{m+1}\,a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*c*d^m*integrate(x^2*x^m/((m + 1)*x^4 - c^2*(m + 1)), x) + (d^m*x*x^m
*log(x^2 + c) - d^m*x*x^m*log(x^2 - c))/(m + 1))*b + (d*x)^(m + 1)*a/(d*(m
+ 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{m}\,\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^{2}}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c/x^2)),x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c/x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{m}\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^{2}}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c/x**2)),x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c/x**2)), x)
```

3.185 $\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c/x^2)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c/x^2]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c/x^2]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx$$

Mathematica [A] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2]), x]
```

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b\,\text{artanh}\left(\frac{c}{x^2}\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctanh(c/x^2) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\,\text{artanh}\left(\frac{c}{x^2}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c/x^2) + a), x)
```

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c/x^2)),x)
[Out] int((d*x)^m/(a+b*arctanh(c/x^2)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\,\mathrm{artanh}\left(\frac{c}{x^2}\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctanh(c/x^2) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c/x^2)),x)
[Out] int((d*x)^m/(a + b*atanh(c/x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c/x**2)),x)
[Out] Integral((d*x)**m/(a + b*atanh(c/x**2)), x)
```

3.186 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2} \, dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2},x\right)$$

[Out] Unintegrable((d*x)^m/(a+b*arctanh(c/x^2))^2,x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2} \, dx$$

Verification is Not applicable to the result.

[In] Int[(d*x)^m/(a + b*ArcTanh[c/x^2])^2,x]

[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c/x^2])^2, x]

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2} \, dx = \int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2} \, dx$$

Mathematica [A] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2} \, dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2])^2,x]

[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2])^2, x]

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(dx)^m}{b^2\,\mathrm{artanh}\left(\frac{c}{x^2}\right)^2+2\,ab\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")

[Out] integral((d*x)^m/(b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+a\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c/x^2) + a)^2, x)
```

maple [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a + b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c/x^2))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(d^m x^4 - c^2 d^m\right)x^m}{b^2 cx \log\left(x^2 + c\right) - b^2 cx \log\left(x^2 - c\right) + 2\,abcx} + \int -\frac{\left(d^m(m+3)x^4 - c^2 d^m(m-1)\right)x^m}{b^2 cx^2 \log\left(x^2 + c\right) - b^2 cx^2 \log\left(x^2 - c\right) + 2\,abcx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] (d^m*x^4 - c^2*d^m)*x^m/(b^2*c*x*log(x^2 + c) - b^2*c*x*log(x^2 - c) + 2*a*
b*c*x) + integrate(-(d^m*(m + 3)*x^4 - c^2*d^m*(m - 1))*x^m/(b^2*c*x^2*log(
x^2 + c) - b^2*c*x^2*log(x^2 - c) + 2*a*b*c*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{\left(a + b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c/x^2))^2,x)
```

```
[Out] int((d*x)^m/(a + b*atanh(c/x^2))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Timed out
```

3.187 $\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right) dx$

Optimal. Leaf size=88

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{4c^8}+\frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}$$

```
[Out] 1/12*b*x^(3/2)/c^5+1/20*b*x^(5/2)/c^3+1/28*b*x^(7/2)/c-1/4*b*arctanh(c*x^(1
/2))/c^8+1/4*x^4*(a+b*arctanh(c*x^(1/2)))+1/4*b*x^(1/2)/c^7
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6097, 50, 63, 206}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+\frac{bx^{5/2}}{20c^3}+\frac{bx^{3/2}}{12c^5}+\frac{b\sqrt{x}}{4c^7}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{4c^8}+\frac{bx^{7/2}}{28c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(4*c^7) + (b*x^(3/2))/(12*c^5) + (b*x^(5/2))/(20*c^3) + (b*x^(7
/2))/(28*c) - (b*ArcTanh[c*Sqrt[x]])/(4*c^8) + (x^4*(a + b*ArcTanh[c*Sqrt[x
]]))/4
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right) dx = \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{1}{8}(bc)\int \frac{x^{7/2}}{1-c^2x}dx$$
$$=\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int \frac{x^{5/2}}{1-c^2x}dx}{8c}$$
$$=\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int \frac{x^{3/2}}{1-c^2x}dx}{8c^3}$$
$$=\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int \frac{\sqrt{x}}{1-c^2x}dx}{8c^5}$$
$$=\frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}dx}{8c^7}$$
$$=\frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\,\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,c\right.}{4c^7}$$
$$=\frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{4c^8}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)$$

Mathematica [A] time = 0.03, size = 114, normalized size = 1.30

$$\frac{ax^4}{4}+\frac{b\log\left(1-c\sqrt{x}\right)}{8c^8}-\frac{b\log\left(c\sqrt{x}+1\right)}{8c^8}+\frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}bx^4\tanh^{-1}\left(c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(4*c^7) + (b*x^(3/2))/(12*c^5) + (b*x^(5/2))/(20*c^3) + (b*x^(7
/2))/(28*c) + (a*x^4)/4 + (b*x^4*ArcTanh[c*Sqrt[x]])/4 + (b*Log[1 - c*Sqrt[
x]])/(8*c^8) - (b*Log[1 + c*Sqrt[x]])/(8*c^8)
```

fricas [A] time = 0.64, size = 89, normalized size = 1.01

$$\frac{210\,ac^8x^4+105\left(bc^8x^4-b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)+2\left(15\,bc^7x^3+21\,bc^5x^2+35\,bc^3x+105\,bc\right)\sqrt{x}}{840\,c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/840*(210*a*c^8*x^4 + 105*(b*c^8*x^4 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(
c^2*x - 1)) + 2*(15*b*c^7*x^3 + 21*b*c^5*x^2 + 35*b*c^3*x + 105*b*c)*sqrt(x
))/c^8
```

giac [B] time = 0.21, size = 359, normalized size = 4.08

$$\frac{1}{4}ax^4+\frac{2}{105}bc\left(\frac{\frac{105\left(c\sqrt{x}+1\right)^6}{\left(c\sqrt{x}-1\right)^6}-\frac{315\left(c\sqrt{x}+1\right)^5}{\left(c\sqrt{x}-1\right)^5}+\frac{770\left(c\sqrt{x}+1\right)^4}{\left(c\sqrt{x}-1\right)^4}-\frac{770\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{609\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}-\frac{203\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+44}{c^9\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^7}+\frac{105\left(\frac{(c\sqrt{}}{(c\sqrt{}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")
```

```
[Out] 1/4*a*x^4 + 2/105*b*c*((105*(c*sqrt(x) + 1)^6/(c*sqrt(x) - 1)^6 - 315*(c*sq
rt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 770*(c*sqrt(x) + 1)^4/(c*sqrt(x) - 1)^4 -
770*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 609*(c*sqrt(x) + 1)^2/(c*sqrt(x)
- 1)^2 - 203*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 44)/(c^9*((c*sqrt(x) + 1)/(c
*sqrt(x) - 1) - 1)^7) + 105*((c*sqrt(x) + 1)^7/(c*sqrt(x) - 1)^7 + 7*(c*sqr
t(x) + 1)^5/(c*sqrt(x) - 1)^5 + 7*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + (c*
sqrt(x) + 1)/(c*sqrt(x) - 1))*log(-(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)
/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1) - c) + 1)/(c*((c*sqrt(x) + 1)/(c*sqrt(x
) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1) - c) - 1))/(c^9*((c*sqrt(x)
+ 1)/(c*sqrt(x) - 1) - 1)^8))
```

maple [A] time = 0.03, size = 84, normalized size = 0.95

$$\frac{x^4 a}{4}+\frac{b\,x^4\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{4}+\frac{b\,x^{\frac{7}{2}}}{28c}+\frac{b\,x^{\frac{5}{2}}}{20c^3}+\frac{b\,x^{\frac{3}{2}}}{12c^5}+\frac{b\sqrt{x}}{4c^7}+\frac{b\ln\left(c\sqrt{x}-1\right)}{8c^8}-\frac{b\ln\left(1+c\sqrt{x}\right)}{8c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(1/2))),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x^(1/2))+1/28*b*x^(7/2)/c+1/20*b*x^(5/2)/c^3+
1/12*b*x^(3/2)/c^5+1/4*b*x^(1/2)/c^7+1/8/c^8*b*ln(c*x^(1/2)-1)-1/8/c^8*b*ln
(1+c*x^(1/2))
```

maxima [A] time = 0.32, size = 86, normalized size = 0.98

$$\frac{1}{4}\,ax^4+\frac{1}{840}\left(210\,x^4\,\mathrm{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(15\,c^6x^{\frac{7}{2}}+21\,c^4x^{\frac{5}{2}}+35\,c^2x^{\frac{3}{2}}+105\,\sqrt{x}\right)}{c^8}-\frac{105\,\log\left(c\sqrt{x}+1\right)}{c^9}+\frac{105}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/840*(210*x^4*arctanh(c*sqrt(x)) + c*(2*(15*c^6*x^(7/2) + 21*c
^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sqrt(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9
 + 105*log(c*sqrt(x) - 1)/c^9))*b
```

mupad [B] time = 1.43, size = 86, normalized size = 0.98

$$\frac{\frac{b\,c^3\,x^{3/2}}{12}-\frac{b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{4}+\frac{b\,c^5\,x^{5/2}}{20}+\frac{b\,c^7\,x^{7/2}}{28}+\frac{b\,c\,\sqrt{x}}{4}}{c^8}+\frac{b\,\left(105\,x^4\,\ln\left(c\,\sqrt{x}+1\right)-105\,x^4\,\ln\left(1-c\,\sqrt{x}\right)\right)}{840}+\frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^(1/2))),x)
```

```
[Out] ((b*c^3*x^(3/2))/12 - (b*atanh(c*x^(1/2)))/4 + (b*c^5*x^(5/2))/20 + (b*c^7*
x^(7/2))/28 + (b*c*x^(1/2))/4)/c^8 + (b*(105*x^4*log(c*x^(1/2) + 1) - 105*x
^4*log(1 - c*x^(1/2))))/840 + (a*x^4)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(1/2))),x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*sqrt(x))), x)
```

3.188 $\int x^2 \left(a + b \tanh^{-1}\left(c\sqrt{x}\right)\right) dx$

Optimal. Leaf size=75

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3c^6}+\frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}$$

```
[Out] 1/9*b*x^(3/2)/c^3+1/15*b*x^(5/2)/c-1/3*b*arctanh(c*x^(1/2))/c^6+1/3*x^3*(a+
b*arctanh(c*x^(1/2)))+1/3*b*x^(1/2)/c^5
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6097, 50, 63, 206}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+\frac{bx^{3/2}}{9c^3}+\frac{b\sqrt{x}}{3c^5}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3c^6}+\frac{bx^{5/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(3*c^5) + (b*x^(3/2))/(9*c^3) + (b*x^(5/2))/(15*c) - (b*ArcTanh
[c*Sqrt[x]])/(3*c^6) + (x^3*(a + b*ArcTanh[c*Sqrt[x]]))/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{1}{6}(bc)\int\frac{x^{5/2}}{1-c^2x}\,dx\\
&= \frac{bx^{5/2}}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{x^{3/2}}{1-c^2x}\,dx}{6c}\\
&= \frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{\sqrt{x}}{1-c^2x}\,dx}{6c^3}\\
&= \frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{1}{\sqrt{x}(1-c^2x)}\,dx}{6c^5}\\
&= \frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\,\mathrm{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\right.}{3c^5}\\
&= \frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3c^6}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 101, normalized size = 1.35

$$
\frac{ax^3}{3}+\frac{b\log\left(1-c\sqrt{x}\right)}{6c^6}-\frac{b\log\left(c\sqrt{x}+1\right)}{6c^6}+\frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}+\frac{1}{3}bx^3\tanh^{-1}\left(c\sqrt{x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(3*c^5) + (b*x^(3/2))/(9*c^3) + (b*x^(5/2))/(15*c) + (a*x^3)/3
+ (b*x^3*ArcTanh[c*Sqrt[x]])/3 + (b*Log[1 - c*Sqrt[x]])/(6*c^6) - (b*Log[1
+ c*Sqrt[x]])/(6*c^6)
```

fricas [A] time = 0.68, size = 80, normalized size = 1.07

$$
\frac{30\,ac^6x^3+15\left(bc^6x^3-b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)+2\left(3\,bc^5x^2+5\,bc^3x+15\,bc\right)\sqrt{x}}{90\,c^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/90*(30*a*c^6*x^3 + 15*(b*c^6*x^3 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2
*x - 1)) + 2*(3*b*c^5*x^2 + 5*b*c^3*x + 15*b*c)*sqrt(x))/c^6
```

giac [B] time = 0.20, size = 301, normalized size = 4.01

$$
\frac{1}{3}ax^3+\frac{2}{45}bc\left(\frac{\frac{45\left(c\sqrt{x}+1\right)^4}{\left(c\sqrt{x}-1\right)^4}-\frac{90\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{140\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}-\frac{70\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+23}{c^7\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^5}+\frac{15\left(\frac{3\left(c\sqrt{x}+1\right)^5}{\left(c\sqrt{x}-1\right)^5}+\frac{10\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{3\left(c\sqrt{x}+1\right.}{c\sqrt{x}-1}\right.}{c^7\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^6}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 + 2/45*b*c*((45*(c*sqrt(x) + 1)^4/(c*sqrt(x) - 1)^4 - 90*(c*sqrt(
x) + 1)^3/(c*sqrt(x) - 1)^3 + 140*(c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 - 70*
(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 23)/(c^7*((c*sqrt(x) + 1)/(c*sqrt(x) - 1)
 - 1)^5) + 15*(3*(c*sqrt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 10*(c*sqrt(x) + 1)^3
/(c*sqrt(x) - 1)^3 + 3*(c*sqrt(x) + 1)/(c*sqrt(x) - 1))*log(-(c*((c*sqrt(x)
 + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1) - c) + 1)/(c*
((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1) -
c) - 1))/(c^7*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^6))
```

maple [A] time = 0.03, size = 75, normalized size = 1.00

$$\frac{x^3 a}{3}+\frac{b\,x^3\operatorname{arctanh}\left(c\sqrt{x}\right)}{3}+\frac{b\,x^{\frac{5}{2}}}{15c}+\frac{b\,x^{\frac{3}{2}}}{9c^3}+\frac{b\sqrt{x}}{3c^5}+\frac{b\ln\left(c\sqrt{x}-1\right)}{6c^6}-\frac{b\ln\left(1+c\sqrt{x}\right)}{6c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(1/2))),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c*x^(1/2))+1/15*b*x^(5/2)/c+1/9*b*x^(3/2)/c^3+1
/3*b*x^(1/2)/c^5+1/6/c^6*b*ln(c*x^(1/2)-1)-1/6/c^6*b*ln(1+c*x^(1/2))
```

maxima [A] time = 0.32, size = 78, normalized size = 1.04

$$\frac{1}{3}\,ax^3+\frac{1}{90}\left(30\,x^3\operatorname{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(3\,c^4x^{\frac{5}{2}}+5\,c^2x^{\frac{3}{2}}+15\,\sqrt{x}\right)}{c^6}-\frac{15\,\log\left(c\sqrt{x}+1\right)}{c^7}+\frac{15\,\log\left(c\sqrt{x}-1\right)}{c^7}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/90*(30*x^3*arctanh(c*sqrt(x)) + c*(2*(3*c^4*x^(5/2) + 5*c^2*x
^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*sqrt(x) + 1)/c^7 + 15*log(c*sqrt(x) - 1
)/c^7))*b
```

mupad [B] time = 1.22, size = 58, normalized size = 0.77

$$\frac{a\,x^3}{3}+\frac{\frac{b\,c^3\,x^{3/2}}{9}-\frac{b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{3}+\frac{b\,c^5\,x^{5/2}}{15}+\frac{b\,c\,\sqrt{x}}{3}}{c^6}+\frac{b\,x^3\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^(1/2))),x)
```

```
[Out] (a*x^3)/3 + ((b*c^3*x^(3/2))/9 - (b*atanh(c*x^(1/2)))/3 + (b*c^5*x^(5/2))/1
5 + (b*c*x^(1/2))/3)/c^6 + (b*x^3*atanh(c*x^(1/2)))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(1/2))),x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*sqrt(x))), x)
```

3.189 $\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=62

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4}+\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}$$

```
[Out] 1/6*b*x^(3/2)/c-1/2*b*arctanh(c*x^(1/2))/c^4+1/2*x^2*(a+b*arctanh(c*x^(1/2)
))+1/2*b*x^(1/2)/c^3
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 50, 63, 206}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+\frac{b\sqrt{x}}{2c^3}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4}+\frac{bx^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(2*c^3) + (b*x^(3/2))/(6*c) - (b*ArcTanh[c*Sqrt[x]])/(2*c^4) +
(x^2*(a + b*ArcTanh[c*Sqrt[x]]))/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)dx &= \frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{1}{4}(bc)\int\frac{x^{3/2}}{1-c^2x}\,dx\\ &= \frac{bx^{3/2}}{6c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{\sqrt{x}}{1-c^2x}\,dx}{4c}\\ &= \frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{1}{\sqrt{x}\left(1-c^2x\right)}\,dx}{4c^3}\\ &= \frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\sqrt{x}\right)}{2c^3}\\ &= \frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 88, normalized size = 1.42

$$\frac{ax^2}{2}+\frac{b\log\left(1-c\sqrt{x}\right)}{4c^4}-\frac{b\log\left(c\sqrt{x}+1\right)}{4c^4}+\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}+\frac{1}{2}bx^2\tanh^{-1}\left(c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(2*c^3) + (b*x^(3/2))/(6*c) + (a*x^2)/2 + (b*x^2*ArcTanh[c*Sqrt
[x]])/2 + (b*Log[1 - c*Sqrt[x]])/(4*c^4) - (b*Log[1 + c*Sqrt[x]])/(4*c^4)
```

fricas [A] time = 0.61, size = 70, normalized size = 1.13

$$\frac{6\,ac^4x^2+3\left(bc^4x^2-b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)+2\left(bc^3x+3\,bc\right)\sqrt{x}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*c^4*x^2 + 3*(b*c^4*x^2 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x
 - 1)) + 2*(b*c^3*x + 3*b*c)*sqrt(x))/c^4
```

giac [B] time = 0.21, size = 239, normalized size = 3.85

$$\frac{1}{2}\,ax^2+\frac{2}{3}\,bc\left(\frac{\frac{3\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}-\frac{3\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+2}{c^5\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^3}+\frac{3\left(\frac{\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)\log\left(-\frac{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{\left(c\sqrt{x}+1\right)c}{c\sqrt{x}-1}-c}+1}{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{\left(c\sqrt{x}+1\right)c}{c\sqrt{x}-1}-c}-1}\right)}{c^5\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2 + 2/3*b*c*((3*(c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 - 3*(c*sqrt(x)
+ 1)/(c*sqrt(x) - 1) + 2)/(c^5*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^3) + 3
*((c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + (c*sqrt(x) + 1)/(c*sqrt(x) - 1))*lo
g(-(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) -
 1) - c) + 1)/(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(
c*sqrt(x) - 1) - c) - 1))/(c^5*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^4))
```

maple [A] time = 0.03, size = 66, normalized size = 1.06

$$\frac{a\,x^{2}}{2}+\frac{b\,x^{2}\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{2}+\frac{b\,x^{\frac{3}{2}}}{6c}+\frac{b\sqrt{x}}{2c^{3}}+\frac{b\ln\left(c\sqrt{x}-1\right)}{4c^{4}}-\frac{b\ln\left(1+c\sqrt{x}\right)}{4c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(1/2))),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c*x^(1/2))+1/6*b*x^(3/2)/c+1/2*b*x^(1/2)/c^3+1/
4/c^4*b*ln(c*x^(1/2)-1)-1/4/c^4*b*ln(1+c*x^(1/2))
```

maxima [A] time = 0.31, size = 69, normalized size = 1.11

$$\frac{1}{2}\,ax^{2}+\frac{1}{12}\left(6\,x^{2}\,\mathrm{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(c^{2}x^{\frac{3}{2}}+3\,\sqrt{x}\right)}{c^{4}}-\frac{3\,\log\left(c\sqrt{x}+1\right)}{c^{5}}+\frac{3\,\log\left(c\sqrt{x}-1\right)}{c^{5}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/12*(6*x^2*arctanh(c*sqrt(x)) + c*(2*(c^2*x^(3/2) + 3*sqrt(x))
/c^4 - 3*log(c*sqrt(x) + 1)/c^5 + 3*log(c*sqrt(x) - 1)/c^5))*b
```

mupad [B] time = 1.17, size = 49, normalized size = 0.79

$$\frac{\frac{b\,c^3\,x^{3/2}}{6}-\frac{b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{2}+\frac{b\,c\,\sqrt{x}}{2}}{c^4}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^(1/2))),x)
```

```
[Out] ((b*c^3*x^(3/2))/6 - (b*atanh(c*x^(1/2)))/2 + (b*c*x^(1/2))/2)/c^4 + (a*x^2
)/2 + (b*x^2*atanh(c*x^(1/2)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**(1/2))),x)
```

```
[Out] Integral(x*(a + b*atanh(c*sqrt(x))), x)
```

3.190 $\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=39

$$ax-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^2}+\frac{b\sqrt{x}}{c}+bx\tanh^{-1}\left(c\sqrt{x}\right)$$

```
[Out] a*x-b*arctanh(c*x^(1/2))/c^2+b*x*arctanh(c*x^(1/2))+b*x^(1/2)/c
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6091, 50, 63, 206}

$$ax-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^2}+\frac{b\sqrt{x}}{c}+bx\tanh^{-1}\left(c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*Sqrt[x]],x]
```

```
[Out] (b*Sqrt[x])/c + a*x - (b*ArcTanh[c*Sqrt[x]])/c^2 + b*x*ArcTanh[c*Sqrt[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)dx &= ax+b\int \tanh^{-1}\left(c\sqrt{x}\right)dx\\
&= ax+bx\tanh^{-1}\left(c\sqrt{x}\right)-\frac{1}{2}(bc)\int \frac{\sqrt{x}}{1-c^2x}\,dx\\
&= \frac{b\sqrt{x}}{c}+ax+bx\tanh^{-1}\left(c\sqrt{x}\right)-\frac{b\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}\,dx}{2c}\\
&= \frac{b\sqrt{x}}{c}+ax+bx\tanh^{-1}\left(c\sqrt{x}\right)-\frac{b\,\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,\sqrt{x}\right)}{c}\\
&= \frac{b\sqrt{x}}{c}+ax-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^2}+bx\tanh^{-1}\left(c\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 1.08

$$ax-bc\left(\frac{\tanh^{-1}\left(c\sqrt{x}\right)}{c^3}-\frac{\sqrt{x}}{c^2}\right)+bx\tanh^{-1}\left(c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*Sqrt[x]],x]
```

```
[Out] a*x + b*x*ArcTanh[c*Sqrt[x]] - b*c*(-(Sqrt[x]/c^2) + ArcTanh[c*Sqrt[x]]/c^3
)
```

fricas [A] time = 1.21, size = 56, normalized size = 1.44

$$\frac{2\,ac^2x+2\,bc\sqrt{x}+\left(bc^2x-b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*c^2*x + 2*b*c*sqrt(x) + (b*c^2*x - b)*log(-(c^2*x + 2*c*sqrt(x) +
1)/(c^2*x - 1)))/c^2
```

giac [B] time = 0.20, size = 174, normalized size = 4.46

$$2\,bc\left(\frac{1}{c^3\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)}+\frac{\left(c\sqrt{x}+1\right)\log\left(-\frac{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{\left(c\sqrt{x}+1\right)c}{c\sqrt{x}-1}-c}+1}{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{\left(c\sqrt{x}+1\right)c}{c\sqrt{x}-1}-c}-1}\right)}{\left(c\sqrt{x}-1\right)c^3\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^2}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*b*c*(1/(c^3*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)) + (c*sqrt(x) + 1)*log(
-(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1
) - c) + 1)/(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*
sqrt(x) - 1) - c) - 1))/((c*sqrt(x) - 1)*c^3*((c*sqrt(x) + 1)/(c*sqrt(x) -
1) - 1)^2)) + a*x
```

maple [A] time = 0.03, size = 50, normalized size = 1.28

$$ax + bx \operatorname{arctanh}\left(c\sqrt{x}\right) + \frac{b\sqrt{x}}{c} + \frac{b\ln\left(c\sqrt{x}-1\right)}{2c^2} - \frac{b\ln\left(1+c\sqrt{x}\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^(1/2)),x)
```

```
[Out] a*x+b*x*arctanh(c*x^(1/2))+b*x^(1/2)/c+1/2*b/c^2*ln(c*x^(1/2)-1)-1/2*b/c^2*
ln(1+c*x^(1/2))
```

maxima [A] time = 0.32, size = 53, normalized size = 1.36

$$\frac{1}{2}\left(c\left(\frac{2\sqrt{x}}{c^2} - \frac{\log\left(c\sqrt{x}+1\right)}{c^3} + \frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right) + 2x \operatorname{artanh}\left(c\sqrt{x}\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) +
2*x*arctanh(c*sqrt(x)))*b + a*x
```

mupad [B] time = 0.90, size = 32, normalized size = 0.82

$$a\,x + b\,x\,\operatorname{atanh}\left(c\,\sqrt{x}\right) - \frac{b\,\left(\operatorname{atanh}\left(c\,\sqrt{x}\right) - c\,\sqrt{x}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c*x^(1/2)),x)
```

```
[Out] a*x + b*x*atanh(c*x^(1/2)) - (b*(atanh(c*x^(1/2)) - c*x^(1/2)))/c^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**(1/2)),x)
```

```
[Out] Integral(a + b*atanh(c*sqrt(x)), x)
```

3.191 $\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}\,dx$

Optimal. Leaf size=29

$$a\log(x) - b\text{Li}_2\left(-c\sqrt{x}\right) + b\text{Li}_2\left(c\sqrt{x}\right)$$

```
[Out] a*ln(x)-b*polylog(2,-c*x^(1/2))+b*polylog(2,c*x^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6095, 5912}

$$-b\text{PolyLog}\left(2,-c\sqrt{x}\right) + b\text{PolyLog}\left(2,c\sqrt{x}\right) + a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/x,x]
```

```
[Out] a*Log[x] - b*PolyLog[2, -(c*Sqrt[x])] + b*PolyLog[2, c*Sqrt[x]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx, x, \sqrt{x}\right)\\ &= a\log(x) - b\text{Li}_2\left(-c\sqrt{x}\right) + b\text{Li}_2\left(c\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$a\log(x) - b\text{Li}_2\left(-c\sqrt{x}\right) + b\text{Li}_2\left(c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x,x]
```

```
[Out] a*Log[x] - b*PolyLog[2, -(c*Sqrt[x])] + b*PolyLog[2, c*Sqrt[x]]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(c\sqrt{x}\right)+a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*sqrt(x)) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\left(c\sqrt{x}\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x,x, algorithm="giac")
[Out] integrate((b*arctanh(c*sqrt(x)) + a)/x, x)
```

maple [B] time = 0.04, size = 63, normalized size = 2.17

$$2a\ln\left(c\sqrt{x}\right)+2b\ln\left(c\sqrt{x}\right)\text{arctanh}\left(c\sqrt{x}\right)-b\,\text{dilog}\left(c\sqrt{x}\right)-b\,\text{dilog}\left(1+c\sqrt{x}\right)-b\ln\left(c\sqrt{x}\right)\ln\left(1+c\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x,x)
[Out] 2*a*ln(c*x^(1/2))+2*b*ln(c*x^(1/2))*arctanh(c*x^(1/2))-b*dilog(c*x^(1/2))-b
*dilog(1+c*x^(1/2))-b*ln(c*x^(1/2))*ln(1+c*x^(1/2))
```

maxima [B] time = 0.46, size = 61, normalized size = 2.10

$$-\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\text{Li}_2\left(-c\sqrt{x}+1\right)\right)b+\left(\log\left(c\sqrt{x}+1\right)\log\left(-c\sqrt{x}\right)+\text{Li}_2\left(c\sqrt{x}+1\right)\right)b+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x,x, algorithm="maxima")
[Out] -(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1))*b + (log(c*sq
rt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b + a*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\,\text{atanh}\left(c\,\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))/x,x)
[Out] int((a + b*atanh(c*x^(1/2)))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{atanh}\left(c\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x,x)
[Out] Integral((a + b*atanh(c*sqrt(x)))/x, x)
```

3.192 $\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^2}\,dx$

Optimal. Leaf size=40

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}+bc^2\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc}{\sqrt{x}}$$

```
[Out] b*c^2*arctanh(c*x^(1/2))+(-a-b*arctanh(c*x^(1/2)))/x-b*c/x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6097, 51, 63, 206}

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}+bc^2\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/x^2,x]
```

```
[Out] -((b*c)/Sqrt[x]) + b*c^2*ArcTanh[c*Sqrt[x]] - (a + b*ArcTanh[c*Sqrt[x]])/x
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^2}\,dx = -\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x} + \frac{1}{2}(bc)\int \frac{1}{x^{3/2}\left(1-c^2x\right)}\,dx$$
$$= -\frac{bc}{\sqrt{x}} - \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x} + \frac{1}{2}\left(bc^3\right)\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}\,dx$$
$$= -\frac{bc}{\sqrt{x}} - \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x} + \left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{bc}{\sqrt{x}} + bc^2\tanh^{-1}\left(c\sqrt{x}\right) - \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 1.68

$$-\frac{a}{x} - \frac{1}{2}bc^2\log\left(1-c\sqrt{x}\right) + \frac{1}{2}bc^2\log\left(c\sqrt{x}+1\right) - \frac{bc}{\sqrt{x}} - \frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^2,x]
```

```
[Out] -(a/x) - (b*c)/Sqrt[x] - (b*ArcTanh[c*Sqrt[x]])/x - (b*c^2*Log[1 - c*Sqrt[x
]])/2 + (b*c^2*Log[1 + c*Sqrt[x]])/2
```

fricas [A] time = 0.80, size = 53, normalized size = 1.32

$$-\frac{2\,bc\sqrt{x} - \left(bc^2x - b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right) + 2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*sqrt(x) - (b*c^2*x - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x -
 1)) + 2*a)/x
```

giac [B] time = 0.43, size = 168, normalized size = 4.20

$$2\left(\frac{\left(c\sqrt{x}+1\right)bc\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{\left(c\sqrt{x}-1\right)\left(\frac{\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2} + \frac{2\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1} + 1\right)} + \frac{\frac{2\left(c\sqrt{x}+1\right)ac}{c\sqrt{x}-1} + \frac{\left(c\sqrt{x}+1\right)bc}{c\sqrt{x}-1} + bc}{\frac{\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2} + \frac{2\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1} + 1}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2,x, algorithm="giac")
```

```
[Out] 2*((c*sqrt(x) + 1)*b*c*log(-(c*sqrt(x) + 1)/(c*sqrt(x) - 1))/((c*sqrt(x) -
1)*((c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 + 2*(c*sqrt(x) + 1)/(c*sqrt(x) - 1)
 + 1)) + (2*(c*sqrt(x) + 1)*a*c/(c*sqrt(x) - 1) + (c*sqrt(x) + 1)*b*c/(c*sq
rt(x) - 1) + b*c)/((c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 + 2*(c*sqrt(x) + 1)/
(c*sqrt(x) - 1) + 1))*c
```

maple [A] time = 0.03, size = 55, normalized size = 1.38

$$-\frac{a}{x} - \frac{b\,\text{arctanh}\left(c\sqrt{x}\right)}{x} - \frac{bc}{\sqrt{x}} - \frac{c^2b\ln\left(c\sqrt{x}-1\right)}{2} + \frac{c^2b\ln\left(1+c\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c*x^(1/2))-b*c/x^(1/2)-1/2*c^2*b*ln(c*x^(1/2)-1)+1/2*c^2*b
*ln(1+c*x^(1/2))
```

maxima [A] time = 0.32, size = 51, normalized size = 1.28

$$\frac{1}{2}\left(\left(c\log\left(c\sqrt{x}+1\right)-c\log\left(c\sqrt{x}-1\right)-\frac{2}{\sqrt{x}}\right)c-\frac{2\,\mathrm{artanh}\left(c\sqrt{x}\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*((c*log(c*sqrt(x) + 1) - c*log(c*sqrt(x) - 1) - 2/sqrt(x))*c - 2*arctan
h(c*sqrt(x))/x)*b - a/x
```

mupad [B] time = 1.12, size = 52, normalized size = 1.30

$$b\,c\,\mathrm{atan}\left(\frac{c^2\,\sqrt{x}}{\sqrt{-c^2}}\right)\,\sqrt{-c^2}-\frac{a}{x}-\frac{b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)+b\,c\,\sqrt{x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))/x^2,x)
```

```
[Out] b*c*atan((c^2*x^(1/2))/(-c^2)^(1/2))*(-c^2)^(1/2) - a/x - (b*atanh(c*x^(1/2
)) + b*c*x^(1/2))/x
```

sympy [A] time = 21.87, size = 231, normalized size = 5.78

$$\begin{cases} -\frac{a}{x}+\frac{b\,\mathrm{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} & \text{for } c=-\sqrt{\frac{1}{x}} \\ -\frac{a}{x}-\frac{b\,\mathrm{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} & \text{for } c=\sqrt{\frac{1}{x}} \\ -\frac{ac^2x^{\frac{3}{2}}}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{a\sqrt{x}}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{bc^4x^{\frac{5}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{bc^3x^2}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{2bc^2x^{\frac{3}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{bcx}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{b\sqrt{x}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*atanh(sqrt(x)*sqrt(1/x))/x, Eq(c, -sqrt(1/x))), (-a/x -
 b*atanh(sqrt(x)*sqrt(1/x))/x, Eq(c, sqrt(1/x))), (-a*c**2*x**(3/2)/(c**2*x
**(5/2) - x**(3/2)) + a*sqrt(x)/(c**2*x**(5/2) - x**(3/2)) + b*c**4*x**(5/2
)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) - b*c**3*x**2/(c**2*x**(5/2)
- x**(3/2)) - 2*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2))
 + b*c*x/(c**2*x**(5/2) - x**(3/2)) + b*sqrt(x)*atanh(c*sqrt(x))/(c**2*x**(
5/2) - x**(3/2)), True))
```

3.193 $\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^3}\,dx$

Optimal. Leaf size=60

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}+\frac{1}{2}bc^4\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc^3}{2\sqrt{x}}-\frac{bc}{6x^{3/2}}$$

```
[Out] -1/6*b*c/x^(3/2)+1/2*b*c^4*arctanh(c*x^(1/2))+1/2*(-a-b*arctanh(c*x^(1/2)))
/x^2-1/2*b*c^3/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6097, 51, 63, 206}

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}-\frac{bc^3}{2\sqrt{x}}+\frac{1}{2}bc^4\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc}{6x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/x^3,x]
```

```
[Out] -(b*c)/(6*x^(3/2)) - (b*c^3)/(2*Sqrt[x]) + (b*c^4*ArcTanh[c*Sqrt[x]])/2 - (
a + b*ArcTanh[c*Sqrt[x]])/(2*x^2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}+\frac{1}{4}(bc)\int \frac{1}{x^{5/2}\left(1-c^2x\right)}\,dx\\ &= -\frac{bc}{6x^{3/2}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}+\frac{1}{4}\left(bc^3\right)\int \frac{1}{x^{3/2}\left(1-c^2x\right)}\,dx\\ &= -\frac{bc}{6x^{3/2}}-\frac{bc^3}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}+\frac{1}{4}\left(bc^5\right)\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}\,dx\\ &= -\frac{bc}{6x^{3/2}}-\frac{bc^3}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}+\frac{1}{2}\left(bc^5\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,\sqrt{x}\right)\\ &= -\frac{bc}{6x^{3/2}}-\frac{bc^3}{2\sqrt{x}}+\frac{1}{2}bc^4\tanh^{-1}\left(c\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 86, normalized size = 1.43

$$-\frac{a}{2x^2}-\frac{1}{4}bc^4\log\left(1-c\sqrt{x}\right)+\frac{1}{4}bc^4\log\left(c\sqrt{x}+1\right)-\frac{bc^3}{2\sqrt{x}}-\frac{bc}{6x^{3/2}}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*c)/(6*x^(3/2)) - (b*c^3)/(2*Sqrt[x]) - (b*ArcTanh[c*Sqrt[x]
])/(2*x^2) - (b*c^4*Log[1 - c*Sqrt[x]])/4 + (b*c^4*Log[1 + c*Sqrt[x]])/4
```

fricas [A] time = 0.56, size = 64, normalized size = 1.07

$$\frac{3\left(bc^4x^2-b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)-2\left(3\,bc^3x+bc\right)\sqrt{x}-6\,a}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(b*c^4*x^2 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) - 2*(3*
b*c^3*x + b*c)*sqrt(x) - 6*a)/x^2
```

giac [B] time = 0.20, size = 356, normalized size = 5.93

$$\frac{2}{3}c\left(\frac{3\left(\frac{\left(c\sqrt{x}+1\right)^3bc^3}{\left(c\sqrt{x}-1\right)^3}+\frac{\left(c\sqrt{x}+1\right)bc^3}{c\sqrt{x}-1}\right)\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{\frac{\left(c\sqrt{x}+1\right)^4}{\left(c\sqrt{x}-1\right)^4}+\frac{4\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{6\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}+\frac{4\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+1}+\frac{\frac{6\left(c\sqrt{x}+1\right)^3ac^3}{\left(c\sqrt{x}-1\right)^3}+\frac{6\left(c\sqrt{x}+1\right)ac^3}{c\sqrt{x}-1}+\frac{3\left(c\sqrt{x}+1\right)^3bc^3}{\left(c\sqrt{x}-1\right)^3}+\frac{6\left(c\sqrt{x}+1\right)^2b}{\left(c\sqrt{x}-1\right)^2}}{\frac{\left(c\sqrt{x}+1\right)^4}{\left(c\sqrt{x}-1\right)^4}+\frac{4\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{6\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}+\frac{4(c}{c\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="giac")
```

```
[Out] 2/3*c*(3*((c*sqrt(x) + 1)^3*b*c^3/(c*sqrt(x) - 1)^3 + (c*sqrt(x) + 1)*b*c^3
/(c*sqrt(x) - 1))*log(-(c*sqrt(x) + 1)/(c*sqrt(x) - 1))/((c*sqrt(x) + 1)^4/
(c*sqrt(x) - 1)^4 + 4*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 6*(c*sqrt(x) +
1)^2/(c*sqrt(x) - 1)^2 + 4*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1) + (6*(c*sqr
t(x) + 1)^3*a*c^3/(c*sqrt(x) - 1)^3 + 6*(c*sqrt(x) + 1)*a*c^3/(c*sqrt(x) -
1) + 3*(c*sqrt(x) + 1)^3*b*c^3/(c*sqrt(x) - 1)^3 + 6*(c*sqrt(x) + 1)^2*b*c^
```

```
3/(c*sqrt(x) - 1)^2 + 5*(c*sqrt(x) + 1)*b*c^3/(c*sqrt(x) - 1) + 2*b*c^3)/((
c*sqrt(x) + 1)^4/(c*sqrt(x) - 1)^4 + 4*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3
+ 6*(c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 + 4*(c*sqrt(x) + 1)/(c*sqrt(x) - 1)
 + 1))
```

maple [A] time = 0.04, size = 64, normalized size = 1.07

$$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}\left(c\sqrt{x}\right)}{2x^2}-\frac{bc}{6x^{\frac{3}{2}}}-\frac{b\,c^3}{2\sqrt{x}}-\frac{c^4 b\ln\left(c\sqrt{x}-1\right)}{4}+\frac{c^4 b\ln\left(1+c\sqrt{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x^(1/2))-1/6*b*c/x^(3/2)-1/2*b*c^3/x^(1/2)-1
/4*c^4*b*ln(c*x^(1/2)-1)+1/4*c^4*b*ln(1+c*x^(1/2))
```

maxima [A] time = 0.31, size = 64, normalized size = 1.07

$$\frac{1}{12}\left(\left(3\,c^3\log\left(c\sqrt{x}+1\right)-3\,c^3\log\left(c\sqrt{x}-1\right)-\frac{2\left(3\,c^2x+1\right)}{x^{\frac{3}{2}}}\right)c-\frac{6\,\mathrm{artanh}\left(c\sqrt{x}\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*(3*c^2*x + 1
)/x^(3/2))*c - 6*arctanh(c*sqrt(x))/x^2)*b - 1/2*a/x^2
```

mupad [B] time = 1.36, size = 61, normalized size = 1.02

$$\frac{b\,c^4\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{2}-\frac{b\,\left(3\,\ln\left(c\,\sqrt{x}+1\right)-3\,\ln\left(1-c\,\sqrt{x}\right)+2\,c\,\sqrt{x}+6\,c^3\,x^{3/2}\right)}{12\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))/x^3,x)
```

```
[Out] (b*c^4*atanh(c*x^(1/2)))/2 - (b*(3*log(c*x^(1/2) + 1) - 3*log(1 - c*x^(1/2)
) + 2*c*x^(1/2) + 6*c^3*x^(3/2)))/(12*x^2) - a/(2*x^2)
```

sympy [A] time = 89.35, size = 342, normalized size = 5.70

$$\begin{cases}-\frac{a}{2x^2}+\frac{b\,\mathrm{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{2x^2} \\ -\frac{a}{2x^2}-\frac{b\,\mathrm{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{2x^2} \\ -\frac{3ac^2x^{\frac{3}{2}}}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{3a\sqrt{x}}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{3bc^6x^{\frac{7}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3bc^5x^3}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3bc^4x^{\frac{5}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{2bc^3x^2}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3bc^2x^{\frac{3}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{bc}{6c^2x^{\frac{7}{2}}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) + b*atanh(sqrt(x)*sqrt(1/x))/(2*x**2), Eq(c, -sqrt(1
/x))), (-a/(2*x**2) - b*atanh(sqrt(x)*sqrt(1/x))/(2*x**2), Eq(c, sqrt(1/x))
), (-3*a*c**2*x**(3/2)/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*a*sqrt(x)/(6*c**2
*x**(7/2) - 6*x**(5/2)) + 3*b*c**6*x**(7/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/
```

```
2) - 6*x**(5/2)) - 3*b*c**5*x**3/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b*c**4*
x**(5/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 2*b*c**3*x**2/(6
*c**2*x**(7/2) - 6*x**(5/2)) - 3*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(6*c**2*x
**(7/2) - 6*x**(5/2)) + b*c*x/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*b*sqrt(x)*
atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)), True))
```

3.194 $\int \frac{a+b\tanh^{-1}(c\sqrt{x})}{x^4} dx$

Optimal. Leaf size=73

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}+\frac{1}{3}bc^6\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc^5}{3\sqrt{x}}-\frac{bc^3}{9x^{3/2}}-\frac{bc}{15x^{5/2}}$$

```
[Out] -1/15*b*c/x^(5/2)-1/9*b*c^3/x^(3/2)+1/3*b*c^6*arctanh(c*x^(1/2))+1/3*(-a-b*
arctanh(c*x^(1/2)))/x^3-1/3*b*c^5/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6097, 51, 63, 206}

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}+\frac{1}{3}bc^6\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/x^4,x]
```

```
[Out] -(b*c)/(15*x^(5/2)) - (b*c^3)/(9*x^(3/2)) - (b*c^5)/(3*Sqrt[x]) + (b*c^6*Ar
cTanh[c*Sqrt[x]])/3 - (a + b*ArcTanh[c*Sqrt[x]])/(3*x^3)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^4}\,dx = -\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}+\frac{1}{6}(bc)\int \frac{1}{x^{7/2}\left(1-c^2x\right)}\,dx$$

$$= -\frac{bc}{15x^{5/2}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}+\frac{1}{6}\left(bc^3\right)\int \frac{1}{x^{5/2}\left(1-c^2x\right)}\,dx$$

$$= -\frac{bc}{15x^{5/2}}-\frac{bc^3}{9x^{3/2}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}+\frac{1}{6}\left(bc^5\right)\int \frac{1}{x^{3/2}\left(1-c^2x\right)}\,dx$$

$$= -\frac{bc}{15x^{5/2}}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}+\frac{1}{6}\left(bc^7\right)\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}\,dx$$

$$= -\frac{bc}{15x^{5/2}}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}+\frac{1}{3}\left(bc^7\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x\right.$$

$$= -\frac{bc}{15x^{5/2}}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}+\frac{1}{3}bc^6\tanh^{-1}\left(c\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}$$

Mathematica [A] time = 0.03, size = 99, normalized size = 1.36

$$-\frac{a}{3x^3}-\frac{1}{6}bc^6\log\left(1-c\sqrt{x}\right)+\frac{1}{6}bc^6\log\left(c\sqrt{x}+1\right)-\frac{bc^5}{3\sqrt{x}}-\frac{bc^3}{9x^{3/2}}-\frac{bc}{15x^{5/2}}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*c)/(15*x^(5/2)) - (b*c^3)/(9*x^(3/2)) - (b*c^5)/(3*Sqrt[x])
 - (b*ArcTanh[c*Sqrt[x]])/(3*x^3) - (b*c^6*Log[1 - c*Sqrt[x]])/6 + (b*c^6*L
og[1 + c*Sqrt[x]])/6
```

fricas [A] time = 0.77, size = 74, normalized size = 1.01

$$\frac{15\left(bc^6x^3-b\right)\log\left(-\frac{c^2x+2c\sqrt{x}+1}{c^2x-1}\right)-2\left(15\,bc^5x^2+5\,bc^3x+3\,bc\right)\sqrt{x}-30\,a}{90\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="fricas")
```

```
[Out] 1/90*(15*(b*c^6*x^3 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) - 2*(1
5*b*c^5*x^2 + 5*b*c^3*x + 3*b*c)*sqrt(x) - 30*a)/x^3
```

giac [B] time = 0.22, size = 534, normalized size = 7.32

$$\frac{2}{45}c\left(\frac{15\left(\frac{3\left(c\sqrt{x}+1\right)^5bc^5}{\left(c\sqrt{x}-1\right)^5}+\frac{10\left(c\sqrt{x}+1\right)^3bc^5}{\left(c\sqrt{x}-1\right)^3}+\frac{3\left(c\sqrt{x}+1\right)bc^5}{c\sqrt{x}-1}\right)\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{\frac{\left(c\sqrt{x}+1\right)^6}{\left(c\sqrt{x}-1\right)^6}+\frac{6\left(c\sqrt{x}+1\right)^5}{\left(c\sqrt{x}-1\right)^5}+\frac{15\left(c\sqrt{x}+1\right)^4}{\left(c\sqrt{x}-1\right)^4}+\frac{20\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{15\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}+\frac{6\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+1}+\frac{\frac{90\left(c\sqrt{x}+1\right)^5ac^5}{\left(c\sqrt{x}-1\right)^5}+\frac{300\left(c\sqrt{x}+1\right.}{\left(c\sqrt{x}-1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="giac")
```

```
[Out] 2/45*c*(15*(3*(c*sqrt(x) + 1)^5*b*c^5/(c*sqrt(x) - 1)^5 + 10*(c*sqrt(x) + 1
)^3*b*c^5/(c*sqrt(x) - 1)^3 + 3*(c*sqrt(x) + 1)*b*c^5/(c*sqrt(x) - 1))*log(
```

```
-(c*sqrt(x) + 1)/(c*sqrt(x) - 1))/((c*sqrt(x) + 1)^6/(c*sqrt(x) - 1)^6 + 6*
(c*sqrt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 15*(c*sqrt(x) + 1)^4/(c*sqrt(x) - 1)^
4 + 20*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 15*(c*sqrt(x) + 1)^2/(c*sqrt(x
) - 1)^2 + 6*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1) + (90*(c*sqrt(x) + 1)^5*a
*c^5/(c*sqrt(x) - 1)^5 + 300*(c*sqrt(x) + 1)^3*a*c^5/(c*sqrt(x) - 1)^3 + 90
*(c*sqrt(x) + 1)*a*c^5/(c*sqrt(x) - 1) + 45*(c*sqrt(x) + 1)^5*b*c^5/(c*sqrt
(x) - 1)^5 + 135*(c*sqrt(x) + 1)^4*b*c^5/(c*sqrt(x) - 1)^4 + 230*(c*sqrt(x)
 + 1)^3*b*c^5/(c*sqrt(x) - 1)^3 + 210*(c*sqrt(x) + 1)^2*b*c^5/(c*sqrt(x) -
1)^2 + 93*(c*sqrt(x) + 1)*b*c^5/(c*sqrt(x) - 1) + 23*b*c^5)/((c*sqrt(x) + 1
)^6/(c*sqrt(x) - 1)^6 + 6*(c*sqrt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 15*(c*sqrt(
x) + 1)^4/(c*sqrt(x) - 1)^4 + 20*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 15*(
c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 + 6*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)
)
```

maple [A] time = 0.04, size = 73, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{b\operatorname{arctanh}\left(c\sqrt{x}\right)}{3x^3}-\frac{bc}{15x^{\frac{5}{2}}}-\frac{b\,c^3}{9x^{\frac{3}{2}}}-\frac{b\,c^5}{3\sqrt{x}}-\frac{c^6b\ln\left(c\sqrt{x}-1\right)}{6}+\frac{c^6b\ln\left(1+c\sqrt{x}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x^(1/2))-1/15*b*c/x^(5/2)-1/9*b*c^3/x^(3/2)-
1/3*b*c^5/x^(1/2)-1/6*c^6*b*ln(c*x^(1/2)-1)+1/6*c^6*b*ln(1+c*x^(1/2))
```

maxima [A] time = 0.32, size = 72, normalized size = 0.99

$$\frac{1}{90}\left(\left(15\,c^5\log\left(c\sqrt{x}+1\right)-15\,c^5\log\left(c\sqrt{x}-1\right)-\frac{2\left(15\,c^4x^2+5\,c^2x+3\right)}{x^{\frac{5}{2}}}\right)c-\frac{30\,\operatorname{artanh}\left(c\sqrt{x}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="maxima")
```

```
[Out] 1/90*((15*c^5*log(c*sqrt(x) + 1) - 15*c^5*log(c*sqrt(x) - 1) - 2*(15*c^4*x^
2 + 5*c^2*x + 3)/x^(5/2))*c - 30*arctanh(c*sqrt(x))/x^3)*b - 1/3*a/x^3
```

mupad [B] time = 1.39, size = 69, normalized size = 0.95

$$\frac{b\,c^6\operatorname{atanh}\left(c\,\sqrt{x}\right)}{3}-\frac{b\,\left(15\,\ln\left(c\,\sqrt{x}+1\right)-15\,\ln\left(1-c\,\sqrt{x}\right)+6\,c\,\sqrt{x}+10\,c^3\,x^{3/2}+30\,c^5\,x^{5/2}\right)}{90\,x^3}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))/x^4,x)
```

```
[Out] (b*c^6*atanh(c*x^(1/2)))/3 - (b*(15*log(c*x^(1/2) + 1) - 15*log(1 - c*x^(1/
2)) + 6*c*x^(1/2) + 10*c^3*x^(3/2) + 30*c^5*x^(5/2)))/(90*x^3) - a/(3*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**4,x)
```

```
[Out] Timed out
```

3.195 $\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$

Optimal. Leaf size=211

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{4c^8}+\frac{ab\sqrt{x}}{2c^7}+\frac{bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{6c^5}+\frac{bx^{5/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{10c^3}+\frac{bx^{7/2}\left(a+b\tanh\right.}{14c}$$

```
[Out] 71/420*b^2*x/c^6+3/70*b^2*x^2/c^4+1/84*b^2*x^3/c^2+1/6*b*x^(3/2)*(a+b*arcta
nh(c*x^(1/2)))/c^5+1/10*b*x^(5/2)*(a+b*arctanh(c*x^(1/2)))/c^3+1/14*b*x^(7/
2)*(a+b*arctanh(c*x^(1/2)))/c-1/4*(a+b*arctanh(c*x^(1/2)))^2/c^8+1/4*x^4*(a
+b*arctanh(c*x^(1/2)))^2+44/105*b^2*ln(-c^2*x+1)/c^8+1/2*a*b*x^(1/2)/c^7+1/
2*b^2*arctanh(c*x^(1/2))*x^(1/2)/c^7
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] Defer[Int][x^3*(a + b*ArcTanh[c*Sqrt[x]])^2, x]
```

Rubi steps

$$\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx=\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$$

Mathematica [A] time = 0.12, size = 224, normalized size = 1.06

$$\frac{105a^2c^8x^4+30abc^7x^{7/2}+42abc^5x^{5/2}+70abc^3x^{3/2}+2bc\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)\left(105ac^7x^{7/2}+b\left(15c^6x^3+21c^4x^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (210*a*b*c*Sqrt[x] + 71*b^2*c^2*x + 70*a*b*c^3*x^(3/2) + 18*b^2*c^4*x^2 + 4
2*a*b*c^5*x^(5/2) + 5*b^2*c^6*x^3 + 30*a*b*c^7*x^(7/2) + 105*a^2*c^8*x^4 +
2*b*c*Sqrt[x]*(105*a*c^7*x^(7/2) + b*(105 + 35*c^2*x + 21*c^4*x^2 + 15*c^6*
x^3))*ArcTanh[c*Sqrt[x]] + 105*b^2*(-1 + c^8*x^4)*ArcTanh[c*Sqrt[x]]^2 + b*
(105*a + 176*b)*Log[1 - c*Sqrt[x]] - 105*a*b*Log[1 + c*Sqrt[x]] + 176*b^2*L
og[1 + c*Sqrt[x]])/(420*c^8)
```

fricas [A] time = 1.52, size = 273, normalized size = 1.29

$$\frac{420\,a^2c^8x^4+20\,b^2c^6x^3+72\,b^2c^4x^2+284\,b^2c^2x+105\left(b^2c^8x^4-b^2\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2+4\left(105\,abc^8-105\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/1680*(420*a^2*c^8*x^4 + 20*b^2*c^6*x^3 + 72*b^2*c^4*x^2 + 284*b^2*c^2*x +
 105*(b^2*c^8*x^4 - b^2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 + 4*
(105*a*b*c^8 - 105*a*b + 176*b^2)*log(c*sqrt(x) + 1) - 4*(105*a*b*c^8 - 105
```

```
*a*b - 176*b^2)*log(c*sqrt(x) - 1) + 4*(105*a*b*c^8*x^4 - 105*a*b*c^8 + (15
*b^2*c^7*x^3 + 21*b^2*c^5*x^2 + 35*b^2*c^3*x + 105*b^2*c)*sqrt(x))*log(-(c^
2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) + 8*(15*a*b*c^7*x^3 + 21*a*b*c^5*x^2 +
35*a*b*c^3*x + 105*a*b*c)*sqrt(x))/c^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^2 x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2*x^3, x)
```

maple [B] time = 0.06, size = 396, normalized size = 1.88

$$\frac{b^2 \operatorname{arctanh}\left(c\sqrt{x}\right)\sqrt{x}}{2c^7} - \frac{ab\ln\left(1+c\sqrt{x}\right)}{4c^8} + \frac{ab\ln\left(c\sqrt{x}-1\right)}{4c^8} + \frac{x^{\frac{5}{2}}ab}{10c^3} + \frac{ab\,x^{\frac{3}{2}}}{6c^5} + \frac{x^{\frac{7}{2}}ab}{14c} + \frac{b^2 \operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)}{4c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(1/2)))^2,x)
```

```
[Out] -1/4/c^8*a*b*ln(1+c*x^(1/2))+1/4/c^8*a*b*ln(c*x^(1/2)-1)+1/10/c^3*x^(5/2)*a
*b+1/6/c^5*a*b*x^(3/2)+1/14/c*x^(7/2)*a*b+1/4/c^8*b^2*arctanh(c*x^(1/2))*ln
(c*x^(1/2)-1)-1/4/c^8*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/8/c^8*b^2*ln
(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))-1/8/c^8*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1
+c*x^(1/2))+1/8/c^8*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))+1/6/c^
5*b^2*arctanh(c*x^(1/2))*x^(3/2)+1/10/c^3*b^2*arctanh(c*x^(1/2))*x^(5/2)+1/
14/c*b^2*arctanh(c*x^(1/2))*x^(7/2)+1/2*a*b*x^4*arctanh(c*x^(1/2))+1/2*a*b*
x^(1/2)/c^7+1/2*b^2*arctanh(c*x^(1/2))*x^(1/2)/c^7+71/420*b^2*x/c^6+1/4*a^2
*x^4+3/70*b^2*x^2/c^4+1/84*b^2*x^3/c^2+44/105/c^8*b^2*ln(1+c*x^(1/2))+1/16/
c^8*b^2*ln(c*x^(1/2)-1)^2+1/16/c^8*b^2*ln(1+c*x^(1/2))^2+1/4*b^2*x^4*arctan
h(c*x^(1/2))^2+44/105/c^8*b^2*ln(c*x^(1/2)-1)
```

maxima [A] time = 0.34, size = 265, normalized size = 1.26

$$\frac{1}{4}b^2x^4\operatorname{artanh}\left(c\sqrt{x}\right)^2 + \frac{1}{4}a^2x^4 + \frac{1}{420}\left(210\,x^4\operatorname{artanh}\left(c\sqrt{x}\right) + c\left(\frac{2\left(15\,c^6x^{\frac{7}{2}} + 21\,c^4x^{\frac{5}{2}} + 35\,c^2x^{\frac{3}{2}} + 105\sqrt{x}\right)}{c^8} - \frac{105}{\ }\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c*sqrt(x))^2 + 1/4*a^2*x^4 + 1/420*(210*x^4*arctanh(c*s
qrt(x)) + c*(2*(15*c^6*x^(7/2) + 21*c^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sqrt
(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9 + 105*log(c*sqrt(x) - 1)/c^9))*a*b +
1/1680*(4*c*(2*(15*c^6*x^(7/2) + 21*c^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sqrt
(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9 + 105*log(c*sqrt(x) - 1)/c^9)*arctanh
(c*sqrt(x)) + (20*c^6*x^3 + 72*c^4*x^2 + 284*c^2*x - 2*(105*log(c*sqrt(x) -
 1) - 352)*log(c*sqrt(x) + 1) + 105*log(c*sqrt(x) + 1)^2 + 105*log(c*sqrt(x
) - 1)^2 + 704*log(c*sqrt(x) - 1))/c^8)*b^2
```

mupad [B] time = 4.87, size = 453, normalized size = 2.15

$$\frac{a^2\,x^4}{4} + \frac{44\,b^2\,\ln\left(c\,\sqrt{x}-1\right)}{105\,c^8} + \frac{44\,b^2\,\ln\left(c\,\sqrt{x}+1\right)}{105\,c^8} + \frac{71\,b^2\,x}{420\,c^6} - \frac{b^2\ln\left(c\,\sqrt{x}+1\right)^2}{16\,c^8} - \frac{b^2\ln\left(1-c\,\sqrt{x}\right)^2}{16\,c^8} + \frac{b^2\,x^3}{84\,c^2} + \frac{3\,b^2\,x^2}{70\,c^4} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^(1/2)))^2,x)
```

```
[Out] (a^2*x^4)/4 + (44*b^2*log(c*x^(1/2) - 1))/(105*c^8) + (44*b^2*log(c*x^(1/2)
 + 1))/(105*c^8) + (71*b^2*x)/(420*c^6) - (b^2*log(c*x^(1/2) + 1)^2)/(16*c^
8) - (b^2*log(1 - c*x^(1/2))^2)/(16*c^8) + (b^2*x^3)/(84*c^2) + (3*b^2*x^2)
/(70*c^4) + (b^2*x^4*log(c*x^(1/2) + 1)^2)/16 + (b^2*x^4*log(1 - c*x^(1/2))
^2)/16 + (b^2*x^(7/2)*log(c*x^(1/2) + 1))/(28*c) + (b^2*x^(5/2)*log(c*x^(1/
2) + 1))/(20*c^3) + (b^2*x^(3/2)*log(c*x^(1/2) + 1))/(12*c^5) + (b^2*x^(1/2
)*log(c*x^(1/2) + 1))/(4*c^7) - (b^2*x^(7/2)*log(1 - c*x^(1/2)))/(28*c) - (
b^2*x^(5/2)*log(1 - c*x^(1/2)))/(20*c^3) - (b^2*x^(3/2)*log(1 - c*x^(1/2)))
/(12*c^5) - (b^2*x^(1/2)*log(1 - c*x^(1/2)))/(4*c^7) + (a*b*log(c*x^(1/2) -
 1))/(4*c^8) - (a*b*log(c*x^(1/2) + 1))/(4*c^8) + (a*b*x^4*log(c*x^(1/2) +
1))/4 - (a*b*x^4*log(1 - c*x^(1/2)))/4 + (b^2*log(c*x^(1/2) + 1)*log(1 - c*
x^(1/2)))/(8*c^8) + (a*b*x^(7/2))/(14*c) + (a*b*x^(5/2))/(10*c^3) + (a*b*x^
(3/2))/(6*c^5) + (a*b*x^(1/2))/(2*c^7) - (b^2*x^4*log(c*x^(1/2) + 1)*log(1
- c*x^(1/2)))/8
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b \operatorname{atanh}\left(c\sqrt{x}\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(1/2)))**2,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*sqrt(x)))**2, x)
```

3.196 $\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$

Optimal. Leaf size=173

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{3c^6}+\frac{2ab\sqrt{x}}{3c^5}+\frac{2bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{9c^3}+\frac{2bx^{5/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{15c}+\frac{1}{3}x^3\left(a+b\tanh\right.$$

```
[Out] 8/45*b^2*x/c^4+1/30*b^2*x^2/c^2+2/9*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))/c^3+
2/15*b*x^(5/2)*(a+b*arctanh(c*x^(1/2)))/c-1/3*(a+b*arctanh(c*x^(1/2)))^2/c^
6+1/3*x^3*(a+b*arctanh(c*x^(1/2)))^2+23/45*b^2*ln(-c^2*x+1)/c^6+2/3*a*b*x^(
1/2)/c^5+2/3*b^2*arctanh(c*x^(1/2))*x^(1/2)/c^5
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] Defer[Int][x^2*(a + b*ArcTanh[c*Sqrt[x]])^2, x]
```

Rubi steps

$$\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx=\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$$

Mathematica [A] time = 0.11, size = 194, normalized size = 1.12

$$\frac{30a^2c^6x^3+12abc^5x^{5/2}+20abc^3x^{3/2}+4bc\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)\left(15ac^5x^{5/2}+b\left(3c^4x^2+5c^2x+15\right)\right)+60abc\sqrt{x}+2b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (60*a*b*c*Sqrt[x] + 16*b^2*c^2*x + 20*a*b*c^3*x^(3/2) + 3*b^2*c^4*x^2 + 12*
a*b*c^5*x^(5/2) + 30*a^2*c^6*x^3 + 4*b*c*Sqrt[x]*(15*a*c^5*x^(5/2) + b*(15
+ 5*c^2*x + 3*c^4*x^2))*ArcTanh[c*Sqrt[x]] + 30*b^2*(-1 + c^6*x^3)*ArcTanh[
c*Sqrt[x]]^2 + 2*b*(15*a + 23*b)*Log[1 - c*Sqrt[x]] - 30*a*b*Log[1 + c*Sqrt
[x]] + 46*b^2*Log[1 + c*Sqrt[x]])/(90*c^6)
```

fricas [A] time = 1.21, size = 241, normalized size = 1.39

$$\frac{60a^2c^6x^3+6b^2c^4x^2+32b^2c^2x+15\left(b^2c^6x^3-b^2\right)\log\left(-\frac{c^2x+2c\sqrt{x}+1}{c^2x-1}\right)^2+4\left(15abc^6-15ab+23b^2\right)\log\left(c\sqrt{x}+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/180*(60*a^2*c^6*x^3 + 6*b^2*c^4*x^2 + 32*b^2*c^2*x + 15*(b^2*c^6*x^3 - b^
2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 + 4*(15*a*b*c^6 - 15*a*b +
 23*b^2)*log(c*sqrt(x) + 1) - 4*(15*a*b*c^6 - 15*a*b - 23*b^2)*log(c*sqrt(x
) - 1) + 4*(15*a*b*c^6*x^3 - 15*a*b*c^6 + (3*b^2*c^5*x^2 + 5*b^2*c^3*x + 15
```

```
*b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) + 8*(3*a*b*c^5
*x^2 + 5*a*b*c^3*x + 15*a*b*c)*sqrt(x))/c^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\operatorname{artanh}\left(c\sqrt{x}\right)+a\right)^2 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2*x^2, x)
```

maple [B] time = 0.06, size = 358, normalized size = 2.07

$$\frac{x^3a^2}{3}+\frac{b^2x^3\operatorname{arctanh}\left(c\sqrt{x}\right)^2}{3}+\frac{2b^2\operatorname{arctanh}\left(c\sqrt{x}\right)x^{\frac{5}{2}}}{15c}+\frac{2b^2\operatorname{arctanh}\left(c\sqrt{x}\right)x^{\frac{3}{2}}}{9c^3}+\frac{2b^2\operatorname{arctanh}\left(c\sqrt{x}\right)\sqrt{x}}{3c^5}+\frac{b^2\operatorname{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(1/2)))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arctanh(c*x^(1/2))^2+2/15/c*b^2*arctanh(c*x^(1/2))*
x^(5/2)+2/9/c^3*b^2*arctanh(c*x^(1/2))*x^(3/2)+2/3*b^2*arctanh(c*x^(1/2))*x
^(1/2)/c^5+1/3/c^6*b^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/3/c^6*b^2*arcta
nh(c*x^(1/2))*ln(1+c*x^(1/2))+1/12/c^6*b^2*ln(c*x^(1/2)-1)^2-1/6/c^6*b^2*ln
(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+1/12/c^6*b^2*ln(1+c*x^(1/2))^2-1/6/c^6*
b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+1/6/c^6*b^2*ln(-1/2*c*x^(1/2)+1/
2)*ln(1/2+1/2*c*x^(1/2))+1/30*b^2*x^2/c^2+8/45*b^2*x/c^4+23/45/c^6*b^2*ln(c
*x^(1/2)-1)+23/45/c^6*b^2*ln(1+c*x^(1/2))+2/3*a*b*x^3*arctanh(c*x^(1/2))+2/
15/c*x^(5/2)*a*b+2/9/c^3*a*b*x^(3/2)+2/3*a*b*x^(1/2)/c^5+1/3/c^6*a*b*ln(c*x
^(1/2)-1)-1/3/c^6*a*b*ln(1+c*x^(1/2))
```

maxima [A] time = 0.34, size = 241, normalized size = 1.39

$$\frac{1}{3}b^2x^3\operatorname{artanh}\left(c\sqrt{x}\right)^2+\frac{1}{3}a^2x^3+\frac{1}{45}\left(30\,x^3\operatorname{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(3\,c^4x^{\frac{5}{2}}+5\,c^2x^{\frac{3}{2}}+15\,\sqrt{x}\right)}{c^6}-\frac{15\,\log\left(c\sqrt{x}+1\right)}{c^7}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*arctanh(c*sqrt(x))^2 + 1/3*a^2*x^3 + 1/45*(30*x^3*arctanh(c*sqr
t(x)) + c*(2*(3*c^4*x^(5/2) + 5*c^2*x^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*sq
rt(x) + 1)/c^7 + 15*log(c*sqrt(x) - 1)/c^7))*a*b + 1/180*(4*c*(2*(3*c^4*x^(
5/2) + 5*c^2*x^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*sqrt(x) + 1)/c^7 + 15*log
(c*sqrt(x) - 1)/c^7)*arctanh(c*sqrt(x)) + (6*c^4*x^2 + 32*c^2*x - 2*(15*log
(c*sqrt(x) - 1) - 46)*log(c*sqrt(x) + 1) + 15*log(c*sqrt(x) + 1)^2 + 15*log
(c*sqrt(x) - 1)^2 + 92*log(c*sqrt(x) - 1))/c^6)*b^2
```

mupad [B] time = 1.57, size = 185, normalized size = 1.07

$$\frac{46\,b^2\,\ln\left(c^2\,x-1\right)-30\,b^2\,\operatorname{atanh}\left(c\,\sqrt{x}\right)^2-60\,a\,b\,\operatorname{atanh}\left(c\,\sqrt{x}\right)+16\,b^2\,c^2\,x+30\,a^2\,c^6\,x^3+3\,b^2\,c^4\,x^2+30\,b^2\,c^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^(1/2)))^2,x)
```

```
[Out] (46*b^2*log(c^2*x - 1) - 30*b^2*atanh(c*x^(1/2))^2 - 60*a*b*atanh(c*x^(1/2)
) + 16*b^2*c^2*x + 30*a^2*c^6*x^3 + 3*b^2*c^4*x^2 + 30*b^2*c^6*x^3*atanh(c*
x^(1/2))^2 + 60*b^2*c*x^(1/2)*atanh(c*x^(1/2)) + 60*a*b*c*x^(1/2) + 20*b^2*
c^3*x^(3/2)*atanh(c*x^(1/2)) + 12*b^2*c^5*x^(5/2)*atanh(c*x^(1/2)) + 20*a*b
*c^3*x^(3/2) + 12*a*b*c^5*x^(5/2) + 60*a*b*c^6*x^3*atanh(c*x^(1/2)))/(90*c^
6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{atanh}\left(c\sqrt{x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(1/2)))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*sqrt(x)))**2, x)
```

3.197 $\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$

Optimal. Leaf size=129

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{2c^4}+\frac{ab\sqrt{x}}{c^3}+\frac{bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{3c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+\frac{b^2\sqrt{x}\tanh^{-1}\left(c\sqrt{\right.}\right.}{c^3}$$

```
[Out] 1/6*b^2*x/c^2+1/3*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))/c-1/2*(a+b*arctanh(c*x
^(1/2)))^2/c^4+1/2*x^2*(a+b*arctanh(c*x^(1/2)))^2+2/3*b^2*ln(-c^2*x+1)/c^4+
a*b*x^(1/2)/c^3+b^2*arctanh(c*x^(1/2))*x^(1/2)/c^3
```

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] Defer[Int][x*(a + b*ArcTanh[c*Sqrt[x]])^2, x]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx=\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$$

Mathematica [A] time = 0.09, size = 160, normalized size = 1.24

$$\frac{3a^2c^4x^2+2abc^3x^{3/2}+2bc\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)\left(3ac^3x^{3/2}+b\left(c^2x+3\right)\right)+6abc\sqrt{x}+b(3a+4b)\log\left(1-c\sqrt{x}\right)-}{6c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (6*a*b*c*Sqrt[x] + b^2*c^2*x + 2*a*b*c^3*x^(3/2) + 3*a^2*c^4*x^2 + 2*b*c*Sq
rt[x]*(3*a*c^3*x^(3/2) + b*(3 + c^2*x))*ArcTanh[c*Sqrt[x]] + 3*b^2*(-1 + c^
4*x^2)*ArcTanh[c*Sqrt[x]]^2 + b*(3*a + 4*b)*Log[1 - c*Sqrt[x]] - 3*a*b*Log[
1 + c*Sqrt[x]] + 4*b^2*Log[1 + c*Sqrt[x]])/(6*c^4)
```

fricas [A] time = 0.92, size = 207, normalized size = 1.60

$$12\,a^2c^4x^2+4\,b^2c^2x+3\left(b^2c^4x^2-b^2\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2+4\left(3\,abc^4-3\,ab+4\,b^2\right)\log\left(c\sqrt{x}+1\right)-4\left(3\,abc^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*a^2*c^4*x^2 + 4*b^2*c^2*x + 3*(b^2*c^4*x^2 - b^2)*log(-(c^2*x + 2*
c*sqrt(x) + 1)/(c^2*x - 1))^2 + 4*(3*a*b*c^4 - 3*a*b + 4*b^2)*log(c*sqrt(x)
 + 1) - 4*(3*a*b*c^4 - 3*a*b - 4*b^2)*log(c*sqrt(x) - 1) + 4*(3*a*b*c^4*x^2
 - 3*a*b*c^4 + (b^2*c^3*x + 3*b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1
)/(c^2*x - 1)) + 8*(a*b*c^3*x + 3*a*b*c)*sqrt(x))/c^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\mathrm{artanh}\left(c\sqrt{x}\right)+a\right)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2*x, x)
```

maple [B] time = 0.05, size = 317, normalized size = 2.46

$$\frac{a^2x^2}{2}+\frac{b^2x^2\,\mathrm{arctanh}\left(c\sqrt{x}\right)^2}{2}+\frac{b^2\,\mathrm{arctanh}\left(c\sqrt{x}\right)x^{\frac{3}{2}}}{3c}+\frac{b^2\,\mathrm{arctanh}\left(c\sqrt{x}\right)\sqrt{x}}{c^3}+\frac{b^2\,\mathrm{arctanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)}{2c^4}-\frac{b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(1/2)))^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*b^2*x^2*arctanh(c*x^(1/2))^2+1/3/c*b^2*arctanh(c*x^(1/2))*x
^(3/2)+b^2*arctanh(c*x^(1/2))*x^(1/2)/c^3+1/2/c^4*b^2*arctanh(c*x^(1/2))*ln
(c*x^(1/2)-1)-1/2/c^4*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))+1/8/c^4*b^2*ln
(c*x^(1/2)-1)^2-1/4/c^4*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+1/8/c^4*b
^2*ln(1+c*x^(1/2))^2-1/4/c^4*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+1/4
/c^4*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))+1/6*b^2*x/c^2+2/3/c^4
*b^2*ln(c*x^(1/2)-1)+2/3/c^4*b^2*ln(1+c*x^(1/2))+x^2*a*b*arctanh(c*x^(1/2))
+1/3*a*b*x^(3/2)/c+a*b*x^(1/2)/c^3+1/2/c^4*a*b*ln(c*x^(1/2)-1)-1/2/c^4*a*b*
ln(1+c*x^(1/2))
```

maxima [B] time = 0.33, size = 215, normalized size = 1.67

$$\frac{1}{2}b^2x^2\,\mathrm{artanh}\left(c\sqrt{x}\right)^2+\frac{1}{2}a^2x^2+\frac{1}{6}\left(6\,x^2\,\mathrm{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(c^2x^{\frac{3}{2}}+3\sqrt{x}\right)}{c^4}-\frac{3\,\log\left(c\sqrt{x}+1\right)}{c^5}+\frac{3\,\log\left(c\sqrt{x}-1\right)}{c^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctanh(c*sqrt(x))^2 + 1/2*a^2*x^2 + 1/6*(6*x^2*arctanh(c*sqrt(
x)) + c*(2*(c^2*x^(3/2) + 3*sqrt(x))/c^4 - 3*log(c*sqrt(x) + 1)/c^5 + 3*log
(c*sqrt(x) - 1)/c^5))*a*b + 1/24*(4*c*(2*(c^2*x^(3/2) + 3*sqrt(x))/c^4 - 3*
log(c*sqrt(x) + 1)/c^5 + 3*log(c*sqrt(x) - 1)/c^5)*arctanh(c*sqrt(x)) + (4*
c^2*x - 2*(3*log(c*sqrt(x) - 1) - 8)*log(c*sqrt(x) + 1) + 3*log(c*sqrt(x) +
 1)^2 + 3*log(c*sqrt(x) - 1)^2 + 16*log(c*sqrt(x) - 1))/c^4)*b^2
```

mupad [B] time = 1.28, size = 143, normalized size = 1.11

$$\frac{a^2\,x^2}{2}-\frac{b^2\,\mathrm{atanh}\left(c\,\sqrt{x}\right)^2}{2\,c^4}+\frac{2\,b^2\,\ln\left(c^2\,x-1\right)}{3\,c^4}+\frac{b^2\,x^2\,\mathrm{atanh}\left(c\,\sqrt{x}\right)^2}{2}+\frac{b^2\,x}{6\,c^2}+\frac{b^2\,x^{3/2}\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{3\,c}+\frac{b^2\,\sqrt{x}\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^(1/2)))^2,x)
```

```
[Out] (a^2*x^2)/2 - (b^2*atanh(c*x^(1/2))^2)/(2*c^4) + (2*b^2*log(c^2*x - 1))/(3*
c^4) + (b^2*x^2*atanh(c*x^(1/2))^2)/2 + (b^2*x)/(6*c^2) + (b^2*x^(3/2)*atan
h(c*x^(1/2)))/(3*c) + (b^2*x^(1/2)*atanh(c*x^(1/2)))/c^3 + (a*b*x^(3/2))/(3
*c) + (a*b*x^(1/2))/c^3 - (a*b*atanh(c*x^(1/2)))/c^4 + a*b*x^2*atanh(c*x^(1
/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**(1/2)))**2,x)
```

```
[Out] Integral(x*(a + b*atanh(c*sqrt(x)))**2, x)
```

3.198 $\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$

Optimal. Leaf size=85

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c^2}+\frac{2ab\sqrt{x}}{c}+x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+\frac{b^2\log\left(1-c^2x\right)}{c^2}+\frac{2b^2\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)}{c}$$

```
[Out] -(a+b*arctanh(c*x^(1/2)))^2/c^2+x*(a+b*arctanh(c*x^(1/2)))^2+b^2*ln(-c^2*x+
1)/c^2+2*a*b*x^(1/2)/c+2*b^2*arctanh(c*x^(1/2))*x^(1/2)/c
```

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^2, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx=\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$$

Mathematica [A] time = 0.06, size = 115, normalized size = 1.35

$$\frac{a^2c^2x+2abc\sqrt{x}+b(a+b)\log\left(1-c\sqrt{x}\right)-ab\log\left(c\sqrt{x}+1\right)+2bc\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)\left(ac\sqrt{x}+b\right)+b^2\left(c^2x-1\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (2*a*b*c*Sqrt[x] + a^2*c^2*x + 2*b*c*(b + a*c*Sqrt[x])*Sqrt[x]*ArcTanh[c*Sq
rt[x]] + b^2*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^2 + b*(a + b)*Log[1 - c*Sqrt[x
]] - a*b*Log[1 + c*Sqrt[x]] + b^2*Log[1 + c*Sqrt[x]])/c^2
```

fricas [B] time = 1.13, size = 165, normalized size = 1.94

$$\frac{4a^2c^2x+8abc\sqrt{x}+\left(b^2c^2x-b^2\right)\log\left(-\frac{c^2x+2c\sqrt{x}+1}{c^2x-1}\right)^2+4\left(abc^2-ab+b^2\right)\log\left(c\sqrt{x}+1\right)-4\left(abc^2-ab-b^2\right)\log}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*c^2*x + 8*a*b*c*sqrt(x) + (b^2*c^2*x - b^2)*log(-(c^2*x + 2*c*sq
rt(x) + 1)/(c^2*x - 1))^2 + 4*(a*b*c^2 - a*b + b^2)*log(c*sqrt(x) + 1) - 4*
(a*b*c^2 - a*b - b^2)*log(c*sqrt(x) - 1) + 4*(a*b*c^2*x - a*b*c^2 + b^2*c*s
qrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)))/c^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\operatorname{artanh}\left(c\sqrt{x}\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2, x)
```

maple [B] time = 0.05, size = 272, normalized size = 3.20

$$a^2x+x\,b^2\operatorname{arctanh}\left(c\sqrt{x}\right)^2+\frac{2b^2\operatorname{arctanh}\left(c\sqrt{x}\right)\sqrt{x}}{c}+\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)}{c^2}-\frac{b^2\operatorname{arctanh}\left(c\sqrt{x}\right)\ln\left(1\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2,x)
```

```
[Out] a^2*x+x*b^2*arctanh(c*x^(1/2))^2+2*b^2*arctanh(c*x^(1/2))*x^(1/2)/c+1/c^2*b
^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/c^2*b^2*arctanh(c*x^(1/2))*ln(1+c*x
^(1/2))+1/4/c^2*b^2*ln(c*x^(1/2)-1)^2-1/2/c^2*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/
2*c*x^(1/2))+1/c^2*b^2*ln(c*x^(1/2)-1)+1/c^2*b^2*ln(1+c*x^(1/2))+1/4/c^2*b^
2*ln(1+c*x^(1/2))^2+1/2/c^2*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2)
)-1/2/c^2*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+2*a*b*x*arctanh(c*x^(1
/2))+2*a*b*x^(1/2)/c+1/c^2*a*b*ln(c*x^(1/2)-1)-1/c^2*a*b*ln(1+c*x^(1/2))
```

maxima [B] time = 0.34, size = 175, normalized size = 2.06

$$\left(c\left(\frac{2\sqrt{x}}{c^2}-\frac{\log\left(c\sqrt{x}+1\right)}{c^3}+\frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right)+2\,x\operatorname{artanh}\left(c\sqrt{x}\right)\right)ab+\frac{1}{4}\left(4\,c\left(\frac{2\sqrt{x}}{c^2}-\frac{\log\left(c\sqrt{x}+1\right)}{c^3}+\frac{\log\left(c\right.}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] (c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) + 2*x*
arctanh(c*sqrt(x)))*a*b + 1/4*(4*c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3
+ log(c*sqrt(x) - 1)/c^3)*arctanh(c*sqrt(x)) + 4*x*arctanh(c*sqrt(x))^2 - (
2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log(c*sqrt(x) + 1)^2 - log(
c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1))/c^2)*b^2 + a^2*x
```

mupad [B] time = 1.06, size = 94, normalized size = 1.11

$$a^2\,x+\frac{c\,\left(2\,b^2\,\sqrt{x}\,\mathrm{atanh}\left(c\,\sqrt{x}\right)+2\,a\,b\,\sqrt{x}\right)-b^2\,\mathrm{atanh}\left(c\,\sqrt{x}\right)^2+b^2\,\ln\left(c^2\,x-1\right)-2\,a\,b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{c^2}+b^2\,x\,\mathrm{at}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^2,x)
```

```
[Out] a^2*x + (c*(2*b^2*x^(1/2)*atanh(c*x^(1/2)) + 2*a*b*x^(1/2)) - b^2*atanh(c*x
^(1/2))^2 + b^2*log(c^2*x - 1) - 2*a*b*atanh(c*x^(1/2)))/c^2 + b^2*x*atanh(
c*x^(1/2))^2 + 2*a*b*x*atanh(c*x^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**2, x)
```

3.199 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=145

$$-2b\text{Li}_2\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+2b\text{Li}_2\left(\frac{2}{1-c\sqrt{x}}-1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right.$$

```
[Out] -4*(a+b*arctanh(c*x^(1/2)))^2*arctanh(-1+2/(1-c*x^(1/2)))-2*b*(a+b*arctanh(
c*x^(1/2)))*polylog(2,1-2/(1-c*x^(1/2)))+2*b*(a+b*arctanh(c*x^(1/2)))*polyl
og(2,-1+2/(1-c*x^(1/2)))+b^2*polylog(3,1-2/(1-c*x^(1/2)))-b^2*polylog(3,-1+
2/(1-c*x^(1/2)))
```

Rubi [A] time = 0.32, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-2b\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+2b\text{PolyLog}\left(2,\frac{2}{1-c\sqrt{x}}-1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+b^2\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x,x]
```

```
[Out] 4*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^2 - 2*b*(a + b*
ArcTanh[c*Sqrt[x]])*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])] + 2*b*(a + b*ArcTanh[
c*Sqrt[x]])*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])] + b^2*PolyLog[3, 1 - 2/(1 -
c*Sqrt[x])] - b^2*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, \sqrt{x}\right)$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-(8bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh\right.}{}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+(4bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh\right.}{1}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-2b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\text{Li}_2$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-2b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\text{Li}_2$$

Mathematica [A] time = 0.09, size = 164, normalized size = 1.13

$$4\tanh^{-1}\left(\frac{2}{c\sqrt{x}-1}+1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-b\left(-2\text{Li}_2\left(\frac{\sqrt{x}c+1}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+2\text{Li}_2\left(\frac{\sqrt{x}c+1}{c\sqrt{x}-1}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2/x,x]
```

```
[Out] 4*ArcTanh[1 + 2/(-1 + c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^2 - b*(-2*(a +
 b*ArcTanh[c*Sqrt[x]])*PolyLog[2, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] + 2*(a +
 b*ArcTanh[c*Sqrt[x]])*PolyLog[2, (1 + c*Sqrt[x])/(-1 + c*Sqrt[x])] + b*(Po
lyLog[3, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] - PolyLog[3, (1 + c*Sqrt[x])/(-1
+ c*Sqrt[x])]))
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(c\sqrt{x}\right)^2+2\,ab\,\text{artanh}\left(c\sqrt{x}\right)+a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*sqrt(x))^2 + 2*a*b*arctanh(c*sqrt(x)) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2/x, x)
```

maple [C] time = 0.23, size = 742, normalized size = 5.12

$$2a^2\ln\left(c\sqrt{x}\right)+2b^2\ln\left(c\sqrt{x}\right)\operatorname{arctanh}\left(c\sqrt{x}\right)^2-2b^2\operatorname{arctanh}\left(c\sqrt{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+c\sqrt{x}\right)^2}{-c^2x+1}\right)+b^2\operatorname{polylog}\left(3,-\frac{(1}{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2/x,x)
```

```
[Out] 2*a^2*ln(c*x^(1/2))+2*b^2*ln(c*x^(1/2))*arctanh(c*x^(1/2))^2-2*b^2*arctanh(
c*x^(1/2))*polylog(2,-(1+c*x^(1/2))^2/(-c^2*x+1))+b^2*polylog(3,-(1+c*x^(1/
2))^2/(-c^2*x+1))-2*b^2*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))^2/(-c^2*x+1)-
1)+2*b^2*arctanh(c*x^(1/2))^2*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+4*b^2*ar
ctanh(c*x^(1/2))*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4*b^2*polylog(3,
(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+2*b^2*arctanh(c*x^(1/2))^2*ln(1+(1+c*x^(1/2
))/(-c^2*x+1)^(1/2))+4*b^2*arctanh(c*x^(1/2))*polylog(2,-(1+c*x^(1/2))/(-c^
2*x+1)^(1/2))-4*b^2*polylog(3,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-I*b^2*Pi*csg
n(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(
1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^2-I*b^2*Pi*csgn(I/(1+(1
+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^
(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^2+I*b^2*Pi*csgn(I*((1+c*x^(1/2))
^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*arctanh(c*x^(1/2))^2+I*b
^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I/(1+(1+c*x^(1/2))^2/(-c^
2*x+1)))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1
)))*arctanh(c*x^(1/2))^2+4*a*b*ln(c*x^(1/2))*arctanh(c*x^(1/2))-2*a*b*ln(c*
x^(1/2))*ln(1+c*x^(1/2))-2*a*b*dilog(c*x^(1/2))-2*a*b*dilog(1+c*x^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}b^2\int\frac{\log\left(c\sqrt{x}+1\right)^2}{x}\,dx-\frac{1}{2}b^2\int\frac{\log\left(c\sqrt{x}+1\right)\log\left(-c\sqrt{x}+1\right)}{x}\,dx+\frac{1}{4}b^2\int\frac{\log\left(-c\sqrt{x}+1\right)^2}{x}\,dx+ab\int\frac{\log\left(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*integrate(log(c*sqrt(x) + 1)^2/x, x) - 1/2*b^2*integrate(log(c*sqrt
(x) + 1)*log(-c*sqrt(x) + 1)/x, x) + 1/4*b^2*integrate(log(-c*sqrt(x) + 1)^
2/x, x) + a*b*integrate(log(c*sqrt(x) + 1)/x, x) - a*b*integrate(log(-c*sqr
t(x) + 1)/x, x) + a^2*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^2/x,x)
```

```
[Out] int((a + b*atanh(c*x^(1/2)))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**2/x, x)
```

3.200 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=85

$$c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-\frac{2bc\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{\sqrt{x}}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x}+b^2c^2\log(x)-b^2c^2\log\left(1-c^2x\right)$$

```
[Out] c^2*(a+b*arctanh(c*x^(1/2)))^2-(a+b*arctanh(c*x^(1/2)))^2/x+b^2*c^2*ln(x)-b
^2*c^2*ln(-c^2*x+1)-2*b*c*(a+b*arctanh(c*x^(1/2)))/x^(1/2)
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^2/x^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx$$

Mathematica [A] time = 0.11, size = 129, normalized size = 1.52

$$-\frac{a^2-abc^2x\log\left(c\sqrt{x}+1\right)+bc^2x(a+b)\log\left(1-c\sqrt{x}\right)+2abc\sqrt{x}+2b\tanh^{-1}\left(c\sqrt{x}\right)\left(a+bc\sqrt{x}\right)+b^2c^2x\log\left(c\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2/x^2,x]
```

```
[Out] -((a^2 + 2*a*b*c*Sqrt[x] + 2*b*(a + b*c*Sqrt[x])*ArcTanh[c*Sqrt[x]] - b^2*(
-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^2 + b*(a + b)*c^2*x*Log[1 - c*Sqrt[x]] - a*b
*c^2*x*Log[1 + c*Sqrt[x]] + b^2*c^2*x*Log[1 + c*Sqrt[x]] - b^2*c^2*x*Log[x]
)/x)
```

fricas [B] time = 0.72, size = 157, normalized size = 1.85

$$\frac{8\,b^2c^2x\log\left(\sqrt{x}\right)+4\left(ab-b^2\right)c^2x\log\left(c\sqrt{x}+1\right)-4\left(ab+b^2\right)c^2x\log\left(c\sqrt{x}-1\right)-8\,abc\sqrt{x}+\left(b^2c^2x-b^2\right)\log\left(-\right.}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(8*b^2*c^2*x*log(sqrt(x)) + 4*(a*b - b^2)*c^2*x*log(c*sqrt(x) + 1) - 4*
(a*b + b^2)*c^2*x*log(c*sqrt(x) - 1) - 8*a*b*c*sqrt(x) + (b^2*c^2*x - b^2)*
log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 - 4*a^2 - 4*(b^2*c*sqrt(x) +
a*b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(c\sqrt{x}\right)+a\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2/x^2, x)
```

maple [B] time = 0.07, size = 292, normalized size = 3.44

$$-\frac{a^{2}}{x}-\frac{b^{2} \operatorname{arctanh}\left(c\sqrt{x}\right)^{2}}{x}-\frac{2 c\, b^{2} \operatorname{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}}-c^{2} b^{2} \operatorname{arctanh}\left(c\sqrt{x}\right) \ln\left(c\sqrt{x}-1\right)+c^{2} b^{2} \operatorname{arctanh}\left(c\sqrt{x}\right) \ln\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2/x^2,x)
```

```
[Out] -a^2/x-b^2/x*arctanh(c*x^(1/2))^2-2*c*b^2*arctanh(c*x^(1/2))/x^(1/2)-c^2*b^
2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)+c^2*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1
/2))-1/4*c^2*b^2*ln(c*x^(1/2)-1)^2+1/2*c^2*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c
*x^(1/2))+2*c^2*b^2*ln(c*x^(1/2))-c^2*b^2*ln(c*x^(1/2)-1)-c^2*b^2*ln(1+c*x^
(1/2))-1/4*c^2*b^2*ln(1+c*x^(1/2))^2+1/2*c^2*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(
1+c*x^(1/2))-1/2*c^2*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))-2*a*b
/x*arctanh(c*x^(1/2))-2*c*a*b/x^(1/2)-c^2*a*b*ln(c*x^(1/2)-1)+c^2*a*b*ln(1+
c*x^(1/2))
```

maxima [B] time = 0.33, size = 174, normalized size = 2.05

$$\left(\left(c \log\left(c\sqrt{x}+1\right)-c \log\left(c\sqrt{x}-1\right)-\frac{2}{\sqrt{x}}\right)c-\frac{2 \operatorname{artanh}\left(c\sqrt{x}\right)}{x}\right)ab+\frac{1}{4}\left(\left(2\left(\log\left(c\sqrt{x}-1\right)-2\right)\log\left(c\sqrt{x}+1\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="maxima")
```

```
[Out] ((c*log(c*sqrt(x) + 1) - c*log(c*sqrt(x) - 1) - 2/sqrt(x))*c - 2*arctanh(c*
sqrt(x))/x)*a*b + 1/4*((2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log
(c*sqrt(x) + 1)^2 - log(c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1) + 4*log(x))
*c^2 + 4*(c*log(c*sqrt(x) + 1) - c*log(c*sqrt(x) - 1) - 2/sqrt(x))*c*arctan
h(c*sqrt(x)))*b^2 - b^2*arctanh(c*sqrt(x))^2/x - a^2/x
```

mupad [B] time = 1.79, size = 278, normalized size = 3.27

$$2 b^{2} c^{2} \ln\left(\sqrt{x}\right)-\frac{a^{2}}{x}-b^{2} c^{2} \ln\left(c\sqrt{x}-1\right)-b^{2} c^{2} \ln\left(c\sqrt{x}+1\right)+\frac{b^{2} c^{2} \ln\left(c\sqrt{x}+1\right)^{2}}{4}+\frac{b^{2} c^{2} \ln\left(1-c\sqrt{x}\right)^{2}}{4}-\frac{b^{2} \ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^2/x^2,x)
```

```
[Out] 2*b^2*c^2*log(x^(1/2)) - a^2/x - b^2*c^2*log(c*x^(1/2) - 1) - b^2*c^2*log(c
*x^(1/2) + 1) + (b^2*c^2*log(c*x^(1/2) + 1)^2)/4 + (b^2*c^2*log(1 - c*x^(1/
2))^2)/4 - (b^2*log(c*x^(1/2) + 1)^2)/(4*x) - (b^2*log(1 - c*x^(1/2))^2)/(4
*x) - a*b*c^2*log(c*x^(1/2) - 1) + a*b*c^2*log(c*x^(1/2) + 1) - (2*a*b*c)/x
^(1/2) - (a*b*log(c*x^(1/2) + 1))/x + (a*b*log(1 - c*x^(1/2)))/x - (b^2*c^2
*log(c*x^(1/2) + 1)*log(1 - c*x^(1/2)))/2 - (b^2*c*log(c*x^(1/2) + 1))/x^(1
```

```
/2) + (b^2*c*log(1 - c*x^(1/2)))/x^(1/2) + (b^2*log(c*x^(1/2) + 1)*log(1 -
c*x^(1/2)))/(2*x)
```

sympy [A] time = 22.59, size = 680, normalized size = 8.00

$$\begin{cases} -\frac{a^2}{x} + \frac{2ab\,\text{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} - \frac{b^2\,\text{atanh}^2\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} \\ -\frac{a^2}{x} - \frac{2ab\,\text{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} - \frac{b^2\,\text{atanh}^2\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} \\ -\frac{a^2}{x} \\ -\frac{a^2c^2x^{\frac{3}{2}}}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} + \frac{a^2\sqrt{x}}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} + \frac{2abc^4x^{\frac{5}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} - \frac{2abc^3x^2}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} - \frac{4abc^2x^{\frac{3}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} + \frac{2abcx}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} + \frac{2ab\sqrt{x}\,\text{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} + \frac{b^2c^4x^{\frac{5}{2}}\log(x}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2/x**2,x)
```

```
[Out] Piecewise((-a**2/x + 2*a*b*atanh(sqrt(x)*sqrt(1/x))/x - b**2*atanh(sqrt(x)*
sqrt(1/x))**2/x, Eq(c, -sqrt(1/x))), (-a**2/x - 2*a*b*atanh(sqrt(x)*sqrt(1/
x))/x - b**2*atanh(sqrt(x)*sqrt(1/x))**2/x, Eq(c, sqrt(1/x))), (-a**2/x, Eq
(c, 0)), (-a**2*c**2*x**(3/2)/(c**2*x**(5/2) - x**(3/2)) + a**2*sqrt(x)/(c*
*2*x**(5/2) - x**(3/2)) + 2*a*b*c**4*x**(5/2)*atanh(c*sqrt(x))/(c**2*x**(5/
2) - x**(3/2)) - 2*a*b*c**3*x**2/(c**2*x**(5/2) - x**(3/2)) - 4*a*b*c**2*x*
*(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + 2*a*b*c*x/(c**2*x**(5/
2) - x**(3/2)) + 2*a*b*sqrt(x)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2))
+ b**2*c**4*x**(5/2)*log(x)/(c**2*x**(5/2) - x**(3/2)) - 2*b**2*c**4*x**(5/
2)*log(sqrt(x) - 1/c)/(c**2*x**(5/2) - x**(3/2)) + b**2*c**4*x**(5/2)*atanh
(c*sqrt(x))**2/(c**2*x**(5/2) - x**(3/2)) - 2*b**2*c**4*x**(5/2)*atanh(c*sq
rt(x))/(c**2*x**(5/2) - x**(3/2)) - 2*b**2*c**3*x**2*atanh(c*sqrt(x))/(c**2
*x**(5/2) - x**(3/2)) - b**2*c**2*x**(3/2)*log(x)/(c**2*x**(5/2) - x**(3/2)
) + 2*b**2*c**2*x**(3/2)*log(sqrt(x) - 1/c)/(c**2*x**(5/2) - x**(3/2)) - 2*
b**2*c**2*x**(3/2)*atanh(c*sqrt(x))**2/(c**2*x**(5/2) - x**(3/2)) + 2*b**2*
c**2*x**(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + 2*b**2*c*x*atan
h(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + b**2*sqrt(x)*atanh(c*sqrt(x))**2/
(c**2*x**(5/2) - x**(3/2)), True))
```

3.201 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=133

$$\frac{1}{2}c^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-\frac{bc^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{\sqrt{x}}-\frac{bc\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{3x^{3/2}}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{2x^2}+\frac{2}{3}b^2$$

```
[Out] -1/6*b^2*c^2/x-1/3*b*c*(a+b*arctanh(c*x^(1/2)))/x^(3/2)+1/2*c^4*(a+b*arctan
h(c*x^(1/2)))^2-1/2*(a+b*arctanh(c*x^(1/2)))^2/x^2+2/3*b^2*c^4*ln(x)-2/3*b^
2*c^4*ln(-c^2*x+1)-b*c^3*(a+b*arctanh(c*x^(1/2)))/x^(1/2)
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x^3,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^2/x^3, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx$$

Mathematica [A] time = 0.13, size = 178, normalized size = 1.34

$$-\frac{3a^2+bc^4x^2(3a+4b)\log\left(1-c\sqrt{x}\right)-3abc^4x^2\log\left(c\sqrt{x}+1\right)+6abc^3x^{3/2}+2b\tanh^{-1}\left(c\sqrt{x}\right)\left(3a+bc\sqrt{x}\left(3c\right.\right.}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2/x^3,x]
```

```
[Out] -1/6*(3*a^2 + 2*a*b*c*Sqrt[x] + b^2*c^2*x + 6*a*b*c^3*x^(3/2) + 2*b*(3*a +
b*c*Sqrt[x]*(1 + 3*c^2*x))*ArcTanh[c*Sqrt[x]] - 3*b^2*(-1 + c^4*x^2)*ArcTan
h[c*Sqrt[x]]^2 + b*(3*a + 4*b)*c^4*x^2*Log[1 - c*Sqrt[x]] - 3*a*b*c^4*x^2*L
og[1 + c*Sqrt[x]] + 4*b^2*c^4*x^2*Log[1 + c*Sqrt[x]] - 4*b^2*c^4*x^2*Log[x]
)/x^2
```

fricas [A] time = 0.74, size = 201, normalized size = 1.51

$$\frac{32\,b^2c^4x^2\log\left(\sqrt{x}\right)+4\left(3\,ab-4\,b^2\right)c^4x^2\log\left(c\sqrt{x}+1\right)-4\left(3\,ab+4\,b^2\right)c^4x^2\log\left(c\sqrt{x}-1\right)-4\,b^2c^2x+3\left(b^2c^4\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(32*b^2*c^4*x^2*log(sqrt(x)) + 4*(3*a*b - 4*b^2)*c^4*x^2*log(c*sqrt(x)
 + 1) - 4*(3*a*b + 4*b^2)*c^4*x^2*log(c*sqrt(x) - 1) - 4*b^2*c^2*x + 3*(b^2
*c^4*x^2 - b^2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 - 12*a^2 - 4*
```

```
(3*a*b + (3*b^2*c^3*x + b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2
*x - 1)) - 8*(3*a*b*c^3*x + a*b*c)*sqrt(x))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2/x^3, x)
```

maple [B] time = 0.07, size = 332, normalized size = 2.50

$$-\frac{a^2}{2x^2} - \frac{b^2 \operatorname{arctanh}\left(c\sqrt{x}\right)^2}{2x^2} - \frac{c\,b^2 \operatorname{arctanh}\left(c\sqrt{x}\right)}{3x^{\frac{3}{2}}} - \frac{c^3 b^2 \operatorname{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}} - \frac{c^4 b^2 \operatorname{arctanh}\left(c\sqrt{x}\right) \ln\left(c\sqrt{x}-1\right)}{2} + \frac{c^4 b^2 \text{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2-1/2*b^2/x^2*arctanh(c*x^(1/2))^2-1/3*c*b^2*arctanh(c*x^(1/2))/
x^(3/2)-c^3*b^2*arctanh(c*x^(1/2))/x^(1/2)-1/2*c^4*b^2*arctanh(c*x^(1/2))*l
n(c*x^(1/2)-1)+1/2*c^4*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/8*c^4*b^2*l
n(c*x^(1/2)-1)^2+1/4*c^4*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))-1/8*c^4*
b^2*ln(1+c*x^(1/2))^2-1/4*c^4*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/
2))+1/4*c^4*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))-1/6*b^2*c^2/x+4/3*c^
4*b^2*ln(c*x^(1/2))-2/3*c^4*b^2*ln(c*x^(1/2)-1)-2/3*c^4*b^2*ln(1+c*x^(1/2))
-a*b/x^2*arctanh(c*x^(1/2))-1/3*c*a*b/x^(3/2)-c^3*a*b/x^(1/2)-1/2*c^4*a*b*l
n(c*x^(1/2)-1)+1/2*c^4*a*b*ln(1+c*x^(1/2))
```

maxima [B] time = 0.33, size = 234, normalized size = 1.76

$$\frac{1}{6}\left(\left(3c^3\log\left(c\sqrt{x}+1\right) - 3c^3\log\left(c\sqrt{x}-1\right) - \frac{2\left(3c^2x+1\right)}{x^{\frac{3}{2}}}\right)c - \frac{6 \operatorname{artanh}\left(c\sqrt{x}\right)}{x^2}\right)ab + \frac{1}{24}\left(\left(16c^2\log(x) - \frac{3c^2x\,\text{lo}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*((3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*(3*c^2*x + 1)
/x^(3/2))*c - 6*arctanh(c*sqrt(x))/x^2)*a*b + 1/24*((16*c^2*log(x) - (3*c^2
*x*log(c*sqrt(x) + 1)^2 + 3*c^2*x*log(c*sqrt(x) - 1)^2 + 16*c^2*x*log(c*sqr
t(x) - 1) - 2*(3*c^2*x*log(c*sqrt(x) - 1) - 8*c^2*x)*log(c*sqrt(x) + 1) + 4
)/x)*c^2 + 4*(3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*(3*c^
2*x + 1)/x^(3/2))*c*arctanh(c*sqrt(x)))*b^2 - 1/2*b^2*arctanh(c*sqrt(x))^2/
x^2 - 1/2*a^2/x^2
```

mupad [B] time = 2.71, size = 341, normalized size = 2.56

$$\frac{4\,b^2\,c^4\,\ln\left(\sqrt{x}\right)}{3} - \frac{a^2}{2\,x^2} - \frac{2\,b^2\,c^4\,\ln\left(c\,\sqrt{x}-1\right)}{3} - \frac{2\,b^2\,c^4\,\ln\left(c\,\sqrt{x}+1\right)}{3} + \frac{b^2\,c^4\ln\left(c\,\sqrt{x}+1\right)^2}{8} + \frac{b^2\,c^4\ln\left(1-c\,\sqrt{x}\right)^2}{8} - \frac{b}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^2/x^3,x)
```

```
[Out] (4*b^2*c^4*log(x^(1/2)))/3 - a^2/(2*x^2) - (2*b^2*c^4*log(c*x^(1/2) - 1))/3
 - (2*b^2*c^4*log(c*x^(1/2) + 1))/3 + (b^2*c^4*log(c*x^(1/2) + 1)^2)/8 + (b
^2*c^4*log(1 - c*x^(1/2))^2)/8 - (b^2*c^2)/(6*x) - (b^2*log(c*x^(1/2) + 1)^
2)/(8*x^2) - (b^2*log(1 - c*x^(1/2))^2)/(8*x^2) - (b^2*c^3*log(c*x^(1/2) +
1))/(2*x^(1/2)) + (b^2*c^3*log(1 - c*x^(1/2)))/(2*x^(1/2)) - (a*b*c^4*log(c
*x^(1/2) - 1))/2 + (a*b*c^4*log(c*x^(1/2) + 1))/2 - (a*b*c)/(3*x^(3/2)) - (
a*b*log(c*x^(1/2) + 1))/(2*x^2) + (a*b*log(1 - c*x^(1/2)))/(2*x^2) - (b^2*c
^4*log(c*x^(1/2) + 1)*log(1 - c*x^(1/2)))/4 - (a*b*c^3)/x^(1/2) - (b^2*c*lo
g(c*x^(1/2) + 1))/(6*x^(3/2)) + (b^2*c*log(1 - c*x^(1/2)))/(6*x^(3/2)) + (b
^2*log(c*x^(1/2) + 1)*log(1 - c*x^(1/2)))/(4*x^2)
```

<u>**sympy**</u> **[A]** time = 91.04, size = 972, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2), Eq(c, 0)), (-a**2/(2*x**2) + a*b*atanh(sqrt(x)*s
qrt(1/x))/x**2 - b**2*atanh(sqrt(x)*sqrt(1/x))**2/(2*x**2), Eq(c, -sqrt(1/x
))), (-a**2/(2*x**2) - a*b*atanh(sqrt(x)*sqrt(1/x))/x**2 - b**2*atanh(sqrt(
x)*sqrt(1/x))**2/(2*x**2), Eq(c, sqrt(1/x))), (-3*a**2*c**2*x**(3/2)/(6*c**
2*x**(7/2) - 6*x**(5/2)) + 3*a**2*sqrt(x)/(6*c**2*x**(7/2) - 6*x**(5/2)) +
6*a*b*c**6*x**(7/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) - 6*a*b
*c**5*x**3/(6*c**2*x**(7/2) - 6*x**(5/2)) - 6*a*b*c**4*x**(5/2)*atanh(c*sqr
t(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*a*b*c**3*x**2/(6*c**2*x**(7/2) - 6
*x**(5/2)) - 6*a*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(
5/2)) + 2*a*b*c*x/(6*c**2*x**(7/2) - 6*x**(5/2)) + 6*a*b*sqrt(x)*atanh(c*sq
rt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*b**2*c**6*x**(7/2)*log(x)/(6*c**2
*x**(7/2) - 6*x**(5/2)) - 8*b**2*c**6*x**(7/2)*log(sqrt(x) - 1/c)/(6*c**2*x
**(7/2) - 6*x**(5/2)) + 3*b**2*c**6*x**(7/2)*atanh(c*sqrt(x))**2/(6*c**2*x*
*(7/2) - 6*x**(5/2)) - 8*b**2*c**6*x**(7/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/
2) - 6*x**(5/2)) - 6*b**2*c**5*x**3*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x
**(5/2)) - 4*b**2*c**4*x**(5/2)*log(x)/(6*c**2*x**(7/2) - 6*x**(5/2)) + 8*b
**2*c**4*x**(5/2)*log(sqrt(x) - 1/c)/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b**
2*c**4*x**(5/2)*atanh(c*sqrt(x))**2/(6*c**2*x**(7/2) - 6*x**(5/2)) + 8*b**2
*c**4*x**(5/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) - b**2*c**4*
x**(5/2)/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*b**2*c**3*x**2*atanh(c*sqrt(x))
/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b**2*c**2*x**(3/2)*atanh(c*sqrt(x))**2/
(6*c**2*x**(7/2) - 6*x**(5/2)) + b**2*c**2*x**(3/2)/(6*c**2*x**(7/2) - 6*x*
*(5/2)) + 2*b**2*c*x*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*b*
*2*sqrt(x)*atanh(c*sqrt(x))**2/(6*c**2*x**(7/2) - 6*x**(5/2)), True))
```

3.202 $\int x^3 \left(a+b \tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx$

Optimal. Leaf size=374

$$-\frac{88b^2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{35c^8}+\frac{71b^2x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{140c^6}+\frac{9b^2x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{70c^4}+\frac{b^2x^3\left(a+\right.}{}$$

```
[Out] 23/420*b^3*x^(3/2)/c^5+1/140*b^3*x^(5/2)/c^3-47/70*b^3*arctanh(c*x^(1/2))/c
^8+71/140*b^2*x*(a+b*arctanh(c*x^(1/2)))/c^6+9/70*b^2*x^2*(a+b*arctanh(c*x^
(1/2)))/c^4+1/28*b^2*x^3*(a+b*arctanh(c*x^(1/2)))/c^2+44/35*b*(a+b*arctanh(
c*x^(1/2)))^2/c^8+1/4*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))^2/c^5+3/20*b*x^(5/
2)*(a+b*arctanh(c*x^(1/2)))^2/c^3+3/28*b*x^(7/2)*(a+b*arctanh(c*x^(1/2)))^2
/c-1/4*(a+b*arctanh(c*x^(1/2)))^3/c^8+1/4*x^4*(a+b*arctanh(c*x^(1/2)))^3-88
/35*b^2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^8-44/35*b^3*polylog(
2,1-2/(1-c*x^(1/2)))/c^8+47/70*b^3*x^(1/2)/c^7+3/4*b*(a+b*arctanh(c*x^(1/2)
))^2*x^(1/2)/c^7
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] Defer[Int][x^3*(a + b*ArcTanh[c*Sqrt[x]])^3, x]
```

Rubi steps

$$\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx=\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx$$

Mathematica [A] time = 1.27, size = 418, normalized size = 1.12

$$\frac{210a^3c^8x^4+6b\tanh^{-1}\left(c\sqrt{x}\right)\left(105a^2c^8x^4+2abc\sqrt{x}\left(15c^6x^3+21c^4x^2+35c^2x+105\right)+b^2\left(5c^6x^3+18c^4x^2+71c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (-564*a*b^2 + 630*a^2*b*c*Sqrt[x] + 564*b^3*c*Sqrt[x] + 426*a*b^2*c^2*x + 2
10*a^2*b*c^3*x^(3/2) + 46*b^3*c^3*x^(3/2) + 108*a*b^2*c^4*x^2 + 126*a^2*b*c
^5*x^(5/2) + 6*b^3*c^5*x^(5/2) + 30*a*b^2*c^6*x^3 + 90*a^2*b*c^7*x^(7/2) +
210*a^3*c^8*x^4 + 6*b^2*(b*(-176 + 105*c*Sqrt[x] + 35*c^3*x^(3/2) + 21*c^5*
x^(5/2) + 15*c^7*x^(7/2)) + 105*a*(-1 + c^8*x^4))*ArcTanh[c*Sqrt[x]]^2 + 21
0*b^3*(-1 + c^8*x^4)*ArcTanh[c*Sqrt[x]]^3 + 6*b*ArcTanh[c*Sqrt[x]]*(105*a^2
*c^8*x^4 + b^2*(-94 + 71*c^2*x + 18*c^4*x^2 + 5*c^6*x^3) + 2*a*b*c*Sqrt[x]*
(105 + 35*c^2*x + 21*c^4*x^2 + 15*c^6*x^3) - 352*b^2*Log[1 + E^(-2*ArcTanh[
c*Sqrt[x]])]) + 315*a^2*b*Log[1 - c*Sqrt[x]] - 315*a^2*b*Log[1 + c*Sqrt[x]]
 + 1056*a*b^2*Log[1 - c^2*x] + 1056*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]
])])/(840*c^8)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^3\,\text{artanh}\left(c\sqrt{x}\right)^3+3\,ab^2x^3\,\text{artanh}\left(c\sqrt{x}\right)^2+3\,a^2bx^3\,\text{artanh}\left(c\sqrt{x}\right)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*x^3*arctanh(c*sqrt(x))^2 +
3*a^2*b*x^3*arctanh(c*sqrt(x)) + a^3*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^3 x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3*x^3, x)
```

maple [C] time = 3.66, size = 1518, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(1/2)))^3,x)
```

```
[Out] -47/70*b^3*arctanh(c*x^(1/2))/c^8+47/70*b^3*x^(1/2)/c^7+3/16*I/c^8*b^3*Pi*c
sgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))^2*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*a
rctanh(c*x^(1/2))^2-3/16*I/c^8*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csg
n(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x
^(1/2))^2-3/8/c^8*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+3/4/c^8*a*b^
2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-3/4/c^8*a*b^2*arctanh(c*x^(1/2))*ln(1+
c*x^(1/2))-3/8/c^8*a*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+3/8/c^8*a*b^
2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))+3/10/c^3*a*b^2*x^(5/2)*arcta
nh(c*x^(1/2))+1/2/c^5*a*b^2*arctanh(c*x^(1/2))*x^(3/2)+3/2/c^7*a*b^2*arctan
h(c*x^(1/2))*x^(1/2)+3/14/c*a*b^2*arctanh(c*x^(1/2))*x^(7/2)-3/8*I/c^8*b^3*
Pi*arctanh(c*x^(1/2))^2+1/28*a*b^2*x^3/c^2+23/420*b^3*x^(3/2)/c^5+1/140*b^3
*x^(5/2)/c^3+9/70/c^4*a*b^2*x^2+71/140/c^6*x*a*b^2+3/20/c^3*b^3*arctanh(c*x
^(1/2))^2*x^(5/2)+1/4/c^5*b^3*arctanh(c*x^(1/2))^2*x^(3/2)+3/4/c^7*b^3*arct
anh(c*x^(1/2))^2*x^(1/2)+9/70/c^4*b^3*arctanh(c*x^(1/2))*x^2+71/140/c^6*b^3
*arctanh(c*x^(1/2))*x+3/28/c*b^3*arctanh(c*x^(1/2))^2*x^(7/2)+1/28/c^2*b^3*
arctanh(c*x^(1/2))*x^3+1/4/c^5*a^2*b*x^(3/2)+3/4/c^7*x^(1/2)*a^2*b+3/20/c^3
*x^(5/2)*a^2*b+3/28/c*x^(7/2)*a^2*b+3/16/c^8*a*b^2*ln(c*x^(1/2)-1)^2+3/16/c
^8*a*b^2*ln(1+c*x^(1/2))^2+44/35/c^8*a*b^2*ln(c*x^(1/2)-1)+44/35/c^8*a*b^2*
ln(1+c*x^(1/2))+3/8/c^8*a^2*b*ln(c*x^(1/2)-1)-3/8/c^8*a^2*b*ln(1+c*x^(1/2))
+3/8/c^8*b^3*arctanh(c*x^(1/2))^2*ln(c*x^(1/2)-1)-3/8/c^8*b^3*arctanh(c*x^(
1/2))^2*ln(1+c*x^(1/2))-88/35/c^8*b^3*arctanh(c*x^(1/2))*ln(1+I*(1+c*x^(1/2
))/(-c^2*x+1)^(1/2))-88/35/c^8*b^3*arctanh(c*x^(1/2))*ln(1-I*(1+c*x^(1/2))/
(-c^2*x+1)^(1/2))+3/4/c^8*b^3*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))/(-c^2*x
+1)^(1/2))+3/4*a*b^2*x^4*arctanh(c*x^(1/2))^2+3/4*a^2*b*x^4*arctanh(c*x^(1/
2))+1/4*x^4*a^3-11/15/c^8*b^3+3/16*I/c^8*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(
-c^2*x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1))
)^2*arctanh(c*x^(1/2))^2+3/8*I/c^8*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(
1/2))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^2*arctanh(c*x^(1/2))^2+1/4*b^3*x^4*
arctanh(c*x^(1/2))^3-88/35/c^8*b^3*dilog(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2)
)-88/35/c^8*b^3*dilog(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+44/35/c^8*b^3*arc
tanh(c*x^(1/2))^2-1/4/c^8*b^3*arctanh(c*x^(1/2))^3-3/16*I/c^8*b^3*arctanh(c
*x^(1/2))^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*(1+c*x^(1/2))^
2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)
))+3/16*I/c^8*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-
c^2*x+1)))^3*arctanh(c*x^(1/2))^2+3/16*I/c^8*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/
(c^2*x-1))^3*arctanh(c*x^(1/2))^2-3/8*I/c^8*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^
```

```
2/(-c^2*x+1)))^3*arctanh(c*x^(1/2))^2+3/8*I/c^8*b^3*Pi*csgn(I/(1+(1+c*x^(1/
2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^2
```

maxima [B] time = 1.28, size = 1972, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*x^4 - 1/26880*a*b^2*c*((315*c^7*x^4 + 500*c^5*x^3 + 1002*c^3*x^2 +
3684*c*x - 12*(105*c^7*x^4 + 120*c^6*x^(7/2) + 140*c^5*x^3 + 168*c^4*x^(5/2
) + 210*c^3*x^2 + 280*c^2*x^(3/2) + 420*c*x + 840*sqrt(x))*log(c*sqrt(x) +
1))/c^8 - 6396*log(c*sqrt(x) + 1)/c^9 - 6396*log(c*sqrt(x) - 1)/c^9) - 1/22
40*(840*x^4*log(c*sqrt(x) + 1) - c*((105*c^7*x^4 - 120*c^6*x^(7/2) + 140*c^
5*x^3 - 168*c^4*x^(5/2) + 210*c^3*x^2 - 280*c^2*x^(3/2) + 420*c*x - 840*sqr
t(x))/c^8 + 840*log(c*sqrt(x) + 1)/c^9))*a*b^2*log(-c*sqrt(x) + 1) + 1/2240
*(840*x^4*log(c*sqrt(x) + 1) - c*((105*c^7*x^4 - 120*c^6*x^(7/2) + 140*c^5*
x^3 - 168*c^4*x^(5/2) + 210*c^3*x^2 - 280*c^2*x^(3/2) + 420*c*x - 840*sqrt(
x))/c^8 + 840*log(c*sqrt(x) + 1)/c^9))*a^2*b - 1/2240*(840*x^4*log(-c*sqrt(
x) + 1) - c*((105*c^7*x^4 + 120*c^6*x^(7/2) + 140*c^5*x^3 + 168*c^4*x^(5/2)
 + 210*c^3*x^2 + 280*c^2*x^(3/2) + 420*c*x + 840*sqrt(x))/c^8 + 840*log(c*s
qrt(x) - 1)/c^9))*a^2*b + 1/1881600*(11025*(32*log(-c*sqrt(x) + 1)^2 - 8*lo
g(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^8 + 57600*(49*log(-c*sqrt(x) + 1)^2
 - 14*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^7 + 548800*(18*log(-c*sqrt(x)
 + 1)^2 - 6*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^6 + 790272*(25*log(-c*
sqrt(x) + 1)^2 - 10*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^5 + 3087000*(8
*log(-c*sqrt(x) + 1)^2 - 4*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^4 + 219
5200*(9*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^
3 + 4939200*(2*log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x
) - 1)^2 + 2822400*(log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 2)*(c*s
qrt(x) - 1))*a*b^2/c^8 - 1/3161088000*(385875*(256*log(-c*sqrt(x) + 1)^3 -
96*log(-c*sqrt(x) + 1)^2 + 24*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^8 +
2304000*(343*log(-c*sqrt(x) + 1)^3 - 147*log(-c*sqrt(x) + 1)^2 + 42*log(-c*
sqrt(x) + 1) - 6)*(c*sqrt(x) - 1)^7 + 76832000*(36*log(-c*sqrt(x) + 1)^3 -
18*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 1)*(c*sqrt(x) - 1)^6 + 4
4255232*(125*log(-c*sqrt(x) + 1)^3 - 75*log(-c*sqrt(x) + 1)^2 + 30*log(-c*s
qrt(x) + 1) - 6)*(c*sqrt(x) - 1)^5 + 216090000*(32*log(-c*sqrt(x) + 1)^3 -
24*log(-c*sqrt(x) + 1)^2 + 12*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^4 +
614656000*(9*log(-c*sqrt(x) + 1)^3 - 9*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqr
t(x) + 1) - 2)*(c*sqrt(x) - 1)^3 + 691488000*(4*log(-c*sqrt(x) + 1)^3 - 6*l
og(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^2 + 79027
2000*(log(-c*sqrt(x) + 1)^3 - 3*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) +
1) - 6)*(c*sqrt(x) - 1))*b^3/c^8 + 44/35*(log(c*sqrt(x) + 1)*log(-1/2*c*sqr
t(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2))*b^3/c^8 - 1881559/3763200*b^3*log
(c*sqrt(x) - 1)/c^8 + 1/2240*(2283*a*b^2 - 752*b^3)*log(c*sqrt(x) + 1)/c^8
+ 1/3161088000*(1157625*(16*a*b^2*c^8 - b^3*c^8)*x^4 - 27000*(1680*a*b^2*c^
7 + 169*b^3*c^7)*x^(7/2) + 3500*(24528*a*b^2*c^6 - 3565*b^3*c^6)*x^3 + 9878
4000*(b^3*c^8*x^4 - b^3)*log(c*sqrt(x) + 1)^3 - 168*(895440*a*b^2*c^5 + 442
69*b^3*c^5)*x^(5/2) + 210*(1248240*a*b^2*c^4 - 334699*b^3*c^4)*x^2 + 564480
0*(105*a*b^2*c^8*x^4 + 15*b^3*c^7*x^(7/2) + 21*b^3*c^5*x^(5/2) + 35*b^3*c^3
*x^(3/2) + 105*b^3*c*sqrt(x) - 105*a*b^2 + 176*b^3)*log(c*sqrt(x) + 1)^2 -
352800*(105*b^3*c^8*x^4 - 120*b^3*c^7*x^(7/2) + 140*b^3*c^6*x^3 - 168*b^3*c
^5*x^(5/2) + 210*b^3*c^4*x^2 - 280*b^3*c^3*x^(3/2) + 420*b^3*c^2*x - 840*b^
3*c*sqrt(x) + 533*b^3 - 840*(b^3*c^8*x^4 - b^3)*log(c*sqrt(x) + 1))*log(-c*
sqrt(x) + 1)^2 - 280*(1718640*a*b^2*c^3 + 2899*b^3*c^3)*x^(3/2) + 420*(2424
240*a*b^2*c^2 - 1227199*b^3*c^2)*x - 1411200*(105*a*b^2*c^8*x^4 - 120*a*b^2
*c^7*x^(7/2) - 168*a*b^2*c^5*x^(5/2) - 280*a*b^2*c^3*x^(3/2) - 840*a*b^2*c*
sqrt(x) + 20*(7*a*b^2*c^6 - 2*b^3*c^6)*x^3 + 6*(35*a*b^2*c^4 - 24*b^3*c^4)*
x^2 + 4*(105*a*b^2*c^2 - 142*b^3*c^2)*x)*log(c*sqrt(x) + 1) + 840*(11025*b^
```

```
3*c^8*x^4 + 27000*b^3*c^7*x^(7/2) - 16100*b^3*c^6*x^3 + 89544*b^3*c^5*x^(5/
2) - 85890*b^3*c^4*x^2 + 286440*b^3*c^3*x^(3/2) - 348180*b^3*c^2*x + 191772
0*b^3*c*sqrt(x) - 352800*(b^3*c^8*x^4 - b^3)*log(c*sqrt(x) + 1)^2 - 13440*(
15*b^3*c^7*x^(7/2) + 21*b^3*c^5*x^(5/2) + 35*b^3*c^3*x^(3/2) + 105*b^3*c*sq
rt(x) + 176*b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1) - 840*(3835440*a*b
^2*c + 618199*b^3*c)*sqrt(x))/c^8
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b \operatorname{atanh}\left(c \sqrt{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^(1/2)))^3,x)
```

```
[Out] int(x^3*(a + b*atanh(c*x^(1/2)))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b \operatorname{atanh}\left(c\sqrt{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(1/2)))**3,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*sqrt(x)))**3, x)
```

3.203 $\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx$

Optimal. Leaf size=304

$$-\frac{46b^2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{15c^6}+\frac{8b^2x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{15c^4}+\frac{b^2x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{10c^2}+\frac{23b\left(a+b\tanh\right.}{1}$$

```
[Out] 1/30*b^3*x^(3/2)/c^3-19/30*b^3*arctanh(c*x^(1/2))/c^6+8/15*b^2*x*(a+b*arcta
nh(c*x^(1/2)))/c^4+1/10*b^2*x^2*(a+b*arctanh(c*x^(1/2)))/c^2+23/15*b*(a+b*a
rctanh(c*x^(1/2)))^2/c^6+1/3*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))^2/c^3+1/5*b
*x^(5/2)*(a+b*arctanh(c*x^(1/2)))^2/c-1/3*(a+b*arctanh(c*x^(1/2)))^3/c^6+1/
3*x^3*(a+b*arctanh(c*x^(1/2)))^3-46/15*b^2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1
-c*x^(1/2)))/c^6-23/15*b^3*polylog(2,1-2/(1-c*x^(1/2)))/c^6+19/30*b^3*x^(1/
2)/c^5+b*(a+b*arctanh(c*x^(1/2)))^2*x^(1/2)/c^5
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] Defer[Int][x^2*(a + b*ArcTanh[c*Sqrt[x]])^3, x]
```

Rubi steps

$$\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx=\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx$$

Mathematica [A] time = 0.86, size = 351, normalized size = 1.15

$$\frac{10a^3c^6x^3+b\tanh^{-1}\left(c\sqrt{x}\right)\left(30a^2c^6x^3+4abc\sqrt{x}\left(3c^4x^2+5c^2x+15\right)+b^2\left(3c^4x^2+16c^2x-19\right)-92b^2\log\left(e^{-2\tanh}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (-19*a*b^2 + 30*a^2*b*c*Sqrt[x] + 19*b^3*c*Sqrt[x] + 16*a*b^2*c^2*x + 10*a^
2*b*c^3*x^(3/2) + b^3*c^3*x^(3/2) + 3*a*b^2*c^4*x^2 + 6*a^2*b*c^5*x^(5/2) +
 10*a^3*c^6*x^3 + 2*b^2*(b*(-23 + 15*c*Sqrt[x] + 5*c^3*x^(3/2) + 3*c^5*x^(5
/2)) + 15*a*(-1 + c^6*x^3))*ArcTanh[c*Sqrt[x]]^2 + 10*b^3*(-1 + c^6*x^3)*Ar
cTanh[c*Sqrt[x]]^3 + b*ArcTanh[c*Sqrt[x]]*(30*a^2*c^6*x^3 + 4*a*b*c*Sqrt[x]
*(15 + 5*c^2*x + 3*c^4*x^2) + b^2*(-19 + 16*c^2*x + 3*c^4*x^2) - 92*b^2*Log
[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + 15*a^2*b*Log[1 - c*Sqrt[x]] - 15*a^2*b*L
og[1 + c*Sqrt[x]] + 46*a*b^2*Log[1 - c^2*x] + 46*b^3*PolyLog[2, -E^(-2*ArcT
anh[c*Sqrt[x]])])/(30*c^6)
```

fricas [F] time = 1.72, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^3x^2\operatorname{artanh}\left(c\sqrt{x}\right)^3+3\,ab^2x^2\operatorname{artanh}\left(c\sqrt{x}\right)^2+3\,a^2bx^2\operatorname{artanh}\left(c\sqrt{x}\right)+a^3x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctanh(c*sqrt(x))^3 + 3*a*b^2*x^2*arctanh(c*sqrt(x))^2 +
3*a^2*b*x^2*arctanh(c*sqrt(x)) + a^3*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^3 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3*x^2, x)
```

maple [C] time = 0.58, size = 1423, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(1/2)))^3,x)
```

```
[Out] 1/3*x^3*a^3+8/15*a*b^2*x/c^4+1/10/c^2*b^3*arctanh(c*x^(1/2))*x^2+8/15/c^4*b
^3*arctanh(c*x^(1/2))*x+1/5/c*b^3*arctanh(c*x^(1/2))^2*x^(5/2)+1/3/c^3*a^2*
b*x^(3/2)+1/c^5*x^(1/2)*a^2*b+1/5/c*x^(5/2)*a^2*b+23/15/c^6*a*b^2*ln(c*x^(1
/2)-1)+23/15/c^6*a*b^2*ln(1+c*x^(1/2))+1/2/c^6*a^2*b*ln(c*x^(1/2)-1)-1/2/c^
6*a^2*b*ln(1+c*x^(1/2))+1/2/c^6*b^3*arctanh(c*x^(1/2))^2*ln(c*x^(1/2)-1)-1/
2/c^6*b^3*arctanh(c*x^(1/2))^2*ln(1+c*x^(1/2))-46/15/c^6*b^3*arctanh(c*x^(1
/2))*ln(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-46/15/c^6*b^3*arctanh(c*x^(1/2)
)*ln(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+1/c^6*b^3*arctanh(c*x^(1/2))^2*ln(
(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+a^2*b*x^3*arctanh(c*x^(1/2))+a*b^2*x^3*arct
anh(c*x^(1/2))^2+1/10/c^2*a*b^2*x^2+1/4/c^6*a*b^2*ln(c*x^(1/2)-1)^2+1/4/c^6
*a*b^2*ln(1+c*x^(1/2))^2+1/3/c^3*b^3*arctanh(c*x^(1/2))^2*x^(3/2)+1/c^5*b^3
*arctanh(c*x^(1/2))^2*x^(1/2)+1/4*I/c^6*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x
+1)^(1/2))^2*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*arctanh(c*x^(1/2))^2+1/2*I/c
^6*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))*csgn(I*(1+c*x^(1/2))^2/(c^
2*x-1))^2*arctanh(c*x^(1/2))^2+1/4*I/c^6*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(
-c^2*x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1))
)^2*arctanh(c*x^(1/2))^2-1/4*I/c^6*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))
*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh
(c*x^(1/2))^2-19/30*b^3*arctanh(c*x^(1/2))/c^6+19/30*b^3*x^(1/2)/c^5+1/2*I/
c^6*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^2-1/
4*I/c^6*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*(1+c*x^(1/2))^
2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)
))*arctanh(c*x^(1/2))^2+1/30*b^3*x^(3/2)/c^3+1/3*b^3*x^3*arctanh(c*x^(1/2))
^3-46/15/c^6*b^3*dilog(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-46/15/c^6*b^3*di
log(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+23/15/c^6*b^3*arctanh(c*x^(1/2))^2-
1/3/c^6*b^3*arctanh(c*x^(1/2))^3+1/4*I/c^6*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c
^2*x-1))^3*arctanh(c*x^(1/2))^2+1/4*I/c^6*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^
2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*arctanh(c*x^(1/2))^2-1/2*I/c^6*b^3
*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*arctanh(c*x^(1/2))^2+1/c^6*a*b
^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/c^6*a*b^2*arctanh(c*x^(1/2))*ln(1+c
*x^(1/2))-1/2/c^6*a*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+1/2/c^6*a*b^2
*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))-1/2/c^6*a*b^2*ln(-1/2*c*x^(1/
2)+1/2)*ln(1+c*x^(1/2))+2/5/c*a*b^2*x^(5/2)*arctanh(c*x^(1/2))+2/3/c^3*a*b^
2*arctanh(c*x^(1/2))*x^(3/2)+2/c^5*a*b^2*arctanh(c*x^(1/2))*x^(1/2)-1/2*I/c
^6*b^3*Pi*arctanh(c*x^(1/2))^2-2/3/c^6*b^3
```

maxima [B] time = 1.14, size = 1579, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 - 1/720*a*b^2*c*((20*c^5*x^3 + 39*c^3*x^2 + 138*c*x - 6*(10*c^5
*x^3 + 12*c^4*x^(5/2) + 15*c^3*x^2 + 20*c^2*x^(3/2) + 30*c*x + 60*sqrt(x))*
log(c*sqrt(x) + 1))/c^6 - 222*log(c*sqrt(x) + 1)/c^7 - 222*log(c*sqrt(x) -
1)/c^7) - 1/120*(60*x^3*log(c*sqrt(x) + 1) - c*((10*c^5*x^3 - 12*c^4*x^(5/2
) + 15*c^3*x^2 - 20*c^2*x^(3/2) + 30*c*x - 60*sqrt(x))/c^6 + 60*log(c*sqrt(
x) + 1)/c^7))*a*b^2*log(-c*sqrt(x) + 1) + 1/120*(60*x^3*log(c*sqrt(x) + 1)
- c*((10*c^5*x^3 - 12*c^4*x^(5/2) + 15*c^3*x^2 - 20*c^2*x^(3/2) + 30*c*x -
60*sqrt(x))/c^6 + 60*log(c*sqrt(x) + 1)/c^7))*a^2*b - 1/120*(60*x^3*log(-c*
sqrt(x) + 1) - c*((10*c^5*x^3 + 12*c^4*x^(5/2) + 15*c^3*x^2 + 20*c^2*x^(3/2
) + 30*c*x + 60*sqrt(x))/c^6 + 60*log(c*sqrt(x) - 1)/c^7))*a^2*b + 1/7200*(
100*(18*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^
6 + 432*(25*log(-c*sqrt(x) + 1)^2 - 10*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x)
- 1)^5 + 3375*(8*log(-c*sqrt(x) + 1)^2 - 4*log(-c*sqrt(x) + 1) + 1)*(c*sqrt
(x) - 1)^4 + 4000*(9*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 2)*(c*
sqrt(x) - 1)^3 + 13500*(2*log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 1
)*(c*sqrt(x) - 1)^2 + 10800*(log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1)
+ 2)*(c*sqrt(x) - 1))*a*b^2/c^6 - 1/864000*(1000*(36*log(-c*sqrt(x) + 1)^3
- 18*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 1)*(c*sqrt(x) - 1)^6 +
 1728*(125*log(-c*sqrt(x) + 1)^3 - 75*log(-c*sqrt(x) + 1)^2 + 30*log(-c*sqr
t(x) + 1) - 6)*(c*sqrt(x) - 1)^5 + 16875*(32*log(-c*sqrt(x) + 1)^3 - 24*log
(-c*sqrt(x) + 1)^2 + 12*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^4 + 80000*
(9*log(-c*sqrt(x) + 1)^3 - 9*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1)
- 2)*(c*sqrt(x) - 1)^3 + 135000*(4*log(-c*sqrt(x) + 1)^3 - 6*log(-c*sqrt(x)
 + 1)^2 + 6*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^2 + 216000*(log(-c*sqr
t(x) + 1)^3 - 3*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(
x) - 1))*b^3/c^6 + 23/15*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + di
log(1/2*c*sqrt(x) + 1/2))*b^3/c^6 - 8929/14400*b^3*log(c*sqrt(x) - 1)/c^6 +
 1/120*(147*a*b^2 - 38*b^3)*log(c*sqrt(x) + 1)/c^6 + 1/864000*(1000*(12*a*b
^2*c^6 - b^3*c^6)*x^3 + 36000*(b^3*c^6*x^3 - b^3)*log(c*sqrt(x) + 1)^3 - 48
*(660*a*b^2*c^5 + 91*b^3*c^5)*x^(5/2) + 15*(4440*a*b^2*c^4 - 919*b^3*c^4)*x
^2 + 14400*(15*a*b^2*c^6*x^3 + 3*b^3*c^5*x^(5/2) + 5*b^3*c^3*x^(3/2) + 15*b
^3*c*sqrt(x) - 15*a*b^2 + 23*b^3)*log(c*sqrt(x) + 1)^2 - 1800*(10*b^3*c^6*x
^3 - 12*b^3*c^5*x^(5/2) + 15*b^3*c^4*x^2 - 20*b^3*c^3*x^(3/2) + 30*b^3*c^2*
x - 60*b^3*c*sqrt(x) + 37*b^3 - 60*(b^3*c^6*x^3 - b^3)*log(c*sqrt(x) + 1))*
log(-c*sqrt(x) + 1)^2 - 20*(6840*a*b^2*c^3 + 619*b^3*c^3)*x^(3/2) + 870*(36
0*a*b^2*c^2 - 161*b^3*c^2)*x - 7200*(10*a*b^2*c^6*x^3 - 12*a*b^2*c^5*x^(5/2
) - 20*a*b^2*c^3*x^(3/2) - 60*a*b^2*c*sqrt(x) + 3*(5*a*b^2*c^4 - 2*b^3*c^4)
*x^2 + 2*(15*a*b^2*c^2 - 16*b^3*c^2)*x)*log(c*sqrt(x) + 1) + 60*(100*b^3*c^
6*x^3 + 264*b^3*c^5*x^(5/2) - 165*b^3*c^4*x^2 + 1140*b^3*c^3*x^(3/2) - 1230
*b^3*c^2*x + 8820*b^3*c*sqrt(x) - 1800*(b^3*c^6*x^3 - b^3)*log(c*sqrt(x) +
1)^2 - 480*(3*b^3*c^5*x^(5/2) + 5*b^3*c^3*x^(3/2) + 15*b^3*c*sqrt(x) + 23*b
^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1) - 60*(17640*a*b^2*c + 4369*b^3*
c)*sqrt(x))/c^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \operatorname{atanh}\left(c \sqrt{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^(1/2)))^3,x)
```

```
[Out] int(x^2*(a + b*atanh(c*x^(1/2)))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{atanh}\left(c\sqrt{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(1/2)))**3,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*sqrt(x)))**3, x)
```

3.204 $\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$

Optimal. Leaf size=234

$$-\frac{4b^2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^4}+\frac{b^2x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^2}+\frac{2b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c^4}-\frac{\left(a+b\tanh^{-1}\left(c\right.\right.}{2c^4}$$

```
[Out] -1/2*b^3*arctanh(c*x^(1/2))/c^4+1/2*b^2*x*(a+b*arctanh(c*x^(1/2)))/c^2+2*b*
(a+b*arctanh(c*x^(1/2)))^2/c^4+1/2*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))^2/c-1
/2*(a+b*arctanh(c*x^(1/2)))^3/c^4+1/2*x^2*(a+b*arctanh(c*x^(1/2)))^3-4*b^2*
(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^4-2*b^3*polylog(2,1-2/(1-c*x
^(1/2)))/c^4+1/2*b^3*x^(1/2)/c^3+3/2*b*(a+b*arctanh(c*x^(1/2)))^2*x^(1/2)/c
^3
```

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] Defer[Int][x*(a + b*ArcTanh[c*Sqrt[x]])^3, x]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx=\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$$

Mathematica [A] time = 0.56, size = 285, normalized size = 1.22

$$\frac{2a^3c^4x^2+2b\tanh^{-1}\left(c\sqrt{x}\right)\left(3a^2c^4x^2+2abc\sqrt{x}\left(c^2x+3\right)+b^2\left(c^2x-1\right)-8b^2\log\left(e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}+1\right)\right)+2a^2bc^3}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (-2*a*b^2 + 6*a^2*b*c*Sqrt[x] + 2*b^3*c*Sqrt[x] + 2*a*b^2*c^2*x + 2*a^2*b*c
^3*x^(3/2) + 2*a^3*c^4*x^2 + 2*b^2*(b*(-4 + 3*c*Sqrt[x] + c^3*x^(3/2)) + 3*
a*(-1 + c^4*x^2))*ArcTanh[c*Sqrt[x]]^2 + 2*b^3*(-1 + c^4*x^2)*ArcTanh[c*Sqr
t[x]]^3 + 2*b*ArcTanh[c*Sqrt[x]]*(3*a^2*c^4*x^2 + b^2*(-1 + c^2*x) + 2*a*b*
c*Sqrt[x]*(3 + c^2*x) - 8*b^2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + 3*a^2*b
*Log[1 - c*Sqrt[x]] - 3*a^2*b*Log[1 + c*Sqrt[x]] + 8*a*b^2*Log[1 - c^2*x] +
 8*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])])/(4*c^4)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^3x\,\mathrm{artanh}\left(c\sqrt{x}\right)^3+3\,ab^2x\,\mathrm{artanh}\left(c\sqrt{x}\right)^2+3\,a^2bx\,\mathrm{artanh}\left(c\sqrt{x}\right)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctanh(c*sqrt(x))^3 + 3*a*b^2*x*arctanh(c*sqrt(x))^2 + 3*a^
2*b*x*arctanh(c*sqrt(x)) + a^3*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^3 x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3*x, x)
```

maple [C] time = 0.42, size = 1339, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(1/2)))^3,x)
```

```
[Out] 1/2*a*b^2*x/c^2-1/2/c^4*b^3+2/c^4*a*b^2*ln(1+c*x^(1/2))+3/4/c^4*a^2*b*ln(c*
x^(1/2)-1)-3/4/c^4*a^2*b*ln(1+c*x^(1/2))+3/4/c^4*b^3*arctanh(c*x^(1/2))^2*l
n(c*x^(1/2)-1)-3/4/c^4*b^3*arctanh(c*x^(1/2))^2*ln(1+c*x^(1/2))-4/c^4*b^3*a
rctanh(c*x^(1/2))*ln(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4/c^4*b^3*arctanh(
c*x^(1/2))*ln(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/2/c^4*b^3*arctanh(c*x^(
1/2))^2*ln((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+1/2/c*b^3*arctanh(c*x^(1/2))^2*x
^(3/2)+3/2/c^3*b^3*arctanh(c*x^(1/2))^2*x^(1/2)+1/2/c^2*b^3*arctanh(c*x^(1/
2))*x+1/2/c*a^2*b*x^(3/2)+3/2/c^3*x^(1/2)*a^2*b+3/2*a*b^2*x^2*arctanh(c*x^(
1/2))^2+3/2*a^2*b*x^2*arctanh(c*x^(1/2))+3/8/c^4*a*b^2*ln(c*x^(1/2)-1)^2+3/
8/c^4*a*b^2*ln(1+c*x^(1/2))^2+2/c^4*a*b^2*ln(c*x^(1/2)-1)-1/2*b^3*arctanh(c
*x^(1/2))/c^4+1/2*b^3*x^(1/2)/c^3+1/c*a*b^2*arctanh(c*x^(1/2))*x^(3/2)+3/c^
3*a*b^2*arctanh(c*x^(1/2))*x^(1/2)-3/4*I/c^4*b^3*Pi*arctanh(c*x^(1/2))^2-3/
4/c^4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+3/2/c^4*a*b^2*arctanh(c*
x^(1/2))*ln(c*x^(1/2)-1)-3/2/c^4*a*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-3
/4/c^4*a*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+3/4/c^4*a*b^2*ln(-1/2*c*
x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))+3/8*I/c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/
(c^2*x-1))^3*arctanh(c*x^(1/2))^2+1/2*b^3*x^2*arctanh(c*x^(1/2))^3-4/c^4*b^
3*dilog(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4/c^4*b^3*dilog(1-I*(1+c*x^(1/2
))/(-c^2*x+1)^(1/2))+2/c^4*b^3*arctanh(c*x^(1/2))^2-1/2/c^4*b^3*arctanh(c*x
^(1/2))^3-3/8*I/c^4*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*(1
+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^
2/(-c^2*x+1)))*arctanh(c*x^(1/2))^2+1/2*x^2*a^3-3/4*I/c^4*b^3*Pi*csgn(I/(1+
(1+c*x^(1/2))^2/(-c^2*x+1)))^3*arctanh(c*x^(1/2))^2+3/8*I/c^4*b^3*Pi*csgn(I
*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*arctanh(c*x^(1
/2))^2+3/4*I/c^4*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*
x^(1/2))^2+3/8*I/c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))^2*csgn(I
*(1+c*x^(1/2))^2/(c^2*x-1))*arctanh(c*x^(1/2))^2+3/4*I/c^4*b^3*Pi*csgn(I*(1
+c*x^(1/2))/(-c^2*x+1)^(1/2))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^2*arctanh(c
*x^(1/2))^2-3/8*I/c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*
x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^2
+3/8*I/c^4*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*csgn(I*(1+c*x^(1/2
))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^2
```

maxima [B] time = 1.04, size = 1184, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x^2 - 1/32*a*b^2*c*((3*c^3*x^2 + 10*c*x - 2*(3*c^3*x^2 + 4*c^2*x^(3
/2) + 6*c*x + 12*sqrt(x))*log(c*sqrt(x) + 1))/c^4 - 14*log(c*sqrt(x) + 1)/c
```

```
^5 - 14*log(c*sqrt(x) - 1)/c^5) - 1/16*(12*x^2*log(c*sqrt(x) + 1) - c*((3*c
^3*x^2 - 4*c^2*x^(3/2) + 6*c*x - 12*sqrt(x))/c^4 + 12*log(c*sqrt(x) + 1)/c^
5))*a*b^2*log(-c*sqrt(x) + 1) + 1/16*(12*x^2*log(c*sqrt(x) + 1) - c*((3*c^3
*x^2 - 4*c^2*x^(3/2) + 6*c*x - 12*sqrt(x))/c^4 + 12*log(c*sqrt(x) + 1)/c^5)
)*a^2*b - 1/16*(12*x^2*log(-c*sqrt(x) + 1) - c*((3*c^3*x^2 + 4*c^2*x^(3/2)
+ 6*c*x + 12*sqrt(x))/c^4 + 12*log(c*sqrt(x) - 1)/c^5))*a^2*b + 1/192*(9*(8
*log(-c*sqrt(x) + 1)^2 - 4*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^4 + 32*
(9*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^3 + 2
16*(2*log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^2
+ 288*(log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1))*
a*b^2/c^4 - 1/4608*(9*(32*log(-c*sqrt(x) + 1)^3 - 24*log(-c*sqrt(x) + 1)^2
+ 12*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^4 + 128*(9*log(-c*sqrt(x) + 1
)^3 - 9*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 2)*(c*sqrt(x) - 1)^
3 + 432*(4*log(-c*sqrt(x) + 1)^3 - 6*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(
x) + 1) - 3)*(c*sqrt(x) - 1)^2 + 1152*(log(-c*sqrt(x) + 1)^3 - 3*log(-c*sqr
t(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(x) - 1))*b^3/c^4 + 2*(log(
c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2))*b^3/
c^4 - 319/384*b^3*log(c*sqrt(x) - 1)/c^4 + 1/16*(25*a*b^2 - 4*b^3)*log(c*sq
rt(x) + 1)/c^4 + 1/4608*(288*(b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1)^3 + 27*
(8*a*b^2*c^4 - b^3*c^4)*x^2 + 576*(3*a*b^2*c^4*x^2 + b^3*c^3*x^(3/2) + 3*b^
3*c*sqrt(x) - 3*a*b^2 + 4*b^3)*log(c*sqrt(x) + 1)^2 - 72*(3*b^3*c^4*x^2 - 4
*b^3*c^3*x^(3/2) + 6*b^3*c^2*x - 12*b^3*c*sqrt(x) + 7*b^3 - 12*(b^3*c^4*x^2
 - b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1)^2 - 4*(168*a*b^2*c^3 + 37*b
^3*c^3)*x^(3/2) + 6*(312*a*b^2*c^2 - 115*b^3*c^2)*x - 288*(3*a*b^2*c^4*x^2
- 4*a*b^2*c^3*x^(3/2) - 12*a*b^2*c*sqrt(x) + 2*(3*a*b^2*c^2 - 2*b^3*c^2)*x)
*log(c*sqrt(x) + 1) + 12*(9*b^3*c^4*x^2 + 28*b^3*c^3*x^(3/2) - 18*b^3*c^2*x
 + 300*b^3*c*sqrt(x) - 72*(b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1)^2 - 96*(b^
3*c^3*x^(3/2) + 3*b^3*c*sqrt(x) + 4*b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x)
 + 1) - 12*(600*a*b^2*c + 223*b^3*c)*sqrt(x))/c^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^(1/2)))^3,x)
```

```
[Out] int(x*(a + b*atanh(c*x^(1/2)))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**(1/2)))**3,x)
```

```
[Out] Integral(x*(a + b*atanh(c*sqrt(x)))**3, x)
```

3.205 $\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx$

Optimal. Leaf size=142

$$-\frac{6b^2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^2}+\frac{3b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c^2}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{c^2}+\frac{3b\sqrt{x}\left(a+b\tan\right.}{c}$$

[Out] 3*b*(a+b*arctanh(c*x^(1/2)))^2/c^2-(a+b*arctanh(c*x^(1/2)))^3/c^2+x*(a+b*arctanh(c*x^(1/2)))^3-6*b^2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^2-3*b^3*polylog(2,1-2/(1-c*x^(1/2)))/c^2+3*b*(a+b*arctanh(c*x^(1/2)))^2*x^(1/2)/c

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3,x]

[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^3, x]

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx=\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx$$

Mathematica [A] time = 0.29, size = 201, normalized size = 1.42

$$\frac{a\left(2a^2c^2x+6abc\sqrt{x}+3ab\log\left(1-c\sqrt{x}\right)-3ab\log\left(c\sqrt{x}+1\right)+6b^2\log\left(1-c^2x\right)\right)+6b\tanh^{-1}\left(c\sqrt{x}\right)\left(a^2c^2x\right.}{}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3,x]

[Out] (6*b^2*(-1 + c*Sqrt[x])*(a + b + a*c*Sqrt[x])*ArcTanh[c*Sqrt[x]]^2 + 2*b^3*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^3 + 6*b*ArcTanh[c*Sqrt[x]]*(2*a*b*c*Sqrt[x] + a^2*c^2*x - 2*b^2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + a*(6*a*b*c*Sqrt[x] + 2*a^2*c^2*x + 3*a*b*Log[1 - c*Sqrt[x]] - 3*a*b*Log[1 + c*Sqrt[x]] + 6*b^2*Log[1 - c^2*x]) + 6*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])])/(2*c^2)

fricas [F] time = 1.35, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3\,\text{artanh}\left(c\sqrt{x}\right)^3+3\,ab^2\,\text{artanh}\left(c\sqrt{x}\right)^2+3\,a^2b\,\text{artanh}\left(c\sqrt{x}\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")

[Out] integral(b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b*arctanh(c*sqrt(x)) + a^3, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3, x)
```

maple [C] time = 0.32, size = 6235, normalized size = 43.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2}\left(c\left(\frac{2\sqrt{x}}{c^2}-\frac{\log\left(c\sqrt{x}+1\right)}{c^3}+\frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right)+2x\,\text{artanh}\left(c\sqrt{x}\right)\right)a^2b+\frac{3}{4}\left(4c\left(\frac{2\sqrt{x}}{c^2}-\frac{\log\left(c\sqrt{x}+1\right)}{c^3}+\frac{\log\left(c\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 3/2*(c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) +
2*x*arctanh(c*sqrt(x)))*a^2*b + 3/4*(4*c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1
)/c^3 + log(c*sqrt(x) - 1)/c^3)*arctanh(c*sqrt(x)) + 4*x*arctanh(c*sqrt(x))
^2 - (2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log(c*sqrt(x) + 1)^2
- log(c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1))/c^2)*a*b^2 + a^3*x - 1/32*b^
3*(((4*log(-c*sqrt(x) + 1)^3 - 6*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) +
 1) - 3)*(c*sqrt(x) - 1)^2 + 8*(log(-c*sqrt(x) + 1)^3 - 3*log(-c*sqrt(x) +
1)^2 + 6*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(x) - 1))/c^2 - 4*integrate(log(c*
sqrt(x) + 1)^3 - 3*log(c*sqrt(x) + 1)^2*log(-c*sqrt(x) + 1) + 3*log(c*sqrt(
x) + 1)*log(-c*sqrt(x) + 1)^2, x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\text{atanh}\left(c\,\sqrt{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^3,x)
```

```
[Out] int((a + b*atanh(c*x^(1/2)))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**3,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**3, x)
```

3.206 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x}\,dx$

Optimal. Leaf size=224

$$3b^2\text{Li}_3\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-3b^2\text{Li}_3\left(\frac{2}{1-c\sqrt{x}}-1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-3b\text{Li}_2\left(1-\frac{2}{1-c\sqrt{x}}\right.$$

```
[Out] -4*(a+b*arctanh(c*x^(1/2)))^3*arctanh(-1+2/(1-c*x^(1/2)))-3*b*(a+b*arctanh(
c*x^(1/2)))^2*polylog(2,1-2/(1-c*x^(1/2)))+3*b*(a+b*arctanh(c*x^(1/2)))^2*p
olylog(2,-1+2/(1-c*x^(1/2)))+3*b^2*(a+b*arctanh(c*x^(1/2)))*polylog(3,1-2/(
1-c*x^(1/2)))-3*b^2*(a+b*arctanh(c*x^(1/2)))*polylog(3,-1+2/(1-c*x^(1/2)))-
3/2*b^3*polylog(4,1-2/(1-c*x^(1/2)))+3/2*b^3*polylog(4,-1+2/(1-c*x^(1/2)))
```

Rubi [A] time = 0.51, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6095, 5914, 6052, 5948, 6058, 6062, 6610}

$$3b^2\text{PolyLog}\left(3,1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-3b^2\text{PolyLog}\left(3,\frac{2}{1-c\sqrt{x}}-1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-3b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x,x]
```

```
[Out] 4*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^3 - 3*b*(a + b*
ArcTanh[c*Sqrt[x]])^2*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])] + 3*b*(a + b*ArcTan
h[c*Sqrt[x]])^2*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])] + 3*b^2*(a + b*ArcTanh[c
*Sqrt[x]])*PolyLog[3, 1 - 2/(1 - c*Sqrt[x])] - 3*b^2*(a + b*ArcTanh[c*Sqrt[
x]])*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])] - (3*b^3*PolyLog[4, 1 - 2/(1 - c*Sq
rt[x])])/2 + (3*b^3*PolyLog[4, -1 + 2/(1 - c*Sqrt[x])])/2
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x}\,dx = 2\,\mathrm{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx, x, \sqrt{x}\right)$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-(12bc)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tanh^{-1}\right.}{}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3+(6bc)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(\right.}{1-}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-3b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\mathrm{Li}_2\left(1\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-3b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\mathrm{Li}_2\left(1\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-3b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\mathrm{Li}_2\left(1\right.$$

Mathematica [A] time = 0.21, size = 248, normalized size = 1.11

$$\frac{3}{2}b\left(2\mathrm{Li}_2\left(\frac{\sqrt{x}c+1}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-2\mathrm{Li}_2\left(\frac{\sqrt{x}c+1}{c\sqrt{x}-1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+b\left(-2\mathrm{Li}_3\left(\frac{\sqrt{x}c+1}{1-c\sqrt{x}}\right)\left(a+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x,x]
```

```
[Out] 4*ArcTanh[1 + 2/(-1 + c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^3 + (3*b*(2*(a
 + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] - 2*
(a + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, (1 + c*Sqrt[x])/(-1 + c*Sqrt[x])] +
 b*(-2*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, (1 + c*Sqrt[x])/(1 - c*Sqrt[x]
)] + 2*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, (1 + c*Sqrt[x])/(-1 + c*Sqrt[x
```

```
])] + b*(PolyLog[4, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] - PolyLog[4, (1 + c*Sq
rt[x])/(-1 + c*Sqrt[x])]))))/2
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}\left(c\sqrt{x}\right)^3 + 3\,ab^2 \operatorname{artanh}\left(c\sqrt{x}\right)^2 + 3\,a^2 b \operatorname{artanh}\left(c\sqrt{x}\right) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b
*arctanh(c*sqrt(x)) + a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3/x, x)
```

maple [C] time = 0.18, size = 1542, normalized size = 6.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3/x,x)
```

```
[Out] I*b^3*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1
)))^3*arctanh(c*x^(1/2))^3+6*a*b^2*ln(c*x^(1/2))*arctanh(c*x^(1/2))^2-6*a*b
^2*arctanh(c*x^(1/2))*polylog(2,-(1+c*x^(1/2))^2/(-c^2*x+1))-6*a*b^2*arctan
h(c*x^(1/2))^2*ln((1+c*x^(1/2))^2/(-c^2*x+1)-1)+6*a^2*b*ln(c*x^(1/2))*arcta
nh(c*x^(1/2))-3*a^2*b*ln(c*x^(1/2))*ln(1+c*x^(1/2))+6*a*b^2*arctanh(c*x^(1/
2))^2*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+12*a*b^2*arctanh(c*x^(1/2))*poly
log(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+6*a*b^2*arctanh(c*x^(1/2))^2*ln(1+(1+
c*x^(1/2))/(-c^2*x+1)^(1/2))+12*a*b^2*arctanh(c*x^(1/2))*polylog(2,-(1+c*x^
(1/2))/(-c^2*x+1)^(1/2))-3/2*b^3*polylog(4,-(1+c*x^(1/2))^2/(-c^2*x+1))+12*
b^3*polylog(4,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+12*b^3*polylog(4,-(1+c*x^(1/2
))/(-c^2*x+1)^(1/2))+2*a^3*ln(c*x^(1/2))-I*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2
/(-c^2*x+1)))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^
2*x+1)))^2*arctanh(c*x^(1/2))^3-I*b^3*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)
-1))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^
2*arctanh(c*x^(1/2))^3+3*I*a*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(
1+(1+c*x^(1/2))^2/(-c^2*x+1)))^3*arctanh(c*x^(1/2))^2+3*a*b^2*polylog(3,-(1
+c*x^(1/2))^2/(-c^2*x+1))-12*a*b^2*polylog(3,(1+c*x^(1/2))/(-c^2*x+1)^(1/2)
)-12*a*b^2*polylog(3,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3*a^2*b*dilog(c*x^(1/
2))-3*a^2*b*dilog(1+c*x^(1/2))+2*b^3*arctanh(c*x^(1/2))^3*ln(1+(1+c*x^(1/2)
)/(-c^2*x+1)^(1/2))+6*b^3*arctanh(c*x^(1/2))^2*polylog(2,-(1+c*x^(1/2))/(-c
^2*x+1)^(1/2))-12*b^3*arctanh(c*x^(1/2))*polylog(3,-(1+c*x^(1/2))/(-c^2*x+1
)^(1/2))+2*b^3*ln(c*x^(1/2))*arctanh(c*x^(1/2))^3-2*b^3*arctanh(c*x^(1/2))^
3*ln((1+c*x^(1/2))^2/(-c^2*x+1)-1)-3*b^3*arctanh(c*x^(1/2))^2*polylog(2,-(1
+c*x^(1/2))^2/(-c^2*x+1))+3*b^3*arctanh(c*x^(1/2))*polylog(3,-(1+c*x^(1/2))
^2/(-c^2*x+1))+2*b^3*arctanh(c*x^(1/2))^3*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/
2))+6*b^3*arctanh(c*x^(1/2))^2*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-12
*b^3*arctanh(c*x^(1/2))*polylog(3,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3*I*a*b^2
```

```
*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*
x+1)))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1))
)*arctanh(c*x^(1/2))^2-3*I*a*b^2*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*
csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arc
tanh(c*x^(1/2))^2-3*I*a*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(
I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(
c*x^(1/2))^2+I*b^3*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I/(1+(1+c
*x^(1/2))^2/(-c^2*x+1)))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(1+(1+c*x^(1
/2))^2/(-c^2*x+1)))*arctanh(c*x^(1/2))^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}b^3\int\frac{\log\left(c\sqrt{x}+1\right)^3}{x}\,dx-\frac{3}{8}b^3\int\frac{\log\left(c\sqrt{x}+1\right)^2\log\left(-c\sqrt{x}+1\right)}{x}\,dx+\frac{3}{8}b^3\int\frac{\log\left(c\sqrt{x}+1\right)\log\left(-c\sqrt{x}+1\right)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^3*integrate(log(c*sqrt(x) + 1)^3/x, x) - 3/8*b^3*integrate(log(c*sqrt
(x) + 1)^2*log(-c*sqrt(x) + 1)/x, x) + 3/8*b^3*integrate(log(c*sqrt(x) + 1)
*log(-c*sqrt(x) + 1)^2/x, x) - 1/8*b^3*integrate(log(-c*sqrt(x) + 1)^3/x, x
) + 3/4*a*b^2*integrate(log(c*sqrt(x) + 1)^2/x, x) - 3/2*a*b^2*integrate(lo
g(c*sqrt(x) + 1)*log(-c*sqrt(x) + 1)/x, x) + 3/4*a*b^2*integrate(log(-c*sqr
t(x) + 1)^2/x, x) + 3/2*a^2*b*integrate(log(c*sqrt(x) + 1)/x, x) - 3/2*a^2*
b*integrate(log(-c*sqrt(x) + 1)/x, x) + a^3*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^3/x,x)
```

```
[Out] int((a + b*atanh(c*x^(1/2)))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**3/x,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**3/x, x)
```

3.207 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=142

$$6b^2c^2\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+3bc^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3-\frac{3bc\left(\right.}{}$$

```
[Out] 3*b*c^2*(a+b*arctanh(c*x^(1/2)))^2+c^2*(a+b*arctanh(c*x^(1/2)))^3-(a+b*arct
anh(c*x^(1/2)))^3/x+6*b^2*c^2*(a+b*arctanh(c*x^(1/2)))*ln(2-2/(1+c*x^(1/2))
)-3*b^3*c^2*polylog(2,-1+2/(1+c*x^(1/2)))-3*b*c*(a+b*arctanh(c*x^(1/2)))^2/
x^(1/2)
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x^2,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^3/x^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$$

Mathematica [A] time = 0.32, size = 230, normalized size = 1.62

$$\frac{a\left(-2a^2-3abc^2x\log\left(1-c\sqrt{x}\right)+3abc^2x\log\left(c\sqrt{x}+1\right)-6abc\sqrt{x}+12b^2c^2x\log\left(\frac{c\sqrt{x}}{\sqrt{1-c^2x}}\right)\right)-6b\tanh^{-1}\left(c\sqrt{x}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x^2,x]
```

```
[Out] (6*b^2*(-1 + c*Sqrt[x])*(a + a*c*Sqrt[x] + b*c*Sqrt[x])*ArcTanh[c*Sqrt[x]]^
2 + 2*b^3*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^3 - 6*b*ArcTanh[c*Sqrt[x]]*(a^2 +
 2*a*b*c*Sqrt[x] - 2*b^2*c^2*x*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])]) + a*(-2*
a^2 - 6*a*b*c*Sqrt[x] - 3*a*b*c^2*x*Log[1 - c*Sqrt[x]] + 3*a*b*c^2*x*Log[1
+ c*Sqrt[x]] + 12*b^2*c^2*x*Log[(c*Sqrt[x])/Sqrt[1 - c^2*x]]) - 6*b^3*c^2*x
*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])])/(2*x)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{artanh}\left(c\sqrt{x}\right)^3+3\,ab^2\,\text{artanh}\left(c\sqrt{x}\right)^2+3\,a^2b\,\text{artanh}\left(c\sqrt{x}\right)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b
*arctanh(c*sqrt(x)) + a^3)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3/x^2, x)
```

maple [C] time = 0.63, size = 5199, normalized size = 36.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3/x^2,x)
```

```
[Out] result too large to display
```

maxima [B] time = 1.73, size = 528, normalized size = 3.72

$$-3\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)\right)b^3c^2-3\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\operatorname{Li}_2\left(-c\sqrt{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/
2))*b^3*c^2 - 3*(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1)
)*b^3*c^2 + 3*(log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b
^3*c^2 - 3*a*b^2*c^2*log(c*sqrt(x) - 1) - 3/4*((2*c*log(c*sqrt(x) - 1) - c*
log(x) + 2/sqrt(x))*c - 2*log(-c*sqrt(x) + 1)/x)*a^2*b - a^3/x + 3/2*(a^2*b
*c^2 - 2*a*b^2*c^2)*log(c*sqrt(x) + 1) - 3/4*(a^2*b*c^2 - 4*a*b^2*c^2)*log(
x) - 1/8*(12*a^2*b*c*sqrt(x) - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1)^3 + (b^
3*c^2*x - b^3)*log(-c*sqrt(x) + 1)^3 + 6*(b^3*c*sqrt(x) + a*b^2 - (a*b^2*c^
2 - b^3*c^2)*x)*log(c*sqrt(x) + 1)^2 + 3*(2*b^3*c*sqrt(x) + 2*a*b^2 - 2*(a*
b^2*c^2 + b^3*c^2)*x - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x)
 + 1)^2 + 12*(2*a*b^2*c*sqrt(x) + a^2*b)*log(c*sqrt(x) + 1) - 3*(8*a*b^2*c*
sqrt(x) - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1)^2 + 4*(b^3*c*sqrt(x) + a*b^2
 - (a*b^2*c^2 - b^3*c^2)*x)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1))/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{atanh}\left(c\,\sqrt{x}\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^3/x^2,x)
```

```
[Out] int((a + b*atanh(c*x^(1/2)))^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**3/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**3/x**2, x)
```

3.208 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=234

$$4b^2c^4\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b^2c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2x}+\frac{1}{2}c^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3+2bc^4\left(a\right.$$

[Out] 1/2*b^3*c^4*arctanh(c*x^(1/2))-1/2*b^2*c^2*(a+b*arctanh(c*x^(1/2)))/x+2*b*c^4*(a+b*arctanh(c*x^(1/2)))^2-1/2*b*c*(a+b*arctanh(c*x^(1/2)))^2/x^(3/2)+1/2*c^4*(a+b*arctanh(c*x^(1/2)))^3-1/2*(a+b*arctanh(c*x^(1/2)))^3/x^2+4*b^2*c^4*(a+b*arctanh(c*x^(1/2)))*ln(2-2/(1+c*x^(1/2)))-2*b^3*c^4*polylog(2,-1+2/(1+c*x^(1/2)))-1/2*b^3*c^3/x^(1/2)-3/2*b*c^3*(a+b*arctanh(c*x^(1/2)))^2/x^(1/2)

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x^3,x]

[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^3/x^3, x]

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$$

Mathematica [A] time = 0.72, size = 333, normalized size = 1.42

$$-\frac{2a^3+2b\tanh^{-1}\left(c\sqrt{x}\right)\left(3a^2+2abc\sqrt{x}\left(3c^2x+1\right)-8b^2c^4x^2\log\left(1-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)+b^2c^2x\left(1-c^2x\right)\right)+3a^2bc}{}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x^3,x]

[Out] -1/4*(2*a^3 + 2*a^2*b*c*Sqrt[x] + 2*a*b^2*c^2*x + 6*a^2*b*c^3*x^(3/2) + 2*b^3*c^3*x^(3/2) - 2*a*b^2*c^4*x^2 - 2*b^2*(b*c*Sqrt[x]*(-1 - 3*c^2*x + 4*c^3*x^(3/2)) + 3*a*(-1 + c^4*x^2))*ArcTanh[c*Sqrt[x]]^2 - 2*b^3*(-1 + c^4*x^2)*ArcTanh[c*Sqrt[x]]^3 + 2*b*ArcTanh[c*Sqrt[x]]*(3*a^2 + b^2*c^2*x*(1 - c^2*x) + 2*a*b*c*Sqrt[x]*(1 + 3*c^2*x) - 8*b^2*c^4*x^2*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])]) + 3*a^2*b*c^4*x^2*Log[1 - c*Sqrt[x]] - 3*a^2*b*c^4*x^2*Log[1 + c*Sqrt[x]] - 16*a*b^2*c^4*x^2*Log[(c*Sqrt[x])/Sqrt[1 - c^2*x]] + 8*b^3*c^4*x^2*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])])/x^2

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{artanh}\left(c\sqrt{x}\right)^3+3\,ab^2\,\text{artanh}\left(c\sqrt{x}\right)^2+3\,a^2b\,\text{artanh}\left(c\sqrt{x}\right)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b
*arctanh(c*sqrt(x)) + a^3)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3/x^3, x)
```

maple [C] time = 0.59, size = 1365, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3/x^3,x)
```

```
[Out] -3/8*I*c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2
*x+1)))^3*arctanh(c*x^(1/2))^2+3/4*I*c^4*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(
-c^2*x+1)))^3*arctanh(c*x^(1/2))^2-3/8*I*c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/
(c^2*x-1))^3*arctanh(c*x^(1/2))^2-3/4*I*c^4*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^
2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^2-c*a*b^2*arctanh(c*x^(1/2))/x^(3/2)-3*
c^3*a*b^2*arctanh(c*x^(1/2))/x^(1/2)+3/4*c^4*a*b^2*ln(c*x^(1/2)-1)*ln(1/2+1
/2*c*x^(1/2))-3/2*c^4*a*b^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)+3/2*c^4*a*b^
2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))+3/4*c^4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*l
n(1+c*x^(1/2))-3/4*c^4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))+3
/4*I*c^4*b^3*Pi*arctanh(c*x^(1/2))^2-1/2*b^3/x^2*arctanh(c*x^(1/2))^3-4*c^4
*b^3*dilog((1+c*x^(1/2))/(-c^2*x+1)^(1/2))-2*c^4*b^3*arctanh(c*x^(1/2))^2+1
/2*c^4*b^3*arctanh(c*x^(1/2))^3+4*c^4*b^3*dilog(1+(1+c*x^(1/2))/(-c^2*x+1)^
(1/2))+1/2*b^3*c^4*arctanh(c*x^(1/2))+3/8*I*c^4*b^3*Pi*csgn(I/(1+(1+c*x^(1/
2))^2/(-c^2*x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2
/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))*arctanh(c*x^(1/2))^2-3/8*I*c^4*b
^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))^2*csgn(I*(1+c*x^(1/2))^2/(c^2*
x-1))*arctanh(c*x^(1/2))^2-3/8*I*c^4*b^3*Pi*csgn(I/(1+(1+c*x^(1/2))^2/(-c^2
*x+1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c*x^(1/2))^2/(-c^2*x+1)))^2*
arctanh(c*x^(1/2))^2-3/4*I*c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2)
)*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^2*arctanh(c*x^(1/2))^2+3/8*I*c^4*b^3*Pi
*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1+(1+c
*x^(1/2))^2/(-c^2*x+1)))^2*arctanh(c*x^(1/2))^2+4*c^4*b^3*arctanh(c*x^(1/2)
)*ln(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-1/2*c^2*b^3*arctanh(c*x^(1/2))/x-3/2
*c^3*b^3*arctanh(c*x^(1/2))^2/x^(1/2)-1/2*c*b^3*arctanh(c*x^(1/2))^2/x^(3/2
)-1/2*c^4*b^3/(c*x^(1/2)+1-(-c^2*x+1)^(1/2))*(-c^2*x+1)^(1/2)-1/2*c^2*a*b^2
/x-3/2*c^4*b^3*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+1/2*
c^4*b^3/((-c^2*x+1)^(1/2)+c*x^(1/2)+1)*(-c^2*x+1)^(1/2)-2*c^4*a*b^2*ln(c*x^
(1/2)-1)-2*c^4*a*b^2*ln(1+c*x^(1/2))-1/2*c*a^2*b/x^(3/2)-3/4*c^4*a^2*b*ln(c
*x^(1/2)-1)+3/4*c^4*b^3*arctanh(c*x^(1/2))^2*ln(1+c*x^(1/2))+3/4*c^4*a^2*b*
ln(1+c*x^(1/2))+4*c^4*a*b^2*ln(c*x^(1/2))-3/2*a^2*b/x^2*arctanh(c*x^(1/2))-
3/2*a*b^2/x^2*arctanh(c*x^(1/2))^2-3/2*c^3*a^2*b/x^(1/2)-3/8*c^4*a*b^2*ln(c
*x^(1/2)-1)^2-3/8*c^4*a*b^2*ln(1+c*x^(1/2))^2-3/4*c^4*b^3*arctanh(c*x^(1/2)
)^2*ln(c*x^(1/2)-1)-1/2*a^3/x^2
```

maxima [B] time = 1.96, size = 703, normalized size = 3.00

$$-2\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)\right)b^3c^4-2\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\mathrm{Li}_2\left(-c\sqrt{x}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^3,x, algorithm="maxima")
```

```
[Out] -2*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/
2))*b^3*c^4 - 2*(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1)
)*b^3*c^4 + 2*(log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b
^3*c^4 - 1/8*((6*c^3*log(c*sqrt(x) - 1) - 3*c^3*log(x) + (6*c^2*x + 3*c*sqr
t(x) + 2)/x^(3/2))*c - 6*log(-c*sqrt(x) + 1)/x^2)*a^2*b + 1/4*(3*a^2*b*c^4
- 8*a*b^2*c^4 + b^3*c^4)*log(c*sqrt(x) + 1) - 1/4*(8*a*b^2*c^4 + b^3*c^4)*l
og(c*sqrt(x) - 1) - 1/8*(3*a^2*b*c^4 - 16*a*b^2*c^4)*log(x) - 1/2*a^3/x^2 -
 1/16*(4*a^2*b*c*sqrt(x) - (b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1)^3 + (b^3*
c^4*x^2 - b^3)*log(-c*sqrt(x) + 1)^3 + 2*(3*b^3*c^3*x^(3/2) + b^3*c*sqrt(x)
 + 3*a*b^2 - (3*a*b^2*c^4 - 4*b^3*c^4)*x^2)*log(c*sqrt(x) + 1)^2 + (6*b^3*c
^3*x^(3/2) + 2*b^3*c*sqrt(x) + 6*a*b^2 - 2*(3*a*b^2*c^4 + 4*b^3*c^4)*x^2 -
3*(b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1)^2 + 4*(3*a^2*
b*c^3 + 2*b^3*c^3)*x^(3/2) - 2*(3*a^2*b*c^2 - 4*a*b^2*c^2)*x + 4*(6*a*b^2*c
^3*x^(3/2) + b^3*c^2*x + 2*a*b^2*c*sqrt(x) + 3*a^2*b)*log(c*sqrt(x) + 1) -
(24*a*b^2*c^3*x^(3/2) + 4*b^3*c^2*x + 8*a*b^2*c*sqrt(x) - 3*(b^3*c^4*x^2 -
b^3)*log(c*sqrt(x) + 1)^2 + 4*(3*b^3*c^3*x^(3/2) + b^3*c*sqrt(x) + 3*a*b^2
- (3*a*b^2*c^4 - 4*b^3*c^4)*x^2)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1))/x
^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(1/2)))^3/x^3,x)
```

```
[Out] int((a + b*atanh(c*x^(1/2)))^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**3/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**3/x**3, x)
```

3.209 $\int x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=38

$$\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{x^2}{10}+\frac{x}{5}+\frac{1}{5}\log(1-x)$$

```
[Out] 1/5*x+1/10*x^2+2/5*x^(5/2)*arctanh(x^(1/2))+1/5*ln(1-x)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6097, 43}

$$\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{x}{5}+\frac{1}{5}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Sqrt[x]],x]
[Out] x/5 + x^2/10 + (2*x^(5/2)*ArcTanh[Sqrt[x]])/5 + Log[1 - x]/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)\,dx&=\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\frac{x^2}{1-x}\,dx\\&=\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\left(-1+\frac{1}{1-x}-x\right)dx\\&=\frac{x}{5}+\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.82

$$\frac{1}{10}\left(4x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)+(x+2)x+2\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Sqrt[x]],x]
[Out] (x*(2 + x) + 4*x^(5/2)*ArcTanh[Sqrt[x]] + 2*Log[1 - x])/10
```

fricas [A] time = 0.78, size = 36, normalized size = 0.95

$$\frac{1}{5}x^{\frac{5}{2}}\log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right)+\frac{1}{10}x^2+\frac{1}{5}x+\frac{1}{5}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*x^(5/2)*log(-(x + 2*sqrt(x) + 1)/(x - 1)) + 1/10*x^2 + 1/5*x + 1/5*log(
x - 1)
```

giac [B] time = 0.20, size = 170, normalized size = 4.47

$$\frac{8\left(\frac{(\sqrt{x}+1)^3}{(\sqrt{x}-1)^3}-\frac{(\sqrt{x}+1)^2}{(\sqrt{x}-1)^2}+\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{5\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^4}+\frac{2\left(\frac{5(\sqrt{x}+1)^4}{(\sqrt{x}-1)^4}+\frac{10(\sqrt{x}+1)^2}{(\sqrt{x}-1)^2}+1\right)\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{5\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^5}+\frac{2}{5}\log\left(\frac{\sqrt{x}+1}{|\sqrt{x}-1|}\right)-\frac{2}{5}\log\left(\left|-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 8/5*((sqrt(x) + 1)^3/(sqrt(x) - 1)^3 - (sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + (s
qrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^4 + 2/5*(5*(sq
rt(x) + 1)^4/(sqrt(x) - 1)^4 + 10*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + 1)*log(
-(sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^5 + 2/5*log
((sqrt(x) + 1)/abs(sqrt(x) - 1)) - 2/5*log(abs(-(sqrt(x) + 1)/(sqrt(x) - 1)
 + 1))
```

maple [A] time = 0.02, size = 35, normalized size = 0.92

$$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}+\frac{x}{5}+\frac{\ln\left(-1+\sqrt{x}\right)}{5}+\frac{\ln\left(1+\sqrt{x}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(x^(1/2)),x)
```

```
[Out] 2/5*x^(5/2)*arctanh(x^(1/2))+1/10*x^2+1/5*x+1/5*ln(-1+x^(1/2))+1/5*ln(1+x^(
1/2))
```

maxima [A] time = 0.31, size = 24, normalized size = 0.63

$$\frac{2}{5}x^{\frac{5}{2}}\operatorname{artanh}\left(\sqrt{x}\right)+\frac{1}{10}x^2+\frac{1}{5}x+\frac{1}{5}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)*arctanh(sqrt(x)) + 1/10*x^2 + 1/5*x + 1/5*log(x - 1)
```

mupad [B] time = 0.86, size = 24, normalized size = 0.63

$$\frac{x}{5}+\frac{\ln\left(x-1\right)}{5}+\frac{2\,x^{5/2}\,\operatorname{atanh}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh(x^(1/2)),x)
```

```
[Out] x/5 + log(x - 1)/5 + (2*x^(5/2)*atanh(x^(1/2)))/5 + x^2/10
```

sympy [B] time = 5.35, size = 121, normalized size = 3.18

$$\frac{4x^{\frac{7}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{10x-10}-\frac{4x^{\frac{5}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{10x-10}+\frac{x^3}{10x-10}+\frac{x^2}{10x-10}+\frac{4x\log\left(\sqrt{x}+1\right)}{10x-10}-\frac{4x\operatorname{atanh}\left(\sqrt{x}\right)}{10x-10}-\frac{4\log\left(\sqrt{x}+1\right)}{10x-10}+\frac{4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(x**(1/2)),x)
```

```
[Out] 4*x**(7/2)*atanh(sqrt(x))/(10*x - 10) - 4*x**(5/2)*atanh(sqrt(x))/(10*x - 1
0) + x**3/(10*x - 10) + x**2/(10*x - 10) + 4*x*log(sqrt(x) + 1)/(10*x - 10)
 - 4*x*atanh(sqrt(x))/(10*x - 10) - 4*log(sqrt(x) + 1)/(10*x - 10) + 4*atan
h(sqrt(x))/(10*x - 10) - 2/(10*x - 10)
```

3.210 $\int \sqrt{x} \tanh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=31

$$\frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{x}{3}+\frac{1}{3}\log(1-x)$$

```
[Out] 1/3*x+2/3*x^(3/2)*arctanh(x^(1/2))+1/3*ln(1-x)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6097, 43}

$$\frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{x}{3}+\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Sqrt[x]],x]
```

```
[Out] x/3 + (2*x^(3/2)*ArcTanh[Sqrt[x]])/3 + Log[1 - x]/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\tanh^{-1}\left(\sqrt{x}\right) dx &= \frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int \frac{x}{1-x}\,dx\\ &= \frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int \left(-1+\frac{1}{1-x}\right) dx\\ &= \frac{x}{3}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.81

$$\frac{1}{3}\left(2x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)+x+\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Sqrt[x]],x]
```

```
[Out] (x + 2*x^(3/2)*ArcTanh[Sqrt[x]] + Log[1 - x])/3
```

fricas [A] time = 1.00, size = 31, normalized size = 1.00

$$\frac{1}{3}\,x^{\frac{3}{2}}\log\left(-\frac{x+2\,\sqrt{x}+1}{x-1}\right)+\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*x^(3/2)*log(-(x + 2*sqrt(x) + 1)/(x - 1)) + 1/3*x + 1/3*log(x - 1)
```

giac [B] time = 0.15, size = 121, normalized size = 3.90

$$\frac{2\left(\frac{3\left(\sqrt{x}+1\right)^2}{\left(\sqrt{x}-1\right)^2}+1\right)\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{3\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^3}+\frac{4\left(\sqrt{x}+1\right)}{3\left(\sqrt{x}-1\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^2}+\frac{2}{3}\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right)-\frac{2}{3}\log\left(\left|-\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + 1)*log(-(sqrt(x) + 1)/(sqrt(x) - 1
))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^3 + 4/3*(sqrt(x) + 1)/((sqrt(x) - 1)*(
(sqrt(x) + 1)/(sqrt(x) - 1) - 1)^2) + 2/3*log((sqrt(x) + 1)/abs(sqrt(x) - 1
)) - 2/3*log(abs(-(sqrt(x) + 1)/(sqrt(x) - 1) + 1))
```

maple [A] time = 0.02, size = 30, normalized size = 0.97

$$\frac{2x^{\frac{3}{2}}\operatorname{arctanh}\left(\sqrt{x}\right)}{3}+\frac{x}{3}+\frac{\ln\left(-1+\sqrt{x}\right)}{3}+\frac{\ln\left(1+\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x^(1/2))*x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)*arctanh(x^(1/2))+1/3*x+1/3*ln(-1+x^(1/2))+1/3*ln(1+x^(1/2))
```

maxima [A] time = 0.32, size = 19, normalized size = 0.61

$$\frac{2}{3}x^{\frac{3}{2}}\operatorname{artanh}\left(\sqrt{x}\right)+\frac{1}{3}x+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*arctanh(sqrt(x)) + 1/3*x + 1/3*log(x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\sqrt{x}\operatorname{atanh}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(x^(1/2)),x)
```

```
[Out] int(x^(1/2)*atanh(x^(1/2)), x)
```

sympy [A] time = 1.06, size = 39, normalized size = 1.26

$$\frac{2x^{\frac{3}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{3}+\frac{x}{3}+\frac{2\log\left(\sqrt{x}+1\right)}{3}-\frac{2\operatorname{atanh}\left(\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x**(1/2))*x**(1/2),x)
```

```
[Out] 2*x**(3/2)*atanh(sqrt(x))/3 + x/3 + 2*log(sqrt(x) + 1)/3 - 2*atanh(sqrt(x))
/3
```

3.211 $\int \frac{\tanh^{-1}(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$\log(1-x)+2\sqrt{x}\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] ln(1-x)+2*arctanh(x^(1/2))*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{6097, 31\}$

$$\log(1-x)+2\sqrt{x}\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTanh[Sqrt[x]] + Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\sqrt{x}\tanh^{-1}\left(\sqrt{x}\right)-\int \frac{1}{1-x}\,dx\\ &= 2\sqrt{x}\tanh^{-1}\left(\sqrt{x}\right)+\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\log(1-x)+2\sqrt{x}\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTanh[Sqrt[x]] + Log[1 - x]
```

fricas [A] time = 0.58, size = 25, normalized size = 1.25

$$\sqrt{x}\log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x)*log(-(x + 2*sqrt(x) + 1)/(x - 1)) + log(x - 1)
```

giac [B] time = 0.24, size = 72, normalized size = 3.60

$$\frac{2\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\frac{\sqrt{x}+1}{\sqrt{x}-1}-1}+2\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right)-2\log\left(\left|-\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(-(sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1) + 2*l
og((sqrt(x) + 1)/abs(sqrt(x) - 1)) - 2*log(abs(-(sqrt(x) + 1)/(sqrt(x) - 1)
 + 1))
```

maple [A] time = 0.02, size = 17, normalized size = 0.85

$$\ln(1-x)+2\,\mathrm{arctanh}\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x^(1/2))/x^(1/2),x)
```

```
[Out] ln(1-x)+2*arctanh(x^(1/2))*x^(1/2)
```

maxima [A] time = 0.31, size = 16, normalized size = 0.80

$$2\sqrt{x}\ \mathrm{artanh}\left(\sqrt{x}\right)+\log\left(-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*arctanh(sqrt(x)) + log(-x + 1)
```

mupad [B] time = 0.80, size = 14, normalized size = 0.70

$$\ln\left(x-1\right)+2\sqrt{x}\ \mathrm{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x^(1/2))/x^(1/2),x)
```

```
[Out] log(x - 1) + 2*x^(1/2)*atanh(x^(1/2))
```

sympy [B] time = 0.54, size = 87, normalized size = 4.35

$$\frac{2x^{\frac{3}{2}}\,\mathrm{atanh}\left(\sqrt{x}\right)}{x-1}-\frac{2\sqrt{x}\ \mathrm{atanh}\left(\sqrt{x}\right)}{x-1}+\frac{2x\log\left(\sqrt{x}+1\right)}{x-1}-\frac{2x\,\mathrm{atanh}\left(\sqrt{x}\right)}{x-1}-\frac{2\log\left(\sqrt{x}+1\right)}{x-1}+\frac{2\,\mathrm{atanh}\left(\sqrt{x}\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*x**(3/2)*atanh(sqrt(x))/(x - 1) - 2*sqrt(x)*atanh(sqrt(x))/(x - 1) + 2*x*
log(sqrt(x) + 1)/(x - 1) - 2*x*atanh(sqrt(x))/(x - 1) - 2*log(sqrt(x) + 1)/
(x - 1) + 2*atanh(sqrt(x))/(x - 1)
```

3.212 $\int \frac{\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=24

$$-\log(1-x)+\log(x)-\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

```
[Out] -ln(1-x)+ln(x)-2*arctanh(x^(1/2))/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6097, 36, 31, 29}

$$-\log(1-x)+\log(x)-\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx &= -\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{(1-x)x}\,dx\\
&= -\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{1-x}\,dx+\int \frac{1}{x}\,dx\\
&= -\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}-\log(1-x)+\log(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$-\log(1-x)+\log(x)-\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

fricas [A] time = 0.96, size = 37, normalized size = 1.54

$$-\frac{x\log\left(x-1\right)-x\log(x)+\sqrt{x}\log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="fricas")
```

```
[Out] -(x*log(x - 1) - x*log(x) + sqrt(x)*log(-(x + 2*sqrt(x) + 1)/(x - 1)))/x
```

giac [B] time = 0.18, size = 72, normalized size = 3.00

$$\frac{2\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\frac{\sqrt{x}+1}{\sqrt{x}-1}+1}-2\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right)+2\log\left(\left|-\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] 2*log(-(sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) + 1) - 2*l
og((sqrt(x) + 1)/abs(sqrt(x) - 1)) + 2*log(abs(-(sqrt(x) + 1)/(sqrt(x) - 1)
 - 1))
```

maple [A] time = 0.03, size = 29, normalized size = 1.21

$$-\frac{2\,\mathrm{arctanh}\left(\sqrt{x}\right)}{\sqrt{x}}+\ln(x)-\ln\left(-1+\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x^(1/2))/x^(3/2),x)
```

```
[Out] -2*arctanh(x^(1/2))/x^(1/2)+ln(x)-ln(-1+x^(1/2))-ln(1+x^(1/2))
```

maxima [A] time = 0.31, size = 18, normalized size = 0.75

$$-\frac{2\,\mathrm{artanh}\left(\sqrt{x}\right)}{\sqrt{x}}-\log\left(x-1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2*arctanh(sqrt(x))/sqrt(x) - log(x - 1) + log(x)
```

mupad [B] time = 0.79, size = 22, normalized size = 0.92

$$2\ln\left(\sqrt{x}\right)-\ln\left(x-1\right)-\frac{2\,\mathrm{atanh}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(x^(1/2))/x^(3/2),x)
```

```
[Out] 2*log(x^(1/2)) - log(x - 1) - (2*atanh(x^(1/2)))/x^(1/2)
```

sympy [B] time = 1.38, size = 126, normalized size = 5.25

$$-\frac{2x^{\frac{3}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{x^2-x}+\frac{2\sqrt{x}\operatorname{atanh}\left(\sqrt{x}\right)}{x^2-x}+\frac{x^2\log\left(x\right)}{x^2-x}-\frac{2x^2\log\left(\sqrt{x}+1\right)}{x^2-x}+\frac{2x^2\operatorname{atanh}\left(\sqrt{x}\right)}{x^2-x}-\frac{x\log\left(x\right)}{x^2-x}+\frac{2x\log\left(\sqrt{x}+\right.}{x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x**(1/2))/x**(3/2),x)
```

```
[Out] -2*x**(3/2)*atanh(sqrt(x))/(x**2 - x) + 2*sqrt(x)*atanh(sqrt(x))/(x**2 - x)
 + x**2*log(x)/(x**2 - x) - 2*x**2*log(sqrt(x) + 1)/(x**2 - x) + 2*x**2*ata
nh(sqrt(x))/(x**2 - x) - x*log(x)/(x**2 - x) + 2*x*log(sqrt(x) + 1)/(x**2 -
 x) - 2*x*atanh(sqrt(x))/(x**2 - x)
```

3.213 $\int x^3 \left(a + b\tanh^{-1}\left(cx^{3/2}\right)\right) dx$

Optimal. Leaf size=190

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{16c^{8/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{16c^{8/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}+$$

```
[Out] 3/20*b*x^(5/2)/c+1/4*x^4*(a+b*arctanh(c*x^(3/2)))-1/4*b*arctanh(c^(1/3)*x^(
1/2))/c^(8/3)+1/16*b*ln(1+c^(2/3)*x-c^(1/3)*x^(1/2))/c^(8/3)-1/16*b*ln(1+c^
(2/3)*x+c^(1/3)*x^(1/2))/c^(8/3)-1/8*b*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(
1/2))*3^(1/2)/c^(8/3)+1/8*b*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/
2)/c^(8/3)
```

Rubi [A] time = 0.30, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6097, 321, 329, 296, 634, 618, 204, 628, 206}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{16c^{8/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{16c^{8/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (3*b*x^(5/2))/(20*c) - (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/
(8*c^(8/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(8*c^(8/3
)) - (b*ArcTanh[c^(1/3)*Sqrt[x]])/(4*c^(8/3)) + (x^4*(a + b*ArcTanh[c*x^(3/
2)]))/4 + (b*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/(16*c^(8/3)) - (b*Log[1
+ c^(1/3)*Sqrt[x] + c^(2/3)*x])/(16*c^(8/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)dx=\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{1}{8}(3bc)\int\frac{x^{9/2}}{1-c^2x^3}\,dx$$
$$=\frac{3bx^{5/2}}{20c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{(3b)\int\frac{x^{3/2}}{1-c^2x^3}\,dx}{8c}$$
$$=\frac{3bx^{5/2}}{20c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{(3b)\operatorname{Subst}\left(\int\frac{x^4}{1-c^2x^6}\,dx,x,\sqrt{x}\right)}{4c}$$
$$=\frac{3bx^{5/2}}{20c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\operatorname{Subst}\left(\int\frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)}{4c^{7/3}}-\frac{b\operatorname{Subst}\left(\right.}{}$$
$$=\frac{3bx^{5/2}}{20c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{4c^{8/3}}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\operatorname{Subst}\left(\int\frac{-\sqrt[3]{c}+2c^{2}}{1-\sqrt[3]{c}x+c^{2}}\right.}{16c^{8/3}}$$
$$=\frac{3bx^{5/2}}{20c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{4c^{8/3}}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+\right.}{16c^{8/3}}$$
$$=\frac{3bx^{5/2}}{20c}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{4c^{8/3}}$$

Mathematica [A] time = 0.06, size = 222, normalized size = 1.17

$$\frac{ax^4}{4}+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}\right)}{8c^{8/3}}-\frac{b\log\left(\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{8/3}}+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{16c^{8/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{16c^{8/3}}+\frac{\sqrt{3}b\tan}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (3*b*x^(5/2))/(20*c) + (a*x^4)/4 + (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*Sqrt[x
])/Sqrt[3]])/(8*c^(8/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3
]])/(8*c^(8/3)) + (b*x^4*ArcTanh[c*x^(3/2)])/4 + (b*Log[1 - c^(1/3)*Sqrt[x]
])/(8*c^(8/3)) - (b*Log[1 + c^(1/3)*Sqrt[x]])/(8*c^(8/3)) + (b*Log[1 - c^(1
/3)*Sqrt[x] + c^(2/3)*x])/(16*c^(8/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] + c^(2/
3)*x])/(16*c^(8/3))
```

fricas [C] time = 3.71, size = 1803, normalized size = 9.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")
```

```
[Out] 1/160*(40*a*c*x^4 + 24*b*x^(5/2) - 20*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^8
 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b
^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*c
*arctan(1/24*(4*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^
8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*b^2*c^5*sqrt(x) + 4*b^4*c^5*sqrt(x) + 4*b
^4*c^2 + 4*b^4*x - 2*(2*b^3*c^5*sqrt(x) + b^3*c^2)*((1/2)^(1/3)*(b^3 - (c^8
 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b))*sqrt(((1/2)^(1/3)*(b
^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)
^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b +
 4*b^2)*c^3 - sqrt(3)*(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/
3)*(I*sqrt(3) + 1) + 2*b)^2*c^8 - 4*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 +
 b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b*c^8 + 4*b^2*c^8 + 8*b^2*c^3*sqrt(x
))*sqrt(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) +
 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*
sqrt(3) + 1) + 2*b)*b + 4*b^2))/b^3) - 10*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3
/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c*log(-1/4*((1/2)^(1/3)*(b^3 -
 (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^5 + ((1/2)^(
1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b*c^5
 - b^2*c^5 + b^2*sqrt(x)) - 20*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 +
 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*c*log((4*(-1/1024*b^3 + 1/1024*
(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^5 + 2*(4
*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3)
 + 1) - b)*b*c^5 + b^2*c^5 + b^2*sqrt(x)) - 40*sqrt(3*(4*(-1/1024*b^3 + 1/1
024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2 + 6*(4
*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3)
 + 1) - b)*b + 3*b^2)*c*arctan(1/3*((4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/
c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^8 + 2*(4*(-1/1024*b^3
+ 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*b*c
^8 + b^2*c^8 - 2*b^2*c^3*sqrt(x) + sqrt(-4*(4*(-1/1024*b^3 + 1/1024*(c^8 -
1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2*b^2*c^5*sqrt(x) -
 4*b^4*c^5*sqrt(x) + 4*b^4*c^2 + 4*b^4*x - 4*(2*b^3*c^5*sqrt(x) - b^3*c^2)*
(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(
3) + 1) - b))*c^3)*sqrt(3*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/10
24*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2 + 6*(4*(-1/1024*b^3 + 1/1024*(c^8
- 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*b + 3*b^2)/b^3) +
 5*(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1)
+ 2*b)*c - 6*b*c)*log(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3
```

```
)*(I*sqrt(3) + 1) + 2*b)^2*b^2*c^5*sqrt(x) + 4*b^4*c^5*sqrt(x) + 4*b^4*c^2
+ 4*b^4*x - 2*(2*b^3*c^5*sqrt(x) + b^3*c^2)*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b
^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)) + 10*((4*(-1/1024*b^3 + 1/1
024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*c + 3*b*
c)*log(-4*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3
)*(I*sqrt(3) + 1) - b)^2*b^2*c^5*sqrt(x) - 4*b^4*c^5*sqrt(x) + 4*b^4*c^2 +
4*b^4*x - 4*(2*b^3*c^5*sqrt(x) - b^3*c^2)*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1
)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)) + 20*(b*c*x^4 - b*c
)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)))/c
```

giac [C] time = 0.37, size = 227, normalized size = 1.19

$$\frac{1}{4}\,ax^4+\frac{1}{320}\left(40\,x^4\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)+c\left(\frac{48\,x^{\frac{5}{2}}}{c^2}-\frac{10\,\sqrt{3}\left(-i\,\sqrt{3}-1\right)^2|c|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,\sqrt{x}+\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{c^5}+\frac{5\left(-i\,\sqrt{3}-1\right)^2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")
```

```
[Out] 1/4*a*x^4 + 1/320*(40*x^4*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1)) + c*(48*x^(
5/2)/c^2 - 10*sqrt(3)*(-I*sqrt(3) - 1)^2*abs(c)^(4/3)*arctan(1/3*sqrt(3)*(2
*sqrt(x) + (-1/c)^(1/3))/(-1/c)^(1/3))/c^5 + 5*(-I*sqrt(3) - 1)^2*abs(c)^(4
/3)*log(x + sqrt(x)*(-1/c)^(1/3) + (-1/c)^(2/3))/c^5 - 40*(-1/c)^(2/3)*log(
abs(sqrt(x) - (-1/c)^(1/3)))/c^3 + 40*sqrt(3)*abs(c)^(4/3)*arctan(1/3*sqrt(
3)*c^(1/3)*(2*sqrt(x) + 1/c^(1/3)))/c^5 - 20*abs(c)^(4/3)*log(x + sqrt(x)/c
^(1/3) + 1/c^(2/3))/c^5 + 40*log(abs(sqrt(x) - 1/c^(1/3)))/c^(11/3)))*b
```

maple [A] time = 0.04, size = 194, normalized size = 1.02

$$\frac{x^4a}{4}+\frac{b\,x^4\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{4}+\frac{3b\,x^{\frac{5}{2}}}{20c}+\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{8c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(3/2))),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x^(3/2))+3/20*b*x^(5/2)/c+1/8*b/c^3/(1/c)^(1/
3)*ln(x^(1/2)-(1/c)^(1/3))-1/16*b/c^3/(1/c)^(1/3)*ln(x+(1/c)^(1/3)*x^(1/2)+
(1/c)^(2/3))+1/8*b/c^3*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3
)*x^(1/2)+1))-1/8*b/c^3/(1/c)^(1/3)*ln(x^(1/2)+(1/c)^(1/3))+1/16*b/c^3/(1/c
)^(1/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/8*b/c^3*3^(1/2)/(1/c)^(1/3)
*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

maxima [A] time = 0.43, size = 172, normalized size = 0.91

$$\frac{1}{4}\,ax^4+\frac{1}{80}\left(20\,x^4\,\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+c\left(\frac{12\,x^{\frac{5}{2}}}{c^2}+\frac{10\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}\,\sqrt{x}+c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}+\frac{10\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}\,\sqrt{x}-c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/80*(20*x^4*arctanh(c*x^(3/2)) + c*(12*x^(5/2)/c^2 + 10*sqrt(3
)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) + c^(1/3))/c^(1/3))/c^(11/3) + 10*s
qrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) - c^(1/3))/c^(1/3))/c^(11/3) -
 5*log(c^(2/3)*x + c^(1/3)*sqrt(x) + 1)/c^(11/3) + 5*log(c^(2/3)*x - c^(1/3
)*sqrt(x) + 1)/c^(11/3) - 10*log((c^(1/3)*sqrt(x) + 1)/c^(1/3))/c^(11/3) +
10*log((c^(1/3)*sqrt(x) - 1)/c^(1/3))/c^(11/3)))*b
```

mupad [B] time = 13.38, size = 231, normalized size = 1.22

$$\frac{a\,x^4}{4}+\frac{3\,b\,x^{5/2}}{20\,c}+\frac{b\,\ln\left(\frac{c^{1/3}\,\sqrt{x}-1}{c^{1/3}\,\sqrt{x}+1}\right)}{8\,c^{8/3}}+\frac{\ln\left(1-c\,x^{3/2}\right)\left(\frac{b\,x^4}{4}-\frac{b\,c^2\,x^7}{4}\right)}{2\,c^2\,x^3-2}+\frac{b\,x^4\,\ln\left(c\,x^{3/2}+1\right)}{8}+\frac{b\,\ln\left(\frac{\sqrt{3}+c^{2/3}\,x\,1\mathrm{i}-c^{1/3}\,\sqrt{x}\,4\mathrm{i}-\sqrt{\ldots}}{2\,c^{2/3}\,x+1+\sqrt{3}\,1\mathrm{i}}\right.}{8\,c^{8/\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atanh(c*x^(3/2))),x)
```

```
[Out] (a*x^4)/4 + (3*b*x^(5/2))/(20*c) + (b*log((c^(1/3)*x^(1/2) - 1)/(c^(1/3)*x^
(1/2) + 1)))/(8*c^(8/3)) + (log(1 - c*x^(3/2))*((b*x^4)/4 - (b*c^2*x^7)/4))
/(2*c^2*x^3 - 2) + (b*x^4*log(c*x^(3/2) + 1))/8 + (b*log((3^(1/2) + c^(2/3)
*x*1i - c^(1/3)*x^(1/2)*4i - 3^(1/2)*c^(2/3)*x + 1i)/(3^(1/2)*1i + 2*c^(2/3
)*x + 1))*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(8*c^(8/3)) + (2^(1/2)*b*log((c^(2/
3)*x*1i - 3^(1/2) + c^(1/3)*x^(1/2)*4i + 3^(1/2)*c^(2/3)*x + 1i)/(2*c^(2/3)
*x - 3^(1/2)*1i + 1))*(3^(1/2)*1i + 1)^(1/2)*1i)/(16*c^(8/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(3/2))),x)
```

```
[Out] Timed out
```

3.214 $\int x^2 \left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\,dx$

Optimal. Leaf size=49

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\tanh^{-1}\left(cx^{3/2}\right)}{3c^2}+\frac{bx^{3/2}}{3c}$$

```
[Out] 1/3*b*x^(3/2)/c-1/3*b*arctanh(c*x^(3/2))/c^2+1/3*x^3*(a+b*arctanh(c*x^(3/2)
))
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6097, 321, 329, 275, 206}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\tanh^{-1}\left(cx^{3/2}\right)}{3c^2}+\frac{bx^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (b*x^(3/2))/(3*c) - (b*ArcTanh[c*x^(3/2)])/(3*c^2) + (x^3*(a + b*ArcTanh[c*
x^(3/2)]))/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)dx&=\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{1}{2}(bc)\int\frac{x^{7/2}}{1-c^2x^3}dx\\&=\frac{bx^{3/2}}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\int\frac{\sqrt{x}}{1-c^2x^3}dx}{2c}\\&=\frac{bx^{3/2}}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{x^2}{1-c^2x^6}dx,x,\sqrt{x}\right)}{c}\\&=\frac{bx^{3/2}}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1-c^2x^2}dx,x,x^{3/2}\right)}{3c}\\&=\frac{bx^{3/2}}{3c}-\frac{b\tanh^{-1}\left(cx^{3/2}\right)}{3c^2}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 1.53

$$\frac{ax^3}{3}+\frac{b\log\left(1-cx^{3/2}\right)}{6c^2}-\frac{b\log\left(cx^{3/2}+1\right)}{6c^2}+\frac{bx^{3/2}}{3c}+\frac{1}{3}bx^3\tanh^{-1}\left(cx^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (b*x^(3/2))/(3*c) + (a*x^3)/3 + (b*x^3*ArcTanh[c*x^(3/2)])/3 + (b*Log[1 - c
*x^(3/2)])/(6*c^2) - (b*Log[1 + c*x^(3/2)])/(6*c^2)
```

fricas [A] time = 0.67, size = 64, normalized size = 1.31

$$\frac{2\,ac^2x^3+2\,bcx^{\frac{3}{2}}+\left(bc^2x^3-b\right)\log\left(-\frac{c^2x^3+2\,cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^2*x^3 + 2*b*c*x^(3/2) + (b*c^2*x^3 - b)*log(-(c^2*x^3 + 2*c*x^(3
/2) + 1)/(c^2*x^3 - 1)))/c^2
```

giac [B] time = 0.17, size = 97, normalized size = 1.98

$$\frac{1}{3}\,ax^3+\frac{2}{3}\,bc\left(\frac{1}{c^3\left(\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}-1\right)}+\frac{\left(cx^{\frac{3}{2}}+1\right)\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)}{\left(cx^{\frac{3}{2}}-1\right)c^3\left(\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 + 2/3*b*c*(1/(c^3*((c*x^(3/2) + 1)/(c*x^(3/2) - 1) - 1)) + (c*x^(
3/2) + 1)*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1))/((c*x^(3/2) - 1)*c^3*((c*x^
(3/2) + 1)/(c*x^(3/2) - 1) - 1)^2))
```

maple [A] time = 0.02, size = 57, normalized size = 1.16

$$\frac{x^3a}{3}+\frac{b\,x^3\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3}+\frac{b\,x^{\frac{3}{2}}}{3c}+\frac{b\ln\left(c\,x^{\frac{3}{2}}-1\right)}{6c^2}-\frac{b\ln\left(c\,x^{\frac{3}{2}}+1\right)}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(3/2))),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c*x^(3/2))+1/3*b*x^(3/2)/c+1/6/c^2*b*ln(c*x^(3/
2)-1)-1/6/c^2*b*ln(c*x^(3/2)+1)
```

maxima [A] time = 0.32, size = 58, normalized size = 1.18

$$\frac{1}{3}\,ax^{3}+\frac{1}{6}\left(2\,x^{3}\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+c\left(\frac{2\,x^{\frac{3}{2}}}{c^{2}}-\frac{\log\left(cx^{\frac{3}{2}}+1\right)}{c^{3}}+\frac{\log\left(cx^{\frac{3}{2}}-1\right)}{c^{3}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c*x^(3/2)) + c*(2*x^(3/2)/c^2 - log(c*x^(3/2
) + 1)/c^3 + log(c*x^(3/2) - 1)/c^3))*b
```

mupad [B] time = 1.76, size = 110, normalized size = 2.24

$$\frac{a\,x^3}{3}+\frac{b\,x^{3/2}}{3\,c}+\frac{b\,\ln\left(\frac{c\,x^{3/2}-1}{c\,x^{3/2}+1}\right)}{6\,c^2}+\frac{b\,x^3\,\ln\left(c\,x^{3/2}+1\right)}{6}+\frac{b\,x^3\,\ln\left(1-c\,x^{3/2}\right)}{3\,\left(2\,c^2\,x^3-2\right)}-\frac{b\,c^2\,x^6\,\ln\left(1-c\,x^{3/2}\right)}{3\,\left(2\,c^2\,x^3-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^(3/2))),x)
```

```
[Out] (a*x^3)/3 + (b*x^(3/2))/(3*c) + (b*log((c*x^(3/2) - 1)/(c*x^(3/2) + 1)))/(6
*c^2) + (b*x^3*log(c*x^(3/2) + 1))/6 + (b*x^3*log(1 - c*x^(3/2)))/(3*(2*c^2
*x^3 - 2)) - (b*c^2*x^6*log(1 - c*x^(3/2)))/(3*(2*c^2*x^3 - 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(3/2))),x)
```

```
[Out] Timed out
```

3.215 $\int x \left(a + b \tanh^{-1}\left(cx^{3/2}\right)\right) dx$

Optimal. Leaf size=190

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{4/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{4/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-$$

```
[Out] 1/2*x^2*(a+b*arctanh(c*x^(3/2)))-1/2*b*arctanh(c^(1/3)*x^(1/2))/c^(4/3)+1/8
*b*ln(1+c^(2/3)*x-c^(1/3)*x^(1/2))/c^(4/3)-1/8*b*ln(1+c^(2/3)*x+c^(1/3)*x^(
1/2))/c^(4/3)+1/4*b*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)/c^(4/
3)-1/4*b*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)/c^(4/3)+3/2*b*x^
(1/2)/c
```

Rubi [A] time = 0.24, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6097, 321, 329, 210, 634, 618, 204, 628, 206}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{4/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{4/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (3*b*Sqrt[x])/(2*c) + (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(
4*c^(4/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(4*c^(4/3)
) - (b*ArcTanh[c^(1/3)*Sqrt[x]])/(2*c^(4/3)) + (x^2*(a + b*ArcTanh[c*x^(3/2
)]))/2 + (b*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/(8*c^(4/3)) - (b*Log[1 +
c^(1/3)*Sqrt[x] + c^(2/3)*x])/(8*c^(4/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)dx=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{1}{4}(3bc)\int\frac{x^{5/2}}{1-c^2x^3}\,dx$$

$$=\frac{3b\sqrt{x}}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{(3b)\int\frac{1}{\sqrt{x}\left(1-c^2x^3\right)}\,dx}{4c}$$

$$=\frac{3b\sqrt{x}}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{(3b)\operatorname{Subst}\left(\int\frac{1}{1-c^2x^6}\,dx,x,\sqrt{x}\right)}{2c}$$

$$=\frac{3b\sqrt{x}}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\operatorname{Subst}\left(\int\frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)}{2c}-\frac{b\operatorname{Subst}\left(\right.}{}$$

$$=\frac{3b\sqrt{x}}{2c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{2c^{4/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\operatorname{Subst}\left(\int\frac{-\sqrt[3]{c}+2c^{2/3}}{1-\sqrt[3]{c}x+c^{2/3}}\right.}{8c^{4/3}}$$

$$=\frac{3b\sqrt{x}}{2c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{2c^{4/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c\right.}{8c^{4/3}}$$

$$=\frac{3b\sqrt{x}}{2c}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{2c^{4/3}}+$$

Mathematica [A] time = 0.04, size = 222, normalized size = 1.17

$$\frac{ax^2}{2}+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}\right)}{4c^{4/3}}-\frac{b\log\left(\sqrt[3]{c}\sqrt{x}+1\right)}{4c^{4/3}}+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{4/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{4/3}}-\frac{\sqrt{3}b\tan}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (3*b*Sqrt[x])/(2*c) + (a*x^2)/2 - (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*Sqrt[x]
)/Sqrt[3]])/(4*c^(4/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]
])/(4*c^(4/3)) + (b*x^2*ArcTanh[c*x^(3/2)])/2 + (b*Log[1 - c^(1/3)*Sqrt[x]]
)/(4*c^(4/3)) - (b*Log[1 + c^(1/3)*Sqrt[x]])/(4*c^(4/3)) + (b*Log[1 - c^(1/
3)*Sqrt[x] + c^(2/3)*x])/(8*c^(4/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)
*x])/(8*c^(4/3))
```

fricas [C] time = 3.95, size = 1848, normalized size = 9.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")
```

```
[Out] 1/16*(8*a*c*x^2 + 4*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^
3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3
/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*c*arctan(-1/24*(2*s
qrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(
3) + 1) + 2*b)^2*c^2 + 4*b^2*c^2 + 4*b^2*c*sqrt(x) + 4*b^2*x - 2*(2*b*c^2 +
 b*c*sqrt(x))*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqr
t(3) + 1) + 2*b))*(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(
I*sqrt(3) + 1) + 2*b)*c^3 - 2*b*c^3)*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3
/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^4
 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2) + sqrt(3)*
(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2
*b)^2*c^4 + 4*b^2*c^4 + 8*b^2*c^3*sqrt(x) - 4*(b*c^4 + b*c^3*sqrt(x))*((1/2
)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b))*s
qrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1)
+ 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt
(3) + 1) + 2*b)*b + 4*b^2))/b^3) - 2*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4
+ b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c*log(1/2*((1/2)^(1/3)*(b^3 - (c^4
- 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c - b*c + b*sqrt(x)) -
 4*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(
3) + 1) - b)*c*log((2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4
)^(1/3)*(I*sqrt(3) + 1) - b)*c + b*c + b*sqrt(x)) - 8*sqrt(3*(2*(-1/128*b^3
 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2 +
6*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3
) + 1) - b)*b + 3*b^2)*c*arctan(-1/3*(2*sqrt((2*(-1/128*b^3 + 1/128*(c^4 -
1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^2 + b^2*c^2 - b^
2*c*sqrt(x) + b^2*x + (2*b*c^2 - b*c*sqrt(x))*(2*(-1/128*b^3 + 1/128*(c^4 -
 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b))*((2*(-1/128*b^3 +
1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*c^3 + b
*c^3)*sqrt(3*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3
)*(I*sqrt(3) + 1) - b)^2 + 6*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/1
28*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*b + 3*b^2) + ((2*(-1/128*b^3 + 1/128
*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^4 + b^2*
c^4 - 2*b^2*c^3*sqrt(x) + 2*(b*c^4 - b*c^3*sqrt(x))*(2*(-1/128*b^3 + 1/128*
(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b))*sqrt(3*(2*(-
1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1)
- b)^2 + 6*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*
(I*sqrt(3) + 1) - b)*b + 3*b^2))/b^3) + (((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/
```

```
c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c - 6*b*c)*log(((1/2)^(1/3)*(b^
3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^2 + 4*b^2
*c^2 + 4*b^2*c*sqrt(x) + 4*b^2*x - 2*(2*b*c^2 + b*c*sqrt(x))*((1/2)^(1/3)*(
b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)) + 2*((2*(-
1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1)
- b)*c + 3*b*c)*log(4*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/
c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^2 + 4*b^2*c^2 - 4*b^2*c*sqrt(x) + 4*b^2
*x + 4*(2*b*c^2 - b*c*sqrt(x))*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1
/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)) + 4*(b*c*x^2 - b*c)*log(-(c^2*x^3
 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)) + 24*b*sqrt(x))/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{artanh}\left(cx^{\frac{3}{2}}\right) + a\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)*x, x)
```

maple [A] time = 0.04, size = 194, normalized size = 1.02

$$\frac{a x^2}{2}+\frac{b x^2 \operatorname{arctanh}\left(c x^{\frac{3}{2}}\right)}{2}+\frac{3b\sqrt{x}}{2c}+\frac{b \ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b \ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{4c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(3/2))),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c*x^(3/2))+3/2*b*x^(1/2)/c+1/4*b/c^2/(1/c)^(2/3
)*ln(x^(1/2)-(1/c)^(1/3))-1/8*b/c^2/(1/c)^(2/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1
/c)^(2/3))-1/4*b/c^2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*
x^(1/2)+1))-1/4*b/c^2/(1/c)^(2/3)*ln(x^(1/2)+(1/c)^(1/3))+1/8*b/c^2/(1/c)^(
2/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))-1/4*b/c^2/(1/c)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

maxima [A] time = 0.42, size = 172, normalized size = 0.91

$$\frac{1}{2}ax^2+\frac{1}{8}\left(4x^2\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)-c\left(\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}+\frac{\log\left(c^{\frac{2}{3}}x+c^{\frac{1}{3}}\cdots\right.}{c^{\frac{7}{3}}}\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/8*(4*x^2*arctanh(c*x^(3/2)) - c*(2*sqrt(3)*arctan(1/3*sqrt(3)
*(2*c^(2/3)*sqrt(x) + c^(1/3))/c^(1/3))/c^(7/3) + 2*sqrt(3)*arctan(1/3*sqrt
(3)*(2*c^(2/3)*sqrt(x) - c^(1/3))/c^(1/3))/c^(7/3) + log(c^(2/3)*x + c^(1/3
```

```
)*sqrt(x) + 1)/c^(7/3) - log(c^(2/3)*x - c^(1/3)*sqrt(x) + 1)/c^(7/3) + 2*l
og((c^(1/3)*sqrt(x) + 1)/c^(1/3))/c^(7/3) - 2*log((c^(1/3)*sqrt(x) - 1)/c^(
1/3))/c^(7/3) - 12*sqrt(x)/c^2))*b
```

mupad [B] time = 11.97, size = 247, normalized size = 1.30

$$\frac{a\,x^2}{2}+\frac{3\,b\,\sqrt{x}}{2\,c}+\frac{b\,\ln\left(\frac{c^{1/3}\,\sqrt{x}-1}{c^{1/3}\,\sqrt{x}+1}\right)}{4\,c^{4/3}}+\frac{\ln\left(1-c\,x^{3/2}\right)\,\left(\frac{b\,x^2}{2}-\frac{b\,c^2\,x^5}{2}\right)}{2\,c^2\,x^3-2}+\frac{b\,x^2\,\ln\left(c\,x^{3/2}+1\right)}{4}+\frac{b\,\ln\left(\frac{\sqrt{3}\,c^{2/3}\,x+c^{2/3}\,x\,1{}\mathrm{i}-c^{1/3}\,\sqrt{x}\,\cdots}{2\,c^{2/3}\,x+1-\sqrt{3}\,1{}\mathrm{i}\,\cdots}\right)\cdots}{4\,c^{4/3}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^(3/2))),x)
```

```
[Out] (a*x^2)/2 + (3*b*x^(1/2))/(2*c) + (b*log((c^(1/3)*x^(1/2) - 1)/(c^(1/3)*x^(
1/2) + 1)))/(4*c^(4/3)) + (log(1 - c*x^(3/2))*((b*x^2)/2 - (b*c^2*x^5)/2))/
(2*c^2*x^3 - 2) + (b*x^2*log(c*x^(3/2) + 1))/4 + (b*log((c^(2/3)*x*1i - 3^(
1/2) - c^(1/3)*x^(1/2)*4i + 3^(1/2)*c^(2/3)*x + 1i)/(2*c^(2/3)*x - 3^(1/2)*
1i + 1))*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(4*c^(4/3)) + (2^(1/2)*b*log((2*2^(1
/2) - c^(1/3)*x^(1/2)*(3^(1/2)*1i + 1)^(5/2)*1i - 2^(1/2)*c^(2/3)*x + 2^(1/
2)*3^(1/2)*c^(2/3)*x*1i)/(3^(1/2)*1i + 2*c^(2/3)*x + 1))*(3^(1/2)*1i + 1)^(
1/2)*1i)/(8*c^(4/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**(3/2))),x)
```

```
[Out] Timed out
```

3.216 $\int \left(a + b\tanh^{-1}\left(cx^{3/2}\right)\right) dx$

Optimal. Leaf size=170

$$ax+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{4c^{2/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{4c^{2/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)}{2c^{2/3}}-b\tan$$

```
[Out] a*x+b*x*arctanh(c*x^(3/2))-b*arctanh(c^(1/3)*x^(1/2))/c^(2/3)+1/4*b*ln(1+c^
(2/3)*x-c^(1/3)*x^(1/2))/c^(2/3)-1/4*b*ln(1+c^(2/3)*x+c^(1/3)*x^(1/2))/c^(2
/3)-1/2*b*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)/c^(2/3)+1/2*b*a
rctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)/c^(2/3)
```

Rubi [A] time = 0.29, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6091, 329, 296, 634, 618, 204, 628, 206}

$$ax+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{4c^{2/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{4c^{2/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)}{2c^{2/3}}-b\tan$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x^(3/2)],x]
```

```
[Out] a*x - (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(2*c^(2/3)) + (Sq
rt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(2*c^(2/3)) - (b*ArcTanh[c
^(1/3)*Sqrt[x]])/c^(2/3) + b*x*ArcTanh[c*x^(3/2)] + (b*Log[1 - c^(1/3)*Sqrt
[x] + c^(2/3)*x])/(4*c^(2/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x])/(4
*c^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)dx &= ax+b\int \tanh^{-1}\left(cx^{3/2}\right)dx \\
&= ax+bx\tanh^{-1}\left(cx^{3/2}\right)-\frac{1}{2}(3bc)\int \frac{x^{3/2}}{1-c^2x^3}\,dx \\
&= ax+bx\tanh^{-1}\left(cx^{3/2}\right)-(3bc)\,\text{Subst}\left(\int \frac{x^4}{1-c^2x^6}\,dx,x,\sqrt{x}\right) \\
&= ax+bx\tanh^{-1}\left(cx^{3/2}\right)-\frac{b\,\text{Subst}\left(\int \frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)}{\sqrt[3]{c}}-\frac{b\,\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt[3]{c}\,x}{2}}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\right.}{\sqrt[3]{c}} \\
&= ax-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}}+bx\tanh^{-1}\left(cx^{3/2}\right)+\frac{b\,\text{Subst}\left(\int \frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx,x,\sqrt{x}\right.}{4c^{2/3}} \\
&= ax-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}}+bx\tanh^{-1}\left(cx^{3/2}\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{4c^{2/3}}-\frac{b\log}{} \\
&= ax-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}}+bx
\end{aligned}$$

Mathematica [A] time = 0.11, size = 141, normalized size = 0.83

$$ax-\frac{b\left(-\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)\right.}{4c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x^(3/2)],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^(3/2)] - (b*(2*Sqrt[3]*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])
/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]] + 4*ArcTanh[c
```

```
^(1/3)*Sqrt[x]] - Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x] + Log[1 + c^(1/3)*Sq
rt[x] + c^(2/3)*x]))/(4*c^(2/3))
```

fricas [C] time = 3.60, size = 1682, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="fricas")
```

```
[Out] a*x - 1/2*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/
3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3
/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*arctan(1/24*(4*sqrt(3)*sqrt((
(1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b
)^2*b^2*c*sqrt(x) + 4*b^4*c*sqrt(x) + 4*b^4*x + 4*b^4 - 2*(2*b^3*c*sqrt(x)
+ b^3)*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) +
1) + 2*b))*sqrt(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*s
qrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(
1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*c - sqrt(3)*(((1/2)^(1/3)*(b^3 - (c^
2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^2 - 4*((1/2)^(1/
3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b*c^2 +
 4*b^2*c^2 + 8*b^2*c*sqrt(x))*sqrt(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 +
b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b
^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2))/b^3) + 1/8*((1/2
)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - 4*b)*lo
g(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) +
2*b)^2*b^2*c*sqrt(x) + 4*b^4*c*sqrt(x) + 4*b^4*x + 4*b^4 - 2*(2*b^3*c*sqrt(
x) + b^3)*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3)
 + 1) + 2*b)) + 1/4*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1
/3)*(I*sqrt(3) + 1) + 2*b)*log(-4*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/
16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2*b^2*c*sqrt(x) - 4*b^4*c*sqrt(x) +
4*b^4*x + 4*b^4 - 4*(2*b^3*c*sqrt(x) - b^3)*((-1/16*b^3 + 1/16*(c^2 - 1)*b^
3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)) - 1/4*((1/2)^(1/3)*(b^3 -
 (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*log(-1/4*((1/2)^
(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c
+ ((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) +
2*b)*b*c - b^2*c + b^2*sqrt(x)) - 1/2*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2
+ 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)*log(((-1/16*b^3 + 1/16*(c^2 - 1)
*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2*c + 2*((-1/16*b^3 + 1
/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)*b*c + b^2*
c + b^2*sqrt(x)) + 1/2*(b*x - b)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3
- 1)) - sqrt(3*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(
I*sqrt(3) + 1) - b)^2 + 6*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c
^2)^(1/3)*(I*sqrt(3) + 1) - b)*b + 3*b^2)*arctan(1/3*(((-1/16*b^3 + 1/16*(c
^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^2 + 2*((-1/1
6*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)*b
*c^2 + b^2*c^2 - 2*b^2*c*sqrt(x) + sqrt(-4*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3
/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2*b^2*c*sqrt(x) - 4*b^4*c*s
qrt(x) + 4*b^4*x + 4*b^4 - 4*(2*b^3*c*sqrt(x) - b^3)*((-1/16*b^3 + 1/16*(c^
2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b))*c)*sqrt(3*((-1/1
6*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2
 + 6*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3)
+ 1) - b)*b + 3*b^2)/b^3)
```

giac [A] time = 0.25, size = 186, normalized size = 1.09

$$\frac{1}{4}\left(\left(c\left(\frac{2\sqrt{3}\,|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^{2}}+\frac{2\sqrt{3}\,|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^{2}}-\frac{|c|^{\frac{1}{3}}\log\left(x+\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}+\cdots\right.}{c^{2}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="giac")
```

```
[Out] 1/4*(c*(2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)^(1/
3))*abs(c)^(1/3))/c^2 + 2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(x
) - 1/abs(c)^(1/3))*abs(c)^(1/3))/c^2 - abs(c)^(1/3)*log(x + sqrt(x)/abs(c)
^(1/3) + 1/abs(c)^(2/3))/c^2 + abs(c)^(1/3)*log(x - sqrt(x)/abs(c)^(1/3) +
1/abs(c)^(2/3))/c^2 - 2*abs(c)^(1/3)*log(sqrt(x) + 1/abs(c)^(1/3))/c^2 + 2*
abs(c)^(1/3)*log(abs(sqrt(x) - 1/abs(c)^(1/3)))/c^2) + 2*x*log(-(c*x^(3/2)
+ 1)/(c*x^(3/2) - 1)))*b + a*x
```

maple [A] time = 0.03, size = 179, normalized size = 1.05

$$ax+bx\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)+\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(\sqrt{x}\right.}{2c\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^(3/2)),x)
```

```
[Out] a*x+b*x*arctanh(c*x^(3/2))+1/2*b/c/(1/c)^(1/3)*ln(x^(1/2)-(1/c)^(1/3))-1/4*
b/c/(1/c)^(1/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/2*b*3^(1/2)/c/(1/c)
^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)+1))-1/2*b/c/(1/c)^(1/3)*ln
(x^(1/2)+(1/c)^(1/3))+1/4*b/c/(1/c)^(1/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2
/3))+1/2*b*3^(1/2)/c/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-
1))
```

maxima [A] time = 0.42, size = 158, normalized size = 0.93

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}-\frac{\log\left(c^{\frac{2}{3}}x+c^{\frac{1}{3}}\sqrt{x}+1\right)}{c^{\frac{5}{3}}}+\frac{\log\left(c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right.}{c^{\frac{5}{3}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) + c^(1/3))/c^(1/3))
/c^(5/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) - c^(1/3))/c^(1/
3))/c^(5/3) - log(c^(2/3)*x + c^(1/3)*sqrt(x) + 1)/c^(5/3) + log(c^(2/3)*x
- c^(1/3)*sqrt(x) + 1)/c^(5/3) - 2*log((c^(1/3)*sqrt(x) + 1)/c^(1/3))/c^(5/
3) + 2*log((c^(1/3)*sqrt(x) - 1)/c^(1/3))/c^(5/3)) + 4*x*arctanh(c*x^(3/2))
)*b + a*x
```

mupad [B] time = 5.10, size = 107, normalized size = 0.63

$$a\,x+b\,x\,\mathrm{atanh}\left(c\,x^{3/2}\right)-\frac{b\,\mathrm{atanh}\left(c^{1/3}\,\sqrt{x}\right)}{c^{2/3}}+\frac{b\,\mathrm{atanh}\left(\frac{486\,c^8\,\sqrt{x}}{-243\,c^{23/3}+\sqrt{3}\,c^{23/3}\,243\mathrm{i}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,c^{2/3}}+\frac{b\,\mathrm{atanh}\left(\frac{486\,c^8\,\sqrt{x}}{243\,c^{23/3}+\sqrt{3}\,c^{23}\right.}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c*x^(3/2)),x)
```

```
[Out] a*x + b*x*atanh(c*x^(3/2)) - (b*atanh(c^(1/3)*x^(1/2)))/c^(2/3) + (b*atanh(
(486*c^8*x^(1/2))/(3^(1/2)*c^(23/3)*243i - 243*c^(23/3)))*(3^(1/2)*1i + 1))
/(2*c^(2/3)) + (b*atanh((486*c^8*x^(1/2))/(3^(1/2)*c^(23/3)*243i + 243*c^(2
3/3)))*(3^(1/2)*1i - 1))/(2*c^(2/3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**(3/2)),x)
```

```
[Out] Timed out
```

3.217 $\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}\,dx$

Optimal. Leaf size=34

$$a\log(x)-\frac{1}{3}b\text{Li}_2\left(-cx^{3/2}\right)+\frac{1}{3}b\text{Li}_2\left(cx^{3/2}\right)$$

```
[Out] a*ln(x)-1/3*b*polylog(2,-c*x^(3/2))+1/3*b*polylog(2,c*x^(3/2))
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6095, 5912}

$$-\frac{1}{3}b\text{PolyLog}\left(2,-cx^{3/2}\right)+\frac{1}{3}b\text{PolyLog}\left(2,cx^{3/2}\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])/x,x]
```

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^(3/2))])/3 + (b*PolyLog[2, c*x^(3/2)])/3
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}\,dx &= \frac{2}{3}\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,x^{3/2}\right)\\ &= a\log(x)-\frac{1}{3}b\text{Li}_2\left(-cx^{3/2}\right)+\frac{1}{3}b\text{Li}_2\left(cx^{3/2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.94

$$a\log(x)+\frac{1}{3}b\left(\text{Li}_2\left(cx^{3/2}\right)-\text{Li}_2\left(-cx^{3/2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x,x]
```

```
[Out] a*Log[x] + (b*(-PolyLog[2, -(c*x^(3/2))] + PolyLog[2, c*x^(3/2)]))/3
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx^{\frac{3}{2}}\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^(3/2)) + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{artanh}\left(cx^{\frac{3}{2}}\right) + a}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)/x, x)
```

maple [B] time = 0.04, size = 63, normalized size = 1.85

$$\frac{2a \ln\left(c\,x^{\frac{3}{2}}\right)}{3}+\frac{2b \ln\left(c\,x^{\frac{3}{2}}\right) \operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3}-\frac{b \operatorname{dilog}\left(c\,x^{\frac{3}{2}}\right)}{3}-\frac{b \operatorname{dilog}\left(c\,x^{\frac{3}{2}}+1\right)}{3}-\frac{b \ln\left(c\,x^{\frac{3}{2}}\right) \ln\left(c\,x^{\frac{3}{2}}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))/x,x)
```

```
[Out] 2/3*a*ln(c*x^(3/2))+2/3*b*ln(c*x^(3/2))*arctanh(c*x^(3/2))-1/3*b*dilog(c*x^
(3/2))-1/3*b*dilog(c*x^(3/2)+1)-1/3*b*ln(c*x^(3/2))*ln(c*x^(3/2)+1)
```

maxima [B] time = 0.51, size = 62, normalized size = 1.82

$$-\frac{1}{3}\left(\log\left(cx^{\frac{3}{2}}\right)\log\left(-cx^{\frac{3}{2}}+1\right)+\operatorname{Li}_2\left(-cx^{\frac{3}{2}}+1\right)\right)b+\frac{1}{3}\left(\log\left(cx^{\frac{3}{2}}+1\right)\log\left(-cx^{\frac{3}{2}}\right)+\operatorname{Li}_2\left(cx^{\frac{3}{2}}+1\right)\right)b+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x,x, algorithm="maxima")
```

```
[Out] -1/3*(log(c*x^(3/2))*log(-c*x^(3/2) + 1) + dilog(-c*x^(3/2) + 1))*b + 1/3*(
log(c*x^(3/2) + 1)*log(-c*x^(3/2)) + dilog(c*x^(3/2) + 1))*b + a*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{atanh}\left(c\,x^{3/2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(3/2)))/x,x)
```

```
[Out] int((a + b*atanh(c*x^(3/2)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))/x,x)
```

```
[Out] Timed out
```

3.218 $\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^2}\,dx$

Optimal. Leaf size=172

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}-\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)-\frac{1}{2}\sqrt{3}bc^{2/3}\tan^{-1}\left(\frac{1-}{}\right.$$

```
[Out] (-a-b*arctanh(c*x^(3/2)))/x+b*c^(2/3)*arctanh(c^(1/3)*x^(1/2))-1/4*b*c^(2/3
)*ln(1+c^(2/3)*x-c^(1/3)*x^(1/2))+1/4*b*c^(2/3)*ln(1+c^(2/3)*x+c^(1/3)*x^(1
/2))-1/2*b*c^(2/3)*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)+1/2*b*
c^(2/3)*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6097, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}-\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)-\frac{1}{2}\sqrt{3}bc^{2/3}\tan^{-1}\left(\frac{1-}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])/x^2,x]
```

```
[Out] -(Sqrt[3]*b*c^(2/3)*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 + (Sqrt[3]*b
*c^(2/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 + b*c^(2/3)*ArcTanh[c^(
1/3)*Sqrt[x]] - (a + b*ArcTanh[c*x^(3/2)])/x - (b*c^(2/3)*Log[1 - c^(1/3)*S
qrt[x] + c^(2/3)*x])/4 + (b*c^(2/3)*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^2}\,dx = -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}+\frac{1}{2}(3bc)\int \frac{1}{\sqrt{x}\left(1-c^2x^3\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}+(3bc)\,\text{Subst}\left(\int \frac{1}{1-c^2x^6}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}+(bc)\,\text{Subst}\left(\int \frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)+(bc)\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{\cdot}}\right.$$
$$= bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}-\frac{1}{4}\left(bc^{2/3}\right)\text{Subst}\left(\int \frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}\,x+c^{2/3}x^2}\right.$$
$$= bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}-\frac{1}{4}bc^{2/3}\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)+\frac{1}{4}b$$
$$= -\frac{1}{2}\sqrt{3}\,bc^{2/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)+\frac{1}{2}\sqrt{3}\,bc^{2/3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)+bc^{2/3}\tanh^{-1}\left(\right.$$

Mathematica [A] time = 0.03, size = 205, normalized size = 1.19

$$-\frac{a}{x}-\frac{1}{2}bc^{2/3}\log\left(1-\sqrt[3]{c}\sqrt{x}\right)+\frac{1}{2}bc^{2/3}\log\left(\sqrt[3]{c}\sqrt{x}+1\right)-\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^2,x]
```

```
[Out] -(a/x) + (Sqrt[3]*b*c^(2/3)*ArcTan[(-1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 + (
Sqrt[3]*b*c^(2/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 - (b*ArcTanh[c
*x^(3/2)])/x - (b*c^(2/3)*Log[1 - c^(1/3)*Sqrt[x]])/2 + (b*c^(2/3)*Log[1 +
```

```
c^(1/3)*Sqrt[x]])/2 - (b*c^(2/3)*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/4 +
(b*c^(2/3)*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x])/4
```

fricas [A] time = 0.74, size = 234, normalized size = 1.36

$$-\frac{2\sqrt{3}\left(-c^{2}\right)^{\frac{1}{3}}bx\arctan\left(\frac{2\sqrt{3}\left(-c^{2}\right)^{\frac{2}{3}}\sqrt{x}+\sqrt{3}c}{3c}\right)-2\sqrt{3}b\left(c^{2}\right)^{\frac{1}{3}}x\arctan\left(\frac{2\sqrt{3}\left(c^{2}\right)^{\frac{2}{3}}\sqrt{x}-\sqrt{3}c}{3c}\right)+\left(-c^{2}\right)^{\frac{1}{3}}bx\log\left(c^{2}x-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-c^2)^(1/3)*b*x*arctan(1/3*(2*sqrt(3)*(-c^2)^(2/3)*sqrt(x)
 + sqrt(3)*c)/c) - 2*sqrt(3)*b*(c^2)^(1/3)*x*arctan(1/3*(2*sqrt(3)*(c^2)^(2
/3)*sqrt(x) - sqrt(3)*c)/c) + (-c^2)^(1/3)*b*x*log(c^2*x - (-c^2)^(1/3)*c*s
qrt(x) + (-c^2)^(2/3)) + b*(c^2)^(1/3)*x*log(c^2*x - (c^2)^(1/3)*c*sqrt(x)
+ (c^2)^(2/3)) - 2*(-c^2)^(1/3)*b*x*log(c*sqrt(x) + (-c^2)^(1/3)) - 2*b*(c^
2)^(1/3)*x*log(c*sqrt(x) + (c^2)^(1/3)) + 2*b*log(-(c^2*x^3 + 2*c*x^(3/2) +
 1)/(c^2*x^3 - 1)) + 4*a)/x
```

giac [A] time = 0.33, size = 172, normalized size = 1.00

$$\frac{1}{4}\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{\log\left(x+\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}-\frac{\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)^(1/3))*abs(c)^(1/3)
)/abs(c)^(1/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) - 1/abs(c)^(1/3))*
abs(c)^(1/3))/abs(c)^(1/3) + log(x + sqrt(x)/abs(c)^(1/3) + 1/abs(c)^(2/3))
/abs(c)^(1/3) - log(x - sqrt(x)/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3)
 + 2*log(sqrt(x) + 1/abs(c)^(1/3))/abs(c)^(1/3) - 2*log(abs(sqrt(x) - 1/abs
(c)^(1/3)))/abs(c)^(1/3))*b*c - 1/2*b*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1))
/x - a/x
```

maple [A] time = 0.04, size = 167, normalized size = 0.97

$$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{x}-\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(\sqrt{x}\right.}{2\left(\frac{1}{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c*x^(3/2))-1/2*b/(1/c)^(2/3)*ln(x^(1/2)-(1/c)^(1/3))+1/4*b
/(1/c)^(2/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/2*b/(1/c)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)+1))+1/2*b/(1/c)^(2/3)*ln(x^(1/2
```

```
)+(1/c)^(1/3))-1/4*b/(1/c)^(2/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/2*
b/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

maxima [A] time = 0.42, size = 163, normalized size = 0.95

$$\frac{1}{4}\left(\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\frac{\log\left(c^{\frac{2}{3}}x+c^{\frac{1}{3}}\sqrt{x}+1\right)}{c^{\frac{1}{3}}}-\frac{\log\left(c^{\frac{2}{3}}x-c^{\frac{1}{3}}\sqrt{x}\right.}{c^{\frac{1}{3}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) + c^(1/3))/c^(1/3))/c
^(1/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) - c^(1/3))/c^(1/3)
)/c^(1/3) + log(c^(2/3)*x + c^(1/3)*sqrt(x) + 1)/c^(1/3) - log(c^(2/3)*x -
c^(1/3)*sqrt(x) + 1)/c^(1/3) + 2*log((c^(1/3)*sqrt(x) + 1)/c^(1/3))/c^(1/3)
 - 2*log((c^(1/3)*sqrt(x) - 1)/c^(1/3))/c^(1/3))*c - 4*arctanh(c*x^(3/2))/x
)*b - a/x
```

mupad [B] time = 7.46, size = 220, normalized size = 1.28

$$\frac{b\,c^{2/3}\,\ln\left(\frac{c^{1/3}\,\sqrt{x}+1}{c^{1/3}\,\sqrt{x}-1}\right)}{2}-\frac{a}{x}+\frac{\ln\left(1-c\,x^{3/2}\right)\,\left(b\,x-b\,c^2\,x^4\right)}{2\,x^2-2\,c^2\,x^5}-\frac{b\,\ln\left(c\,x^{3/2}+1\right)}{2\,x}+\frac{b\,c^{2/3}\,\ln\left(\frac{\sqrt{3}+c^{2/3}\,x\,1\mathrm{i}-c^{1/3}\,\sqrt{x}\,4\mathrm{i}-\sqrt{3}\,c^{2/3}\,x+1\mathrm{i}}{2\,c^{2/3}\,x+1+\sqrt{3}\,1\mathrm{i}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(3/2)))/x^2,x)
```

```
[Out] (b*c^(2/3)*log((c^(1/3)*x^(1/2) + 1)/(c^(1/3)*x^(1/2) - 1)))/2 - a/x + (log
(1 - c*x^(3/2))*(b*x - b*c^2*x^4))/(2*x^2 - 2*c^2*x^5) - (b*log(c*x^(3/2) +
 1))/(2*x) + (b*c^(2/3)*log((3^(1/2) + c^(2/3)*x*1i - c^(1/3)*x^(1/2)*4i -
3^(1/2)*c^(2/3)*x + 1i)/(3^(1/2)*1i + 2*c^(2/3)*x + 1))*((3^(1/2)*1i)/2 + 1
/2)^(1/2)*1i)/2 + (b*c^(2/3)*log((c^(2/3)*x*1i - 3^(1/2) + c^(1/3)*x^(1/2)*
4i + 3^(1/2)*c^(2/3)*x + 1i)/(2*c^(2/3)*x - 3^(1/2)*1i + 1))*((3^(1/2)*1i)/
2 - 1/2)^(1/2))/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))/x**2,x)
```

```
[Out] Timed out
```

3.219 $\int \frac{a+b\tanh^{-1}(cx^{3/2})}{x^3}\,dx$

Optimal. Leaf size=188

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{4}\sqrt{3}bc^{4/3}\tan^{-1}\left(\frac{1-}{}\right.$$

```
[Out] 1/2*(-a-b*arctanh(c*x^(3/2)))/x^2+1/2*b*c^(4/3)*arctanh(c^(1/3)*x^(1/2))-1/
8*b*c^(4/3)*ln(1+c^(2/3)*x-c^(1/3)*x^(1/2))+1/8*b*c^(4/3)*ln(1+c^(2/3)*x+c^
(1/3)*x^(1/2))+1/4*b*c^(4/3)*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1
/2)-1/4*b*c^(4/3)*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)-3/2*b*c
/x^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6097, 325, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{4}\sqrt{3}bc^{4/3}\tan^{-1}\left(\frac{1-}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])/x^3,x]
```

```
[Out] (-3*b*c)/(2*Sqrt[x]) + (Sqrt[3]*b*c^(4/3)*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sq
rt[3]])/4 - (Sqrt[3]*b*c^(4/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/4 +
 (b*c^(4/3)*ArcTanh[c^(1/3)*Sqrt[x]])/2 - (a + b*ArcTanh[c*x^(3/2)])/(2*x^2
) - (b*c^(4/3)*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/8 + (b*c^(4/3)*Log[1 +
 c^(1/3)*Sqrt[x] + c^(2/3)*x])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{4}(3bc)\int \frac{1}{x^{3/2}\left(1-c^2x^3\right)}\,dx\\
&= -\frac{3bc}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{4}\left(3bc^3\right)\int \frac{x^{3/2}}{1-c^2x^3}\,dx\\
&= -\frac{3bc}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{2}\left(3bc^3\right)\text{Subst}\left(\int \frac{x^4}{1-c^2x^6}\,dx,x,\sqrt{x}\right)\\
&= -\frac{3bc}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{2}\left(bc^{5/3}\right)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{2}\left(bc^{5/3}\right.\\
&= -\frac{3bc}{2\sqrt{x}}+\frac{1}{2}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{8}\left(bc^{4/3}\right)\text{Subst}\left(\int \frac{-\sqrt[3]{c}}{1-\sqrt[3]{}}\right.\\
&= -\frac{3bc}{2\sqrt{x}}+\frac{1}{2}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{8}bc^{4/3}\log\left(1-\sqrt[3]{c}\sqrt{x}+\right.\\
&= -\frac{3bc}{2\sqrt{x}}+\frac{1}{4}\sqrt{3}\,bc^{4/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)-\frac{1}{4}\sqrt{3}\,bc^{4/3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)+\frac{1}{2}bc^{4/3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 220, normalized size = 1.17

$$-\frac{a}{2x^2}-\frac{1}{4}bc^{4/3}\log\left(1-\sqrt[3]{c}\sqrt{x}\right)+\frac{1}{4}bc^{4/3}\log\left(\sqrt[3]{c}\sqrt{x}+1\right)-\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x+\sqrt[3]{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (3*b*c)/(2*Sqrt[x]) - (Sqrt[3]*b*c^(4/3)*ArcTan[(-1 + 2*c^(1/3
)*Sqrt[x])/Sqrt[3]])/4 - (Sqrt[3]*b*c^(4/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/
Sqrt[3]])/4 - (b*ArcTanh[c*x^(3/2)])/(2*x^2) - (b*c^(4/3)*Log[1 - c^(1/3)*S
qrt[x]])/4 + (b*c^(4/3)*Log[1 + c^(1/3)*Sqrt[x]])/4 - (b*c^(4/3)*Log[1 - c^
(1/3)*Sqrt[x] + c^(2/3)*x])/8 + (b*c^(4/3)*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3
)*x])/8
```

fricas [A] time = 0.63, size = 214, normalized size = 1.14

$$-\frac{2\sqrt{3}\,b\,(-c)^{\frac{1}{3}}\,cx^2\arctan\left(\frac{2}{3}\sqrt{3}\,(-c)^{\frac{1}{3}}\sqrt{x}-\frac{1}{3}\sqrt{3}\right)+2\sqrt{3}\,bc^{\frac{4}{3}}x^2\arctan\left(\frac{2}{3}\sqrt{3}\,c^{\frac{1}{3}}\sqrt{x}-\frac{1}{3}\sqrt{3}\right)+b\,(-c)^{\frac{1}{3}}\,cx^2\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(3)*b*(-c)^(1/3)*c*x^2*arctan(2/3*sqrt(3)*(-c)^(1/3)*sqrt(x) -
1/3*sqrt(3)) + 2*sqrt(3)*b*c^(4/3)*x^2*arctan(2/3*sqrt(3)*c^(1/3)*sqrt(x) -
 1/3*sqrt(3)) + b*(-c)^(1/3)*c*x^2*log(c*x + (-c)^(2/3)*sqrt(x) - (-c)^(1/3
)) + b*c^(4/3)*x^2*log(c*x - c^(2/3)*sqrt(x) + c^(1/3)) - 2*b*(-c)^(1/3)*c*
x^2*log(c*sqrt(x) - (-c)^(2/3)) - 2*b*c^(4/3)*x^2*log(c*sqrt(x) + c^(2/3))
+ 12*b*c*x^(3/2) + 2*b*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)) + 4*
a)/x^2
```

giac [A] time = 0.34, size = 194, normalized size = 1.03

$$-\frac{1}{4}\sqrt{3}\,bc|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)-\frac{1}{4}\sqrt{3}\,bc|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)+\frac{bc^3\log\left(x+\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}\right.}{8\,|c|^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*b*c*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)^(1/3
))*abs(c)^(1/3)) - 1/4*sqrt(3)*b*c*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(
x) - 1/abs(c)^(1/3))*abs(c)^(1/3)) + 1/8*b*c^3*log(x + sqrt(x)/abs(c)^(1/3)
 + 1/abs(c)^(2/3))/abs(c)^(5/3) - 1/8*b*c^3*log(x - sqrt(x)/abs(c)^(1/3) +
1/abs(c)^(2/3))/abs(c)^(5/3) + 1/4*b*c*abs(c)^(1/3)*log(sqrt(x) + 1/abs(c)^
(1/3)) - 1/4*b*c^3*log(abs(sqrt(x) - 1/abs(c)^(1/3)))/abs(c)^(5/3) - 1/4*b*
log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1))/x^2 - 1/2*(3*b*c*x^(3/2) + a)/x^2
```

maple [A] time = 0.04, size = 180, normalized size = 0.96

$$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{2x^2}-\frac{3bc}{2\sqrt{x}}-\frac{bc\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{bc\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x^(3/2))-3/2*b*c/x^(1/2)-1/4*b*c/(1/c)^(1/3)
*ln(x^(1/2)-(1/c)^(1/3))+1/8*b*c/(1/c)^(1/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/c)
^(2/3))-1/4*b*c*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/
2)+1))+1/4*b*c/(1/c)^(1/3)*ln(x^(1/2)+(1/c)^(1/3))-1/8*b*c/(1/c)^(1/3)*ln(x
-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))-1/4*b*c*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^
(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

maxima [A] time = 0.42, size = 168, normalized size = 0.89

$$-\frac{1}{8}\left(\left(2\sqrt{3}c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)+2\sqrt{3}c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)-c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}x+c^{\frac{1}{3}}\sqrt{x}+1\right)+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*((2*sqrt(3)*c^(1/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) + c^(1/3))/c
^(1/3)) + 2*sqrt(3)*c^(1/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) - c^(1/3)
)/c^(1/3)) - c^(1/3)*log(c^(2/3)*x + c^(1/3)*sqrt(x) + 1) + c^(1/3)*log(c^(
2/3)*x - c^(1/3)*sqrt(x) + 1) - 2*c^(1/3)*log((c^(1/3)*sqrt(x) + 1)/c^(1/3)
) + 2*c^(1/3)*log((c^(1/3)*sqrt(x) - 1)/c^(1/3)) + 12/sqrt(x))*c + 4*arctan
h(c*x^(3/2))/x^2)*b - 1/2*a/x^2
```

mupad [B] time = 7.37, size = 228, normalized size = 1.21

$$\frac{b\,c^{4/3}\,\ln\left(\frac{c^{1/3}\,\sqrt{x}+1}{c^{1/3}\,\sqrt{x}-1}\right)}{4}-\frac{a}{2\,x^2}+\frac{\ln\left(1-c\,x^{3/2}\right)\,\left(\frac{b\,x}{2}-\frac{b\,c^2\,x^4}{2}\right)}{2\,x^3-2\,c^2\,x^6}-\frac{3\,b\,c}{2\,\sqrt{x}}-\frac{b\,\ln\left(c\,x^{3/2}+1\right)}{4\,x^2}+\frac{b\,c^{4/3}\,\ln\left(\frac{\sqrt{3}+c^{2/3}\,x\,1\mathrm{i}+c^{1/3}\,\sqrt{x}\,4\mathrm{i}-\sqrt{3}}{2\,c^{2/3}\,x+1+\sqrt{3}\,1\mathrm{i}}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(3/2)))/x^3,x)
```

```
[Out] (b*c^(4/3)*log((c^(1/3)*x^(1/2) + 1)/(c^(1/3)*x^(1/2) - 1)))/4 - a/(2*x^2)
+ (log(1 - c*x^(3/2))*((b*x)/2 - (b*c^2*x^4)/2))/(2*x^3 - 2*c^2*x^6) - (3*b
*c)/(2*x^(1/2)) - (b*log(c*x^(3/2) + 1))/(4*x^2) + (b*c^(4/3)*log((3^(1/2)
+ c^(2/3)*x*1i + c^(1/3)*x^(1/2)*4i - 3^(1/2)*c^(2/3)*x + 1i)/(3^(1/2)*1i +
 2*c^(2/3)*x + 1))*((3^(1/2)*1i)/2 - 1/2)^(1/2))/4 + (b*c^(4/3)*log((c^(2/3
)*x*1i - 3^(1/2) - c^(1/3)*x^(1/2)*4i + 3^(1/2)*c^(2/3)*x + 1i)/(2*c^(2/3)*
x - 3^(1/2)*1i + 1))*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))/x**3,x)
```

```
[Out] Timed out
```

3.220 $\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{3}bc^2\tanh^{-1}\left(cx^{3/2}\right)-\frac{bc}{3x^{3/2}}$$

```
[Out] -1/3*b*c/x^(3/2)+1/3*b*c^2*arctanh(c*x^(3/2))+1/3*(-a-b*arctanh(c*x^(3/2)))
/x^3
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6097, 325, 329, 275, 206}

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{3}bc^2\tanh^{-1}\left(cx^{3/2}\right)-\frac{bc}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])/x^4,x]
```

```
[Out] -(b*c)/(3*x^(3/2)) + (b*c^2*ArcTanh[c*x^(3/2)])/3 - (a + b*ArcTanh[c*x^(3/2
)])/(3*x^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^4}\,dx = -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{2}(bc)\int \frac{1}{x^{5/2}\left(1-c^2x^3\right)}\,dx$$
$$= -\frac{bc}{3x^{3/2}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{2}\left(bc^3\right)\int \frac{\sqrt{x}}{1-c^2x^3}\,dx$$
$$= -\frac{bc}{3x^{3/2}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\left(bc^3\right)\text{Subst}\left(\int \frac{x^2}{1-c^2x^6}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{bc}{3x^{3/2}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{3}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,x^{3/2}\right)$$
$$= -\frac{bc}{3x^{3/2}}+\frac{1}{3}bc^2\tanh^{-1}\left(cx^{3/2}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.55

$$-\frac{a}{3x^3}-\frac{1}{6}bc^2\log\left(1-cx^{3/2}\right)+\frac{1}{6}bc^2\log\left(cx^{3/2}+1\right)-\frac{bc}{3x^{3/2}}-\frac{b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*c)/(3*x^(3/2)) - (b*ArcTanh[c*x^(3/2)])/(3*x^3) - (b*c^2*Lo
g[1 - c*x^(3/2)])/6 + (b*c^2*Log[1 + c*x^(3/2)])/6
```

fricas [A] time = 0.71, size = 59, normalized size = 1.26

$$-\frac{2\,bcx^{\frac{3}{2}}-\left(bc^2x^3-b\right)\log\left(-\frac{c^2x^3+2\,cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*c*x^(3/2) - (b*c^2*x^3 - b)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2
*x^3 - 1)) + 2*a)/x^3
```

giac [A] time = 0.23, size = 67, normalized size = 1.43

$$\frac{1}{6}\,bc^2\log\left(cx^{\frac{3}{2}}+1\right)-\frac{1}{6}\,bc^2\log\left(cx^{\frac{3}{2}}-1\right)-\frac{b\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)}{6\,x^3}-\frac{bcx^{\frac{3}{2}}+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b*c^2*log(c*x^(3/2) + 1) - 1/6*b*c^2*log(c*x^(3/2) - 1) - 1/6*b*log(-(c
*x^(3/2) + 1)/(c*x^(3/2) - 1))/x^3 - 1/3*(b*c*x^(3/2) + a)/x^3
```

maple [A] time = 0.04, size = 55, normalized size = 1.17

$$-\frac{a}{3x^3}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3x^3}-\frac{bc}{3x^{\frac{3}{2}}}-\frac{b\,c^2\ln\left(c\,x^{\frac{3}{2}}-1\right)}{6}+\frac{b\,c^2\ln\left(c\,x^{\frac{3}{2}}+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x^(3/2))-1/3*b*c/x^(3/2)-1/6*b*c^2*ln(c*x^(3
/2)-1)+1/6*b*c^2*ln(c*x^(3/2)+1)
```

maxima [A] time = 0.31, size = 51, normalized size = 1.09

$$\frac{1}{6}\left(\left(c\log\left(cx^{\frac{3}{2}}+1\right)-c\log\left(cx^{\frac{3}{2}}-1\right)-\frac{2}{x^{\frac{3}{2}}}\right)c-\frac{2\,\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c*log(c*x^(3/2) + 1) - c*log(c*x^(3/2) - 1) - 2/x^(3/2))*c - 2*arctan
h(c*x^(3/2))/x^3)*b - 1/3*a/x^3
```

mupad [B] time = 1.36, size = 114, normalized size = 2.43

$$\frac{b\,c^2\,\ln\left(\frac{c\,x^{3/2}+1}{c\,x^{3/2}-1}\right)}{6}-\frac{a}{3\,x^3}-\frac{b\,c}{3\,x^{3/2}}-\frac{b\,\ln\left(c\,x^{3/2}+1\right)}{6\,x^3}+\frac{b\,x\,\ln\left(1-c\,x^{3/2}\right)}{3\,\left(2\,x^4-2\,c^2\,x^7\right)}-\frac{b\,c^2\,x^4\,\ln\left(1-c\,x^{3/2}\right)}{3\,\left(2\,x^4-2\,c^2\,x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(3/2)))/x^4,x)
```

```
[Out] (b*c^2*log((c*x^(3/2) + 1)/(c*x^(3/2) - 1)))/6 - a/(3*x^3) - (b*c)/(3*x^(3/
2)) - (b*log(c*x^(3/2) + 1))/(6*x^3) + (b*x*log(1 - c*x^(3/2)))/(3*(2*x^4 -
 2*c^2*x^7)) - (b*c^2*x^4*log(1 - c*x^(3/2)))/(3*(2*x^4 - 2*c^2*x^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))/x**4,x)
```

```
[Out] Timed out
```

3.221 $\int x^2 \left(a+b \tanh^{-1}\left(cx^{3/2}\right)\right)^2 dx$

Optimal. Leaf size=101

$$-\frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{3c^2}+\frac{2abx^{3/2}}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2+\frac{b^2\log\left(1-c^2x^3\right)}{3c^2}+\frac{2b^2x^{3/2}\tanh^{-1}\left(cx^{3/2}\right)}{3c}$$

[Out] 2/3*a*b*x^(3/2)/c+2/3*b^2*x^(3/2)*arctanh(c*x^(3/2))/c-1/3*(a+b*arctanh(c*x^(3/2)))^2/c^2+1/3*x^3*(a+b*arctanh(c*x^(3/2)))^2+1/3*b^2*ln(-c^2*x^3+1)/c^2

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[x^2*(a + b*ArcTanh[c*x^(3/2)])^2,x]

[Out] Defer[Int][x^2*(a + b*ArcTanh[c*x^(3/2)])^2, x]

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2 dx=\int x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2 dx$$

Mathematica [A] time = 0.10, size = 122, normalized size = 1.21

$$\frac{a^2c^2x^3+2abcx^{3/2}+b(a+b)\log\left(1-cx^{3/2}\right)-ab\log\left(cx^{3/2}+1\right)+2bcx^{3/2}\tanh^{-1}\left(cx^{3/2}\right)\left(acx^{3/2}+b\right)+b^2\left(c^2x^3-\right.}{3c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*ArcTanh[c*x^(3/2)])^2,x]

[Out] (2*a*b*c*x^(3/2) + a^2*c^2*x^3 + 2*b*c*x^(3/2)*(b + a*c*x^(3/2))*ArcTanh[c*x^(3/2)] + b^2*(-1 + c^2*x^3)*ArcTanh[c*x^(3/2)]^2 + b*(a + b)*Log[1 - c*x^(3/2)] - a*b*Log[1 + c*x^(3/2)] + b^2*Log[1 + c*x^(3/2)])/(3*c^2)

fricas [B] time = 0.84, size = 179, normalized size = 1.77

$$\frac{4a^2c^2x^3+8abcx^{\frac{3}{2}}+\left(b^2c^2x^3-b^2\right)\log\left(-\frac{c^2x^3+2cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)^2+4\left(abc^2-ab+b^2\right)\log\left(cx^{\frac{3}{2}}+1\right)-4\left(abc^2-ab-b^2\right)\mathrm{lo}}{12c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="fricas")

[Out] 1/12*(4*a^2*c^2*x^3 + 8*a*b*c*x^(3/2) + (b^2*c^2*x^3 - b^2)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1))^2 + 4*(a*b*c^2 - a*b + b^2)*log(c*x^(3/2) + 1) - 4*(a*b*c^2 - a*b - b^2)*log(c*x^(3/2) - 1) + 4*(a*b*c^2*x^3 + b^2*c*x^(3/2) - a*b*c^2)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)))/c^2

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)+a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)^2*x^2, x)
```

maple [B] time = 0.05, size = 284, normalized size = 2.81

$$\frac{x^3a^2}{3}+\frac{x^3b^2\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)^2}{3}+\frac{2b^2x^{\frac{3}{2}}\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3c}+\frac{b^2\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)\ln\left(c\,x^{\frac{3}{2}}-1\right)}{3c^2}-\frac{b^2\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)\ln\left(c\,x\right.}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(3/2)))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*x^3*b^2*arctanh(c*x^(3/2))^2+2/3*b^2*x^(3/2)*arctanh(c*x^(3
/2))/c+1/3/c^2*b^2*arctanh(c*x^(3/2))*ln(c*x^(3/2)-1)-1/3/c^2*b^2*arctanh(c
*x^(3/2))*ln(c*x^(3/2)+1)+1/12/c^2*b^2*ln(c*x^(3/2)-1)^2-1/6/c^2*b^2*ln(c*x
^(3/2)-1)*ln(1/2+1/2*c*x^(3/2))+1/3/c^2*b^2*ln(c*x^(3/2)-1)+1/3/c^2*b^2*ln(
c*x^(3/2)+1)+1/12/c^2*b^2*ln(c*x^(3/2)+1)^2+1/6/c^2*b^2*ln(-1/2*c*x^(3/2)+1
/2)*ln(1/2+1/2*c*x^(3/2))-1/6/c^2*b^2*ln(-1/2*c*x^(3/2)+1/2)*ln(c*x^(3/2)+1
)+2/3*a*b*x^3*arctanh(c*x^(3/2))+2/3*a*b*x^(3/2)/c+1/3/c^2*a*b*ln(c*x^(3/2)
-1)-1/3/c^2*a*b*ln(c*x^(3/2)+1)
```

maxima [B] time = 0.33, size = 186, normalized size = 1.84

$$\frac{1}{3}\,b^2x^3\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)^2+\frac{1}{3}\,a^2x^3+\frac{1}{3}\left(2\,x^3\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+c\left(\frac{2\,x^{\frac{3}{2}}}{c^2}-\frac{\log\left(cx^{\frac{3}{2}}+1\right)}{c^3}+\frac{\log\left(cx^{\frac{3}{2}}-1\right)}{c^3}\right)\right)ab+\frac{1}{12}\left(4\,c\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*arctanh(c*x^(3/2))^2 + 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c*x^(3/
2)) + c*(2*x^(3/2)/c^2 - log(c*x^(3/2) + 1)/c^3 + log(c*x^(3/2) - 1)/c^3))*
a*b + 1/12*(4*c*(2*x^(3/2)/c^2 - log(c*x^(3/2) + 1)/c^3 + log(c*x^(3/2) - 1
)/c^3)*arctanh(c*x^(3/2)) - (2*(log(c*x^(3/2) - 1) - 2)*log(c*x^(3/2) + 1)
- log(c*x^(3/2) + 1)^2 - log(c*x^(3/2) - 1)^2 - 4*log(c*x^(3/2) - 1))/c^2)*
b^2
```

mupad [B] time = 1.31, size = 105, normalized size = 1.04

$$\frac{c\left(\frac{2\,b^2\,x^{3/2}\,\mathrm{atanh}\left(c\,x^{3/2}\right)}{3}+\frac{2\,a\,b\,x^{3/2}}{3}\right)-\frac{b^2\,\mathrm{atanh}\left(c\,x^{3/2}\right)^2}{3}+\frac{b^2\,\ln\left(c^2\,x^3-1\right)}{3}-\frac{2\,a\,b\,\mathrm{atanh}\left(c\,x^{3/2}\right)}{3}}{c^2}+\frac{a^2\,x^3}{3}+\frac{b^2\,x^3\,\mathrm{atanh}\left(c\,x^{3/2}\right)^2}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^(3/2)))^2,x)
```

```
[Out] (c*((2*b^2*x^(3/2)*atanh(c*x^(3/2)))/3 + (2*a*b*x^(3/2))/3) - (b^2*atanh(c*
x^(3/2))^2)/3 + (b^2*log(c^2*x^3 - 1))/3 - (2*a*b*atanh(c*x^(3/2)))/3)/c^2
+ (a^2*x^3)/3 + (b^2*x^3*atanh(c*x^(3/2))^2)/3 + (2*a*b*x^3*atanh(c*x^(3/2)
))/3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(3/2)))**2,x)
```

```
[Out] Timed out
```

3.222 $\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=156

$$-\frac{2}{3}b\text{Li}_2\left(1-\frac{2}{1-cx^{3/2}}\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{2}{3}b\text{Li}_2\left(\frac{2}{1-cx^{3/2}}-1\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{4}{3}\tanh^{-1}\left(1-\frac{}{1-}\right.$$

```
[Out] -4/3*(a+b*arctanh(c*x^(3/2)))^2*arctanh(-1+2/(1-c*x^(3/2)))-2/3*b*(a+b*arct
anh(c*x^(3/2)))*polylog(2,1-2/(1-c*x^(3/2)))+2/3*b*(a+b*arctanh(c*x^(3/2)))
*polylog(2,-1+2/(1-c*x^(3/2)))+1/3*b^2*polylog(3,1-2/(1-c*x^(3/2)))-1/3*b^2
*polylog(3,-1+2/(1-c*x^(3/2)))
```

Rubi [A] time = 0.32, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-\frac{2}{3}b\text{PolyLog}\left(2,1-\frac{2}{1-cx^{3/2}}\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{2}{3}b\text{PolyLog}\left(2,\frac{2}{1-cx^{3/2}}-1\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])^2/x,x]
```

```
[Out] (4*(a + b*ArcTanh[c*x^(3/2)])^2*ArcTanh[1 - 2/(1 - c*x^(3/2))])/3 - (2*b*(a
 + b*ArcTanh[c*x^(3/2)])*PolyLog[2, 1 - 2/(1 - c*x^(3/2))])/3 + (2*b*(a + b
*ArcTanh[c*x^(3/2)])*PolyLog[2, -1 + 2/(1 - c*x^(3/2))])/3 + (b^2*PolyLog[3
, 1 - 2/(1 - c*x^(3/2))])/3 - (b^2*PolyLog[3, -1 + 2/(1 - c*x^(3/2))])/3
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x}\,dx = \frac{2}{3}\,\mathrm{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, x^{3/2}\right)$$

$$= \frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right)-\frac{1}{3}(8bc)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tanh^{-1}\right.}{}\right.$$

$$= \frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right)+\frac{1}{3}(4bc)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tanh^{-1}\right.}{1}\right.$$

$$= \frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right)-\frac{2}{3}b\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\mathrm{Li}_2\left(\right.$$

$$= \frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right)-\frac{2}{3}b\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\mathrm{Li}_2\left(\right.$$

Mathematica [A] time = 0.14, size = 167, normalized size = 1.07

$$\frac{1}{3}\left(b\left(2\mathrm{Li}_2\left(\frac{cx^{3/2}+1}{1-cx^{3/2}}\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-2\mathrm{Li}_2\left(\frac{cx^{3/2}+1}{cx^{3/2}-1}\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+b\left(\mathrm{Li}_3\left(\frac{cx^{3/2}+1}{cx^{3/2}-1}\right)-\mathrm{Li}_3\left(\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])^2/x,x]
```

```
[Out] (4*(a + b*ArcTanh[c*x^(3/2)])^2*ArcTanh[1 + 2/(-1 + c*x^(3/2))] + b*(2*(a +
 b*ArcTanh[c*x^(3/2)])*PolyLog[2, (1 + c*x^(3/2))/(1 - c*x^(3/2))] - 2*(a +
 b*ArcTanh[c*x^(3/2)])*PolyLog[2, (1 + c*x^(3/2))/(-1 + c*x^(3/2))] + b*(-P
olyLog[3, (1 + c*x^(3/2))/(1 - c*x^(3/2))] + PolyLog[3, (1 + c*x^(3/2))/(-1
 + c*x^(3/2))])))/3
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)^2+2\,ab\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)+a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^(3/2))^2 + 2*a*b*arctanh(c*x^(3/2)) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(cx^{\frac{3}{2}}\right) + a\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)^2/x, x)
```

maple [C] time = 0.25, size = 785, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))^2/x,x)
```

```
[Out] 2/3*a^2*ln(c*x^(3/2))+2/3*b^2*ln(c*x^(3/2))*arctanh(c*x^(3/2))^2-2/3*b^2*ar
ctanh(c*x^(3/2))*polylog(2,-(c*x^(3/2)+1)^2/(-c^2*x^3+1))+1/3*b^2*polylog(3
,-(c*x^(3/2)+1)^2/(-c^2*x^3+1))-2/3*b^2*arctanh(c*x^(3/2))^2*ln((c*x^(3/2)+
1)^2/(-c^2*x^3+1)-1)+2/3*b^2*arctanh(c*x^(3/2))^2*ln(1-(c*x^(3/2)+1)/(-c^2*
x^3+1)^(1/2))+4/3*b^2*arctanh(c*x^(3/2))*polylog(2,(c*x^(3/2)+1)/(-c^2*x^3+
1)^(1/2))-4/3*b^2*polylog(3,(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))+2/3*b^2*arcta
nh(c*x^(3/2))^2*ln(1+(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))+4/3*b^2*arctanh(c*x^
(3/2))*polylog(2,-(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))-4/3*b^2*polylog(3,-(c*x
^(3/2)+1)/(-c^2*x^3+1)^(1/2))-1/3*I*b^2*Pi*csgn(I/(1+(c*x^(3/2)+1)^2/(-c^2*
x^3+1)))*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1)/(1+(c*x^(3/2)+1)^2/(-c^2*x
^3+1)))^2*arctanh(c*x^(3/2))^2+1/3*I*b^2*Pi*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x
^3+1)-1))*csgn(I/(1+(c*x^(3/2)+1)^2/(-c^2*x^3+1)))*csgn(I*((c*x^(3/2)+1)^2/
(-c^2*x^3+1)-1)/(1+(c*x^(3/2)+1)^2/(-c^2*x^3+1)))*arctanh(c*x^(3/2))^2+1/3*
I*b^2*Pi*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1)/(1+(c*x^(3/2)+1)^2/(-c^2*x
^3+1)))^3*arctanh(c*x^(3/2))^2-1/3*I*b^2*Pi*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x
^3+1)-1))*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1)/(1+(c*x^(3/2)+1)^2/(-c^2*
x^3+1)))^2*arctanh(c*x^(3/2))^2+4/3*a*b*ln(c*x^(3/2))*arctanh(c*x^(3/2))-2/
3*a*b*ln(c*x^(3/2))*ln(c*x^(3/2)+1)-2/3*a*b*dilog(c*x^(3/2))-2/3*a*b*dilog(
c*x^(3/2)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} b^2 \int \frac{\log\left(cx^{\frac{3}{2}} + 1\right)^2}{x}\, dx - \frac{1}{2} b^2 \int \frac{\log\left(cx^{\frac{3}{2}} + 1\right)\log\left(-cx^{\frac{3}{2}} + 1\right)}{x}\, dx + \frac{1}{4} b^2 \int \frac{\log\left(-cx^{\frac{3}{2}} + 1\right)^2}{x}\, dx + ab \int \frac{\log\left(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*integrate(log(c*x^(3/2) + 1)^2/x, x) - 1/2*b^2*integrate(log(c*x^(3
/2) + 1)*log(-c*x^(3/2) + 1)/x, x) + 1/4*b^2*integrate(log(-c*x^(3/2) + 1)^
2/x, x) + a*b*integrate(log(c*x^(3/2) + 1)/x, x) - a*b*integrate(log(-c*x^(
3/2) + 1)/x, x) + a^2*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a + b \operatorname{atanh}\left(c\, x^{3/2}\right)\right)}^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(3/2)))^2/x,x)
```

```
[Out] int((a + b*atanh(c*x^(3/2)))^2/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))**2/x,x)
```

```
[Out] Timed out
```

3.223 $\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=96

$$\frac{1}{3}c^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2-\frac{2bc\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)}{3x^{3/2}}-\frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{3x^3}-\frac{1}{3}b^2c^2\log\left(1-c^2x^3\right)+b^2c^2\log$$

```
[Out] -2/3*b*c*(a+b*arctanh(c*x^(3/2)))/x^(3/2)+1/3*c^2*(a+b*arctanh(c*x^(3/2)))^
2-1/3*(a+b*arctanh(c*x^(3/2)))^2/x^3+b^2*c^2*ln(x)-1/3*b^2*c^2*ln(-c^2*x^3+
1)
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])^2/x^4,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*x^(3/2)])^2/x^4, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx$$

Mathematica [A] time = 0.14, size = 123, normalized size = 1.28

$$\frac{1}{3}\left(-\frac{a^2}{x^3}-bc^2(a+b)\log\left(1-cx^{3/2}\right)+bc^2(a-b)\log\left(cx^{3/2}+1\right)-\frac{2abc}{x^{3/2}}-\frac{2b\tanh^{-1}\left(cx^{3/2}\right)\left(a+bcx^{3/2}\right)}{x^3}+\frac{b^2(c$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])^2/x^4,x]
```

```
[Out] (-(a^2/x^3) - (2*a*b*c)/x^(3/2) - (2*b*(a + b*c*x^(3/2))*ArcTanh[c*x^(3/2)]
)/x^3 + (b^2*(-1 + c^2*x^3)*ArcTanh[c*x^(3/2)]^2)/x^3 + 3*b^2*c^2*Log[x] -
b*(a + b)*c^2*Log[1 - c*x^(3/2)] + (a - b)*b*c^2*Log[1 + c*x^(3/2)])/3
```

fricas [B] time = 1.46, size = 173, normalized size = 1.80

$$\frac{24\,b^2c^2x^3\log\left(\sqrt{x}\right)+4\left(ab-b^2\right)c^2x^3\log\left(cx^{\frac{3}{2}}+1\right)-4\left(ab+b^2\right)c^2x^3\log\left(cx^{\frac{3}{2}}-1\right)-8\,abcx^{\frac{3}{2}}+\left(b^2c^2x^3-b^2\right)}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(24*b^2*c^2*x^3*log(sqrt(x)) + 4*(a*b - b^2)*c^2*x^3*log(c*x^(3/2) + 1
) - 4*(a*b + b^2)*c^2*x^3*log(c*x^(3/2) - 1) - 8*a*b*c*x^(3/2) + (b^2*c^2*x
^3 - b^2)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1))^2 - 4*a^2 - 4*(b^
2*c*x^(3/2) + a*b)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(cx^{\frac{3}{2}}\right)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)^2/x^4, x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arctanh}\left(c\,x^{\frac{3}{2}}\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))^2/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x^(3/2)))^2/x^4,x)
```

maxima [B] time = 0.33, size = 175, normalized size = 1.82

$$\frac{1}{3}\left(\left(c\log\left(cx^{\frac{3}{2}}+1\right)-c\log\left(cx^{\frac{3}{2}}-1\right)-\frac{2}{x^{\frac{3}{2}}}\right)c-\frac{2\,\text{artanh}\left(cx^{\frac{3}{2}}\right)}{x^3}\right)ab+\frac{1}{12}\left(\left(2\left(\log\left(cx^{\frac{3}{2}}-1\right)-2\right)\log\left(cx^{\frac{3}{2}}+1\right)-\text{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*((c*log(c*x^(3/2) + 1) - c*log(c*x^(3/2) - 1) - 2/x^(3/2))*c - 2*arctan
h(c*x^(3/2))/x^3)*a*b + 1/12*((2*(log(c*x^(3/2) - 1) - 2)*log(c*x^(3/2) + 1
) - log(c*x^(3/2) + 1)^2 - log(c*x^(3/2) - 1)^2 - 4*log(c*x^(3/2) - 1) + 12
*log(x))*c^2 + 4*(c*log(c*x^(3/2) + 1) - c*log(c*x^(3/2) - 1) - 2/x^(3/2))*
c*arctanh(c*x^(3/2)))*b^2 - 1/3*b^2*arctanh(c*x^(3/2))^2/x^3 - 1/3*a^2/x^3
```

mupad [B] time = 2.04, size = 281, normalized size = 2.93

$$\frac{2\,b^2\,c^2\,\ln\left(x^{3/2}\right)}{3}-\frac{a^2}{3\,x^3}-\frac{b^2\,c^2\,\ln\left(c\,x^{3/2}-1\right)}{3}-\frac{b^2\,c^2\,\ln\left(c\,x^{3/2}+1\right)}{3}+\frac{b^2\,c^2\ln\left(c\,x^{3/2}+1\right)^2}{12}+\frac{b^2\,c^2\ln\left(1-c\,x^{3/2}\right)^2}{12}-\frac{b^2\,\text{ln}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^(3/2)))^2/x^4,x)
```

```
[Out] (2*b^2*c^2*log(x^(3/2)))/3 - a^2/(3*x^3) - (b^2*c^2*log(c*x^(3/2) - 1))/3 -
 (b^2*c^2*log(c*x^(3/2) + 1))/3 + (b^2*c^2*log(c*x^(3/2) + 1)^2)/12 + (b^2*
c^2*log(1 - c*x^(3/2))^2)/12 - (b^2*log(c*x^(3/2) + 1)^2)/(12*x^3) - (b^2*l
og(1 - c*x^(3/2))^2)/(12*x^3) - (a*b*c^2*log(c*x^(3/2) - 1))/3 + (a*b*c^2*l
og(c*x^(3/2) + 1))/3 - (2*a*b*c)/(3*x^(3/2)) - (a*b*log(c*x^(3/2) + 1))/(3*
x^3) + (a*b*log(1 - c*x^(3/2)))/(3*x^3) - (b^2*c^2*log(c*x^(3/2) + 1)*log(1
 - c*x^(3/2)))/6 - (b^2*c*log(c*x^(3/2) + 1))/(3*x^(3/2)) + (b^2*c*log(1 -
c*x^(3/2)))/(3*x^(3/2)) + (b^2*log(c*x^(3/2) + 1)*log(1 - c*x^(3/2)))/(6*x^
3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))**2/x**4,x)
```

```
[Out] Timed out
```

3.224 $\int x^2 \left(a + b \tanh^{-1}(cx^n)\right) dx$

Optimal. Leaf size=66

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx^n)\right)-\frac{bcnx^{n+3}\,{}_2F_1\left(1,\frac{n+3}{2n};\frac{3(n+1)}{2n};c^2x^{2n}\right)}{3(n+3)}$$

```
[Out] 1/3*x^3*(a+b*arctanh(c*x^n))-1/3*b*c*n*x^(3+n)*hypergeom([1, 1/2*(3+n)/n],[
3/2*(1+n)/n],c^2*x^(2*n))/(3+n)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6097, 364}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx^n)\right)-\frac{bcnx^{n+3}\,{}_2F_1\left(1,\frac{n+3}{2n};\frac{3(n+1)}{2n};c^2x^{2n}\right)}{3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (x^3*(a + b*ArcTanh[c*x^n]))/3 - (b*c*n*x^(3 + n)*Hypergeometric2F1[1, (3 +
 n)/(2*n), (3*(1 + n))/(2*n), c^2*x^(2*n)])/(3*(3 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tanh^{-1}(cx^n)\right)dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx^n)\right)-\frac{1}{3}(bcn)\int\frac{x^{2+n}}{1-c^2x^{2n}}\,dx\\ &= \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx^n)\right)-\frac{bcnx^{3+n}\,{}_2F_1\left(1,\frac{3+n}{2n};\frac{3(1+n)}{2n};c^2x^{2n}\right)}{3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 1.11

$$\frac{ax^3}{3}-\frac{bcnx^{n+3}\,{}_2F_1\left(1,\frac{n+3}{2n};\frac{n+3}{2n}+1;c^2x^{2n}\right)}{3(n+3)}+\frac{1}{3}bx^3\tanh^{-1}(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*ArcTanh[c*x^n])/3 - (b*c*n*x^(3 + n)*Hypergeometric2F1[1
, (3 + n)/(2*n), 1 + (3 + n)/(2*n), c^2*x^(2*n)])/(3*(3 + n))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(bx^2 \operatorname{artanh}\left(cx^n\right) + ax^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x^2*arctanh(c*x^n) + a*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{artanh}\left(cx^n\right) + a)x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)*x^2, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{arctanh}\left(c\, x^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^n)),x)
```

```
[Out] int(x^2*(a+b*arctanh(c*x^n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}ax^3 + \frac{1}{6}\left(x^3 \log\left(cx^n + 1\right) - x^3 \log\left(-cx^n + 1\right) + 3\,n \int \frac{x^2}{3\left(cx^n + 1\right)}\, dx + 3\,n \int \frac{x^2}{3\left(cx^n - 1\right)}\, dx\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(x^3*log(c*x^n + 1) - x^3*log(-c*x^n + 1) + 3*n*integrate(1
/3*x^2/(c*x^n + 1), x) + 3*n*integrate(1/3*x^2/(c*x^n - 1), x))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\, \left(a + b \operatorname{atanh}\left(c\, x^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atanh(c*x^n)),x)
```

```
[Out] int(x^2*(a + b*atanh(c*x^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{atanh}\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**n)),x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x**n)), x)
```

3.225 $\int x \left(a+b \tanh^{-1}(c x^n)\right) dx$

Optimal. Leaf size=67

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx^n)\right)-\frac{bcnx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);c^2x^{2n}\right)}{2(n+2)}$$

```
[Out] 1/2*x^2*(a+b*arctanh(c*x^n))-1/2*b*c*n*x^(2+n)*hypergeom([1, 1/2*(2+n)/n],[
3/2+1/n],c^2*x^(2*n))/(2+n)
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6097, 364}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx^n)\right)-\frac{bcnx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);c^2x^{2n}\right)}{2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (x^2*(a + b*ArcTanh[c*x^n]))/2 - (b*c*n*x^(2 + n)*Hypergeometric2F1[1, (2 +
 n)/(2*n), (3 + 2/n)/2, c^2*x^(2*n)])/(2*(2 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tanh^{-1}(cx^n)\right)dx &= \frac{1}{2}x^2\left(a+b\tanh^{-1}(cx^n)\right)-\frac{1}{2}(bcn)\int\frac{x^{1+n}}{1-c^2x^{2n}}\,dx\\ &= \frac{1}{2}x^2\left(a+b\tanh^{-1}(cx^n)\right)-\frac{bcnx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);c^2x^{2n}\right)}{2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 73, normalized size = 1.09

$$\frac{ax^2}{2}-\frac{bcnx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2n};\frac{n+2}{2n}+1;c^2x^{2n}\right)}{2(n+2)}+\frac{1}{2}bx^2\tanh^{-1}(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcTanh[c*x^n])/2 - (b*c*n*x^(2 + n)*Hypergeometric2F1[1
, (2 + n)/(2*n), 1 + (2 + n)/(2*n), c^2*x^(2*n)])/(2*(2 + n))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(bx\,\text{artanh}\left(cx^n\right) + ax, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x*arctanh(c*x^n) + a*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{artanh}\left(cx^n\right) + a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)*x, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x\left(a + b\,\text{arctanh}\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^n)),x)
```

```
[Out] int(x*(a+b*arctanh(c*x^n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,ax^2 + \frac{1}{4}\left(x^2\log\left(cx^n + 1\right) - x^2\log\left(-cx^n + 1\right) + 2\,n\int\frac{x}{2\left(cx^n + 1\right)}\,dx + 2\,n\int\frac{x}{2\left(cx^n - 1\right)}\,dx\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(x^2*log(c*x^n + 1) - x^2*log(-c*x^n + 1) + 2*n*integrate(1
/2*x/(c*x^n + 1), x) + 2*n*integrate(1/2*x/(c*x^n - 1), x))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a + b\,\text{atanh}\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^n)),x)
```

```
[Out] int(x*(a + b*atanh(c*x^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a + b\,\text{atanh}\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**n)),x)
```

```
[Out] Integral(x*(a + b*atanh(c*x**n)), x)
```

3.226 $\int \left(a + b\tanh^{-1}(cx^n)\right) dx$

Optimal. Leaf size=58

$$ax - \frac{bcnx^{n+1}\,{}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); c^2x^{2n}\right)}{n+1} + bx\tanh^{-1}(cx^n)$$

```
[Out] a*x+b*x*arctanh(c*x^n)-b*c*n*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n]
,c^2*x^(2*n))/(1+n)
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6091, 364}

$$ax - \frac{bcnx^{n+1}\,{}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); c^2x^{2n}\right)}{n+1} + bx\tanh^{-1}(cx^n)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x^n],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^n] - (b*c*n*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2
*n), (3 + n^(-1))/2, c^2*x^(2*n)])/(1 + n)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\tanh^{-1}(cx^n)\right) dx &= ax + b\int \tanh^{-1}(cx^n)\, dx \\
&= ax + bx\tanh^{-1}(cx^n) - (bcn)\int \frac{x^n}{1 - c^2x^{2n}}\, dx \\
&= ax + bx\tanh^{-1}(cx^n) - \frac{bcnx^{1+n}\,{}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); c^2x^{2n}\right)}{1+n}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.00

$$ax - \frac{bcnx^{n+1}\,{}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); c^2x^{2n}\right)}{n+1} + bx\tanh^{-1}(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x^n],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^n] - (b*c*n*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2
*n), (3 + n^(-1))/2, c^2*x^(2*n)])/(1 + n)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(b\,\text{artanh}\left(cx^{n}\right)+a,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^n),x, algorithm="fricas")
[Out] integral(b*arctanh(c*x^n) + a, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int b\,\text{artanh}\left(cx^{n}\right)+a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^n),x, algorithm="giac")
[Out] integrate(b*arctanh(c*x^n) + a, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int a+b\,\text{arctanh}\left(c\,x^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^n),x)
[Out] int(a+b*arctanh(c*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(n\int\frac{1}{cx^{n}+1}\,dx+n\int\frac{1}{cx^{n}-1}\,dx+x\log\left(cx^{n}+1\right)-x\log\left(-cx^{n}+1\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^n),x, algorithm="maxima")
[Out] 1/2*(n*integrate(1/(c*x^n + 1), x) + n*integrate(1/(c*x^n - 1), x) + x*log(
c*x^n + 1) - x*log(-c*x^n + 1))*b + a*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int a+b\,\text{atanh}\left(c\,x^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atanh(c*x^n),x)
[Out] int(a + b*atanh(c*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\text{atanh}\left(cx^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**n),x)
[Out] Integral(a + b*atanh(c*x**n), x)
```

3.227 $\int \frac{a+b\tanh^{-1}(cx^n)}{x}\,dx$

Optimal. Leaf size=36

$$a\log(x)-\frac{b\text{Li}_2(-cx^n)}{2n}+\frac{b\text{Li}_2(cx^n)}{2n}$$

```
[Out] a*ln(x)-1/2*b*polylog(2,-c*x^n)/n+1/2*b*polylog(2,c*x^n)/n
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6095, 5912}

$$-\frac{b\text{PolyLog}(2,-cx^n)}{2n}+\frac{b\text{PolyLog}(2,cx^n)}{2n}+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])/x,x]
```

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^n)])/(2*n) + (b*PolyLog[2, c*x^n])/(2*n)
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^n)}{x}\,dx&=\frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,x^n\right)}{n}\\&=a\log(x)-\frac{b\text{Li}_2(-cx^n)}{2n}+\frac{b\text{Li}_2(cx^n)}{2n}\end{aligned}$$

Mathematica [C] time = 0.08, size = 39, normalized size = 1.08

$$\frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)}{n}+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])/x,x]
```

```
[Out] (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2*x^(2*n)])/n + a*L
og[x]
```

fricas [B] time = 0.67, size = 141, normalized size = 3.92

$$-\frac{bn\log\left(c\cosh\left(n\log(x)\right)+c\sinh\left(n\log(x)\right)+1\right)\log(x)-bn\log\left(-c\cosh\left(n\log(x)\right)-c\sinh\left(n\log(x)\right)+1\right)\text{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] -1/2*(b*n*log(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1)*log(x) - b*n*log(-c*
cosh(n*log(x)) - c*sinh(n*log(x)) + 1)*log(x) - b*n*log(x)*log(-(c*cosh(n*l
og(x)) + c*sinh(n*log(x)) + 1)/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)) -
 2*a*n*log(x) - b*dilog(c*cosh(n*log(x)) + c*sinh(n*log(x))) + b*dilog(-c*c
osh(n*log(x)) - c*sinh(n*log(x))))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{artanh}(cx^n) + a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)/x, x)
```

maple [B] time = 0.04, size = 76, normalized size = 2.11

$$\frac{a\ln(cx^n)}{n}+\frac{b\ln(cx^n)\operatorname{arctanh}(cx^n)}{n}-\frac{b\operatorname{dilog}(cx^n)}{2n}-\frac{b\operatorname{dilog}(cx^n+1)}{2n}-\frac{b\ln(cx^n)\ln(cx^n+1)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))/x,x)
```

```
[Out] 1/n*a*ln(c*x^n)+1/n*b*ln(c*x^n)*arctanh(c*x^n)-1/2/n*b*dilog(c*x^n)-1/2/n*b
*dilog(c*x^n+1)-1/2/n*b*ln(c*x^n)*ln(c*x^n+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(n\int\frac{\log(x)}{cxx^n+x}\,dx+n\int\frac{\log(x)}{cxx^n-x}\,dx+\log(cx^n+1)\log(x)-\log(-cx^n+1)\log(x)\right)b+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] 1/2*(n*integrate(log(x)/(c*x*x^n + x), x) + n*integrate(log(x)/(c*x*x^n - x
), x) + log(c*x^n + 1)*log(x) - log(-c*x^n + 1)*log(x))*b + a*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\operatorname{atanh}(cx^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))/x,x)
```

```
[Out] int((a + b*atanh(c*x^n))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{atanh}(cx^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))/x, x)
```

3.228 $\int \frac{a+b\tanh^{-1}(cx^n)}{x^2}\,dx$

Optimal. Leaf size=67

$$-\frac{a+b\tanh^{-1}(cx^n)}{x}-\frac{bcnx^{n-1}\,{}_2F_1\left(1,-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);c^2x^{2n}\right)}{1-n}$$

```
[Out] (-a-b*arctanh(c*x^n))/x-b*c*n*x^(-1+n)*hypergeom([1, 1/2*(-1+n)/n],[3/2-1/2
/n],c^2*x^(2*n))/(1-n)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 364}

$$-\frac{a+b\tanh^{-1}(cx^n)}{x}-\frac{bcnx^{n-1}\,{}_2F_1\left(1,-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);c^2x^{2n}\right)}{1-n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])/x^2,x]
```

```
[Out] -((a + b*ArcTanh[c*x^n])/x) - (b*c*n*x^(-1 + n)*Hypergeometric2F1[1, -(1 -
n)/(2*n), (3 - n^(-1))/2, c^2*x^(2*n)])/(1 - n)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^n)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}(cx^n)}{x}+(bcn)\int \frac{x^{-2+n}}{1-c^2x^{2n}}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx^n)}{x}-\frac{bcnx^{-1+n}\,{}_2F_1\left(1,-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);c^2x^{2n}\right)}{1-n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.99

$$-\frac{a}{x}+\frac{bcnx^{n-1}\,{}_2F_1\left(1,\frac{n-1}{2n};\frac{n-1}{2n}+1;c^2x^{2n}\right)}{n-1}-\frac{b\tanh^{-1}(cx^n)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTanh[c*x^n])/x + (b*c*n*x^(-1 + n)*Hypergeometric2F1[1, (-1
+ n)/(2*n), 1 + (-1 + n)/(2*n), c^2*x^(2*n)])/(-1 + n)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx^n\right)+a}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="fricas")
[Out] integral((b*arctanh(c*x^n) + a)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\left(cx^n\right)+a}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^n) + a)/x^2, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{a+b\arctan\text{h}\left(c\,x^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))/x^2,x)
[Out] int((a+b*arctanh(c*x^n))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(n\int \frac{1}{cx^2x^n+x^2}\,dx+n\int \frac{1}{cx^2x^n-x^2}\,dx+\frac{\log\left(cx^n+1\right)-\log\left(-cx^n+1\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="maxima")
[Out] -1/2*(n*integrate(1/(c*x^2*x^n + x^2), x) + n*integrate(1/(c*x^2*x^n - x^2)
, x) + (log(c*x^n + 1) - log(-c*x^n + 1))/x)*b - a/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\text{atanh}\left(c\,x^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))/x^2,x)
[Out] int((a + b*atanh(c*x^n))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{atanh}\left(cx^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))/x**2,x)
[Out] Integral((a + b*atanh(c*x**n))/x**2, x)
```

3.229 $\int \frac{a+b\tanh^{-1}(cx^n)}{x^3}\,dx$

Optimal. Leaf size=70

$$-\frac{a+b\tanh^{-1}(cx^n)}{2x^2}-\frac{bcnx^{n-2}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);c^2x^{2n}\right)}{2(2-n)}$$

```
[Out] 1/2*(-a-b*arctanh(c*x^n))/x^2-1/2*b*c*n*x^(-2+n)*hypergeom([1, 1/2-1/n],[3/
2-1/n],c^2*x^(2*n))/(2-n)
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 364}

$$-\frac{a+b\tanh^{-1}(cx^n)}{2x^2}-\frac{bcnx^{n-2}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);c^2x^{2n}\right)}{2(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])/x^3,x]
```

```
[Out] -(a + b*ArcTanh[c*x^n])/(2*x^2) - (b*c*n*x^(-2 + n)*Hypergeometric2F1[1, (1
 - 2/n)/2, (3 - 2/n)/2, c^2*x^(2*n)])/(2*(2 - n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^n)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}(cx^n)}{2x^2}+\frac{1}{2}(bcn)\int \frac{x^{-3+n}}{1-c^2x^{2n}}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx^n)}{2x^2}-\frac{bcnx^{-2+n}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);c^2x^{2n}\right)}{2(2-n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 1.04

$$-\frac{a}{2x^2}+\frac{bcnx^{n-2}\,{}_2F_1\left(1,\frac{n-2}{2n};\frac{n-2}{2n}+1;c^2x^{2n}\right)}{2(n-2)}-\frac{b\tanh^{-1}(cx^n)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*ArcTanh[c*x^n])/(2*x^2) + (b*c*n*x^(-2 + n)*Hypergeometric2
F1[1, (-2 + n)/(2*n), 1 + (-2 + n)/(2*n), c^2*x^(2*n)])/(2*(-2 + n))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{artanh}(cx^n) + a}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^n) + a)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{artanh}(cx^n) + a}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)/x^3, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arctanh}(c\,x^n)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x^n))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\left(2\,n\int \frac{1}{2\left(cx^3x^n + x^3\right)}\,dx + 2\,n\int \frac{1}{2\left(cx^3x^n - x^3\right)}\,dx + \frac{\log(cx^n + 1) - \log(-cx^n + 1)}{x^2}\right)b - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*n*integrate(1/2/(c*x^3*x^n + x^3), x) + 2*n*integrate(1/2/(c*x^3*x^
n - x^3), x) + (log(c*x^n + 1) - log(-c*x^n + 1))/x^2)*b - 1/2*a/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{atanh}(c\,x^n)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))/x^3,x)
```

```
[Out] int((a + b*atanh(c*x^n))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atanh}(cx^n)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))/x**3, x)
```

3.230 $\int \frac{a+b\tanh^{-1}(cx^n)}{x^4}\,dx$

Optimal. Leaf size=72

$$-\frac{a+b\tanh^{-1}(cx^n)}{3x^3}-\frac{bcnx^{n-3}\,{}_2F_1\left(1,-\frac{3-n}{2n};-\frac{3(1-n)}{2n};c^2x^{2n}\right)}{3(3-n)}$$

```
[Out] 1/3*(-a-b*arctanh(c*x^n))/x^3-1/3*b*c*n*x^(-3+n)*hypergeom([1, 1/2*(-3+n)/n
],[-3/2*(1-n)/n],c^2*x^(2*n))/(3-n)
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 364}

$$-\frac{a+b\tanh^{-1}(cx^n)}{3x^3}-\frac{bcnx^{n-3}\,{}_2F_1\left(1,-\frac{3-n}{2n};-\frac{3(1-n)}{2n};c^2x^{2n}\right)}{3(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])/x^4,x]
```

```
[Out] -(a + b*ArcTanh[c*x^n])/(3*x^3) - (b*c*n*x^(-3 + n)*Hypergeometric2F1[1, -(
3 - n)/(2*n), (-3*(1 - n))/(2*n), c^2*x^(2*n)])/(3*(3 - n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^n)}{x^4}\,dx&=-\frac{a+b\tanh^{-1}(cx^n)}{3x^3}+\frac{1}{3}(bcn)\int \frac{x^{-4+n}}{1-c^2x^{2n}}\,dx\\&=-\frac{a+b\tanh^{-1}(cx^n)}{3x^3}-\frac{bcnx^{-3+n}\,{}_2F_1\left(1,-\frac{3-n}{2n};-\frac{3(1-n)}{2n};c^2x^{2n}\right)}{3(3-n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 1.01

$$-\frac{a}{3x^3}+\frac{bcnx^{n-3}\,{}_2F_1\left(1,\frac{n-3}{2n};\frac{n-3}{2n}+1;c^2x^{2n}\right)}{3(n-3)}-\frac{b\tanh^{-1}(cx^n)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*ArcTanh[c*x^n])/(3*x^3) + (b*c*n*x^(-3 + n)*Hypergeometric2
F1[1, (-3 + n)/(2*n), 1 + (-3 + n)/(2*n), c^2*x^(2*n)])/(3*(-3 + n))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx^{n}\right)+a}{x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^n) + a)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{artanh}\left(cx^{n}\right)+a}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)/x^4, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arctanh}\left(c\,x^{n}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x^n))/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\left(3\,n\int \frac{1}{3\left(cx^{4}x^{n}+x^{4}\right)}\,dx+3\,n\int \frac{1}{3\left(cx^{4}x^{n}-x^{4}\right)}\,dx+\frac{\log\left(cx^{n}+1\right)-\log\left(-cx^{n}+1\right)}{x^{3}}\right)b-\frac{a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*n*integrate(1/3/(c*x^4*x^n + x^4), x) + 3*n*integrate(1/3/(c*x^4*x^
n - x^4), x) + (log(c*x^n + 1) - log(-c*x^n + 1))/x^3)*b - 1/3*a/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\text{atanh}\left(c\,x^{n}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))/x^4,x)
```

```
[Out] int((a + b*atanh(c*x^n))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{atanh}\left(cx^{n}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))/x**4,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))/x**4, x)
```

3.231 $\int x\left(a+b\tanh^{-1}(cx^n)\right)^2\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(x\left(a+b\tanh^{-1}(cx^n)\right)^2,x\right)$$

```
[Out] Unintegrable(x*(a+b*arctanh(c*x^n))^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\tanh^{-1}(cx^n)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*ArcTanh[c*x^n])^2,x]
[Out] Defer[Int][x*(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}(cx^n)\right)^2\,dx=\int x\left(a+b\tanh^{-1}(cx^n)\right)^2\,dx$$

Mathematica [A] time = 13.83, size = 0, normalized size = 0.00

$$\int x\left(a+b\tanh^{-1}(cx^n)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^n])^2,x]
[Out] Integrate[x*(a + b*ArcTanh[c*x^n])^2, x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^2x\,\mathrm{artanh}(cx^n)^2+2\,abx\,\mathrm{artanh}(cx^n)+a^2x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
[Out] integral(b^2*x*arctanh(c*x^n)^2 + 2*a*b*x*arctanh(c*x^n) + a^2*x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\mathrm{artanh}(cx^n)+a)^2x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
[Out] integrate((b*arctanh(c*x^n) + a)^2*x, x)
```

maple [A] time = 0.19, size = 0, normalized size = 0.00

$$\int x\,(a+b\,\mathrm{arctanh}(c\,x^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^n))^2,x)
```

```
[Out] int(x*(a+b*arctanh(c*x^n))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\,b^2x^2\log\left(-cx^n+1\right)^2+\frac{1}{2}\,a^2x^2-\int-\frac{\left(b^2cxx^n-b^2x\right)\log\left(cx^n+1\right)^2+4\left(abcxx^n-abx\right)\log\left(cx^n+1\right)+\left(4\,abx-\right.}{4\left(cx^n-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^2*log(-c*x^n + 1)^2 + 1/2*a^2*x^2 - integrate(-1/4*((b^2*c*x*x^n
- b^2*x)*log(c*x^n + 1)^2 + 4*(a*b*c*x*x^n - a*b*x)*log(c*x^n + 1) + (4*a*b
*x - (b^2*c*n + 4*a*b*c)*x*x^n - 2*(b^2*c*x*x^n - b^2*x)*log(c*x^n + 1))*lo
g(-c*x^n + 1))/(c*x^n - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int x\,{\left(a+b\,\mathrm{atanh}\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atanh(c*x^n))^2,x)
```

```
[Out] int(x*(a + b*atanh(c*x^n))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{atanh}\left(cx^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**n))**2,x)
```

```
[Out] Integral(x*(a + b*atanh(c*x**n))**2, x)
```

3.232 $\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2,x\right)$$

```
[Out] Unintegrable((a+b*arctanh(c*x^n))^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx=\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx$$

Mathematica [A] time = 1.97, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^n])^2, x]
```

fricas [A] time = 0.98, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(b^2\,\mathrm{artanh}\left(cx^n\right)^2+2\,ab\,\mathrm{artanh}\left(cx^n\right)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\mathrm{artanh}\left(cx^n\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2, x)
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arctanh}\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))^2,x)
```

```
[Out] int((a+b*arctanh(c*x^n))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,b^2x\log\left(-cx^n+1\right)^2+a^2x-\int -\frac{\left(b^2cx^n-b^2\right)\log\left(cx^n+1\right)^2+4\left(abcx^n-ab\right)\log\left(cx^n+1\right)+2\left(2\,ab-\left(b^2cn+\right.\right.}{4\left(cx^n-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x*log(-c*x^n + 1)^2 + a^2*x - integrate(-1/4*((b^2*c*x^n - b^2)*log
(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b - (b^2*c*n +
2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1))/(c*x^n -
1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int {\left(a+b\,\mathrm{atanh}\left(c\,x^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))^2,x)
```

```
[Out] int((a + b*atanh(c*x^n))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{atanh}\left(cx^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))**2, x)
```

3.233 $\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=148

$$-\frac{b\text{Li}_2\left(1-\frac{2}{1-cx^n}\right)\left(a+b\tanh^{-1}(cx^n)\right)}{n}+\frac{b\text{Li}_2\left(\frac{2}{1-cx^n}-1\right)\left(a+b\tanh^{-1}(cx^n)\right)}{n}+\frac{2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)\left(a+b\tanh^{-1}\right.}{n}$$

```
[Out] -2*(a+b*arctanh(c*x^n))^2*arctanh(-1+2/(1-c*x^n))/n-b*(a+b*arctanh(c*x^n))*
polylog(2,1-2/(1-c*x^n))/n+b*(a+b*arctanh(c*x^n))*polylog(2,-1+2/(1-c*x^n))
/n+1/2*b^2*polylog(3,1-2/(1-c*x^n))/n-1/2*b^2*polylog(3,-1+2/(1-c*x^n))/n
```

Rubi [A] time = 0.31, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx^n}\right)\left(a+b\tanh^{-1}(cx^n)\right)}{n}+\frac{b\text{PolyLog}\left(2,\frac{2}{1-cx^n}-1\right)\left(a+b\tanh^{-1}(cx^n)\right)}{n}+\frac{b^2\text{PolyLog}\left(3,1-\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])^2/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^n])^2*ArcTanh[1 - 2/(1 - c*x^n)])/n - (b*(a + b*ArcTa
nh[c*x^n])*PolyLog[2, 1 - 2/(1 - c*x^n)])/n + (b*(a + b*ArcTanh[c*x^n])*Pol
yLog[2, -1 + 2/(1 - c*x^n)])/n + (b^2*PolyLog[3, 1 - 2/(1 - c*x^n)])/(2*n)
- (b^2*PolyLog[3, -1 + 2/(1 - c*x^n)])/(2*n)
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n} - \frac{(4bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\tanh^{-1}(}{1-c^2x^2}\right.}{n}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n} + \frac{(2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-c}\right.}{1-c^2x^2}\right.}{n}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n} - \frac{b\left(a+b\tanh^{-1}(cx^n)\right)\text{Li}_2\left(1-\frac{2}{1-cx}\right.}{n}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n} - \frac{b\left(a+b\tanh^{-1}(cx^n)\right)\text{Li}_2\left(1-\frac{2}{1-cx}\right.}{n}$$

Mathematica [A] time = 0.10, size = 183, normalized size = 1.24

$$\frac{2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)\left(a+b\tanh^{-1}(cx^n)\right)^2-4bc\left(\frac{1}{2}\left(\frac{\text{Li}_2\left(\frac{-cx^n-1}{cx^n-1}\right)\left(-a-b\tanh^{-1}(cx^n)\right)}{2c}+\frac{b\text{Li}_3\left(\frac{-cx^n-1}{cx^n-1}\right)}{4c}\right)+\frac{1}{2}\left(-\frac{\text{Li}_2\left(\frac{cx^n+1}{cx^n-1}\right)(-a\right.}{}\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])^2/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^n])^2*ArcTanh[1 - 2/(1 - c*x^n)] - 4*b*c*((((-a - b*A
rcTanh[c*x^n])*PolyLog[2, (-1 - c*x^n)/(-1 + c*x^n)])/(2*c) + (b*PolyLog[3,
 (-1 - c*x^n)/(-1 + c*x^n)])/(4*c))/2 + (-1/2*((-a - b*ArcTanh[c*x^n])*Poly
Log[2, (1 + c*x^n)/(-1 + c*x^n)])/c - (b*PolyLog[3, (1 + c*x^n)/(-1 + c*x^n
)])/(4*c))/2))/n
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(cx^n\right)^2+2\,ab\,\text{artanh}\left(cx^n\right)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{artanh}(cx^n) + a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2/x, x)
```

maple [C] time = 0.24, size = 880, normalized size = 5.95

$$\frac{a^2\ln(c\,x^n)}{n}+\frac{b^2\ln(c\,x^n)\operatorname{arctanh}(c\,x^n)^2}{n}-\frac{b^2\operatorname{arctanh}(c\,x^n)\operatorname{polylog}\left(2,-\frac{(c\,x^n+1)^2}{-c^2x^{2n}+1}\right)}{n}+\frac{b^2\operatorname{polylog}\left(3,-\frac{(c\,x^n+1)^2}{-c^2x^{2n}+1}\right)}{2n}-\frac{b^2}{\ }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))^2/x,x)
```

```
[Out] 1/n*a^2*ln(c*x^n)+1/n*b^2*ln(c*x^n)*arctanh(c*x^n)^2-1/n*b^2*arctanh(c*x^n)
*polylog(2,-(c*x^n+1)^2/(-c^2*(x^n)^2+1))+1/2/n*b^2*polylog(3,-(c*x^n+1)^2/
(-c^2*(x^n)^2+1))-1/n*b^2*arctanh(c*x^n)^2*ln((c*x^n+1)^2/(-c^2*(x^n)^2+1)-
1)+1/n*b^2*arctanh(c*x^n)^2*ln(1-(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+2/n*b^2*
arctanh(c*x^n)*polylog(2,(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))-2/n*b^2*polylog(
3,(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+1/n*b^2*arctanh(c*x^n)^2*ln(1+(c*x^n+1)
/(-c^2*(x^n)^2+1)^(1/2))+2/n*b^2*arctanh(c*x^n)*polylog(2,-(c*x^n+1)/(-c^2*
(x^n)^2+1)^(1/2))-2/n*b^2*polylog(3,-(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))-1/2*
I/n*b^2*Pi*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)^2+1)-1))*csgn(I*((c*x^n+1)^2/(-c
^2*(x^n)^2+1)-1)/(1+(c*x^n+1)^2/(-c^2*(x^n)^2+1)))^2*arctanh(c*x^n)^2+1/2*I
/n*b^2*Pi*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)^2+1)-1)/(1+(c*x^n+1)^2/(-c^2*(x^n
)^2+1)))^3*arctanh(c*x^n)^2+1/2*I/n*b^2*Pi*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)^
2+1)-1))*csgn(I/(1+(c*x^n+1)^2/(-c^2*(x^n)^2+1)))*csgn(I*((c*x^n+1)^2/(-c^2
*(x^n)^2+1)-1)/(1+(c*x^n+1)^2/(-c^2*(x^n)^2+1)))*arctanh(c*x^n)^2-1/2*I/n*b
^2*Pi*csgn(I/(1+(c*x^n+1)^2/(-c^2*(x^n)^2+1)))*csgn(I*((c*x^n+1)^2/(-c^2*(x
^n)^2+1)-1)/(1+(c*x^n+1)^2/(-c^2*(x^n)^2+1)))^2*arctanh(c*x^n)^2+2/n*a*b*ln
(c*x^n)*arctanh(c*x^n)-1/n*a*b*ln(c*x^n)*ln(c*x^n+1)-1/n*a*b*dilog(c*x^n)-1
/n*a*b*dilog(c*x^n+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}b^2\log(-cx^n+1)^2\log(x)+a^2\log(x)-\int -\frac{\left(b^2cx^n-b^2\right)\log(cx^n+1)^2+4\,(abcx^n-ab)\log(cx^n+1)+2\left(2\,ab-\left(\right.\right.}{4\,(cxx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*log(-c*x^n + 1)^2*log(x) + a^2*log(x) - integrate(-1/4*((b^2*c*x^n
- b^2)*log(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b - (
b^2*c*n*log(x) + 2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^
n + 1))/(c*x*x^n - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{atanh}(c\,x^n))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))^2/x,x)
```

```
[Out] int((a + b*atanh(c*x^n))^2/x, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atanh}(cx^n))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))**2/x, x)
```

3.234 $\int \frac{\left(a+b \tanh^{-1}(cx^n)\right)^2}{x^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arctanh(c*x^n))^2/x^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^n])^2/x^2,x]
[Out] Defer[Int][(a + b*ArcTanh[c*x^n])^2/x^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2} dx = \int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2} dx$$

Mathematica [A] time = 17.80, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])^2/x^2,x]
[Out] Integrate[(a + b*ArcTanh[c*x^n])^2/x^2, x]
```

fricas [A] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}\left(cx^n\right)^2 + 2\,ab \operatorname{artanh}\left(cx^n\right) + a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{artanh}\left(cx^n\right) + a\right)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2/x^2, x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arctanh}(c\, x^n))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))^2/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x^n))^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2 \log(-cx^n + 1)^2}{4\,x} - \frac{a^2}{x} - \int -\frac{\left(b^2cx^n - b^2\right) \log(cx^n + 1)^2 + 4\,(abcx^n - ab) \log(cx^n + 1) + 2\left(2\,ab + \left(b^2cn - 2\,a\right.\right.}{4\left(cx^2x^n - x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*log(-c*x^n + 1)^2/x - a^2/x - integrate(-1/4*((b^2*c*x^n - b^2)*lo
g(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b + (b^2*c*n -
 2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1))/(c*x^2*x
^n - x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a + b \operatorname{atanh}(c\, x^n))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))^2/x^2,x)
```

```
[Out] int((a + b*atanh(c*x^n))^2/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atanh}(cx^n))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))**2/x**2, x)
```

3.235 $\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x^3}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*arctanh(c*x^n))^2/x^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^n])^2/x^3,x]
[Out] Defer[Int][(a + b*ArcTanh[c*x^n])^2/x^3, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^3}\,dx$$

Mathematica [A] time = 17.65, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])^2/x^3,x]
[Out] Integrate[(a + b*ArcTanh[c*x^n])^2/x^3, x]
```

fricas [A] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(cx^n\right)^2+2\,ab\,\text{artanh}\left(cx^n\right)+a^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="fricas")
[Out] integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{artanh}\left(cx^n\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2/x^3, x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{arctanh}(c\,x^n))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))^2/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x^n))^2/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2\log(-cx^n+1)^2}{8\,x^2}-\frac{a^2}{2\,x^2}-\int -\frac{\left(b^2cx^n-b^2\right)\log(cx^n+1)^2+4\,(abcx^n-ab)\log(cx^n+1)+\left(4\,ab+\left(b^2cn-4\,a\right.\right.}{4\left(cx^3x^n-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*log(-c*x^n + 1)^2/x^2 - 1/2*a^2/x^2 - integrate(-1/4*((b^2*c*x^n -
 b^2)*log(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + (4*a*b + (b^2
*c*n - 4*a*b*c)*x^n - 2*(b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1))/
(c*x^3*x^n - x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a + b\operatorname{atanh}(c\,x^n))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atanh(c*x^n))^2/x^3,x)
```

```
[Out] int((a + b*atanh(c*x^n))^2/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atanh}(cx^n))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))**2/x**3, x)
```

3.236 $\int \frac{\tanh^{-1}(ax^n)}{x} \, dx$

Optimal. Leaf size=30

$$\frac{\text{Li}_2(ax^n)}{2n} - \frac{\text{Li}_2(-ax^n)}{2n}$$

```
[Out] -1/2*polylog(2,-a*x^n)/n+1/2*polylog(2,a*x^n)/n
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6095, 5912}

$$\frac{\text{PolyLog}(2, ax^n)}{2n} - \frac{\text{PolyLog}(2, -ax^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x^n]/x,x]
```

```
[Out] -PolyLog[2, -(a*x^n)]/(2*n) + PolyLog[2, a*x^n]/(2*n)
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax^n)}{x} \, dx &= \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(ax)}{x} \, dx, x, x^n\right)}{n} \\
&= -\frac{\text{Li}_2(-ax^n)}{2n} + \frac{\text{Li}_2(ax^n)}{2n}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 33, normalized size = 1.10

$$\frac{ax^n \, {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; a^2x^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x^n]/x,x]
```

```
[Out] (a*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, a^2*x^(2*n)])/n
```

fricas [B] time = 1.32, size = 129, normalized size = 4.30

$$-\frac{n \log\left(a \cosh\left(n \log(x)\right) + a \sinh\left(n \log(x)\right) + 1\right) \log(x) - n \log\left(-a \cosh\left(n \log(x)\right) - a \sinh\left(n \log(x)\right) + 1\right) \log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(n*log(a*cosh(n*log(x)) + a*sinh(n*log(x)) + 1)*log(x) - n*log(-a*cosh
(n*log(x)) - a*sinh(n*log(x)) + 1)*log(x) - n*log(x)*log(-(a*cosh(n*log(x))
 + a*sinh(n*log(x)) + 1)/(a*cosh(n*log(x)) + a*sinh(n*log(x)) - 1)) - dilog
(a*cosh(n*log(x)) + a*sinh(n*log(x))) + dilog(-a*cosh(n*log(x)) - a*sinh(n*
log(x))))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x^n)/x, x)
```

maple [B] time = 0.04, size = 61, normalized size = 2.03

$$\frac{\ln(a\,x^n)\operatorname{arctanh}(a\,x^n)}{n} - \frac{\operatorname{dilog}(a\,x^n)}{2n} - \frac{\operatorname{dilog}(a\,x^n+1)}{2n} - \frac{\ln(a\,x^n)\ln(a\,x^n+1)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x^n)/x,x)
```

```
[Out] 1/n*ln(a*x^n)*arctanh(a*x^n)-1/2/n*dilog(a*x^n)-1/2/n*dilog(a*x^n+1)-1/2/n*
ln(a*x^n)*ln(a*x^n+1)
```

maxima [B] time = 0.42, size = 147, normalized size = 4.90

$$-\frac{1}{2}an\left(\frac{\log\left(\frac{ax^n+1}{a}\right)}{an} - \frac{\log\left(\frac{ax^n-1}{a}\right)}{an}\right)\log(x)+\frac{1}{2}an\left(\frac{\log(ax^n+1)\log(x)-\log(ax^n-1)\log(x)}{an} - \frac{n\log(ax^n+1)\,\mathrm{l}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*n*(log((a*x^n + 1)/a)/(a*n) - log((a*x^n - 1)/a)/(a*n))*log(x) + 1/2
*a*n*((log(a*x^n + 1)*log(x) - log(a*x^n - 1)*log(x))/(a*n) - (n*log(a*x^n
+ 1)*log(x) + dilog(-a*x^n))/(a*n^2) + (n*log(-a*x^n + 1)*log(x) + dilog(a*
x^n))/(a*n^2)) + arctanh(a*x^n)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{atanh}(a\,x^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x^n)/x,x)
```

```
[Out] int(atanh(a*x^n)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x**n)/x,x)
```

```
[Out] Integral(atanh(a*x**n)/x, x)
```

3.237 $\int \frac{\tanh^{-1}(ax^5)}{x} \, dx$

Optimal. Leaf size=24

$$\frac{\text{Li}_2(ax^5)}{10} - \frac{1}{10}\text{Li}_2(-ax^5)$$

```
[Out] -1/10*polylog(2,-a*x^5)+1/10*polylog(2,a*x^5)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6095, 5912}

$$\frac{1}{10}\text{PolyLog}(2, ax^5) - \frac{1}{10}\text{PolyLog}(2, -ax^5)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x^5]/x,x]
```

```
[Out] -PolyLog[2, -(a*x^5)]/10 + PolyLog[2, a*x^5]/10
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax^5)}{x} \, dx &= \frac{1}{5}\text{Subst}\left(\int \frac{\tanh^{-1}(ax)}{x} \, dx, x, x^5\right) \\ &= -\frac{1}{10}\text{Li}_2(-ax^5) + \frac{\text{Li}_2(ax^5)}{10}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$\frac{1}{10}\left(\text{Li}_2(ax^5) - \text{Li}_2(-ax^5)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x^5]/x,x]
```

```
[Out] (-PolyLog[2, -(a*x^5)] + PolyLog[2, a*x^5])/10
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}(ax^5)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x^5)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x^5)/x, x)
```

maple [C] time = 0.10, size = 85, normalized size = 3.54

$$\ln(x)\operatorname{arctanh}\left(a\,x^5\right)-\frac{\left(\sum_{\_R1=\mathrm{RootOf}\left(a\,\_Z^5+1\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}+\frac{\left(\sum_{\_R1=\mathrm{RootOf}\left(a\,\_Z^5-1\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right.\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x^5)/x,x)
```

```
[Out] ln(x)*arctanh(a*x^5)-1/2*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=R
ootOf(_Z^5*a+1))+1/2*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootO
f(_Z^5*a-1))
```

maxima [B] time = 0.31, size = 104, normalized size = 4.33

$$-\frac{1}{2}a\left(\frac{\log\left(ax^5+1\right)}{a}-\frac{\log\left(ax^5-1\right)}{a}\right)\log(x)-\frac{1}{10}a\left(\frac{\log\left(ax^5-1\right)\log\left(ax^5\right)+\mathrm{Li}_2\left(-ax^5+1\right)}{a}-\frac{\log\left(ax^5+1\right)\log\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(a*x^5 + 1)/a - log(a*x^5 - 1)/a)*log(x) - 1/10*a*((log(a*x^5 -
1)*log(a*x^5) + dilog(-a*x^5 + 1))/a - (log(a*x^5 + 1)*log(-a*x^5) + dilog(
a*x^5 + 1))/a) + arctanh(a*x^5)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atanh}\left(a\,x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a*x^5)/x,x)
```

```
[Out] int(atanh(a*x^5)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x**5)/x,x)
```

```
[Out] Timed out
```

3.238 $\int \tanh^{-1}\left(\frac{1}{x}\right)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\log\left(1-x^2\right)+x\tanh^{-1}\left(\frac{1}{x}\right)$$

```
[Out] x*arctanh(1/x)+1/2*ln(-x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6091, 263, 260}

$$\frac{1}{2}\log\left(1-x^2\right)+x\tanh^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x^(-1)],x]
[Out] x*ArcTanh[x^(-1)] + Log[1 - x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}\left(\frac{1}{x}\right)\,dx &= x\tanh^{-1}\left(\frac{1}{x}\right)+\int \frac{1}{\left(1-\frac{1}{x^2}\right)x}\,dx\\ &= x\tanh^{-1}\left(\frac{1}{x}\right)+\int \frac{x}{-1+x^2}\,dx\\ &= x\tanh^{-1}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.89

$$\frac{1}{2}\log\left(x^2-1\right)+x\tanh^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x^(-1)],x]
[Out] x*ArcTanh[x^(-1)] + Log[-1 + x^2]/2
```

fricas [**A**] time = 0.64, size = 22, normalized size = 1.16

$$\frac{1}{2}\,x\log\left(\frac{x+1}{x-1}\right)+\frac{1}{2}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1/x),x, algorithm="fricas")
```

```
[Out] 1/2*x*log((x + 1)/(x - 1)) + 1/2*log(x^2 - 1)
```

giac [**B**] time = 0.16, size = 101, normalized size = 5.32

$$\frac{\log\left(-\frac{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}+1}{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}-1}\right)}{\frac{x+1}{x-1}-1}+\log\left(\frac{|x+1|}{|x-1|}\right)-\log\left(\left|\frac{x+1}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1/x),x, algorithm="giac")
```

```
[Out] log(-(((x + 1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) + 1)/(((x + 1)/(x - 1) -
1)/((x + 1)/(x - 1) + 1) - 1))/((x + 1)/(x - 1) - 1) + log(abs(x + 1)/abs(x
 - 1)) - log(abs((x + 1)/(x - 1) - 1))
```

maple [**A**] time = 0.05, size = 30, normalized size = 1.58

$$x\,\mathrm{arctanh}\left(\frac{1}{x}\right)-\ln\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}-1\right)}{2}+\frac{\ln\left(\frac{1}{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1/x),x)
```

```
[Out] x*arctanh(1/x)-ln(1/x)+1/2*ln(1/x-1)+1/2*ln(1/x+1)
```

maxima [**A**] time = 0.31, size = 15, normalized size = 0.79

$$x\,\mathrm{artanh}\left(\frac{1}{x}\right)+\frac{1}{2}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1/x),x, algorithm="maxima")
```

```
[Out] x*arctanh(1/x) + 1/2*log(x^2 - 1)
```

mupad [**B**] time = 0.07, size = 15, normalized size = 0.79

$$\frac{\ln\left(x^2-1\right)}{2}+x\,\mathrm{atanh}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(1/x),x)
```

```
[Out] log(x^2 - 1)/2 + x*atanh(1/x)
```

sympy [A] time = 0.19, size = 15, normalized size = 0.79

$$x \operatorname{atanh}\left(\frac{1}{x}\right) + \log(x+1) - \operatorname{atanh}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1/x),x)
[Out] x*atanh(1/x) + log(x + 1) - atanh(1/x)
```

3.239 $\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right)^3 dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\tanh^{-1}(cx^n)\right)^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x^n))^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^n])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^n])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right)^3 dx = \int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right)^3 dx$$

Mathematica [A] time = 8.42, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^3, x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left((dx)^m b^3 \operatorname{artanh}(cx^n)^3 + 3\,(dx)^m ab^2 \operatorname{artanh}(cx^n)^2 + 3\,(dx)^m a^2 b \operatorname{artanh}(cx^n) + (dx)^m a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*b^3*arctanh(c*x^n)^3 + 3*(d*x)^m*a*b^2*arctanh(c*x^n)^2 +
3*(d*x)^m*a^2*b*arctanh(c*x^n) + (d*x)^m*a^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{artanh}(cx^n) + a)^3 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^3*(d*x)^m, x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{arctanh}(c\,x^n))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^n))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^n))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3d^mxx^m\log\left(-cx^n+1\right)^3}{8\left(m+1\right)}+\frac{(dx)^{m+1}\,a^3}{d(m+1)}+\int\frac{\left(b^3cd^m(m+1)e^{\left(m\log(x)+n\log(x)\right)}-b^3d^m(m+1)x^m\right)\log\left(cx^n+1\right)^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*d^m*x*x^m*log(-c*x^n + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1)
) + integrate(1/8*((b^3*c*d^m*(m + 1)*e^(m*log(x) + n*log(x)) - b^3*d^m*(m
+ 1)*x^m)*log(c*x^n + 1)^3 + 6*(a*b^2*c*d^m*(m + 1)*e^(m*log(x) + n*log(x))
 - a*b^2*d^m*(m + 1)*x^m)*log(c*x^n + 1)^2 - 3*(2*a*b^2*d^m*(m + 1)*x^m - (
2*a*b^2*c*d^m*(m + 1) + b^3*c*d^m*n)*e^(m*log(x) + n*log(x)) - (b^3*c*d^m*(
m + 1)*e^(m*log(x) + n*log(x)) - b^3*d^m*(m + 1)*x^m)*log(c*x^n + 1))*log(-
c*x^n + 1)^2 + 12*(a^2*b*c*d^m*(m + 1)*e^(m*log(x) + n*log(x)) - a^2*b*d^m*
(m + 1)*x^m)*log(c*x^n + 1) - 3*(4*a^2*b*c*d^m*(m + 1)*e^(m*log(x) + n*log(
x)) - 4*a^2*b*d^m*(m + 1)*x^m + (b^3*c*d^m*(m + 1)*e^(m*log(x) + n*log(x))
- b^3*d^m*(m + 1)*x^m)*log(c*x^n + 1)^2 + 4*(a*b^2*c*d^m*(m + 1)*e^(m*log(x
) + n*log(x)) - a*b^2*d^m*(m + 1)*x^m)*log(c*x^n + 1))*log(-c*x^n + 1))/(c*
(m + 1)*x^n - m - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int {\left(d\,x\right)}^m\,{\left(a+b\,\mathrm{atanh}\left(c\,x^n\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^n))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^n))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**n))**3,x)
```

```
[Out] Timed out
```

3.240 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctanh(c*x^n))^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi steps

$$\int (dx)^m\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx=\int (dx)^m\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx$$

Mathematica [A] time = 5.97, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^2, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left((dx)^m b^2\,\text{artanh}\left(cx^n\right)^2+2\,(dx)^m ab\,\text{artanh}\left(cx^n\right)+(dx)^m a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*b^2*arctanh(c*x^n)^2 + 2*(d*x)^m*a*b*arctanh(c*x^n) + (d*x
)^m*a^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\text{artanh}\left(cx^n\right)+a\right)^2 (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2*(d*x)^m, x)
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a+b\arctan\text{h}\left(c\,x^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^n))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^n))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2 d^m x x^m \log\left(-cx^n+1\right)^2}{4\,(m+1)} + \frac{(dx)^{m+1}\,a^2}{d(m+1)} - \int -\frac{\left(b^2 c d^m (m+1) e^{\left(m\log(x)+n\log(x)\right)} - b^2 d^m (m+1) x^m\right)\log\left(cx^n+1\right)^2 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d^m*x*x^m*log(-c*x^n + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
 - integrate(-1/4*((b^2*c*d^m*(m + 1)*e^(m*log(x) + n*log(x)) - b^2*d^m*(m
+ 1)*x^m)*log(c*x^n + 1)^2 + 4*(a*b*c*d^m*(m + 1)*e^(m*log(x) + n*log(x)) -
 a*b*d^m*(m + 1)*x^m)*log(c*x^n + 1) + 2*(2*a*b*d^m*(m + 1)*x^m - (2*a*b*c*
d^m*(m + 1) + b^2*c*d^m*n)*e^(m*log(x) + n*log(x)) - (b^2*c*d^m*(m + 1)*e^(
m*log(x) + n*log(x)) - b^2*d^m*(m + 1)*x^m)*log(c*x^n + 1))*log(-c*x^n + 1)
)/(c*(m + 1)*x^n - m - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atanh}\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^n))^2,x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**n))**2,x)
```

```
[Out] Timed out
```

3.241 $\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right) dx$

Optimal. Leaf size=84

$$\frac{x(dx)^m \left(a+b\tanh^{-1}(cx^n)\right)}{m+1}-\frac{bcnx^{n+1}(dx)^m {}_2F_1\left(1,\frac{m+n+1}{2n};\frac{m+3n+1}{2n};c^2x^{2n}\right)}{(m+1)(m+n+1)}$$

```
[Out] x*(d*x)^m*(a+b*arctanh(c*x^n))/(1+m)-b*c*n*x^(1+n)*(d*x)^m*hypergeom([1, 1/
2*(1+m+n)/n],[1/2*(1+m+3*n)/n],c^2*x^(2*n))/(1+m)/(1+m+n)
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6097, 20, 364}

$$\frac{(dx)^{m+1} \left(a+b\tanh^{-1}(cx^n)\right)}{d(m+1)}-\frac{bcnx^{n+1}(dx)^m {}_2F_1\left(1,\frac{m+n+1}{2n};\frac{m+3n+1}{2n};c^2x^{2n}\right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x^n]))/(d*(1 + m)) - (b*c*n*x^(1 + n)*(d*x)
^m*Hypergeometric2F1[1, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), c^2*x^(2*n)
])/((1 + m)*(1 + m + n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right) dx &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx^n)\right)}{d(1+m)}-\frac{(bcn)\int \frac{x^{-1+n}(dx)^{1+m}}{1-c^2x^{2n}}\,dx}{d(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx^n)\right)}{d(1+m)}-\frac{\left(bcnx^{-m}(dx)^m\right)\int \frac{x^{m+n}}{1-c^2x^{2n}}\,dx}{1+m}\\
&= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx^n)\right)}{d(1+m)}-\frac{bcnx^{1+n}(dx)^m {}_2F_1\left(1,\frac{1+m+n}{2n};\frac{1+m+3n}{2n};c^2x^{2n}\right)}{(1+m)(1+m+n)}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 77, normalized size = 0.92

$$\frac{x(dx)^m\left((m+n+1)\left(a+b\tanh^{-1}(cx^n)\right)-bcnx^n\,{}_2F_1\left(1,\frac{m+n+1}{2n};\frac{m+3n+1}{2n};c^2x^{2n}\right)\right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (x*(d*x)^m*((1 + m + n)*(a + b*ArcTanh[c*x^n]) - b*c*n*x^n*Hypergeometric2F
1[1, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), c^2*x^(2*n)]))/((1 + m)*(1 + m
 + n))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left((dx)^m\,b\,\text{artanh}\,(cx^n)+(dx)^m\,a,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*b*arctanh(c*x^n) + (d*x)^m*a, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{artanh}\,(cx^n)+a)\,(dx)^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)*(d*x)^m, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (dx)^m\,(a+b\,\text{arctanh}\,(c\,x^n))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^n)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(d^m n\int\frac{x^m}{c(m+1)x^n+m+1}\,dx+d^m n\int\frac{x^m}{c(m+1)x^n-m-1}\,dx+\frac{d^m x x^m\log\left(cx^n+1\right)-d^m x x^m\log\left(-cx^n\right.}{m+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*(d^m*n*integrate(x^m/(c*(m + 1)*x^n + m + 1), x) + d^m*n*integrate(x^m/
(c*(m + 1)*x^n - m - 1), x) + (d^m*x*x^m*log(c*x^n + 1) - d^m*x*x^m*log(-c*
x^n + 1))/(m + 1))*b + (d*x)^(m + 1)*a/(d*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(a+b\,\text{atanh}\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atanh(c*x^n)),x)
```

```
[Out] int((d*x)^m*(a + b*atanh(c*x^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{atanh}(cx^n)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**n)),x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x**n)), x)
```

3.242 $\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{a+b\tanh^{-1}(cx^n)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x^n)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^n]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^n]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx$$

Mathematica [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n]), x]
```

fricas [A] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b\,\text{artanh}(cx^n)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\,\text{artanh}(cx^n)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

maple [A] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\,\mathrm{arctanh}\,(c\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^n)),x)
[Out] int((d*x)^m/(a+b*arctanh(c*x^n)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\,\mathrm{artanh}\,(cx^n) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\,\mathrm{atanh}\,(c\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x^n)),x)
[Out] int((d*x)^m/(a + b*atanh(c*x^n)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\,\mathrm{atanh}\,(cx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**n)),x)
[Out] Integral((d*x)**m/(a + b*atanh(c*x**n)), x)
```

3.243 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx^n)\right)^2}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctanh(c*x^n))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^n])^2,x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}\,dx$$

Mathematica [A] time = 1.83, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n])^2, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(dx)^m}{b^2\,\mathrm{artanh}\left(cx^n\right)^2+2\,ab\,\mathrm{artanh}\left(cx^n\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(b\,\mathrm{artanh}\left(cx^n\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^n) + a)^2, x)
```

maple [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b \operatorname{arctanh}(c x^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^n))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x^n))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(c^2 d^m x e^{\left(m \log(x)+2 n \log(x)\right)} - d^m x x^m\right)}{b^2 c n x^n \log(cx^n + 1) - b^2 c n x^n \log(-cx^n + 1) + 2 abcnx^n} + \int -\frac{2\left(c^2 d^m (m + n + 1) e^{\left(m \log(x)+2 n \log(x)\right)} - d^m (m - n +\right.}{b^2 c n x^n \log(cx^n + 1) - b^2 c n x^n \log(-cx^n + 1) + 2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 2*(c^2*d^m*x*e^(m*log(x) + 2*n*log(x)) - d^m*x*x^m)/(b^2*c*n*x^n*log(c*x^n
+ 1) - b^2*c*n*x^n*log(-c*x^n + 1) + 2*a*b*c*n*x^n) + integrate(-2*(c^2*d^m
*(m + n + 1)*e^(m*log(x) + 2*n*log(x)) - d^m*(m - n + 1)*x^m)/(b^2*c*n*x^n*
log(c*x^n + 1) - b^2*c*n*x^n*log(-c*x^n + 1) + 2*a*b*c*n*x^n), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{(a + b\,\operatorname{atanh}(c\,x^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atanh(c*x^n))^2,x)
```

```
[Out] int((d*x)^m/(a + b*atanh(c*x^n))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**n))**2,x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```